

# Decentralized Stochastic Control of Microgrids to Improve System Frequency Stability

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**Abstract**—The paper proposes a decentralized stochastic control of microgrids aimed at addressing both system frequency stability and microgrids' revenue. With the proposed control strategy, each microgrid decides, based on a set of stochastic rules, a measure of the frequency and microgrid operating conditions, whether it should adopt a cooperative strategy where primary frequency regulation services are provided to the grid or a non-cooperative strategy, where it aims at maximizing its revenues. Simulations, based on the Monte Carlo approach, are performed on the IEEE 39 Bus System in order to validate the robustness of the proposed control strategy.

## I. INTRODUCTION

### A. Motivation

With the purpose to reduce greenhouse gas emissions and improving the reliability of the power system, many countries have invested in the development of the so called *smart grid* [1]. Smart grids will facilitate a higher penetration level of distributed power generators (especially from renewable energy sources), storage units, two-way communications between the utility company and consumers and, most likely, will be partitioned in a number of distributed units in the form of microgrids (MGs). Despite the concept of MG has been object of intense research in the scientific community in the past fifteen years, finding a concise definition has proven to be an elusive task. Broadly speaking, a MG is a cluster of Distributed Energy Resources (DERs) and loads that can work both connected to the grid and in island mode, able to conduct policies of Demand Response (DR) with other MGs [2]. Consistently to this definition, MGs are considered the building blocks of the smart grid [3].

### B. Literature Review

Due to the large interest of the scientific community on the emerging concept of MG, it is urgent to study and define the impact of MGs on the frequency and voltage stability of the grid. A relevant amount of papers have been published on this topic. A review on the impact of low rotational inertia in the power system has been presented in [4]. In [5], angle and voltage stability are analyzed as the MG penetration level increases and, in [6] and [7], the effects of the penetration of wind- and PV solar-based DERs, respectively, is analyzed. These works, however, do not consider the ability of a MG to conduct policies of DR, the interaction with the market, and the impact on the transient response of the transmission

system. It is the authors' opinion that, to fully understand MG operation and dynamics, it is not sufficient to consider an individual MG and ignore the interactions among MGs, the grid and the electrical market. In [8] and [9], the authors investigate the interactions between MGs, the electrical market and the transmission system and the effects of different Energy Management Systems (EMSs) and the size of the storage units. The main result of [8] is that a configuration with few large or several small coordinated MGs, can drastically deteriorate the transient behavior of the grid and reduce its stability margins. On the other hand, a high-granularity and non-coordinated configuration with several small MGs appears to be more convenient for a proper operation of the system. Conclusions from both [8] and [9] envisage the need of designing appropriate control methods to maintain the power grid within safe operational boundaries.

This paper considers a stochastic decentralized control strategy whose purpose is to allow each MG to maintain an acceptable level of operational freedom. The rationale for a stochastic approach, as opposed to a deterministic one, is that the latter would require heavy communication between the various MGs or to a centralized entity, in order to be effective. This requirement, besides the obvious disadvantages of economical costs and robustness [10], might also incur into non feasible privacy issues [11]. On the other hand, in the presence of a reasonably large number of independent units, a stochastic approach ensures that statistically the system will converge to a predetermined, average behaviour without any need for communication. Moreover, the independence among each unit and the introduction of stochasticity in the control actions prevent possibly harmful behaviours from occurring, like synchronization among MGs, that might lead to system instability [12].

### C. Contributions

This work proposes a control strategy that mitigates the negative effects of the penetration of MGs on the frequency deviations of the transmission system. Frequency deviation is a measure of the active power imbalance and should remain within the operational limits in order to avoid transmission line overloads and the triggering of protection devices [13]. Since MGs are expected to buy and sell active power according to their DR policies, the magnitude of frequency deviations can be considered a natural parameter to assess the impact

of the penetration of MGs on the grid. It is therefore crucial to elaborate control strategies that ensure that the power grid works within its boundaries while preserving the operational flexibility of each MG.

The main contribution of this paper is a control strategy that results in a trade-off between the frequency grid stability and the economical convenience of the MGs, thus allowing a larger penetration of units while, at least, partially maintaining their operational independence.

#### D. Organization

The remainder of the paper is organized as follows. Section II describes the modeling of the power system, of the electricity market, as well as of MGs and the control strategy we propose. Section III presents the case study where we compare a deterministic control system with the stochastic approach proposed in this paper. The IEEE 39-bus system is considered in the case study, where the dynamic impact on the grid of the different control strategies of the MGs is thoroughly evaluated and compared through a Monte Carlo approach and stochastic time domain simulations. Main conclusions and future work are outlined in Section IV.

## II. MODELING

In this paper, each MG is modeled taking into account loads, DERs and storage units. These elements are coordinated by an EMS which is responsible, among other tasks (e.g., load shedding, internal power flow management, transition to island mode), to establish the active power set point that the MG sells or buys from the electrical grid [14].

The following assumptions are made:

- The time constants of the control system, generation units and loads of each MG are small compared to the ones of the high voltage transmission system [15]–[17]. On the basis of this consideration their dynamics is neglected and they are treated as algebraic variables.
- Each MG is composed of a different amount of storage units, DERs and loads. To reduce the computational burden, these elements are grouped into aggregated models. This assumption can be relaxed, assuming distributed DERs, storage units and loads at the expense of a higher computational burden.
- The metrics utilized to compare the effects of different control strategies are the Center Of Inertia (COI) of the frequency and the revenue of each MG, defined as

$$\omega_{COI} = \frac{\sum_{i=1}^r H_i \omega_i}{\sum_{i=1}^r H_i}, \quad (1)$$

$$R_i(t) = \int_0^t P_{out_i}(t) \lambda(t) dt, \quad (2)$$

where  $\omega_i$  and  $H_i$  are, respectively, the rotor speed and the inertia constant of the  $i$ -th synchronous machine,  $r$  is the number of conventional generators in the grid and  $P_{out_i}(t)$  and  $\lambda(t)$  are, respectively, the output active power of the  $i$ -th MG and the energy price at time  $t$ .

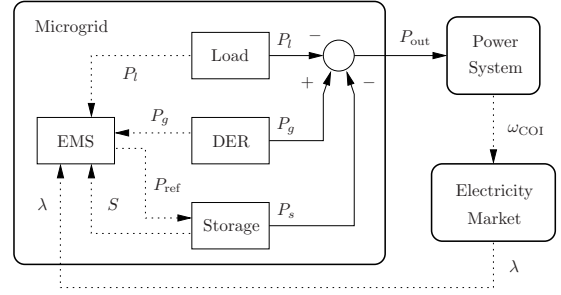


Fig. 1: Structure of the connection between the MG.

The remainder of this section describes first the power system and electricity market models utilized in the simulations. Then, the proposed controller is discussed in detail.

#### A. Power System Model

The model of the power system considered in the case study is based on a set of hybrid differential algebraic equations [18], as follows:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{u}) \\ \mathbf{0} &= \mathbf{g}(\mathbf{x}, \mathbf{y}, \mathbf{u}) \end{aligned} \quad (3)$$

where  $\mathbf{f}$  ( $\mathbf{f} : \mathbb{R}^{p+q+s} \mapsto \mathbb{R}^p$ ) are the differential equations;  $\mathbf{g}$  ( $\mathbf{g} : \mathbb{R}^{p+q+s} \mapsto \mathbb{R}^q$ ) are the algebraic equations;  $\mathbf{x}$  ( $\mathbf{x} \in \mathbb{R}^p$ ) are the state variables;  $\mathbf{y}$  ( $\mathbf{y} \in \mathbb{R}^q$ ) are the algebraic variables; and  $\mathbf{u}$  ( $\mathbf{u} \in \mathbb{R}^s$ ) are discrete events, which mostly model MG EMS logic.

Equations in (3) includes conventional dynamic models of synchronous machines (e.g., 6th order models), their controllers, such as, automatic voltage regulators, turbine governors, and power system stabilizers, as well as lumped models of the transmission system, market dynamics, and the MG components, such as DERs, storage devices and loads. More details on the MG devices are provided in Subsection II-C.

#### B. Electricity Market Model

In this work, we evaluate the performance of a control strategy to mitigate the impact of MG on the COI of the frequency of the grid. The need for such a control strategy is based on the conclusions of [8], where the market model plays a prominent role. Thus, the interested reader is referred to [8] for a detailed model of the system and market dynamics. However, to better understand the results discussed in Section III, we provide a brief outline on the market model used in [8]. Such a model is based on [19], where power system dynamics are assumed to be coupled with a real-time – or *spot* – electricity market, also modeled based on differential equations. These represent an ideal market for which the energy price  $\lambda$ , assumed to be a continuous state variable, is computed and adjusted rapidly enough with respect to the dynamic response of the transmission system, e.g., PJM, California, etc. Further details can be found in [19] and [8].

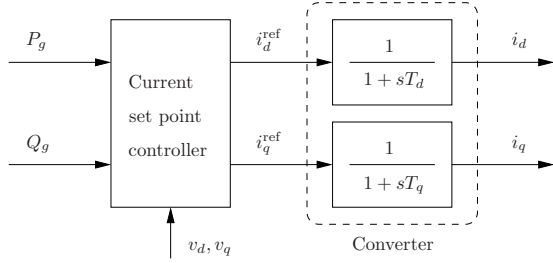


Fig. 2: Control scheme of an converter-based DER.

### C. Microgrid Model

Figure 1 shows the connections of the MG with the power system and the electricity market. The elements that compose the microgrid are the load, the DER, the storage device and the energy management system (EMS). The latter is responsible for the MG active power set point.

The dynamic of the aggregated storage device model is ruled by the following equation, which is the continuous-time equivalent of the model used in [20],

$$T_c \dot{S} = P_s = P_g - P_l - P_{out} \quad (4)$$

where  $S$  is the state of charge of the MG,  $T_c$  is the time constant of the storage active power controller,  $P_s$  is the power generated or absorbed by the storage device ( $P_s > 0$  if the storage is charging);  $P_{out}$  is the power output of the MG; and  $P_g$  and  $P_l$  are the produced active power and the local loads, respectively, of the MG.  $S$  undergoes a anti-windup limiter that model the charged ( $S = 1$ ) and discharged ( $S = 0$ ) conditions.

The dynamic model of the DER that is included in the MG is based on the DER models discussed in [21], [22]. The control scheme included in the DER model is shown in Fig. 2. The power injections into the ac bus are:

$$\begin{aligned} P_g &= v_d i_d + v_q i_q \\ Q_g &= v_q i_d - v_d i_q \end{aligned} \quad (5)$$

where  $i_d$  and  $i_q$  are the ac-side  $dq$ -frame currents of the VSC, respectively and  $v_d$  and  $v_q$  are the  $dq$ -frame components of the bus voltage phasor of the point of connection of the VSC with the ac grid.

Uncertainty and volatility of both generation units and loads are accounted for by modeling the net power produced by the MG as a stochastic process according to

$$P_{net} = P_g - P_l = \bar{P}_{gT} - \bar{P}_{lT} + \eta_M \quad (6)$$

where  $\eta_M$  is a white noise as in [23] with standard deviation  $\sigma_M$ , and  $\bar{P}_{gT}$  and  $\bar{P}_{lT}$  are piece-wise constant functions that account for uncertainty and change randomly with a period  $T$  as discussed in [24]. The noise is modeled as a single stochastic algebraic variable as the behavior of the MG depends on the difference  $P_{net} = P_g - P_l$  and not on their absolute values. Finally,  $P_s$ , the power provided or absorbed by the aggregated storage device included in the MG is the slack variable that allows imposing the desired  $P_{ref}$ , as follows:

$$T_s \dot{P}_s = P_{out} - P_{ref} = -P_s + P_{net} - P_{ref}. \quad (7)$$

where  $T_s$  is the time constant of the storage active power controller and  $P_{ref}$  is the reference power set point as defined by the EMS of the microgrid.

### D. Energy Management System of the Microgrid

As stated in Section I-A, the purpose of this paper is to synthesize a strategy that mitigates the impact of MGs on the power grid while allowing them to continue to operate advantageously on the market. To achieve this result, we propose a stochastic controller for the EMS that, on the basis of the maximum allowed frequency deviation  $\sigma_{\omega_m}$ , and measurements of the COI of the frequency in a time window of length  $T_p$ , switches between two different policies:

- Market based EMS (M-EMS): The MG is free to maximize its revenues (e.g., selling and buying energy without limitations);
- Frequency regulation EMS (F-EMS): The MG participates to the primary frequency regulation of the power grid.

The M-EMS used in this paper follows the same set of if-then rules proposed in [8], while the F-EMS behaves according to the droop control equations classically employed in the primary frequency regulation of the power system [25]. Note that the M-EMS is only one of the possible choices: the same control system can be applied without limitations to other strategies of EMS (e.g., see [9] for an island based EMS).

In order to control the switching between the two EMSs, each MG every  $T_p$  seconds (the time window might change from MG to MG), computes a probability  $P_s \in [0, 1]$ .  $P_s$  is the probability to operate in F-EMS mode in the next time window. The quantity  $P_s(kT_p)$ , at the  $k$ -th time window, is computed as

$$P_s(kT_p) = \frac{3 \left[ \alpha \sigma_{COI}(kT_p) + (1 - \alpha) \sum_{n=k-M}^{k-1} l_n \sigma_{COI}(nT_p) \right]}{\sigma_{\omega_m}}, \quad (8)$$

where  $\sigma_{COI}(kT_p)$  is the standard deviation of the COI of the frequency in the past time window  $[(k-1)T_p, kT_p]$ ,  $\alpha$  is a parameter belonging to  $[0, 1]$ ,  $M$  represents the number of past values of  $\sigma_{COI}$  that are taken in consideration and  $l_n$  are positive coefficients chosen such that  $\sum_{n=1}^{M-1} l_n = 1$ . This control law is similar to a discrete Proportional Integral (PI) controller, as it takes into account the actual value of the controlled variable  $\sigma_{COI}$  and its weighted integral. This choice avoids oscillations in the value of  $P_s$  that might occur if only the last value of  $\sigma_{COI}$  were taken into account (similarly to what would happen to a system with a purely proportional controller). Finally, the parameters  $\alpha$ ,  $M$  and the normalized weights  $l_n$ , can be tuned to ensure that the MGs do not behave too conservatively with respect to the boundary value  $\sigma_{\omega_m}$  and to achieve a desired trade off between the past and the present values of  $\sigma_{COI}$ .

*Remark:* (8) is designed to mitigate the effects of MGs on the power grid while allowing them to maintain, at least partially, their operational freedom. This strategy is, in fact, not

suitable to react instantaneously to exogenous contingencies. Moreover, due to its stochastic nature, especially if the granularity of the system is not large enough (for a more thorough discussion on the granularity, the interested reader can refer to [8]), it might happen that in a particular time window the boundary imposed by  $\sigma_{\omega_m}$  may be violated point-wisely.

### III. CASE STUDY

This section presents the dynamic response of a system with inclusion of MGs, regulated by means of EMSs proposed in the previous Section. The controllers are hereby compared and discussed in various simulations to evaluate their overall performance. Three different scenarios are proposed, as follows:

- 36 MGs without frequency control;
- 36 MGs with the deterministic version of the controller proposed in Section II-D, that switches from M-EMS to F-EMS whenever  $3\sigma_{COI} > \sigma_{\omega_m}$ ;
- 36 MGs with the control system proposed in Section II-D. In this scenario we compare four different sets of parameters to evaluate the controller overall performance as shown in Table II.

In all scenarios above,  $\sigma_{\omega_m}$  is set equal to 0.02 pu.

The performance of the stochastic controllers is evaluated considering the metrics listed in Section II and the the parameter  $T_s$ , that represents the average percentage of time spent in F-EMS mode.  $T_s$  can be interpreted as an indicator of the operational freedom granted to each MG. In fact MGs should be interested in minimizing their own  $T_{s_i}$  and, therefore, minimizing the time spent in providing services to the grid.

Simulations are based on the IEEE 39-bus 10-machine system; this benchmark grid is chosen in order to have both a fairly complex network and reduced state-space dimensions to easily understand the impact of MGs on the system. The state-space of the each case with 36 MG includes 432 state variables and 773 algebraic ones. The results for each scenario are obtained based on a Monte Carlo method (100 simulations are solved for each scenario). Table I shows the parameters for the 36 considered MGs. Three MGs are connected to each of the buses indicated in the table.

All simulations are performed using Dome, a Python-based power system software tool [26]. The Dome version utilized in this case study is based on Python 3.4.1; ATLAS 3.10.1 for dense vector and matrix operations; CVXOPT 1.1.9 for sparse matrix operations; and KLU 1.3.2 for sparse matrix factorization.

#### A. Simulation results

Figure 3 shows the realizations of the COI of the frequency for the different scenarios. The deterministic controllers performances are quite poor as it fails to maintain the COI of the frequency inside the desired range (see Fig. 3b). On the other hand, the stochastic controller exhibits better performances, effectively mitigating the oscillations into the desired range [0.98, 1.02] (see Figs. 3c-3f).

Table III shows the standard deviations, the revenues (values are normalized with respect to the largest revenue, since the

TABLE I: Microgrid parameters. The parameters  $\sigma_{\text{net}}$  are chosen randomly in order to take into account different variations of the load and the DERs energy production of each MG.

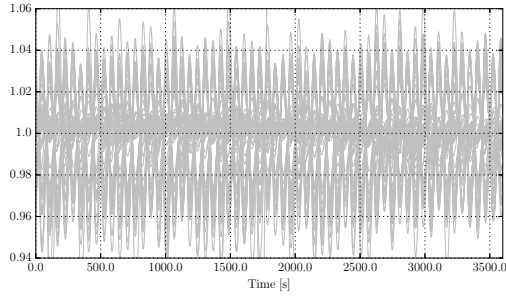
MG	Bus	$\bar{P}_g$ (pu MW)	$\bar{P}_l$ (pu MW)	$\sigma_{\text{net}}$ (pu Hz)	$T_s$ (s)
1	18	0.88	0.54	0.025	18000.0
2	3	0.77	0.20	0.040	25200.0
3	15	0.80	0.10	0.030	23400.0
4	17	0.40	0.20	0.020	28800.0
5	21	0.20	0.10	0.013	18000.0
6	28	0.20	0.40	0.040	25200.0
7	24	0.36	0.84	0.010	23400.0
8	17	0.20	0.50	0.020	28800.0
9	11	0.20	0.30	0.010	14400.0
10	5	0.10	0.80	0.010	18000.0
11	7	0.80	0.10	0.030	26640.0
12	12	0.40	0.40	0.025	24480.0

absolute values depend on the choice of the market model and not on the proposed control scheme and thus, only the relative values are meaningful) and the switching times, averaged over each MG and all realizations. The switching times and the revenues are the parameters chosen to compare the controller performance since the values of the standard deviation are very close to each other, as it can be seen from visual inspection. The smallest switching time is associated with the *Decreasing* set, while the *No integral* set is the one with the highest revenue. As expected, the set of parameters associated with the largest switching time corresponds to the one associated with the largest revenue and vice versa. This is not a surprising result. The larger the switching time, in fact, the more a MG is compelled to assist in the primary frequency regulation instead of taking advantage of market conditions. Also note that the results obtained for each set depend on the particular choice of the market policy of each MG. Different policies, in fact, would lead to a different choice of parameters to achieve the best desired trade-off between economical convenience and operational autonomy.

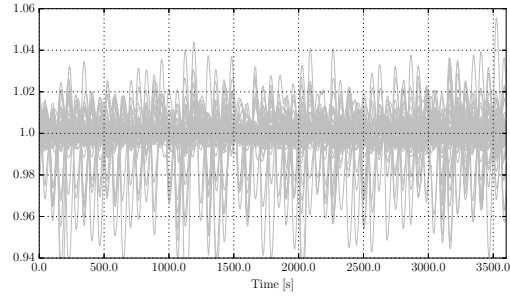
Finally, Table III shows that there is no set of parameters that outperforms the others. Therefore, the choice of the tuning parameters should be based on a trade-off between grid stability and operational freedom.

### IV. CONCLUSIONS

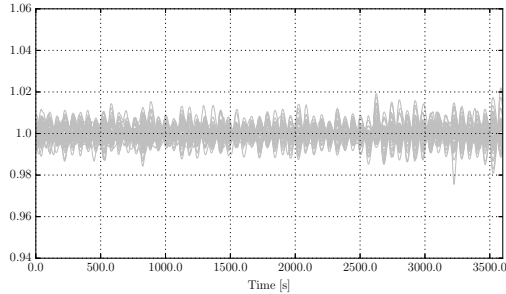
This work proposes a stochastic controller to mitigate the negative impact of the penetration of MGs on the power system. The impact is evaluated by means of the amplitude of the standard deviation of the frequency of the COI. The main result obtained from simulations is that a stochastic approach is able to reduce the fluctuations of the frequency in the desired



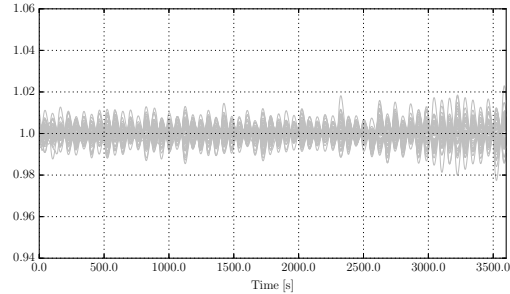
(a) No controller



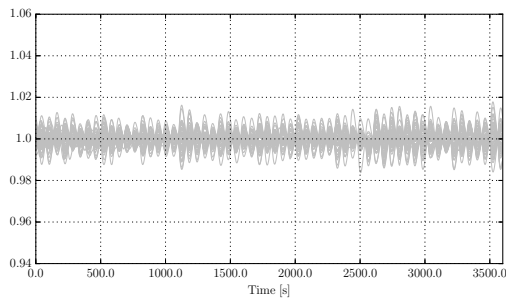
(b) Deterministic controller



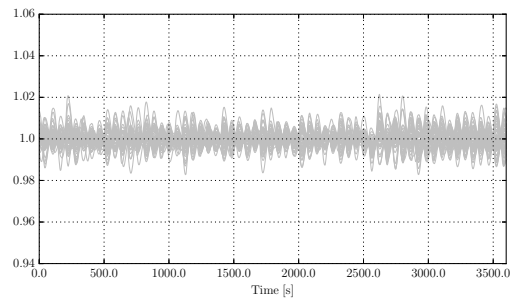
(c) Stochastic controller: no integral action



(d) Stochastic controller: integral action with constant weights



(e) Stochastic controller: integral action with decreasing weights



(f) Stochastic controller: integral action with fast decreasing weights

Fig. 3: COI of the frequency of the 39 bus system. All the realizations of each stochastic process are plotted.

TABLE II: Controller parameters.

Scenario	M	$l_n$	$\alpha$	$T_P$ (s)
No integral	0	-	1	300
Constant	10	0.1	0.7	300
Decreasing	10	$0.3414 \frac{1}{n}$	0.7	300
Fast decreasing	10	$0.6453 \frac{1}{n^2}$	0.7	300

TABLE III: Controller performance.

Scenario	$\bar{\sigma}_{COI}$ (pu Hz)	$\bar{R}(t)$ (pu \$)	$\bar{T}_s$ (%)
No integral	0.0068	1.00	11.60
Constant	0.0065	0.978	6.98
Decreasing	0.0061	0.954	6.35
Fast decreasing	0.0069	0.998	7.66

range and that different trade-offs among economical convenience and operational freedom can be achieved by different choices of the tuning parameters. Due to the stochastic nature of the proposed controller, it should be expected that as the number of MGs increases, the performance of the system, in terms of COI of the frequency, should improve as well.

Finally, we note that this work makes a number of as-

sumptions that may not be regarded as fully realistic. For instance, we use a single parameter (the COI of the frequency) to compute the control strategy. This requires some central entity to broadcast this value to all the MGs that participate to the primary frequency regulation. Accordingly, it might be interesting to relax this assumption to allow single MGs to truly operate in a decentralized way. In future work, we will

evaluate the impact of local frequency measurements instead of the COI. Moreover, the fairness of the stochastic control action, together with the privacy issues, have been neglected and could be addressed in future works to differentiate the switching probability for each MG on the basis of its impact on the system (i.e., the larger the impact the frequency, the larger the probability of participating to the frequency regulation).

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