Tuning of Moving Window Least Squares-based Algorithm for Online Battery Parameter Estimation

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Abstract—Online battery parameter identification algorithms, such as the Moving Window Least Squares, allow model-based state estimators with low computational intensity to be very accurate. This paper presents a procedure for tuning the algorithm parameters by using application-specific current profiles. A gardening application is taken as a case study. The results prove the validity of the proposed procedure and allow us to assess the identification algorithm performance.

I. INTRODUCTION

Lithium-ion batteries are widely used as the energy storage system (ESS) in many applications, such as portable electronic devices, power tools and electric vehicles, because of their high power and energy densities. Each battery cell must work in its safe operating area in order to avoid a degradation of its performance or dangerous situations. A Battery Management System (BMS) guarantees a safe and effective usage of the battery by monitoring and controlling the charging and discharging phases [1].

The BMS control algorithms are based on the knowledge of the state of each cell in the battery. For this reason, the BMS executes a state estimation algorithm to track some useful state variables, such as the state of charge (SOC) and the state of health (SOH). SOC indicates the residual charge stored in the battery, while SOH its degradation in terms of capacity fading and internal resistance increase [2].

Many algorithms for battery state estimation are found in the literature. In applications where high safety levels and high accuracy are required, the preferred choice is the use of model-based algorithms. Examples of widespread model-based algorithms are the Extended Kalman Filter (EKF) [2], [3], the Particle Filter [4] and the Mix Algorithm [5]. They are closed-loop algorithms which use a model to predict the cell voltage and compare it with the measured one, in order to correct the estimates of the state variables. The accuracy of the model-based algorithms depends on the capability of the model to reproduce the cell behaviour.

Many types of lithium-ion cell model have been presented in the literature [6]. Very accurate and reliable models, such as electrochemical ones, have high computational requirements. For this reason, they cannot be implemented in a real-time embedded system, like a BMS. On the other hand, Electrical Circuit Models (ECMs) are suitable for a BMS because they can provide good accuracy with affordable complexity [7]. However, the variation of the ECM parameters with the operating conditions must be considered [8]. A possible solution is the adoption of Look-Up Tables (LUTs) in which

the parameter values are stored and used depending on the actual operating point. This implementation requires low computational resources, but a very extensive offline characterisation, as the model parameters must be extracted using time consuming tests [9]. Furthermore, parameter variations due to manufacturing process tolerances and ageing of the battery can hardly be modelled in this way.

A good approach to face the above issues is to track the parameter variation online, which is the solution adopted in the Dual-EKF and in the Adaptive Mix Algorithm (AMA) techniques [2], [10]. The AMA is an evolution of the Mix Algorithm able to co-estimate the SOC and the ECM parameters using the Moving Window Least Squares (MWLS) method [11]. The MWLS identifies the parameters by applying the Least Squares (LS) technique to a set of cell current and voltage samples in an identification window, which is periodically shifted in time. These samples are previously filtered and decimated in order to reduce the noise influence and to isolate the dynamics of interest. The implementation of the AMA in a BMS and some experimental tests on an e-bike application have been presented in [12].

This paper focuses on the definition of a procedure for tuning the algorithm's parameters, *i.e.*, the length of the identification window and the cut-off frequency of the low-pass filter. This procedure uses a typical current profile of the battery in the considered application. A gardening application is taken as a case study in this paper to verify the tuning procedure and to assess the algorithm performance.

The cell model is presented in Section II and the description of the parameter identification algorithm is reported in Section III, together with its tuning procedure. Section IV describes the case study, while the results obtained are shown in Section V. Finally, conclusions are drawn in Section VI.

II. CELL MODEL

The AMA uses an N-RC equivalent circuit [13], whose general representation is shown in Fig. 1. The left-hand side of the circuit models the cell capacity and the SOC. The latter is calculated as Q/Q_r, where Q is the residual charge and Q_r is the maximum charge that can be stored in the cell. On the other side, the cell terminal voltage v_M is generated as the sum of the voltages v_n, the open-circuit voltage V_{OC} and the voltage across the ohmic resistance R_0, due to the flow of the cell current i_L. The voltages v_n on the RC branches model the relaxation effects [13]. In applications where fast transients are dominant, a single RC branch reduces the complexity of the
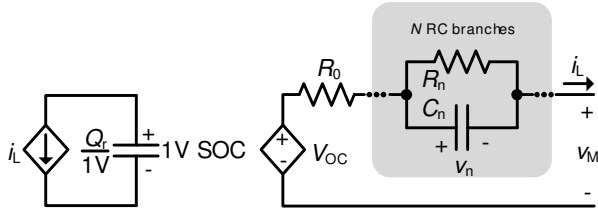
Fig. 1. Electric circuit model.

model, holding a good accuracy. The V_{OC}-SOC non-linear relationship is modelled as a controlled generator, in which the control function is implemented with a LUT. Note that the V_{OC}-SOC relationship is almost invariant with respect to the battery temperature, ageing and manufacturing process tolerances [14].

The values of the model parameters R_0, R_1 and C_1 change over time with the operating conditions. This is because the dynamic cell response depends on battery temperature [9], current rate (C-rate), SOC and ageing. The state space equations of the model are reported in (1), where $\tau_1 = R_1C_1$.

$$\begin{cases} \dfrac{dSOC}{dt} = -\dfrac{i_{\text{L}}}{Q_{\text{r}}} \\[2mm] \dfrac{dv_1}{dt} = -\dfrac{v_1}{\tau_1} + \dfrac{i_{\text{L}}}{C_1} \\[2mm] v_{\text{M}} = V_{\text{OC}} - R_0 i_{\text{L}} - v_1 \end{cases} \tag{1}$$

III. MWLS-BASED PARAMETER IDENTIFICATION ALGORITHM

The objective is to identify the parameters R_0, R_1 and C_1 of the model described in (1). To this end, the model is first linearised around the cell operating point and the parameters are considered constant in the identification window. The V_{OC}-SOC relationship is approximated by a piecewise linear curve $V_{\text{OC}} = \alpha_0 + \alpha_1 SOC$, where α_0 and α_1 change with the operating point. Then, the discrete-time transfer function of the linearised ECM is obtained using the bilinear transform:

$$\frac{Y(z^{-1}) - \alpha_0}{U(z^{-1})} = -\frac{b_2 z^{-2} + b_1 z^{-1} + b_0}{a_2 z^{-2} + a_1 z^{-1} + 1} \tag{2}$$

where $Y(z^{-1})$ and $U(z^{-1})$ are the z-transforms of the voltage output v_{T} and current input i_{L}, respectively [8]. The coefficients of the discrete-time transfer function (2) can be written as follows:

$$a_1 = -\frac{4\tau_1}{2\tau_1 + T} \tag{3}$$

$$a_2 = \frac{2\tau_1 - T}{2\tau_1 + T} \tag{4}$$

$$b_0 = -\left[4R_0 + 2T\left(\frac{\alpha_1}{Q_{\text{r}}} + \frac{R_0}{\tau_1} + \frac{1}{C_1}\right) + \frac{\alpha_1 T^2}{Q_{\text{r}}\tau_1}\right]\gamma \tag{5}$$

$$b_1 = -\left(\frac{2\alpha_1 T^2}{Q_{\text{r}}\tau_1} - 8R_0\right)\gamma \tag{6}$$

$$b_2 = -\left[4R_0 - 2T\left(\frac{\alpha_1}{Q_{\text{r}}} + \frac{R_0}{\tau_1} + \frac{1}{C_1}\right) + \frac{\alpha_1 T^2}{Q_{\text{r}}\tau_1}\right]\gamma \tag{7}$$

where $\gamma = \tau_1/(4\tau_1 + 2T)$ and T is the sampling time.

The second order AutoRegressive eXogenous (ARX) model is the time-domain representation of the discrete-time transfer function (2):

$$\begin{aligned} y(k) = &- a_1 y(k-1) - a_2 y(k-2) + \alpha_0(1 + a_1 + a_2) \\ &+ b_0 u(k) + b_1 u(k-1) + b_2 u(k-2) \end{aligned} \tag{8}$$

Equations (3) and (4) yield $1 + a_1 + a_2 = 0$, thus (8) simplifies as follows:

$$\begin{aligned} y(k) - y(k-2) = &a_1(y(k-2) - y(k-1)) + b_0 u(k) \\ &+ b_1 u(k-1) + b_2 u(k-2) \end{aligned} \tag{9}$$

Eq. (9) is used to build an overdetermined linear system, which is solved by the LS method obtaining the vector $[a_1, b_0, b_1, b_2]$. The latter yields the ECM parameters $[R_0, R_1, C_1]$ by inverting equations (3), (5), (6) and (7). The $y(k)$ and $u(k)$ samples used to build the overdetermined linear system belong to the identification window, which is then shifted in time. These samples are obtained by decimating the voltage and current samples acquired by the BMS. Thus, the sampling time T of the ARX model is related to the number of samples M in the identification window and its length L_{W}, by the relationship $L_{\text{W}} = M \cdot T$. Before decimation, the voltage and current samples are filtered by a third-order Butterworth low-pass filter. This avoids aliasing and allows noise, affecting the measured voltage and current signals, and the dynamics out of interest to be filtered out.

A. Algorithm parameter tuning

The number of samples in the identification window is determined by the affordable complexity in finding the LS solution [12]. The sample time is fixed by the sampling period of the monitoring circuit in the BMS and is typically between $10\,\text{ms}$ and $100\,\text{ms}$. Therefore, the only tunable algorithm's parameters are L_{W} and the cut-off frequency of the filter f_{c}. A good procedure is to first determine a range of reasonable values based on the characteristics of the system or on some general considerations. Then, a tuning phase is carried out by using a load current profile typical of the target application. The best combination of L_{W} and f_{c} is found by evaluating the rms error of the ECM predicted voltage, when L_{W} and f_{c} vary in their defined ranges, and by choosing the couple of values that minimise this error. The latter is computed as the difference between the measured voltage and the one predicted by the ECM, when the parameters R_0, R_1 and C_1 are identified by the MWLS algorithm.

The maximum value of L_{W} has to be chosen so that the assumption of constant model parameter values still holds in the identification window. Regarding its minimum value, we observe that the equations used to compute the vector $[R_0, R_1, C_1]$, obtained from equations (3), (5), (6) and (7), depend on the term $a_1/(2 + a_1)$. The value of a_1 should thus be sufficiently far from -2 to obtain a non ill-conditioned problem.
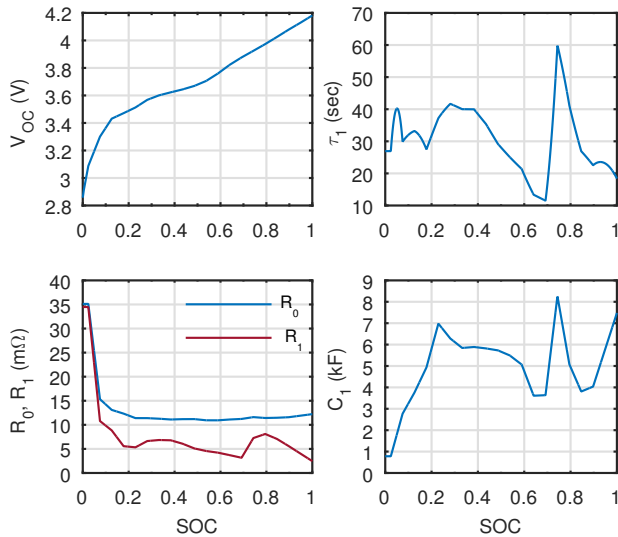
Fig. 2. Cell parameters extracted from a PCT test.

Fig. 3. rms error of the voltage predicted by the ECM, as function of the identification window length L_W and of the cut-off frequency f_c of the filter.

Hence, from (3), the value of T should not be much less than τ_1, i.e., $T \geq \tau_1/10$. This implies that $L_W \geq M \cdot (\tau_1/10)$.

The cut-off frequency of the filter and the sampling time must satisfy the Shannon theorem ($f_c \leq 1/(2T)$), thus $f_c \leq M/(2L_W)$. The minimum value is obtained considering that it cannot be too low in order to preserve the fast dynamics of the battery, so $f_c \geq 0.5/(2\pi\tau_1)$.

IV. CASE STUDY

Battery powered gardening tools represent a valuable case-study to investigate the performance of the MWLS algorithm. This is because they can provide different types of power profiles, which allow us to explore different battery operating conditions. In particular, two tools with different power requirements are considered. The battery consists of 24 lithium-ion cells, manufactured by LG Chem. The cells are arranged in 12 series-connected groups of two parallel-connected cells. The battery is monitored by a BMS, which measures the voltage, current and temperature of each cell group.

A cell group has been characterised using a Pulse Current Test (PCT), which is commonly used to extract the parameters of lithium-ions cells [9]. It consists of a series of current pulses separated by rest times. A current pulse changes the cell SOC and excites the RC branches of the cell model. In the rest time, the voltage evolution is used to extract the parameters of the ECM. The resulting parameters measured at room temperature are shown in Fig. 2.

Four tests have been selected to assess the algorithm in different cases. They all start with a fully charged battery, which is completely discharged operating the tool in its normal use. The first two tests (test 1 and test 2) have been carried out on a tool with a low power requirement of 160 W. The others two (test 3 and test 4) come from the second tool, which has a rated power absorption of 320 W. The tuning procedure has been performed on test 1 and the selected parameters are used for the model parameter identification of the 12 series-connected groups in the four tests.
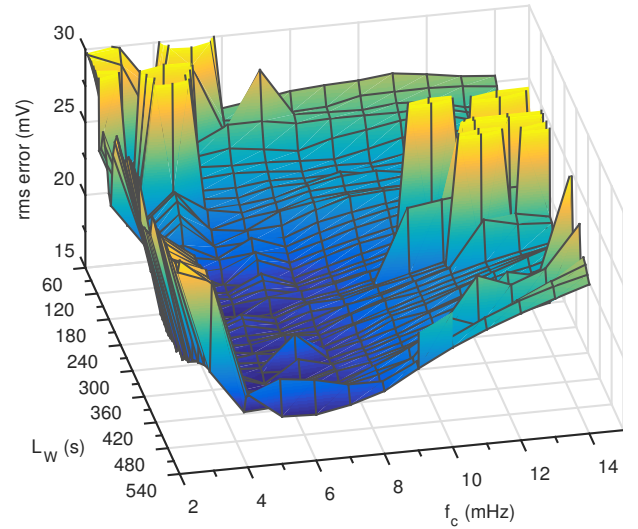
V. RESULTS AND DISCUSSION

The procedure described in Section III-A allows us to define a range for L_W and f_c in which we can found a combination of these two parameters that minimise the ECM error.

The values of L_W and f_c are varied in the ranges of 60–540 s and 2–14 mHz, respectively. The ranges' bounds are calculated considering that a new sample is acquired every 100 ms and that the identification window is composed by $M = 20$ samples. The error computed considering the first group of two parallel-connected cells, when the battery is exerted with test 1, is shown in Fig. 3. This figure highlights an area around $L_W = 210$ s and $f_c = 5$ mHz, where the error is low and the values of the MWLS parameters should be chosen. Therefore, $L_W = 210$ s and $f_c = 5$ mHz has been selected and used to apply the algorithm on all the battery cell groups in each test.

Fig. 4 shows a comparison between the voltage rms errors obtained by using the parameters identified by the MWLS and those extracted offline from the PCT. The errors obtained with online identification are clearly lower in the third and fourth test. As these tests have been executed on the tool with the higher mean power absorption, a battery temperature increase of about 30 °C is observed, as shown in Fig. 5 for test 3. It is worth noting that MWLS is able to take into account the variations of the model parameters due to the temperature change during the use, providing better results in the cell voltage prediction. In more detail, the parameters are close to those extracted by the PCT in the first half of the test, but they are quite different at the end of the test where the battery temperature reaches about 50 °C. This result clearly shows the benefits of using an online parameter identification algorithm.

The situation is slightly different for the other tool (test 1 and test 2). In this case, the online identified parameters are similar to those extracted from the PCT at room temperature and the rms errors are comparable. Indeed, the battery tem-
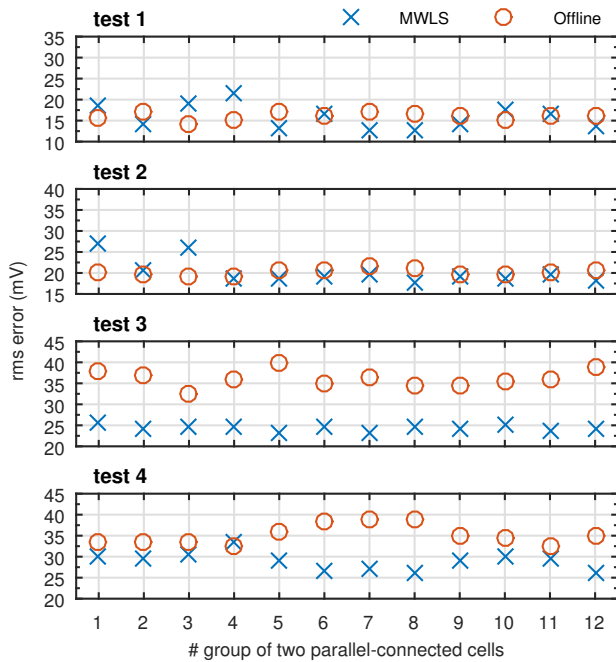
Fig. 4. Comparison of the voltage rms error obtained by using the parameters of the MWLS and those extracted offline. Each point corresponds to a group of two parallel-connected cells in the four tests.
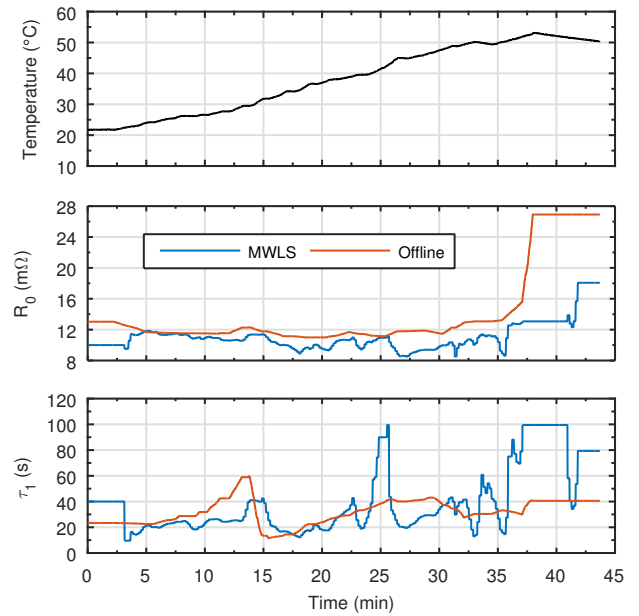

Fig. 5. Temperature and ECM parameters estimated online by the MWLS compared with those extracted offline in test 3 for the first group of two parallel-connected cells.

perature remains almost constant to the ambient value during these tests, as the power absorbed by the tool is relatively low.

VI. CONCLUSIONS

This paper has presented a tuning procedure that guarantees a reliable use of the MWLS algorithm for the parameter identification of a lithium-ion cell ECM in different applications, by simply changing two programmable parameters. The tuning procedure selects the MWLS algorithm parameters that minimise the model voltage prediction error in the considered application, which in this work is battery powered gardening tools. The experimental tests show the effectiveness of this algorithm and how the online model parameter identification improves the accuracy of the ECM, particularly when the battery temperature varies during the use.

ACKNOWLEDGMENT

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