

# Optimization of the geometry of Fresnel linear collectors

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## Abstract

Methods and results concerning the optical optimization of a linear Fresnel collector are presented. The variables considered in the optimization are the positions, widths and focal lengths of the mirrors; the mirrors can be of variable size and focal length, and they can be nonuniformly spaced. The target function to be optimized is the plant cost divided by the collected solar radiation in a year. The computation of the collected radiation and of its average on the year, and the optimization of the cost/radiation function are carried out via suitable mathematical methods and the choice of a plausible cost function. Four different level of optimization (uniformly spaced identical mirrors; nonuniformly spaced identical mirrors; mirrors of the same width with uniform spacing and variable focal lengths; and finally a full optimization) are presented, with a discussion of the resulting gain on the target function (i.e. the reduction of the ratio plant cost / collected radiation). The results show that the application of suitable optimization strategies can lead to an estimated gain around 12% with respect to the initial configuration (all mirrors identical and adjacent), and that a full optimization leads to a gain of almost 5% over a simple uniform optimization. This gain is due in large part to the possibility of regulating the focal lengths (the optimization of focals leads to a 2.8% gain over the uniform case), while only a minor improvement (less than 0.4%) is obtained with nonuniformly spaced identical mirrors.

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## 1. Introduction

Linear Fresnel systems (Di Canio et al., 1979; Feuermann, 1991; Montes et al., 2014; Zhu et al., 2014) are among the most promising technologies for energy production from concentrated sun radiation. In such plants, a linear

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fixed receiver is suspended above a solar field composed by strips of mirrors, flat or slightly concentrating; each strip rotates on a fixed horizontal axis in order to reflect the sun radiation towards the receiver. Several studies have been devoted to plant configurations (Abbas et al., 2013; Grena and Tarquini, 2011; Häberle et al., 2002; Mills and Morrison, 2000; Zhu and Huang, 2014), study and simulation of various aspects of the plant work (Abbas et al., 2012; Munoz-Anton et al., 2014; Pino et al., 2013; Velazquez et al., 2010; Zheng et al., 2014), analysis of different types of receivers (Abbas et al., 2012; Facao and Oliveira, 2011; Lin et al., 2013; Natarajan et al., 2012; Singh et al., 2010), or comparisons with linear trough systems (Giostri et al., 2013; Morin et al., 2012; Schenk et al., 2014); prototypes have been proposed and built (Areva, 2015; Bernhard et al., 2008, 2009; Novatec, 2015; Solar Power Group, 2015). The main advantages with respect to trough systems are the fixed receiver, the larger collection area for each receiver (which reduces the cost of the receivers and simplifies the management of the fluid circulation), the small moving parts (mirrors are far smaller than the single-block mirror of a solar trough) and the lower cost of optical components (mirrors are almost flat, and their construction is simpler). The main disadvantages are the reduced optical efficiency, especially when the sun is far from the focal plane<sup>1</sup>, and the larger susceptibility to optical and tracking errors, due to the larger distances between mirrors and receiver and to the fixed receiver configuration.

Fresnel systems, even from a purely geometric point of view, allow for a large variety of configurations, since the properties of the receiver and the positions, widths and focal distances of the mirrors can all be in principle changed independently. Usually, uniform configurations (with all the mirrors equal, and equally spaced, or not spaced at all) are employed, but this is not mandatory. Uniform configurations, while simpler in design, do not maximize efficiency, since the inclination of the mirrors, the distance from the receiver, the shape of the reflected beams, the shadowing and blocking among the mirror strips, and the shadowing from the receiver change significantly with the distance from the midpoint of the solar field (the point under the receiver). In fact, solutions with varying widths or spacing among the mirrors have been proposed for fixed-frame linear Fresnel collectors<sup>2</sup> (Goswami et al., 1990). The no-blocking condition (total absence of blocking for normal incidence), usually imposed for such systems, can also be applied in the case of independent-mirror Fresnel systems, leading to nonuniformly spaced mirrors. Another possible criterion that has been suggested is the absence of shadowing up to a given incidence angle (Nixon and Davies, 2012). Mirrors with variable sizes and shapes (and even variable heights) are proposed in Chaves and Collares-Pereira (2010), following theoretical principles (etendue-

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<sup>1</sup>The *focal plane* is defined here as the vertical plane containing the receiver.

<sup>2</sup>In a fixed-frame (or “true”) Fresnel collector, the mirror strips are blocked on a common flat frame that moves tracking the sun. Their properties and possible applications are completely different from the case of independent-mirrors Fresnel systems, such as the ones discussed in this work. In this paper, a Fresnel system will always be an independent-mirror system, unless otherwise specified.

matching). An analytical method to build a variable-size, variable-spacing solar field that reduces the degrees of freedom to three variables is described in Abbas and Martines-Val (2015).

If we choose to remove all the uniformity conditions and other prescriptions, the efficiency of a Fresnel plant becomes dependent on a large amount of variables; so, there is space for refined - and nontrivial - optimization, which could potentially lead to significant gains. However, due to the large number of variables, only partial optimization approaches have been tried so far. Studies of the spacing factor for uniform systems, or of the optimal focal length of the mirrors, have been performed for specific systems (Grena and Tarquini, 2011); optimization of the exergy cost with respect to the no-shadowing maximal angle has been performed, in plants adopting the aforementioned no-shadowing criterium (Nixon and Davies, 2012). These studies consider a small number of parameters (3 at most), while no strategies for full optimization have been presented up to now.

In this paper we try to fill this void presenting a method for the full optimization of the configuration of a Fresnel collector. The target function (i.e. the function to minimize) is the ratio (plant cost / collected radiation). The collected radiation is computed considering the geometric optical collection of the system, averaged on the year. The optimized variables are all the listed parameters of a system, except the receiver properties (height and width), which are kept fixed, and the number of mirrors (being discrete, it should be optimized separately, comparing the different cases).

The target function is proportional to the specific cost of the energy produced under two hypotheses (whose validity is discussed in the following): (i) the optical properties of the elements do not change too much with the radiation incidence angle, and (ii) the thermal efficiency does not change too much in working conditions. Methods to refine the analysis removing these two hypotheses are discussed.

Unfortunately, too little data are available to build a general cost function for a Fresnel plant (which will be also strongly dependent on the design choices); here a simple parametrization of the plant cost and an example with arbitrary but plausible cost parameters will be presented.

The optimization will proceed in several steps, in order to evaluate the gain due to different design choices; the gain is defined as the relative reduction of the target function. First, a simple optimization of a uniform system will be made, starting from an initial configuration with all the mirrors equal and adjacent; the optimized variables in this case are only three (width, spacing, focal length). Starting from the optimal uniform configuration, the uniform spacing condition can be removed, maintaining all the mirrors equal: the variables become  $N_m + 2$  (the  $N_m$  positions of the mirrors and the common width and focal length). Alternatively, the condition on the equal focal lengths can be removed, maintaining the mirrors uniformly spaced and with equal widths; in this case there are again  $N_m + 2$  variables. These two special cases of optimization are of interest because they represent plausible engineering choices: mirrors can be mass produced, and then, in the latter case, mechanically bent. The final step is

a full optimization of all the  $3N_m$  variables, removing all the constraints on the solar field. The degrees of freedom can be reduced by a factor of about 2 if the system is assumed to be symmetric, an obvious choice if it is NS oriented. The relative gains due to each type of configuration will be discussed and compared.

The mathematical technique used for optimization is mostly BFGS (Broyden-Fletcher-Goldfarb-Shanno), see e.g., Dennis and Schnabel (1983). With the exception of the uniform optimization, quite straightforward, in the other cases the method will be complemented with simulated annealing cycles (Kirkpatrick et al., 1983) in order to explore the configuration space in search of possible multiple local minima. It must be stressed that, from a practical point of view, the accuracy in finding the optimal configuration may not be very significant, as the reduction on the final cost is the only important aspect. In other words, if a very “flat” minimum is present, two distant configurations may exhibit a very small difference in the target function, and can be considered as equivalent.

Despite the necessary simplifications of the model, the presented methods are quite general and can be easily applied to practical cases, with known cost functions and considering also thermal efficiency.

The paper is organized as follows. In Section 2, the structure of the Fresnel system is described. Section 3 explains how to compute the target function: in particular, it presents the optical simulation of the system, and the method for the computation of the mean efficiency during the year. The model for the cost of the plant is also illustrated, thus defining a suitable target function. Section 4 is devoted to the optimization techniques used in this work. Section 5 shows the results of the optimization, and discusses possible further improvements of the methods to include other effects (more realistic cost functions, thermal efficiency, weather conditions).

## 2. Model of Fresnel system

The optimization only considers the geometric optical collection; for this reason, the only relevant aspects of the Fresnel plant are the geometric properties that determine the optical efficiency. In the model, the receiver is flat, horizontal, and placed at height  $h$  from the ground. The semi-width of the receiver opening will be indicated as  $l$ . The central axis of the receiver opening will be called (somewhat improperly) focal line. The quantities  $h$  and  $l$  are kept fixed (not involved in the optimization).

On the ground, a number  $N_m$  of cylindrical primary mirrors will be placed with rotation axis at ground level. The system is NS oriented. The mirrors will be placed symmetrically around the midline of the solar field (the line directly below the focal line); this means that, in the case of odd  $N_m$ , the central mirror will always be placed exactly below the receiver. This symmetry reduces the total number of degrees of freedom used in the optimization from  $3N_m$  to  $3N_m/2$  in the case of even  $N_m$ , and to  $3(N_m + 1)/2 - 2$  in the case of odd  $N_m$ . The position of the axis of the mirror  $n$  with respect to the midline will be indicated as  $x_n$ , and the semi-width of the mirror will be  $w_n$ . The mirror will have a focal length  $f_n$ . The scheme is shown in Fig. 1.

>>> *approx. position of Figure 1* <<<

For convenience, the list of all the variables defining the system will be indicated as  $X_f$  in the following.

Some comments on the assumptions made here can be useful:

- The assumption of a flat horizontal receiver is the only significant simplification introduced in the model; it is a sound assumption, since almost all the proposed configurations for Fresnel plants adopt a receiver with a secondary reflector or a cavity, with a flat opening that is the effective target;
- Mirrors are assumed to be cylindrical, since this is the most common choice; of course, the same optimization procedure can be applied to parabolic mirrors, adapting the optical simulations (described in Sec. 3);
- Symmetry of the mirrors around the receiver is a straightforward assumption for NS-oriented systems, which will be the orientation considered in the optimizations.

Physical properties of the optical elements (reflectivities, absorbance) are not defined, since only the geometric optical collection is considered. In fact, variations of reflectivities and other surface properties w.r.t. the incidence angle become strong only for high incidence ( $> 60$  deg), and the geometric optical efficiency can be considered to be proportional to the real optical efficiency with very good approximation.

Thermal properties are not considered, either: the energy obtained is assumed to be proportional to the optical collection. A thermal analysis would require a detailed description of the receiver and this is out of the scope of this paper. A short discussion on the possibility of introducing thermal efficiency in the analysis is given in Sec. 5. In fact, the choice of considering only the optical collection does not lead to as large an error as one may think, since the thermal efficiency of a receiver string in stationary state usually does not change too much in working conditions; however, a truly accurate cost optimization would require a thermal model of the receiver.

Beside the plant geometry and properties, two important parameters are the minimum and maximum value of the collected radiation. The existence of these limits is a quite straightforward assumption if a realistic plant must be described, since the system is designed to operate in a certain range of radiation intensity to achieve a determined target temperature. The assumption is also mathematically important for the optimization procedure, since without this hypothesis the optimal system could be completely unrealistic (e.g., a system with a very low concentration can easily have a high optical efficiency, but it is not a realistic solution; on the other side, price parameters that favour extremely high concentrations can be devised, but this is not acceptable). So, the definition of the minimum and maximum working radiation is essential for the optimization procedure. It will be assumed that the plant will not work below the minimum radiation (efficiency 0), and when the collected radiation exceeds the maximum value the plant will work at the maximum value, reducing in some way the incoming radiation (e.g. defocusing some of the mirrors).

### 3. Computation of the target function

#### 3.1. Computation of the mean collected radiation

The computation of the mean collected radiation during the year requires the integration of the collected radiation over a number of different sun positions. For each sun position, the computation of the collected radiation is performed by numerically integrating over the solar cone. So, the basic tool is the computation of the collected radiation for collimated rays. The computational procedure can be summarized in three steps:

1. Given an irradiation of collimated rays with a defined direction (within the solar radiation cone), the amount of radiation collected in the receiver opening is computed, with the tracking position of the plant determined by the sun coordinates;
2. Results of the previous step are averaged on the solar profile;
3. Results of the previous step are averaged on the distribution of the sun position during the year, also considering the change in the extraterrestrial radiation (due to the change in sun-earth distance) and introducing the air mass correction dependent on the sun position.

The first step is performed by computing, for each of the primary mirrors, the fraction of its surface which projects the radiation in the receiver opening (the “active” part): this can be done numerically with high accuracy. The active part will be determined by taking into account shadowing and blocking from other mirrors, out-of-target rays, and the shadow of the receiver itself. This computation is almost exact, except for the small error ( $< 10^{-7}$ ) that arises when finding numerically the limits of the active fraction of the mirrors; the computation does not introduce the discretization effects (such as spurious oscillations of the efficiency) typical of ray-tracing methods.

This computation is repeated for a sample of ray directions (second step) suitably weighted so as to reproduce the solar beam, under the Lambertian assumption of a uniform radiation cone. The geometric cosine effect is included in the computation, as is the enlargement of the projected radiation cone when the sun direction is not orthogonal to the focal line. The result of the computations is the collected radiation (per length unit) given a unitary DNI and a given position for the sun, considering only the geometrical efficiency (all the surface reflectivities and absorbances set to 1). The sun position will be described using as coordinates the declination  $\delta$  and the hour angle  $H$ , for reasons that will be made clear below; such a coordinate system requires the knowledge of the latitude, which will be considered as fixed (and omitted from the list of variables). The results of these first two steps of the computation will be indicated as  $R(\delta, H, X_f)$ . An example of the behaviour of the function  $R(\delta, H, X_f)$  with respect to  $H$ , for three different values of  $\delta$  (summer and winter solstices and equinox), at a latitude of 30 deg N, is shown in Figure 2, for the system that will be used as a starting point for the optimizations.

>>> approx. position of Figure 2 <<<

Air mass correction is then applied to the computed result, in the form (Ineichen, 2008):

$$AM(\delta, H) = \exp\left(-\frac{t}{(\cos z)^d}\right), \quad t = 0.606, \quad d = 0.491, \quad (1)$$

where  $z$  is the Zenith angle of the sun (obtained from  $\delta$  and  $H$ ). The quantity  $AM(z(\delta, H)) \times R(\delta, H, X_f)$  is the radiation (geometrically) collected by the receiver when the sun is at  $(\delta, H)$ , for a DNI equal to  $AM$ . This quantity is proportional to the collected radiation in clear sky average conditions.

Such a quantity should be weighted on the distribution of the extraterrestrial solar radiation with respect to  $(\delta, H)$  during the year (third step), in order to obtain a quantity proportional to the overall collected radiation in the year. A coordinate set given by declination  $\delta$  and hour angle  $H$  is especially well suited to this task, since it can be easily shown that the distribution will be uniform in  $H$ , while the distribution in  $\delta$  can be written as the theoretical distribution for a circular earth orbit, which is easily computable. This is due to the fact that the changes in the radiation due to the variation in earth-sun distance (radiation is proportional to  $1/D^2$ ) are exactly compensated by the change in orbital velocity (the “transit time” of the sun in a given small range of positions is proportional to  $D^2$ , due to the conservation of angular momentum). The domain of the distribution will be limited to  $\delta \in [-\epsilon, +\epsilon]$ , where  $\epsilon$  is the earth axis inclination ( $\epsilon = 23.4393$  deg). The domain of  $H$  will be given by the condition that the sun is above the horizon: it will be dependent on  $\delta$ , and it is given by  $H \in [-H_l(\delta), +H_l(\delta)]$ , with  $H_l = \arccos(-\tan \phi \tan \delta)$ , and  $\phi$  is the latitude. In the following optimizations, a latitude of 30 deg N will be assumed. The normalized distribution will be

$$\rho(\delta, H) = \frac{1}{\pi^2} \frac{\cos \delta}{\sqrt{(\sin \epsilon)^2 - (\sin \delta)^2}}, \quad \delta \in [-\epsilon, +\epsilon], \quad H \in [-H_l(\delta), +H_l(\delta)]. \quad (2)$$

Note that the shape of the distribution does not depend on the latitude  $\phi$ , since  $\delta$  is a global coordinate, but the domain does.

It will be assumed that the plant will work only when  $AM(z(\delta, H))R(\delta, H, X_f) \geq R_{min}$ . This fact restricts the range of  $H$  (since, for  $H = H_l$ ,  $AM * R = 0$ ) to  $[-H_c(\delta, X_f), +H_c(\delta, X_f)]$ , where  $AM(z(\delta, \pm H_c))R(\delta, \pm H_c, X_f) = R_{min}$ <sup>3</sup>. The requirement could also in principle restrict the range of  $\delta$ , but this is unlikely to happen for a real system (it would mean that the system must be completely shut down in winter, and such a working configuration is likely to be far from the optimum).

Given a value of  $\delta$ , the value of  $H_c(\delta, X_f)$  is easily computed with numerical procedures, with a limited number of  $R(\delta, H, X_f)$  evaluations (4-5 for an accuracy of  $10^{-3}$ ).

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<sup>3</sup>In theory, since for a NS system the collected radiation can have maxima that are not at midday, a domain composed of disjoint intervals could be obtained. In practice, this never happens for reasonable  $R_{min}$  values.

The implementation of  $R_{max}$  (maximum geometric collection) is simpler: in the integration, the result of  $AM(z(\delta, H))R(\delta, H, X_f)$  is replaced by  $R_{max}$  when it exceeds the prescribed value.

So, defining the function

$$\bar{R}(\delta, H, X_f) = \begin{cases} AM(z)R(\delta, H, X_f) & \text{when } AM \times R \leq R_{max} \\ R_{max} & \text{otherwise} \end{cases}, \quad (3)$$

the integral to compute in order to obtain a quantity proportional to the collected radiation is

$$R_m(X_f) = \int_{-\epsilon}^{+\epsilon} d\delta \rho(\delta, H) \int_{-H_c(\delta)}^{+H_c(\delta)} dH \bar{R}(\delta, H, X_f). \quad (4)$$

The distribution  $\rho$  depends on  $H$  only in the definition of its domain, so it can be moved outside the integral w.r.t.  $H$ .

The integration is performed first on the variable  $H$ , and then on  $\delta$ . Each integration node requires the numerical computation of  $R(\delta, H, X_f)$ , which is the heaviest computational burden in the procedure; therefore, the number of integration points should be kept as small as possible. Since the values at the endpoints of the integration interval for  $H$  are known (when the sun is at  $\pm H_c$  the collected radiation is  $R = R_{min}$ ), one can use the Gauss-Lobatto quadrature rule; the number of integration nodes can be halved by symmetry (the function does not depend on the sign of  $H$ ). The integral on  $H$  must be computed for each of the nodes for  $\delta$ . One can note that the distribution  $\rho$  diverges as  $\delta$  approaches  $\pm\epsilon$ , with integrable divergences behaving like  $\sim 1/\sqrt{|\epsilon - \delta|}$ . This fact suggests the use of a Gauss-Chebyshev (type I) integration method, which is well-suited for functions with such a behaviour. Using this method (Gauss-Lobatto on  $H$  and Gauss-Chebyshev on  $\delta$ ) the computation of the integral requires a few seconds on a common PC.

This computation only includes the air mass correction, thus assuming that the atmospheric conditions, on average, do not change during the day and during the year. If sufficient meteorological data are available, the analysis can be refined introducing in the radiation distribution a correction factor that considers the measured average radiation when the sun is in a certain position  $(\delta, H)$ .

### 3.2. Plant cost computation

Once the collected radiation has been determined, the computation of a target function for the cost optimization requires a model for the cost of the plant, as a function of the variables to be optimized. The detailed analysis of the cost of Fresnel plants is beyond the scope of this paper, and it is probably premature (Fresnel plants show many different configurations with presumably different cost parameters). For this work, we adopt a plausible and quite general cost model, dependent on few parameters, and then we try to estimate not-too-unrealistic cost parameters from some simple assumptions.

The model used here has four cost components:



1. Since the receiver is not included in the optimization, the receiver cost (including the support structure) will be a fixed component in the cost function.
2. Land preparation and common mirror support structures will have a cost proportional to the occupied area.
3. Each mirror will have a fixed cost component due to the axis, tracking system, mounting costs, and
4. a cost component proportional to its area (cost of the mirror itself).

Assuming that these are the only cost components, the cost function for a linear metre of collector can be written as

$$C(X_f) = c_0 + c_t x_{N_m} + c_m N_m + c_w \sum_{i=1}^{N_m} w_i. \quad (5)$$

where the first term is the receiver component (fixed), the second term is proportional to the occupied area (estimated using the position of the last mirror), the third term is the fixed cost component for each mirror (proportional to the number of mirrors) and the fourth term is the area-dependent cost for the mirrors. Note that the two coefficients  $c_t$  and  $c_w$  are defined in relation to half-widths.

In order to obtain a sufficiently realistic model for the cost of the plant, price parameters were estimated in the following way. We started from a typical cost estimate for linear concentrator plants (200 €/m<sup>2</sup>), assuming that this will be a correct overall estimate for a Fresnel plant with 25 mirror strips of 1 m each, without any spacing between adjacent mirrors. We also assume (somewhat arbitrarily) that the cost will be equally distributed among the four cost component, thus obtaining the price parameters. The result is

$$c_0 = 1250 \text{ €/m}, \quad c_t = 100 \text{ €/m}^2, \quad c_m = 50 \text{ €/m}, \quad c_w = 100 \text{ €/m}^2. \quad (6)$$

The target function that should be minimized will be simply  $C(X_f)/R_m(X_f)$ . This function is proportional to the specific cost of the energy optically collected by the receiver.

The computation of the cost itself does not add any computational difficulties, once the function  $R_m$  is known.

#### 4. Optimization method

The choice of an optimization strategy depends, of course, on the properties of the target function. Here the target function is nonsmooth and nonconvex; moreover, it is not feasible to compute its gradient analytically (where it exists).

If the starting point is comparatively close to a possible minimum, the optimization method employed in the present work is BFGS with numerical computation of the gradient. Although BFGS is conceived, in principle, to minimize functions that are at least twice differentiable, it turns out to be surprisingly robust even in presence of points where the target function is nondifferentiable (Lewis and Overton, 2013).

If the initial guess is potentially far from a minimum, simulated annealing is used to generate a starting point for BFGS.

Four cases are considered here:

- *The uniform case* (UC): the mirrors are all alike (that is, they all have the same width and focal length) and uniformly spaced. Optimization is therefore performed with respect to three variables: distance between adjacent mirrors, mirror semiwidth and mirror focal length. This is a comparatively simple optimization that has already been analyzed by several authors. In our case, several methods, such as BFGS and the Nelder-Mead simplex method, have been applied, and they all converge to the same optimal configuration.
- *Optimization of mirror positions* (OP): the mirrors are all alike, but their positions are free. The efficiency is a function of  $N_m + 2$  variables: the positions of the mirrors ( $N_m$  variables), their common semiwidth and focal length (two more variables). If the system is symmetric with respect to the receiver, the number of degrees of freedom reduces to  $(N_m - 1)/2 + 2$  if  $N_m$  is odd, or to  $N_m/2 + 2$  if  $N_m$  is even.
- *Optimization of focal distances* (OF): this case involves uniformly spaced mirrors of the same size, with varying focal lengths. The degrees of freedom here are again  $N_m + 2$ : the  $N_m$  focal lengths plus the common semiwidth and spacing factor. If the system is symmetric with respect to the receiver, the number of degrees of freedom reduces to  $(N_m + 1)/2 + 2$  if  $N_m$  is odd, or to  $N_m/2 + 2$  if  $N_m$  is even.
- *Nonuniform full optimization* (NU): mirror widths, focal lengths and positions are free. The number of degrees of freedom is  $3N_m$  for a general system, whereas for a symmetric configuration there are  $(3N_m + 1)/2$  free variables if  $N_m$  is odd, or  $3N_m/2$  if  $N_m$  is even.

When dealing with many variables (that is, the OP, OF and NU cases), the target function is likely to have several local minima, which make the optimization task more difficult. In these cases, a first local minimum has been obtained via BFGS starting from the previously obtained optimal configuration. In an effort to explore other minima, simulated annealing has been used to “heat up” the system (that is, move it away from the computed minimum) and then to “cool it down” (that is, find an approximation to a new minimum). At this point, BFGS has been applied to refine the result. This procedure has been repeated five times for OP and NU, thus yielding six local minima, and once for OF, thus obtaining two local minima.

Optimization has been carried out in Matlab, using the following implementations:

- for BFGS, the implementation available with the `fminunc` Matlab function, with default stopping criteria,
- for simulated annealing, a modified version of the Matlab implementation by J. Vandekerhove (available on Matlab File Exchange<sup>4</sup>),

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<sup>4</sup><http://www.mathworks.com/matlabcentral/fileexchange/>

- for Nelder-Mead, implementations available with the `fminsearch` Matlab function and with N. Higham’s Matrix Computations Toolbox (Higham, 2002).

For computational reasons, the inverse focal lengths have been used as variables instead of the actual focal lengths, and the actual target function used in the computation is  $-R_m(X_f)/C(X_f)$ . However, the results shown below (values of variables and percent gains) refer to the original target function  $C(X_f)/R_m(X_f)$ , which is more representative of the specific energy cost.

## 5. Results and discussion

### 5.1. Results

The configuration used in this work contains 25 mirrors and a receiver of width 0.4 m placed 10 m above the central mirror. The configuration is assumed to be symmetric with respect to the receiver.

The limits imposed on the absorbed radiation (normalized with respect to DNI / AM) are 50 (maximum) and 20 (minimum). The choice of 50 as upper bound is motivated by the fact that the maximum radiation computed for the initial configuration (see below) is 50.4.

*Initial configuration.* This is the starting configuration used to initialize the first optimization step. We consider 25 adjacent mirrors, symmetrically arranged with respect to the collector. Each mirror is 1 m wide. The focal length is taken as the arithmetic mean of the distance between the central mirror and the collector (10 m), and of the distance between one of the most external mirrors and the collector (15.62 m), so focal length is  $f = 12.81$  m for all mirrors. The value of the target function for this configuration is  $e_0 = 560.91$ .

*Uniform optimization.* The optimal configuration is given by

- $\Delta x = 1.21$  m (distance between centers of consecutive mirrors),
- $w = 0.50$  m (semi-width of each mirror),
- $f = 16.85$  m.

The target function value for this configuration is  $e_{UC} = 516.00$ , with a gain of 8.01% with respect to  $e_0$ .

*Optimization of mirror positions.* Six local minima were found, with values of the target function in the range [514.17, 515.78]. This range corresponds to 0.3% of the function value, so the six minima can be considered as essentially equivalent. However, the corresponding optimal configurations present nonnegligible differences in the mirror positions; this fact will be discussed below (a similar behavior has also been observed for the NU case). Anyway, even considering the best of the computed minima ( $e_{OP} = 514.17$ ), the gain with respect to the UC case is only 0.36%. This is quite disappointing, since such a small gain is likely not worth the extra effort involved in building a nonuniform solar field, if the mirrors are kept identical.

*Optimization of focal lengths.* Two minima were found, both equal to  $e_{OF} = 501.50$ . Since the two values were identical and the corresponding configurations were quite close, no further minima have been computed. The gain with respect

to UC is 2.81%, a significant result. This shows that variable focal lengths have a bigger impact on the efficiency than variable mirror positions.

Note that such a gain *cannot* be obtained by simply setting the focal lengths equal to the mirrors' distances from the target. This fact can be easily verified by replacing the focal lengths in the OF optimal configuration with the distances from the target, and noting that the gain disappears completely: the value of the target function becomes 517.65. The optimal setting of focal lengths is a more subtle issue.

*Nonuniform full optimization.* Six local minima were found, with values of the target function in the range [492.95, 496.45]. This range corresponds to 0.7% of the function value. Also in this case, even though the variation of the minima is not large, the corresponding configurations are not necessarily close to each other. Considering the best among the minima, i.e.,  $e_{NU} = 492.95$ , the gain with respect to UC is 4.46%, a satisfactory result. The full optimization of a Fresnel plant therefore proves to be effective in improving efficiency and points to the practical interest of choosing nonuniform configurations over uniform ones. However, it should be stressed that a large part of this gain comes from variable focal lengths; the additional gain of NU over OF is “only” 1.65%.

The results of the optimizations are summarized in Tables 1 and 2. In Table 1 the gains with respect to the initial configuration and to the uniform optimization are listed. In Table 2, the complete lists of the geometric parameters for the best minimum found in OP, OF and NU cases are given.

It has been remarked above that, in the OP and NU cases, slightly different local minima are associated with significantly different plant configurations. This is likely due to a “flatness” of the target function in a large neighborhood of the optimal configurations: as a consequence, nonnegligible changes in the input variables may lead to very small variations in the values of the function. This behavior may slow down the optimization process, but it has no negative consequences on the actual realization of the plant, since all the configurations in the neighborhood are equivalent in practice.

>>> *approx. position of Table 1* <<<  
>>> *approx. position of Table 2* <<<

## 5.2. Discussion

When considering the results shown in the present work, one should always keep in mind that the computations are performed for a model with a specific cost function, considering only the geometric optical collection; moreover, our aim here is primarily to illustrate the procedure and its feasibility, rather than give general and conclusive results. Different price functions or different hypotheses on the functioning of the plant may change the conclusions. However, some aspects can be quite safely generalized to generic Fresnel plants working in a similar concentration range, unless drastically different hypotheses on the plant are made. Among these considerations, the following are especially significant:

- the feasibility of the optimization: a few days of work on a common PC were needed;
- the order of magnitude of the obtained gain: a gain of more than 10% can be expected over a configuration with uniform and adjacent mirrors, and a full optimization gives a gain of the order of 4-5% with respect to a simple uniform optimization;
- the relative importance of the different variables to be optimized: independent tuning of the focal lengths is especially relevant, whereas allowing free positions for identical mirrors does not lead to a significant gain;
- the “flatness” of the minimum, with the consequent existence of a large range of configurations that are in practice equivalent;
- the usefulness of the optimization procedure: when planning an investment for a full scale solar field, reducing the cost of some % means significant savings, which are surely worth the time and effort required for a preliminary optimization.

Moreover, the work can be easily extended to more complex and realistic optimizations, when specific data on the planned system are available. Possible improvements for the application to real systems are listed here.

- Cost functions can be defined more accurately, without any additional computational weight.
- Receiver variables can be included in the optimization, defining suitable working conditions (e.g. lower and upper limits on the radiation that depend on the receiver properties).
- Tracking errors, alignment defects, or surface defects of the mirrors (e.g. random deviation from the perfect profile) can be included in the analysis, modifying the optical simulation. For example, defects of the mirrors can be introduced by adopting a degraded sun profile when computing the efficiency, instead of a perfect Lambertian distribution; the effect of tracking errors can be introduced by averaging different configurations, weighed with the expected probability distribution of the tracking error (4-5 samples should be enough if the probability function is smooth and suitable integration techniques are used). The required computational effort is increased, but the optimization remains feasible; use of ray-tracing does not seem to be required, unless one is interested in the details of the distribution on the receiver (in this case, special care should be applied in the optimization, since ray-tracing methods can introduce discontinuities and spurious small oscillations which could hinder convergence of the optimization).
- Optical physical parameters (reflectivities, absorption coefficients) and their dependence on the incidence angle can be also included in the optical simulation, with minimal changes.
- Average meteorological conditions during the year can be included by adding a factor to the radiation distribution, in the same way as the air mass correction.
- If the receiver structure and thermal properties are given, the thermal efficiency can also be considered. This can be done via a steady-state simulation,

computing beforehand the thermal efficiency of the receiver given the value of the absorbed radiation (as proposed in Grena and Tarquini (2011)), and this would not add any computational weight to the optimization; or it can also be made with a transient simulation, given a suitable not-too-complex model of the receiver behaviour. In this last case it is not possible to compute the average on the sun position distribution, and a step-by-step year simulation should be performed. This seems perfectly affordable, given the short time required by the optical computations; the computation of a few hundred integration values requires a few seconds, so it could be expected that the simulation of some thousands of time steps, with the analysis of thermal efficiency, should not require more than a few minutes.

These improvements can be added without changing the optimization scheme, and - with the exception of the transient thermal simulation - without drastically changing the method to compute the target function.

## 6. Conclusions

The feasibility and the usefulness of the cost optimization of a Fresnel plant were analyzed, considering the optimization of all the variables that describe the geometry of the mirror field (position, width, and focal length of each mirror). The optimized quantity was the plant cost divided by the averaged optical collection during a year. The optimization presents no feasibility problem, and the required computing time was reasonable (a few days of work on a common PC). The usefulness of the optimization was investigated by comparing the results obtained for different optimization levels, starting from a uniform adjacent-mirrors configuration, performing a first uniform, simple optimization, and then freeing different parameters up to a full optimization. The gain obtained for a full optimization with respect to the initial configuration is around 12%. The gain obtained passing from a simple, uniform optimization (only three variables involved) to a full optimization is not negligible (almost 5%), and it is surely worth the effort, given the cost of a full-scale solar plant. Among the various configurations under study, the choice of identical mirrors with nonuniform spacing presents no significant gain with respect to the simple uniform optimization, whereas the largest gain (almost 3%) is obtained when the focal distances are allowed to change. The importance of tuning the focal distance according to the mirror position is an aspect (usually not much stressed in the literature) that should be carefully considered.

Even if these results are obtained under some simplifications and for a specific cost function, the methods presented here give a general scheme also suitable for a more accurate optimization that may involve thermal efficiency and adopt more realistic and complex cost functions.

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