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Editors: Ewa Bergqvist, Magnus Österholm,
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ELEMENTARY SCHOOL TEACHERS' IMPLEMENTATION OF DYNAMIC GEOMETRY USING MODEL LESSON VIDEOS

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In this study, a set of activities on line symmetry in a Web Sketchpad environment, published on the website of a Canadian university, were adapted by two Italian researchers for 1st and 2nd grades with an Interactive White Board in the classroom. The activities were proposed to a 2nd grade during two video-recorded lessons conducted by one of the researchers. The videos were viewed by three teachers who then proposed the same activities in their 1st and 2nd grade classes. The study was carried out over a 6-month period. One of its aims, the main focus of this paper, was the following: to study teachers' implementation of the activities and to identify aspects of the study's teacher instructional improvement cycle that were most influential in their implementations of the technology-based activities.

INTRODUCTION

As Crawford and Adler (1996) underline, there is little evidence that the knowledge generated by researchers in mathematics education is embodied in the teaching practices in school. The gap between educational theory and teachers' practice is a major issue in the field of mathematics education (Jaworski, 1998; Mason, 1998). Surely a critical issue concerns the difficulties related to the dissemination of research results in the public domain: many scholars underline the insufficient dissemination of research results for practitioners (Artigue, 2016) and the need for more two-way modes of communication between researchers and practitioners (Venkat, 2016). But this is only one side of the coin. Beyond the dissemination issue, there is also the issue of teachers' interpretation and use of curriculum materials developed by researchers in mathematics education. Indeed, teachers are decision makers, and their decisions are influenced by factors such as knowledge, but also values, beliefs, emotions and previous experiences (Malara & Zan, 2008). Teachers do not approach their professional learning or curriculum materials as blank slates: they have a wide range of experiences, wants, needs, worries affecting their interpretation and use of professional opportunities (Cuoco, 2001). It has been recognised that teachers act as interpreters and mediators of curriculum materials (Remillard, 2005). This reflects a broader pattern in which the unfolding of innovation in education is shaped by the sense making of the agents involved (Spillane et al., 2002). Teachers typically select from and adapt curriculum materials, incorporating these materials into wider systems of classroom practice. Therefore, it is natural to expect adaptations to curriculum material developed by researchers during a teachers' implementation of that material. In the context of technology integration, Ruthven (2009) has identified five structuring features of

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classroom practice that shape the ways in which teachers adapt particular tools to their classroom contexts. Moreover, Ruthven (2016) writes of *interpretative flexibility* to refer to how technology is taken up to aligned with user concerns and adapted to the situations in which use takes place. This opens the way to variation in modalities of use between different user groups and between different settings for use, and to change in these modalities over time. This may lead, in turn, to the product being redesigned, launching a further cycle of adaptation. From a sociocultural perspective, "the conceptualization of instruments [is] an activity distributed between designers and users" (Rabardel & Waern, 2003, p. 643).

In our research, we aim to gain insight into teachers' adaptation decisions by identifying the particular structuring features come into play and which might be the determining factors. Typically, the aims of specific didactical materials are explicit, while the ways of instantiating such materials in the classroom are not. Therefore, in this study, we wanted to flip this point of view, agreeing with the teachers on the content of a set of activities, and then providing a video of an instantiation of the technology-based activities by a researcher acting as teaching in a classroom. At no point were the teachers asked to imitate the researcher. In other words, we designed a *teacher instructional improvement cycle* in which, once the mathematical topic had been chosen and discussed, and a set of technology-based tasks planned, we decided to video-record a researcher as she instantiated the activities in a 2nd grade class. We saw these videos as boundary objects (Star & Griesemer, 1989) useful to study the nature of the teachers' adaptations. We were especially interested in studying the impact of this relatively uncommon design feature on the teachers' implementation decisions. We explicitly chose experienced teachers for two reasons: we conjectured that their consolidated teaching styles and identities would increase the likelihood that adaptations would emerge, and that they would be more aware of their decisions and, therefore, it would be easier for them to express and discuss them.

THE DESIGNED TEACHER INSTRUCTIONAL IMPROVEMENT CYCLE

The mathematical content chosen was line symmetry, a topic with which many primary school teachers do not feel at ease, and that is considered difficult for students, as well; it is typically taught for the first time in 1st and 2nd grade (ages 6-8). We started with a set of activities on line symmetry in Web Sketchpad, published on the Canadian website http://www.sfu.ca/geometry4yl.html. The study was carried out over six months; it involved three elementary school teachers and four classes (three 2nd grades and one 1st grade) in three Italian elementary schools. The teachers' experience was the following: T1 – 20 years of experience and 16 teaching math; T2 – 23 years of experience and 21 teaching math; T3 – 35 years of experience and 25 teaching math. Three researchers (the authors of the papers) were involved in the study: the third author had designed the original digital activities and conducted classroom-based research using them (see Ng & Sinclair, 2015); the other two authors – working at the same Italian university – organized the implementation of the activities and attended the meetings with the teachers. The phases of the study were: (1) Re-design of the lesson plans and interactive files on the site, a priori considerations on such activities; (2) Presentation to the three teachers of the newly designed materials, explaining the changes made; (3) Instantiation of the activities in a 2nd grade class by the first author; (4) T1, T2, T3 reception of the videos and completion of a questionnaire; (5) T1, T2, T3 carried out the activities in their classes (T1 in a 1st grade, T2 and T3 in 2nd grades); these sessions were video-recorded; (6) A posteriori analysis of the video-recordings by all three researchers; and (7) Final questionnaire on how the lessons went, and meeting with the Italian researchers.

In this paper, we report on the questionnaire data and the final meeting (phases 4 and 7). In particular, we discuss the *role played by the video-recordings* of the implementation of the activities in the teachers' processes of decision-making when adapting and implementing the materials. Indeed, the initial instantiation of the activities by a researcher was a distinctive feature of this study.

The activities on line symmetry

The final version of the activities – after the adaptation developed by the Italian researchers – made use of interactive files projected on an IWB. The files contained sets of colored circles symmetrically arranged on two sides of a line that could be continuously dragged on the screen: when a circle is dragged, its corresponding circle moves so as to preserve symmetry. The line is also draggable.

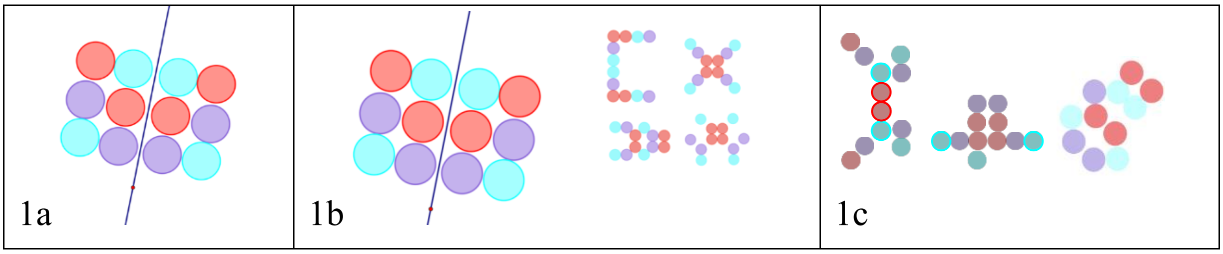

Figure 1a, 1b, 1c: screenshots from the interactive files used for the activities.

The activities consisted of the following five tasks, to be assigned over 3 class periods (about 2.5h): (1) What do you see? What happens when you drag the circles? (Fig. 1a) (2) Make predictions: describe all that will move before dragging to check. (3) In pairs, one student tells the other what to do to reproduce a figure (Fig. 1b). (4) Draw a picture of how the interactive file worked. (5) Which of the pictures can/can't you make with the file? Why? (see Fig. 1c). Tasks 4 and 5 were assigned to all students, at their desks, with paper and pencil. Selected students' answers for Task 5 were discussed by the whole class, using the IWB.

THEORETICAL CONSIDERATIONS

We draw on both Ruthven's (2009) framework to guide our analysis and on the notion of boundary objects, each of which is described below.

Structuring features of classroom practices

In his research analysing the ways in which teachers integrate (or not) digital technologies, Ruthven has identified five structuring features of classroom practices that teachers must often adapt in order to make effective use of the intended affordances of these technologies: working environment (room location, physical layout, use of IWB), resource system (complementing and connecting with existing resources), activity structure (the action and interaction of participants), curriculum script (choosing appropriate tasks, recognising difficulties), and time economy. If any of the structuring features of a teacher's existing classroom practice is challenged by a task and/or tool, the teacher will adapt it, sometimes thereby shifting its intended use (by a researcher or designer).

The videos as boundary objects

Star and Griesemer (1989) describe boundary objects as "scientific objects which both inhabit several intersecting worlds […] *and* satisfy the informational requirements of each of them" (p. 393). Boundary objects are "both plastic enough to adapt to local needs and the constraints of the several parties employing them, yet robust enough to maintain a common identity across sites" (p. 393). They can also "coordinate academic and practitioner perspectives" (Venkat, 2016, p. 187). In the present study, various objects played the role of boundary objects: the activities present on the website; their modifications designed by the Italian researchers; and the videos of the researcher's lessons, that were passed to the teachers and discussed before teachers' implementation of the lessons. All these objects lie at the intersection of different worlds and communities, and have the potential of serving as vehicles to communicate and convey meaning across different communities, even if different communities can define and interpret them in different ways. The activities on the website were at the intersection of the Canadian practices and the Italian ones; the Italian re-designed activities were at the intersection of the researchers' (ideal envisioned classroom and research influence on practices) and teachers' community (actual classrooms and everyday practices); the videos were at the intersection between the single (real) classroom communities and future, potential classrooms communities in which the activities would be realized again.

TEACHERS' REACTIONS TO THE RESEARCHER'S VIDEOS

Question Q1 (What do you see as the major potentials and pitfalls of the activities?) was assigned after the teachers had received only the activities, questions Q2 (What difficulties do you foresee in implementing the activities in your classroom?), Q3 (Having seen the videos, which aspects will you try to replicate and which will you change when teaching yourself?) and Q4 (Are there didactical choices or mathematical considerations in the videos that you did not find clear?) were assigned after they had also viewed the researcher's videos, but still before the teachers' implementations; Q5 (What similarities and differences did you notice in your implementations with respect to that of the researcher?) was assigned after the teachers' implementations.

After receiving only the materials, the teachers (in their answers to Q1) identified a number of potentials in them. These dealt with the resource system, the activity structure and the curriculum script; specifically, the greater ease in working with oblique lines of symmetry compared to previously used materials, and their beliefs – founded in previous experiences – about children's excitement and "surprise" in using the IWB and "moving things around". Interestingly, in their answers to Q2 the main difficulties foreseen by the teachers, after viewing the videos, were only partially related to the pitfalls identified after having viewed only the activities; these had to do with both activity structure and curriculum script. In terms of the former, the teachers were concerned about keeping the children's attention and silence for long periods of time. All three teachers wrote that this kind of activity requires long periods of attention, and with large classes (23-29 children) it may be difficult to maintain silence and concentration. Moreover, they wrote that all children would want to go to the IWB and it would be hard to call them all. T2 also commented on the fact that maintaining silence and order in the classroom would be even harder for her since she is not an "external expert" (unlike the researcher).

In terms of curriculum script, the teachers were concerned about handling students' presumed difficulties in responding to the "creative" drawing request. T2 expressed worry in the prediction task, and T1 wrote: "A pitfall might be the absolute abstractness of the material". They also expressed concern about coordinating discussion about the behaviour of the objects on the screen in a way that would facilitate understanding without putting words in the students' mouths. This was not in an issue in their usual classroom practice, in which they would begin by giving students definitions of objects and then tasks that used these objects. Indeed, speaking about difficulties related to language, T3 wrote: "I think I will have trouble calling 'the objects' in particular the line of symmetry and relationships between this and the balls (parallelism, perpendicularity) with the names given by the students." T2 wrote: "the 'mental experiments' will be maybe the most difficult part but also the most interesting. It will not be easy to manage the lesson when they will have to come up with words to describe the movements without me giving hints." T3 added: "Also with so small and 'ignorant' [in its etymological meaning] children with respect to the language of geometry, I could have trouble using an 'alternative' language that is easy enough to understand."

In response to Q3, the teachers also referred to activity structure and curriculum script analysing critical features of the video. In particular, they appreciated the ideas of: using 'oral descriptions' and words instead of gestures for Tasks 1 and 2; highlighting the terminology used by the children and agreeing on their meanings; using arms and hands to help indicate 'parallel' and 'perpendicular' and seeing if the students tilt their heads to better perceive the line of symmetry. On the other hand, T2 and T3 mentioned that in the researcher's videos they noticed students' difficulties in making up names other that 'rows' and 'columns' to describe perpendicular and parallel alignments of the circles with respect to the line of symmetry. This appeared to them as problematic especially when the line was oblique, so they proposed to modify the activities by asking the students what they meant by ‘rows’ and ‘columns’. We note that this proposed adaptation seems to be in keeping with the curriculum script modeled in the video. In response to Q4, all teachers wrote that they appreciated the didactical choices implemented by the researcher, and they found all mathematical considerations in the videos clear. The only issues that T2 and T3 mentioned related to the resources system in that they preferred to think of these activities as part of a longer sequence that “integrates also the body and manual skills, that is a laboratorial activity”.

Finally, after having implemented the activities, the teachers noticed many similarities between what happened in their classes and in the researcher’s video. The teachers primarily noticed aspects of curriculum script: difficulties in considering the distance from the line of symmetry, usefulness of gestures with arms and hands to indicate parallel and perpendicular alignments with respect to the line of symmetry, use of the word ‘mirror’ to refer to the line of symmetry, difficulties in speaking “with respect to the mirror”, use of arrows to indicate movement in the static drawings, and preference for horizontal or vertical lines of symmetry. However, the teachers also noticed some differences: In T2’s class the children preferred speaking of ‘axis’ when referring to the line of symmetry; moreover, this class had an interesting discussion about whether the line of symmetry was finite or infinite. In T1’s class (the 1st grade) the children seemed to take a longer time to realize they could move the line of symmetry. Finally, T3 remarked, again, how she thought that the children in the researcher’s video were more quiet and attentive than her students, which she believed, depended on the novelty of a different teacher in the classroom.

While many outcomes and comments from the concluding meeting also fit well with Ruthven’s framework, some were more difficult to interpret through such framework. For example, T1 referred to the *content safety* that the videos provided: “It gave me great peace of mind to work next to the researchers, because I knew that what I was going to propose was mathematically sound, and I did not have to worry about appropriateness and depth of the mathematics I was teaching. I knew what properties were important and what to aim for.”

This may be seen as being related to curriculum script, but it emerged because of our novel design, in which the mathematical affordances of the tool use were made explicit for the teachers. T2 also focused on the mathematical dimension of her implementation, saying that the activities helped her engage her students in mathematical discussions: “With dynamic geometry, there was extra support for discussing properties of the line of symmetry, and I could point to the screen and describe properties of a physical object that was coherent with the mathematics.”

DISCUSSION AND CONCLUSIONS

The teachers’ critical analysis of the researcher’s instantiation of the technology-based activities seemed to affect their opinion about potentials and limits of the activity and, in particular, about students’ difficulties. This appears clearly comparing teachers’ answers to Q1 and to Q2. For example, in answering Q1 there was no mention of the language-related difficulties that, instead, appeared heavily in the answers to Q2. The awareness of the delicate issue of how to speak of new mathematical objects and properties characterized many of the teachers' answers to the other questions, as well. So, in a way, the videos planted a new awareness in the teachers, which lead to them paying particular attention to their own words and gestures, as well as to those of their students. This awareness seemed to elicit a new tension in the teachers: in the sense of complex collection of opposing forces of wants, needs and self-assessments of own capabilities that complicate the decision making processes of teachers (Liljedhal et al., 2015). However, the videos also offered helpful suggestions to solve this tension, which were noticed and appreciated by the teachers: how the researcher picked up on students' words and gestured, and how specific language and gestures were agreed upon and used to facilitate discourse on line symmetry. Indeed, the teachers commented on analogies and differences especially on these aspects of the curriculum script. Overall the teachers seemed to appreciate what they saw in the researcher's videos, and decided to implement the activities, seemingly trying to reproduce the activity structure and curriculum script with a very high degree of fidelity. They decided to do this despite their concerns, for example, about the extreme abstractness of the technology-based tasks (an aspect of the curriculum script), or the risky activity structure. In this sense, the researcher's videos seemed to allay the tensions. This may in part be due to the fact that the researcher video was recorded in their school, with students they were familiar with.

Finally, a comment on Ruthven's structuring features. These have provided an insightful tool for analyzing what teachers decided to adapt in order to make effective use of the intended affordances of these technologies. However, some important issues emerged in this study, which do not seem to be properly captured by this framework. The first issue is that of the use of words and gestures in the classroom; this could be seen as part of the curriculum script, or, possibly of the activity structure. But its nature and the strength with which it emerged in this study suggests it should be considered a new feature altogether. We conjecture that this feature may have emerged this strongly in part because of the grade levels involved in the study (attention to words and gestures may play a more major role in early elementary grades than in high school), and in part because of the researcher's video in which particular attention was paid to these aspects of the activities. Also the reference to what we called content safety may be specific to professional development cycles of primary school teachers.

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