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THE FIRST-TIME PHENOMENON:
SUCCESSFUL STUDENTS' MATHEMATICAL CRISIS
IN SECONDARY-TERTIARY TRANSITION

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The huge difficulties related to the transition from secondary to tertiary mathematics are documented by several official data. The analysis of these difficulties is a main issue in educational research at undergraduate level. It is of particular interest the case of the students who choose mathematics as a major. In fact, for the most part, they are students considered excellent in mathematics during secondary school, they seem to have the cognitive resources to succeed, but, in many cases, they encounter several difficulties during their university experience. Therefore, it appears particularly interesting to study also the affective sources and consequences of these difficulties. With this aim, we developed a qualitative and narrative study focused on students' reflections about their mathematical difficulties in the university experience.

INTRODUCTION

A lot of research documents the fact that students encounter huge difficulties in the transition from secondary to tertiary mathematics education. As Niss (2003) observes, students move from one type of institution with its characteristic culture to another type of institution with a completely different (mathematics) culture. This produces marked discontinuities in the transition process and it is the source of several problems. De Guzmán et al. (1998, p. 756) describe tertiary transition as "*a major stumbling block in the teaching of mathematics*". Clark and Lovric (2008), drawing on anthropological theories, theorised a *rite of passage* from secondary to tertiary education. This rite of passage is necessary to handle advanced mathematics and it is characterised by an inevitable crisis: the crisis exists and all the actors of the transition (students, teachers, institutions…) must deal with it. Tall (1991, p. 25) underlines that tertiary transition "involves a struggle (...) and a direct confrontation with inevitable conflicts, which require resolution and reconstruction".

Despite the clear emotional charge of a 'crisis', the research about tertiary transition has been characterised by an almost purely cognitive approach (Artigue, 2016). In particular, the focus of the earliest researches in university mathematics concerned the analysis of the cognitive discontinuity between secondary and tertiary mathematics. Within this frame, Tall (1991) has identified, discussed and analysed significant differences between the approach to mathematics at the secondary level and those at the tertiary level: in the use of symbolism and generalisations, in the role given to the definitions of mathematical object, in the relevance given to the formal reasoning and

proof, in the level of abstraction (Hefendehl-Hebeker et al., 2010, coined the evoking expression 'abstraction shock').

At a cognitive level, this discontinuity requires students to develop new thinking modes and – according to Tall (1991, p. 25) – this is "an immense personal reconstruction". This can be particularly hard for high achievers in mathematics at secondary school because they face with an inexplicable fact: reasoning strategies that worked in their previous mathematical experiences suddenly stop working at university level. In the most of the cases, these students fail in mathematics for the first time in their life: they face with as we call 'the first-time phenomenon'. This is why we believe that the cognitive reconstruction mentioned by Tall strongly involves affective aspects, and they cannot be neglected in the analysis of students' difficulties in tertiary transition. The relevance of the affective component in the transition is confirmed by a recent empirical study developed by Rach and Heinze (2016). They identify five mixed (cognitive and affective) individual variables that affect successful mathematical learning process at the university level: interest, self-concept, specific prior knowledge, prior achievement, and learning strategies.

Therefore, the cognitive reconstruction involves a deep affective reconstruction: it can result particularly hard for high achievers in mathematics at secondary school. In this case, in their university experience, the freshmen can have to reconsider their view of mathematics, their self-perception in mathematics and finally their emotional disposition towards mathematics. In other words, according to the TMA model for attitude (Fig. 1) developed by Di Martino and Zan (2010), they may have to reconsider their attitude towards mathematics.
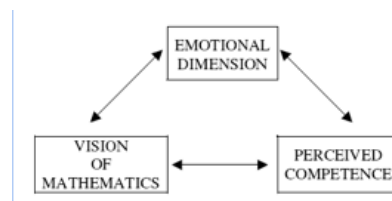

Fig. 1: The TMA-Model for attitude towards mathematics (Di Martino & Zan, 2010)

Within a larger study about students' difficulties in tertiary transition, we focus our attention on the first-time phenomenon. We are interested in analysing its spread and effects. In particular, how the students' view of mathematics and self-perception develop and what role emotions play in the arise and management of this phenomenon.

CONTEXT AND METHODOLOGY

Context

The context of our research is the Bachelor of Mathematics in Pisa. It perfectly fit for our purpose of analysing the tertiary crisis of students considered excellent in mathematics at the secondary school level.

On the one hand, the students of the Bachelor in Mathematics in Pisa were high-rated in the final exam of secondary students from AY 2009/10 to 2012/13 (we define high-rated a mark between 90/100 and 100/100). As shown in Table 1, the percentage of high-rated students enrolled in Pisa is much higher than the national one.

	2009/10	2010/11	2011/12	2012/13
Italy	45.0%	42.4%	44.5%	40.1%
Pisa	73.6%	58.6%	65.7%	60.0%

Table 1: Percentage of high-rated students in the Bachelor of mathematics, in Pisa and national wide.

On the other hand, despite this confluence of high-level students, the dropout rates of the Bachelor in Pisa are in the national average (Table 2).

	2009/10	2010/11	2011/12	2012/13
Italy	24.8%	28.4%	19.4%	20.8%
Pisa	17.8%	34.1%	21.4%	17.8%

Table 2: Dropout rates among first-year students in the Bachelor of Mathematics.

Procedure and rationale

We developed our study with the aim to involve two categories of students: *successful students* – students who passed the exams of the first year of the Bachelor program – and *dropout students* – students that left the Bachelor of Mathematics without getting their degree. The study was conducted in two different stages.

In the first stage, we developed an online questionnaire, structured in sections, different for the two categories of student (successful and dropout). The questionnaire was structured in the following 5 sections: S1 – context information; S2 – perceived differences between school mathematics and university mathematics (for dropout students only); S3 – the personal tertiary experience with mathematics; S4 – ways to overcome difficulties (for successful students) and reasons to quit (for dropout students); S5 – Free conclusive comments. The questionnaires had only a few of multiple-choice questions, for the most part concentrated in S1. We choose to use mainly open questions in order to stimulate narratives (for example the following question in S4: *What memory do you have about your experience with mathematics at university?*). The questionnaire was anonymous and the participation voluntary. 75 successful students and 52 dropout students participated in this first stage.

In S5 of the questionnaire, respondents were invited to share their e-mail to participate in the second, non-anonymous stage of the research. 27 successful students and 10 dropout students replied to the call. In the second stage, the second author conducted a semi-structured interview. The schema of the interview has been developed with the aim to explore in depth the main issues raised by the data collected with the question-

naire and therefore it covered the same five issues of the questionnaire. The interviews were audio-recorded, transcribed and analysed.

The two stages organisation of the study is due to the complementarity and relevance of the two research instruments – questionnaire and interview – (Cohen, Manion & Morrison, 2007). Firstly, we wanted to guarantee anonymity, in order to minimise the social desirability pressure in answering: the anonymous questionnaire allows respondents to freely narrate their university experience. The open items give them the opportunity to talk about facts and emotions that they recognise significant, using the words they consider more appropriate for their memories. Besides, the oral interview maintains this important possibility for respondents, but it also is a non-one-way instrument, permitting the interviewer to ask for more details or clarifications.

The method of analysis

In their study on the analysis of autobiographical interviews, Demazière and Dubar (1997) describe two critical possible approaches to the qualitative data: the *illustrative* approach (collected data are used to illustrate the researcher's theoretical standpoints) and the *restitutory* approach (material is returned to its original form, with no comments). Demazière and Dubar underline the researcher's need to catch the *sense* of a story emerging from an interview, tackling the issue of: 'How to analyse in order to understand?'. About this, Lieblich, Tuval-Mashiach, and Zilber (1998) identify two independent dimensions in the process of reading and analysing stories of life: holistic versus categorical and content versus form. Combining these dimensions, we obtain four different modes of approaching qualitative data, each of them provides different kinds of information. In our study, we carried on (in a different time) two of these modes. Firstly, we developed a holistic and content-oriented approach to the data collected in order to identify recurrent themes and to create categories of answers. In this first phase, we identified the first-time phenomenon as particularly recurrent and significant in the participants' memories. Then, on the basis of the categories created, we developed a categorical-content analysis to describe details of this phenomenon.

In the following, we will focus on the results of the first-time phenomenon. All quotes from the narrative data will be translations. We will quote the data using an alphanumeric code composed of three symbols: Q and I (questionnaire/interview); D or S (dropout/successful student); a serial number.

RESULTS AND DISCUSSION

The most part of successful and dropout students described the cognitive and emotional impact of their first failure in mathematics. We define 'first-time phenomenon' exactly the cognitive and emotional reaction to this first experience of failure in mathematics. This phenomenon is well introduced by the following excerpt:

> I had never encountered difficulties in the study of mathematics during secondary school, so I did not know how to deal with this unexpected failure both concerning the study methods and the emotional reaction. [QS.69]

The categorical-content analysis of the narratives referring to a first-time phenomenon permitted us to recognise the main features of this phenomenon.

Unexpectedness

As it emerges from the previous excerpt, the first-time phenomenon is characterised by an unexpected failure. The unexpectedness is a fundamental component of the phenomenon. On the one hand, it explains the distinction between the *objective* fact (the mathematical failure) and the *subjective* perception of the phenomenon (the interpretation of the fact, in particular, concerning its unexpectedness). On the other hand, it appears to strongly affect the cognitive and emotional reactions to the failure.

There is a tremendous distance between the experienced failure and what happened immediately before the transition period began:

> During secondary school, I was the best one. Here I was considered less than zero. [IS.28]

> I thought it'd be easier. I was the number one in my class, and I found myself failing exams. [IS.16]

This distance provokes strong emotional reactions:

> If you are used to certain things, as to 'be good at school', end up in a completely different world (…) can shock. [QD.35]

> The first time I failed, I felt bad and so I was afraid of experience that feeling again. [IS.35]

During his interview, the student IS.73 generalised and explained this fact:

> Most of the students (…) in Pisa were really good in school and they are not used to failure. Accept failure is not trite. There is a university counselling and I found out that the students who access it are mostly from scientific bachelors than from humanities ones. This fact got me unstuck. Looking back, I would call for help before. [IS.73]

In this situation, the students feel the need to interpret the unexpected failure. According to the attribution theory developed by the psychologist Bernard Weiner (1986), individuals tend to explain their own success and failure in terms of three dimensions: locus (*internal* / *external*); stability (*stable* / *unstable*); controllability (*controlllable* / *uncontrollable*). By the analysis of our data, it emerges that the first attribution of failure is related to the perceived change in the didactic approach:

> University professor presume we know a lot of mathematical facts and they consider 'easy' many arguments that are not so easy to understand for freshmen. [QD.41]

This attribution process causes a clear change in students' view of mathematics and in their perceived competence.

> During secondary school, I had the highest grades in school, here I am mediocre. [IS.65]

> The difficulty of math is definitely increased [in the comparison between the secondary school and university experience] and I understood that I was not so brilliant. [QD.4]

Students have to handle this changing in self-perception in mathematics, and this is not an easy passage.

I started thinking that maybe I was not as good as I thought, that it had been just an illusion. [QD.44]

Sense of impotence

As we can see from the previous extracts, the distance from the secondary school experience doesn't provide means to the students to face difficulties in mathematics. Students are often stuck in this first and unexpected failure experience and they can't find a way to get out of it.

I always did well and I thought that I would make it if I studied. When I arrived, the first impact was terrible. I was on the same course as people who get bored. (…) For the first time, I wanted to do something and I didn't manage to do it. [ID.34]

Strategies which worked during secondary school stop working at the university:

The first impact was hard, I was used to studying the necessary and obtain good marks. Here, I studied a lot and I didn't manage to get enough. [IS.47]

In addition, this changing appears not to be taken in charge by the Institution, students have the perception having to do everything themselves and this is another clear difference with respect to their secondary school experience:

At the first year, it is a shock for everyone, people don't know what's coming, they are not led to what mathematics requires: formalism, precision… You learn it trying and failing but no one shows you how to do. [IS.38]

Students experience a significant sense of impotence, which doesn't let them react appropriately, as if the failure situation were unavoidable and immutable. It is the germ of what Martin Seligman (1975) defines *learned helplessness*: that is, a perceived lack of control over an event, which results from prior exposure to similar negative events perceived as uncontrollable.

Shame

The unexpectedness of the failure elicits strong negative emotional reactions. In particular, the first negative results are often associated with shame:

I was ashamed because sharing my difficulties meant to admit a personal defeat. [ID.14]

Shame is strongly affected by two factors: the fear of having disappointed parents, and, more generally, close people (here the first-time phenomenon plays a role: the failure is unexpected also for other people), and the comparison with peers. In the most of the narratives, students say that they are not the best in mathematics for the first time in their life. Moreover, they often get the impression to be the only ones in difficulty, with important consequences on self-confidence.

I stopped talking about exams with my parents. [QS.73]

I guess I was the worst and the slowest of the class. [QD.21]

I was not happy when I attended math lessons: I had a sense of inferiority and I was ashamed. [QD.48]

The emotion of shame seems to play a key role in the first-time phenomenon, because it often leads students not to share their difficulties and experience with peers. Overcoming this barrier is crucial since it appears to be the principal differences between dropout and successful students. On the one hand, dropout students avoid sharing their difficulties. On the other hand, successful students – overcoming shame – find the courage to share their difficulties with peers.

> So, I made a big improvement when I started asking for help to better students than me (in this respect I was halted by pride and shame). Even now, I'm working on this respect. [QS.6]

> To share difficulties, concerns, passions with other students really helped me. [QS.31]

Realising not to be the only one in difficulty has a double effect: on the emotional level, it alleviates the sense of shame; on the attribution level, it permits them to reconsider the locus of control.

CONCLUSIONS

As it emerges in the students' narratives, the most part of the students who enrol the Bachelor of Mathematics experience failure in mathematics for the first time in their life. For the first time, they have to handle not finding a workable way to study, taking bad marks, not being the best in mathematics. The first-time phenomenon causes a recurrent pattern: the first failure provokes strong cognitive and emotional reactions. Moreover, the failure's unexpectedness undermines the view of mathematics and the self-perception: two of the three dimensions of attitude towards mathematics. But also, the emotional dimension suffers a significant repercussion: the main emotion associated with mathematics after the failure is shame. This change in the attitude towards mathematics has devastating effects: a sense of impotence associated with learned helplessness emerges and shame blocks students in seeking help. In particular, shame is an enormous block in sharing difficulties with peers.

By our data, this appears to be a breakthrough moment between success and dropout students. Overcoming the shame of the unexpected failure in mathematics opens to the crucial exchanges of views with peers: a key element for the redemption in transition. On the hand, it allows students to feel like they are not alone and isolated, but like they are part of a community (the community of mathematics students). On the other hand, it leads to re-evaluate their own difficulties and the attributions they made. This is recognised to be an essential point in overcoming difficulties:

> I try to ask as many questions as I can and, gladly, I see that they are not stupid questions (before instead, I didn't share my doubts with anyone, I was worried that they were stupid doubts). [QS.6]

In the case of dropout students, shame almost always has the upper hand and learned helplessness is reinforced, quickly leading up to dropouts:

> I felt I was wasting my life copying blackboards with symbols (…) I did not understand. So, I preferred to give up and move towards something more accessible. [QD.24]

Our results suggest some implications for practice and some directions for further study (to deepen the knowledge of the phenomenon, its causes and consequences). In particular, it appears necessary to cut off the "first-time chain" giving to the seconddary school students the opportunity to face some mathematical difficulties and also failure in a protect environment in order to work on emotional and cognitive aspects, also at a meta-level.

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