

Application of few-nucleon physics in astrophysics

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Abstract. In this contribution a brief overview of the status and recent developments of *ab-initio* studies of nuclear reactions of astrophysical interest is presented.

1. Introduction

The study of nuclear reactions occurring in the cosmos are one of the primary goal of nuclear physics. These reactions enter in a large variety of phenomena, ranging from Big Bang Nucleosynthesis (BBN) [1], the formation and evolution of stars [2], the structure of neutron stars [3], etc. In particular, the theoretical study of these reactions has been vigorously pursued by many years. The “standard” starting point is to consider the nucleus as composed of nucleons interacting via non-relativistic potentials mediated by pions and other mesons. Also with this “simplification”, there are still several issues not completely understood, in spite of decades of studies. Since the reactions occurring in the cosmos are driven from strong, electro-magnetic (EM), and weak forces, it is necessary, in order to obtain accurate estimates of the reaction rates, to take care of the following issues: *i*) it is necessary to well understand the nature of the strong nucleon-nucleon (NN) and of three-nucleon (3N) interactions; *ii*) a consistent theory for the interaction of nucleons with EM and weak probes (nuclear EM and weak currents) is also required; *iii*) we need to develop reliable and sophisticated methods for taking into account the nuclear structure of the nuclei entering the reactions, and their initial/final state interactions.

Usually, these issues are connected between each other. For example, EM current conservation imposes a strict relation between the Hamiltonian H describing the strong interaction among the nucleons and the nuclear EM current operators, entering in the matrix elements of the EM transition. Analogous relations exist for the weak interaction [4, 5].

Traditionally in nuclear physics the nuclear forces and currents have been constructed adopting some particular meson exchange model, usually limited to consider only one-boson exchange contributions. This picture led to a generation of realistic NN interactions which describe the NN data set with high precision [6, 7, 8]. Also 3N potentials and the EM and weak current operators have been constructed from these meson exchange models [4], but the issue of the consistency between all these quantities still remains to be clarified. Moreover, it is difficult to assess a “theoretical uncertainties” of the results obtained employing these models, as there is not a systematic criterion to select the contributions to be taken into account.

More recently, the advent of effective field theoretical methods and chiral perturbation theory (χ PT) started to provide a more solid basis for a construction of consistent nuclear forces and currents [9, 10]. In the next Section, a brief overview of such a development is presented. In



Section 3, the preliminary results of a preliminary application of this method to the calculation of a reaction rate of astrophysical interest, namely the reaction $p + d \rightarrow {}^3\text{He} + \gamma$, is reported. Finally, in the last section, a brief conclusion is given.

2. *Ab-initio* studies of nuclear reactions of astrophysical interest

Many reactions of astrophysical interest are driven by the EM and weak interactions, as for example the reactions $p + d \rightarrow {}^3\text{He} + \gamma$ or $p + p \rightarrow d + e^+ + \nu_e$. The transition rate is usually well calculated using first order perturbation theory with respect to the “perturbation” Hamiltonian H_I , the part of the Hamiltonian describing the interaction of nucleons with EM and weak external probes. Consequently, it is possible to reduce the problem to the calculation of transition matrix elements of the type $\langle \Psi_f, X_f | H_I | \Psi_i, X_i \rangle$, where $|\Psi_i\rangle$ and $|\Psi_f\rangle$ are the initial and final nuclear states, while $|X_i\rangle$ and $|X_f\rangle$ are the initial and final states of eventual photons and/or leptons appearing in the reaction. The states $|\Psi_i\rangle$ and $|\Psi_f\rangle$ have to be calculated as solutions of $H|\Psi_x\rangle = E_x|\Psi_x\rangle$, where $x = i, f$ (E_i and E_f are energies of the initial and final nuclear states, respectively), namely by fully taking into account the dynamics due to the strong interaction between the nucleons and the distortion induced by the long-range Coulomb interaction between the protons. The latter effect is clearly of paramount importance for studying reactions of astrophysical interest, which usually take place at very low energies.

The “perturbation” H_I can be written in general of the form $H_I = \int d^3x \mathcal{F}_\mu(\mathbf{x}) \mathcal{J}^\mu(\mathbf{x})$, where $\mathcal{J}^\mu \equiv (\rho, \mathbf{j})$ is the so-called nuclear current, consisting of operators acting on the nuclear states. The terms $\mathcal{F}_\mu(\mathbf{x})$ are field-theoretic operators describing the emission/absorption of photons, leptons, etc. This latter part is usually well known, as these operators are derived directly from the Standard Model (SM) EM and weak interactions. On the other hand, the operators $\mathcal{J}^\mu(\mathbf{x})$ have a complicated form due to the effects of the strong force.

There are also reactions not involving the effects of the EM and weak interaction, but driven directly by the strong force and/or the distortion due to the Coulomb interaction. The *ab-initio* study of such reactions requires the direct solution of the Schroedinger equation $H\Psi = E\Psi$ for positive energies, taking into account the dynamics of all nucleons. For example, the reaction ${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + 2p$ involves the dynamics of six nucleons, and the transition amplitude can be calculated solving the Schroedinger equation with appropriate boundary conditions.

In this contribution, we will concentrate on the reactions driven by either the EM or the weak interaction. In the next subsection, a brief introduction to the χ PT for the construction of H will be reported. In Subsect. 2.2 we extend the discussion to the nuclear EM and weak interactions entering H_I . Finally, in the last subsection, some of the most accurate methods used to solve the Schroedinger equation for a system of A nucleons will be briefly recalled.

2.1. Strong forces

Traditionally in nuclear physics the Hamiltonian has been taken in a nonrelativistic form in which pairwise interactions between nucleons are supplemented by 3N forces for systems with more than two nucleons.

As discussed before, in recent years, the problem of the determination of these interactions has been tackled using an effective field theory (EFT) approach, based on the so-called chiral symmetry. This symmetry is an approximate symmetry of Quantum Chromodynamics (QCD), the fundamental theory that describes the interactions of quarks and gluons—the symmetry becomes exact in the limit of vanishing quark masses. At the nuclear level, where the nucleons and pions are the relevant degrees of freedom, this symmetry plays a main role in constraining the forces between nucleons, as well as the interaction of the nucleons with other SM particles as leptons, photons, etc.

In this approach, one starts from the most general Lagrangian describing pions and nucleons which takes into account the (approximate) chiral symmetry of QCD. In particular, chiral

symmetry requires that the pion couple to these baryons, as well as to other pions, by powers of its momentum Q and, as a consequence, the Lagrangian describing their interactions can be expanded in powers of Q/Λ_χ , where $\Lambda_\chi \sim 1$ GeV is the chiral symmetry breaking scale. Classes of Lagrangians emerge, each characterized by a given power of Q/Λ_χ , or equivalently a given order in the derivatives of the pion field and/or pion mass factors, and each containing a certain number of unknown parameters, the so called low-energy constants (LECs), which, in practice, are fixed by comparison with experimental data. The central point of this approach is the definition of a power counting scheme which permits a systematic truncation of the possible contributions to the interaction (the so-called χ PT).

The NN potentials most widely used in the literature are those of Refs. [11] and [12], and they are developed to the next-to-next-to-next-to-leading-order (N^3 LO) in the chiral expansion. Very recently, the potential at one additional order (N^4 LO) has been constructed [13, 14]. The convergence of this chiral expansion, at least for the NN system, seems to be now well established, as follows by comparing the predictions of the N^3 LO and N^4 LO NN forces [13].

In order to be used in the Schroedinger equation these potentials have to be regularized for large relative momenta k (or conversely for small interparticle distances). This is usually performed by multiplying them by a cutoff function, depending on a cutoff parameter Λ , so that for $k \gg \Lambda$ the potentials vanish. This procedure is somewhat arbitrary, and each group has chosen different method of regularization. The dependence of the results on Λ has been often used to establish the “theoretical uncertainty” of the calculation. In fact, it is expected that, at convergence of the expansion over Q/Λ_χ , the results should be independent by the high-energy behavior of the potential, namely by the precise value adopted for the cutoff parameter Λ . Therefore, by varying Λ (within a reasonable range of values), one should have information about the accuracy of the theoretical calculation, in particular that one due to the truncation of the χ PT series. To be noticed that more accurate methods of extracting the theoretical uncertainty of the calculations based on χ PT have been recently proposed [13, 15].

In their current form, these potentials contains 27 parameters (coming from the unknown LECs). These parameters have been fitted to the NN database, with a χ^2 very close to 1. Therefore, these chiral potentials have reached an accuracy comparable to those achieved for the traditional NN potentials.

New mechanisms for the 3N force have also been proposed using the chiral approach. The 3N force appears at N^2 LO and contains two additional LECs [16, 17, 18]. The development of a 3N force at N^3 LO and N^4 LO is currently under way [19].

2.2. Nuclear currents

Also the development of the theory of the nuclear current operators \mathcal{J}^μ was the object of extensive studies. In general, they can be written as sums of one-, two-, and many-body terms that operate on the nucleon degrees of freedom, namely

$$\rho(\mathbf{q}) = \sum_i \rho_i(\mathbf{q}) + \sum_{i<j} \rho_{ij}(\mathbf{q}) + \dots, \quad \mathbf{j}(\mathbf{q}) = \sum_i \mathbf{j}_i(\mathbf{q}) + \sum_{i<j} \mathbf{j}_{ij}(\mathbf{q}) + \dots \quad (1)$$

Above \mathbf{q} is the three-momentum transfer to the nuclear system in the transition.

As stated before, the EM current operator must satisfy the current conservation relation (CCR) $\mathbf{q} \cdot \mathbf{j}_{EM}(\mathbf{q}) = [H, \rho_{EM}(\mathbf{q})]$. Therefore, the CCR imposes rather strict constraints on these EM nuclear currents. The weak current, on the other hand, can be divided in a vector and an axial component. The vector part, from the vector current conservation hypothesis, is directly related to the isovector part of the EM current, and therefore it is in practice constrained by the same CCR given above. The axial current verifies a similar relation in the limit of vanishing pion mass (partially conserved axial current hypothesis), and this also imposes strict constraints on the form of this part [5].

In recent years, the development of the nuclear currents has been brought forth systematically using the EFT approach, both for the EM current [20, 21, 22] and the weak current [23, 24]. In this approach, the EM and weak currents can be obtained directly starting from the same Lagrangian used to construct the nuclear forces. The currents constructed in this way verify the CCR automatically order by order in the χ PT with the corresponding nuclear forces. Also in the construction of the nuclear currents one has to apply cutoff functions in order to regularize them for large relative momenta. An application of the EFT currents to an EM reaction of astrophysical interest is described in Section 3.

2.3. Calculation of the wave functions

Another important aspect for the determination of a reaction rate is the computation of numerically accurate nuclear wave functions. They are obtained as solutions of the associated quantum mechanical problem

$$H\Psi(1, 2, \dots, A) = E\Psi(1, 2, \dots, A) , \quad (2)$$

where H is a nuclear Hamiltonian. In this section, we will limit ourselves to mainly recalling some of the most used techniques. In the Faddeev Equation (FE) approach [25, 26, 27, 28], Eq. (2) is transformed to a set of coupled equations for the Faddeev amplitudes, which are then solved directly (in momentum or coordinate space) after a partial wave expansion.

The Urbana-Argonne group has in recent years developed [29, 30] the Green's Function Monte Carlo (GFMC) method, which has been successfully applied to calculate the spectra of nuclei up to $A = 12$. This technique consists in the computation of $\exp(-\tau H)\Phi(1, 2, \dots, A)$, where $\Phi(1, 2, \dots, A)$ is a trial wave function, using a stochastic procedure to obtain, in the limit of large τ , the exact ground state wave function Ψ [29, 30].

The Stochastic Variational Method (SVM) [31] and the Coupled Rearrangement Channel Gaussian method (CRCG) [32] provide a variational solution of Eq. (2) by expanding the (radial part of the) wave function in Gaussians. Another widely applied variational method is the expansion of the wave function in terms of the Hyperspherical Harmonic (HH) basis [33].

In the no-core shell model (NCSM) method [34, 35] the calculations are performed using a (translationally-invariant) harmonic-oscillator finite basis P and introducing an effective P -dependent Hamiltonian H_P to replace H in Eq. (2). The operator H_P is constructed so that the solution of the equation $H_P\Psi(P) = E_P\Psi(P)$ provides eigenvalues which quickly converge to the exact ones as P is enlarged. The effective interaction hyperspherical harmonic (EIHH) method [36, 37] is based on a similar idea, but the finite basis P is constructed in terms of the HH functions.

In many reaction of astrophysical interest, either the initial or the final wave function corresponds to a scattering state. So far, only the FE and HH methods have been extended to find accurate solutions of Schroedinger equation in the continuum [25, 33]. Recently, also the NCSM has been generalized to treat scattering states using the resonating group method [38].

3. Proton radiative capture on deuterons

The radiative capture $p + d \rightarrow {}^3\text{He} + \gamma$ is a relevant process in many astrophysical environments. For instance, it is the second step in the chain of nuclear reactions which, starting from the proton-proton weak capture, allows to stars like our Sun to shine via pp -chain. Interest in this reaction is also present in the context of BBN [1], since it is one of the main processes through which deuterium can be destroyed and thus affects its eventual yield.

The calculation reported in this Section has been performed as follows. The nuclear Hamiltonian H includes the non-relativistic kinetic energy, the chiral NN potential at N³LO of Ref. [12], the chiral 3N potential at N²LO [18], and the point-Coulomb interaction between the protons.

The “perturbation” Hamiltonian describing the EM interaction of nucleons with photons has been constructed using the chiral technique as well, as described in Ref. [20]. Although both operators in H and H_I have been determined using the same χ PT technique, some details of the construction are not consistent, In particular, the chiral order of the NN force and the EM currents is not consistent, the EM currents do not include a three body term (necessary to verify the CCR with the 3N force), and the cutoff functions used for the NN force and the current are not the same. Consequently, the EM nuclear currents used do not exactly verify CCR. Work is in progress to solve this problem.

The present calculation has been performed by using two values of the cutoff parameter Λ ($= 500$ and 600 MeV) in order to have a hint of the theoretical uncertainty related to the truncation of the chiral expansion.

The initial and final wave functions have been calculated using the HH method, as described in detail in Refs. [33, 39]. The initial wave function is that one of the $p + d$ scattering, while the final wave function is that of the ${}^3\text{He}$ bound state. The theoretical uncertainties arising from the solution of the corresponding Schrodinger equations are very small and can be safely neglected [39].

The preliminary astrophysical S -factor obtained in this way has been reported in Fig. 1, where it is compared with the recent calculation of Ref. [40] performed using the traditional approach, as well as with the existing data of Refs. [41, 42, 44, 43]. The results are presented as a band corresponding to the two selected values of cutoff parameter Λ . The width of the band is rather small, suggesting that the theoretical error due to the truncation of the χ PT is tiny for this process.

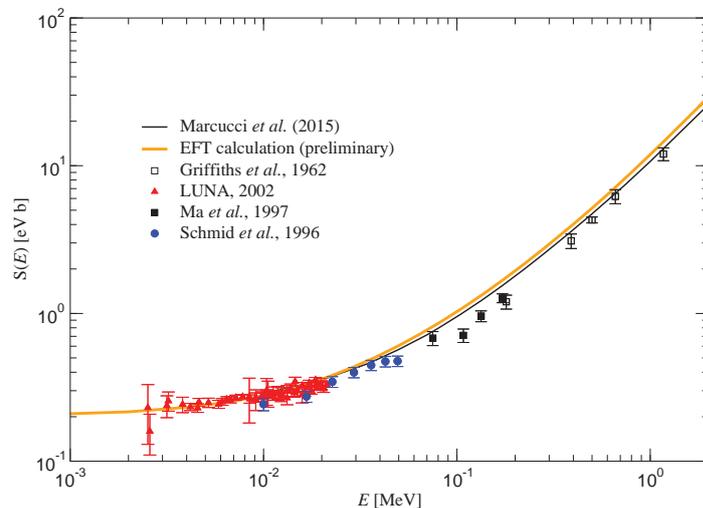


Figure 1. (Color online) Astrophysical S -factor of the reaction $p + d \rightarrow {}^3\text{He} + \gamma$, plotted as function of the center-of-mass kinetic energy E of the $p + d$ initial state. The preliminary results obtained using χ PT as described in the text are given as the orange band. These results are compared with the available experimental data of Refs. [41, 42, 44, 43] and the calculation of Ref. [40] (solid black line).

The present results are in good agreement with the previous calculation and the LUNA data at low energies. Above $E = 50$ keV, the calculated theoretical S -factor is slightly larger than the available experimental data and that of Ref. [40]. Work is in progress to upgrade this χ PT calculation.

4. Conclusions

In recent years, there have been important developments in the construction of the nuclear forces and currents, and in the implementations of techniques for accurately solving (at least for few-nucleon systems) the Schrodinger equation even in presence of complicate NN and 3N force components. In particular, the advent of EFT method, strongly rooted on QCD, now provides a solid basis for the construction of consistent nuclear forces and currents. All these advances should in a near future allow for accurate studies of many reactions of astrophysical interest. Here we have presented only a preliminary study of one of them.

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