Muon capture on the deuteron and the neutron-neutron scattering length

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Background: We consider the muon capture reaction $\mu^- + {}^2\text{H} \rightarrow \nu_\mu + n + n$, which presents a "clean" two-neutron (nn) system in the final state. We study here its capture rate in the doublet hyperfine initial state (Γ^D). The total capture rate for the muon capture $\mu^- + {}^3\text{He} \rightarrow \nu_\mu + {}^3\text{H}$ (Γ_0) is also analyzed, although, in this case, the nn system is not so clean anymore.

Purpose: We investigate whether Γ^D (and Γ_0) could be sensitive to the nn S-wave scattering length (a_{nn}), and we check on the possibility to extract a_{nn} from an accurate measurement of Γ^D.

Method: The muon capture reactions are studied with nuclear potentials and charge-changing weak currents, derived within chiral effective field theory. The next-to-next-to-next-to-leading-order chiral potential with cutoff parameter $\Lambda = 500$ MeV is used, but the low-energy constant (LEC) determining a_{nn} is varied so as to obtain $a_{nn} = -18.95$, -16.0, -22.0, and $+18.22$ fm. The first value is the present empirical one, while the last one is chosen such as to lead to a di-neutron bound system with a binding energy of 139 keV. The LEC's c_D and c_E, present in the three-nucleon potential and axial-vector current (c_D), are constrained to reproduce the $A = 3$ binding energies and the triton Gamow-Teller matrix element.

Results: The capture rate Γ^D is found to be 399(3) s^{-1} for $a_{nn} = -18.95$ and -16.0 fm; and 400(3) s^{-1} for $a_{nn} = -22.0$ fm. However, in the case of $a_{nn} = +18.22$ fm, the result of 275(3) s^{-1} [135(3) s^{-1}] is obtained, when the di-neutron system in the final state is unbound (bound). The total capture rate Γ_0 for muon capture on ${}^3\text{He}$ is found to be 1494(15), 1491(16), 1488(18), and 1475(16) s^{-1} for $a_{nn} = -18.95$, -16.0, -22.0, and $+18.22$ fm, respectively. All the theoretical uncertainties are due to the fitting procedure and radiative corrections.

Conclusions: Our results seem to exclude the possibility of constraining a negative a_{nn} with an uncertainty of less than $\sim \pm 3$ fm through an accurate determination of the muon capture rates, but the uncertainty on the present empirical value will not complicate the interpretation of the (forthcoming) experimental results for Γ^D. Finally, a comparison with the already available experimental data discourages the possibility of a bound di-neutron state (positive a_{nn}).

I. INTRODUCTION

Muon capture reactions on light nuclei, in particular the $\mu^- + {}^2\text{H} \rightarrow \nu_\mu + n + n$ ($\mu-2$) and $\mu^- + {}^3\text{He} \rightarrow \nu_\mu + {}^3\text{H}$ ($\mu-3$) reactions, have recently attracted considerable attention, both theoretically and experimentally [1–8]. One of the reasons for the interest in this issue is the on-going MuSun experiment at the Paul Scherrer Institute (PSI), which is expected to reach a precision of 1.5% in the measurement of the doublet $\mu-2$ capture rate (Γ^D) [3,9]. In fact, the available experimental data for Γ^D are quite inaccurate: Wang *et al.* obtained $\Gamma^D = 365(96)$ s^{-1} [10] more than forty years ago. A few years later, Bertin *et al.* measured $\Gamma^D = 445(60)$ s^{-1} [11], while the measurements performed in the 1980s yielded $\Gamma^D = 470(29)$ s^{-1} [12] and $\Gamma^D = 409(40)$ s^{-1} [13]. Note that all the experiments, except that of Ref. [12], used the neutron detection technique, i.e., they detected a neutron in the final state. On the other hand, for the $\mu-3$ total capture rate (Γ_0), a very accurate measurement is available [14], namely, $\Gamma_0 = 1496(4)$ s^{-1}.

Recent theoretical work on the $\mu-2$ and $\mu-3$ reactions are summarized in Refs. [5–7]. In particular, the work of Ref. [7] represents the first attempt to apply to the considered processes a "consistent" chiral effective field theory (χEFT) approach.

We briefly review it here: The considered two-nucleon (NN) potential is that derived in χEFT up to next-to-next-to-next-to leading order (N3LO) in the chiral expansion by Entem and Machleidt [15,16]. When applied to the $A = 3$ systems, the NN potential is augmented by the three-nucleon (NNN) interaction derived at next-to-next-to leading order (N2LO), in the local form of Ref. [17]. The charge-changing weak current has been derived up to N3LO in Ref. [18]. Its polar-vector part is related, via the conserved-vector-current constraint, to the (isovector) electromagnetic current, which includes, apart from one- and two-pion-exchange terms, two contact terms—one isoscalar and the other isovector—whose strengths are parametrized by the low-energy constants (LECs) g_{4S} and g_{4V}. The two-body axial-vector current includes terms of one-pion range as well as a single contact current, whose strength is parametrized by the LEC d_R. The latter is related to the LEC c_D, which, together with c_E, enters the N2LO NNN potential [19]. The cutoff Λ of the momentum-cutoff function, needed to regularize potentials and currents before they can be used in practical calculations, is taken to be in the range $(500 - 600)$ MeV. The LECs c_D (or d_R) and c_E are determined with the following procedure: (i) The ${}^3\text{H}$ and ${}^3\text{He}$ wave functions are calculated with the hyperspherical harmonics method (see Ref. [20] for a review), using the chiral potentials mentioned above. The corresponding set of LECs,

c_D and c_E, are determined by fitting the $A = 3$ experimental binding energies. (ii) For each set of c_D and c_E, the ^{3}H and ^{3}He wave functions are used to calculate the Gamow-Teller (GT) matrix element in tritium β decay. Comparison with the experimental value leads to a range of values for c_D for each cutoff parameter Λ, from which the corresponding range for c_E is determined. Such a procedure has been widely used by now in a variety of studies, like elastic few-nucleon scattering [21], electromagnetic structure of light nuclei [22], the proton-proton weak capture [23], and the nuclear matter equation of state up to third order in many-body perturbation theory [24]. Finally, after determining the LECs g_{4S} and g_{4V} by reproducing the $A = 3$ magnetic moments, it has been shown in Ref. [7] that the consistent χEFT approach leads to predictions (with an estimated theory uncertainty of about 1%) for the rates of muon capture on deuteron and ^{3}He that are in excellent agreement with the experimental data.

Although extensively studied, a crucial aspect of the $\mu-2$ reaction has not been investigated enough so far: The $\mu-2$ reaction contains in the final state a "clean" two-neutron (nn) system, and therefore the doublet capture rate Γ^D could be sensitive to the nn S-wave scattering length (a_{nn}). In the present work, we check on this possibility, and investigate whether the $\mu-2$ reaction offers the possibility to extract a_{nn} from an accurate measurement of Γ^D, as it will be available soon from the PSI experiment [3,9]. To this aim, we work in the same χEFT framework as in Ref. [7], but apply N3LO NN potentials (with cutoff $\Lambda = 500$ MeV [15]) that predict different values for a_{nn}, i.e., the empirical value $a_{nn} = -18.95$ fm and two more values within a range of $\sim \pm 3$ fm from this empirical one. Note that the empirical value has been obtained from pion capture on the deuteron [25] and neutron-deuteron breakup experiments [26]. We will consider also a case for which $a_{nn} > 0$, which leads to a shallow bound di-neutron state. The reason behind this choice resides in the work of Ref. [27], where it was shown that a hypothetical $^1S_0$ nn bound state would affect the angular distributions of the neutron-deuteron elastic scattering and deuteron breakup cross sections, although a comparison with the available data for the total cross section and angular distributions could not decisively exclude the existence of such a bound state. The analysis was carried out based on the CD Bonn potential [28], where for $a_{nn} = +18.22$ fm a bound nn state was found with a binding energy $B_{nn} = 0.144$ MeV.

The paper is organized as follows: in Sec. II we present the details of the calculation, and in Sec. III we list and discuss the results. Our concluding remark are given in Sec. IV.

II. CALCULATION

We summarize the various steps of our calculations. We consider the NN potential at N3LO of Entem and Machleidt [15], with the cutoff value fixed at $\Lambda = 500$ MeV. The N3LO NN potential includes a charge-symmetry-breaking contact term without derivatives that contributes only in the $^1S_0$ state [16]. This contact is used to create different values for a_{nn}, which are, in particular, -18.95, -16.0, and -22.0 fm. We refer to these different versions of the NN potential as N3LO18, N3LO16, and N3LO22, respectively. The value
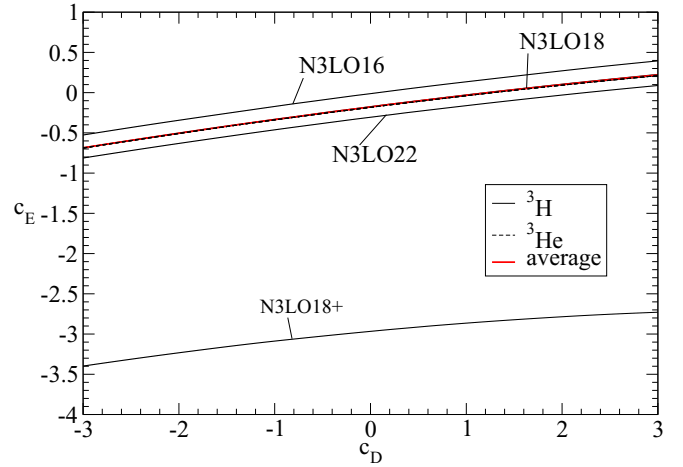

FIG. 1. (Color online) c_D-c_E trajectories fitted to reproduce the experimental ^{3}H and ^{3}He binding energies for the N3LO18, N3LO16, N3LO22, and N3LO18+ NN potentials, augmented by the N2LO NNN interaction model. Note that the curves that can be clearly distinguished in this figure are the ^{3}H trajectories of the respective potentials. The corresponding ^{3}He trajectories cannot be distinguished and are essentially identical to the N3LO18 curve. See text for further explanation.

$a_{nn} = -18.95$ fm corresponds to the empirical one. Finally, we have also constructed a version of the N3LO potential, which produces $a_{nn} = +18.22$ fm (N3LO18+), leading to a two-neutron bound state, with binding energy $B_{nn} = 0.139$ MeV. Then, for each given NN potential, we add the N2LO NNN interaction, and calculate the ^{3}H and ^{3}He binding energies as functions of the LECs c_D and c_E. The corresponding c_D-c_E trajectories are given in Fig. 1. Note that the trajectories which reproduce the ^{3}He binding energy for the various potentials are all on top of each other. Moreover, in the case of the N3LO18 potential, the ^{3}H trajectory is essentially the same as the ^{3}He one. However, for the other NN potentials, the ^{3}H trajectories
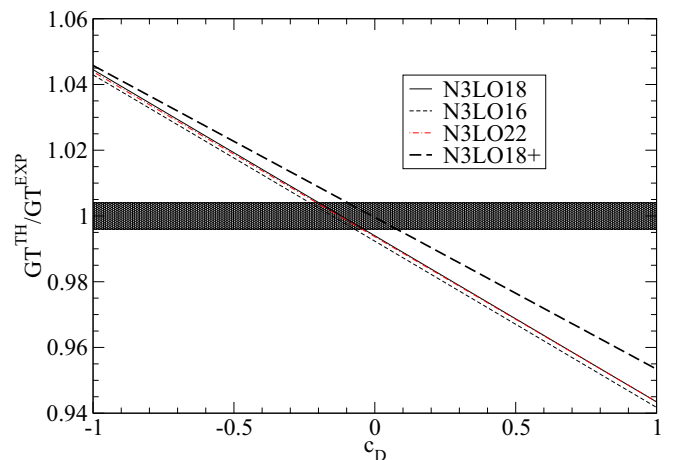

FIG. 2. (Color online) GT^{TH}/GT^{EXP} as a function of the LEC c_D for the N3LO18, N3LO16, N3LO22, and N3LO18+ NN potentials and the N2LO NNN interaction model. The N3LO18 and N3LO22 lines are essentially identical and very close to the N3LO16 one.

TABLE I. LECs c_D and c_E as obtained with the fitting procedure explained in the text.

NN potential model	c_D	$c_E(^3\text{H})$	$c_E(^3\text{He})$
N3LO18	{−0.198, −0.040}	{−0.208, −0.184}	{−0.208, −0.184}
N3LO16	{−0.231, −0.072}	{−0.047, −0.023}	{−0.218, −0.194}
N3LO22	{−0.206, −0.046}	{−0.337, −0.313}	{−0.214, −0.190}
N3LO18+	{−0.096, +0.078}	{−2.978, −2.958}	{−0.197, −0.171}

differ from the corresponding ^{3}He ones and from each other. This is particularly pronounced in the case of N3LO18+. For all cases, except N3LO18, no average curve is displayed, and all the $A = 3$ wave functions have been calculated using, for a given c_D, two different values of c_E, one for ^{3}H and one for ^{3}He, i.e., allowing for charge-symmetry-breaking in the *NNN* interaction. Finally, using the χEFT weak axial current of Ref. [7], as discussed in Sec. I, the GT matrix element of tritium β decay (GT$^{\text{TH}}$) is determined.

The ratio GT$^{\text{TH}}$/GT$^{\text{EXP}}$ is shown in Fig. 2, for all *NN* potentials. The value GT$^{\text{EXP}} = 0.955 \pm 0.004$ has been used, as obtained in Ref. [7]. The range of c_D values for which GT$^{\text{TH}}$ = GT$^{\text{EXP}}$ within the experimental error, and the corresponding ranges for c_E are given in Table I. A few comments are in order. (i) In the N3LO18 case, the ^{3}H and ^{3}He values for c_E are the same, since, as mentioned above, no charge-symmetry-breaking effect is needed in the *NNN* interaction (see Fig. 1). (ii) The ^{3}He values for c_E are all close to each other. This reflects the fact that the *np* and *pp* interactions are not affected by varying the LEC in the *NN* potential to obtain different a_{nn} values. The small difference between the various ^{3}He values is due to the different range of c_D as obtained by the GT fitting procedure. This is again due to the different *nn* interaction, which affects the ^{3}H wave function. (iii) The values for c_E in the ^{3}H case are quite different between each other, especially in the N3LO18+ case. Here we should remark that by using the N3LO18+ potential alone, i.e., without the *NNN* interaction, the triton and ^{3}He binding energies are found to be 9.935 and 7.128 MeV, respectively, with an overbinding in the case of the triton and an underbinding in the case of ^{3}He. A large difference in the range for c_D as well as a strong charge-symmetry-breaking effect is therefore natural. (iv) The values for $c_E(^3\text{H})$ in the case of N3LO18+ are quite large and, according to the general trend of χEFT, should be considered unnatural.

The remaining LECs g_{4V} and g_{4S} entering the *NN* contact terms of the electromagnetic current have been fixed by reproducing the $A = 3$ magnetic moments. Their values are

listed in Table II. Notice that the values of the LECs in the N3LO18+ case are very different from those in the other cases, and although of no relevance in the present study, the isoscalar LEC g_{4S} has even an opposite sign as compared to the N3LO18, N3LO16, and N3LO22 cases. Finally, the numbers in parentheses are the theoretical errors arising from numerics as explained in Ref. [5].

III. RESULTS

The results for the $\mu−2$ doublet capture rate Γ^D, also when only the $^1S_0$ *nn* partial wave is retained [$\Gamma_0(^1S_0)$], calculated with the different *NN* potential models, are listed in Table III. The numbers in parentheses are the theoretical uncertainties obtained by summing, in a very conservative way, those arising from the LECs fitting procedure and those present in the electroweak radiative corrections [29]. By inspection of the table, we can conclude that the $\mu−2$ doublet capture rate is not sensitive to a variation of a_{nn} by $\sim\pm3$ fm, because the change in Γ^D, and $\Gamma^D(^1S_0)$ as well, is smaller than the theoretical uncertainty of 1% or less. On the other hand, a large difference is present for the N3LO18+ results, since $\Gamma^D(^1S_0)$ is a factor of almost 2 smaller than in the other cases. This reflects on Γ^D as well, although the contributions from the waves other than the *S*-wave remain unchanged, and this reduces the difference between the N3LO18+ result and all the others to a factor of ~1.5. Note that the already available experimental data on Γ^D obtained with the neutron detection technique, 365(96) [10], 445(60) [11], and 409(40) s^{-1} [13], although affected by large uncertainties, seem to rule out the N3LO18+ case. For completeness we note that in the case of a bound di-neutron $^1S_0$ state, $(nn)_b$, the reaction $\mu−2$ could go through the channel $\mu^- + {}^2\text{H} \rightarrow \nu_\mu + (nn)_b$ (subsequently denoted as $\mu−2\text{b}$). We have studied the $\mu−2\text{b}$ doublet capture rate and found $\Gamma^D_b = 135(3)$ s^{-1}. By summing this value with the one listed in Table III, we obtain $\Gamma^D = 410(6)$ s^{-1}, where again, in a very conservative way, we have linearly combined the theoretical uncertainties. Notice, however, that this result

TABLE II. LECs g_{4S} and g_{4V} associated with the isoscalar and isovector *NN* contact terms in the electromagnetic current for the different *NN* potentials considered here. The numbers in parentheses are the theoretical errors due to numerics.

NN potential model	c_D	g_{4S}	g_{4V}
N3LO18	{−0.198, −0.040}	{0.207(7), 0.200(7)}	{0.765(4), 0.771(4)}
N3LO16	{−0.231, −0.072}	{0.254(7), 0.257(7)}	{0.801(4), 0.804(4)}
N3LO22	{−0.206, −0.046}	{0.158(7), 0.154(7)}	{0.747(4), 0.745(4)}
N3LO18+	{−0.096, +0.078}	{−0.357(7), −0.358(7)}	{0.463(4), 0.471(4)}

TABLE III. $\mu-2$ doublet capture rate Γ^D and $\mu-3$ total capture rate Γ_0, in s^{-1}, calculated using the *NN* potential models N3LO18, N3LO16, N3LO22 and N3LO18+, augmented, in the $\mu-3$ case, by the N2LO *NNN* interaction. The values for Γ^D when only the $^1S_0 nn$ final state is retained are also listed. The values in parentheses are the theoretical uncertainties.

NN potential model	$\Gamma^D\left(^1S_0\right)$	Γ^D	Γ_0
N3LO18	254(2)	399(3)	1494(15)
N3LO16	254(2)	399(3)	1491(16)
N3LO22	255(2)	400(3)	1488(18)
N3LO18+	130(2)	275(3)	1475(16)

is irrelevant to the experiments of Refs. [10,11,13], because they all used the neutron detection method, which implies that they measured $\mu-2$ without a bound *nn* state contribution.

In Table III we list also the results for the $\mu-3$ total capture rate Γ_0, although in this case the *nn* system is not clean anymore, as in the $\mu-2$ case. From these results, we can conclude that the N3LO18+ case is significantly smaller than all the others, but only slightly in disagreement with the accurate experimental datum of 1496(4) s^{-1} [14], due to our theoretical uncertainty of about 1%.

IV. SUMMARY AND CONCLUSIONS

The muon capture reactions $\mu-2$ and $\mu-3$ have been studied with nuclear potentials and charge-changing weak currents derived within χEFT. The LEC present in the N3LO *NN* potential determining a_{nn} is varied so as to obtain values within a range of $\sim \pm 3$ fm around the empirical one. A positive value for a_{nn} has also been considered, such as to lead to a di-neutron bound system with a binding energy of 139 keV. Our results can be summarized as follows: no significant sensitivity to the *S*-wave *nn* scattering length is found, when this is

changed within $\sim \pm 3$ fm from the present empirical value. The change in a_{nn} affects only the values for the LECs c_D and c_E, requiring a charge-symmetry-breaking *NNN* interaction, which has been taken into account in the present study. The situation is quite different in the case of $a_{nn} = +18.22$ fm (case of *nn* bound state), since Γ^D turns out to be a factor of about 1.5 smaller than in the previous cases, unless the $\mu-2$b reaction is included. It should be mentioned also that in the case of a positive *nn* scattering length, the charge-symmetry-breaking effect in the *NNN* interaction is found to be very large, since the ^{3}He nucleus is underbound, while the triton is overbound.

Therefore, we can conclude that the very accurate determination of Γ^D by the MuSun Collaboration at PSI will not be able to extract a more precise value for a_{nn} in the case of a negative a_{nn} value. On the other hand, the uncertainty in the present empirical value for a_{nn} will not complicate the interpretation of the MuSun result, from which it will be possible to obtain a clear extraction of the LEC d_R (or c_D). Furthermore, the MuSun experiment will most likely be able to confirm or exclude the existence of a bound di-neutron state, if the experimental capture rate will have a sufficiently high accuracy such that a value of 410(6) s^{-1} can be ruled out. However, in our opinion, should the existence of a bound di-neutron state be confirmed, then our current very successful picture of muon capture processes and, more general, of light nuclei would have to be severely revised. This is a similar conclusion to what was obtained in Ref. [30], where the possibility of a bound tetra-neutron system was investigated.

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