Implementation of an economic MPC with robustly optimal steady-state behavior

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Abstract: Designing an economic model predictive control (EMPC) algorithm that asymptotically achieves the optimal performance in presence of plant-model mismatch is still an open problem. Starting from previous work, we elaborate an EMPC algorithm using the offset-free formulation from tracking MPC algorithms in combination with modifier-adaptation technique from the real-time optimization (RTO) field. The augmented state used for offset-free design is estimated using a Moving Horizon Estimator formulation, and we also propose a method to estimate the required plant steady-state gradients using a subspace identification algorithm. Then, we show how the proposed formulation behaves on a simple illustrative example.

Keywords: Economic Model Predictive Control (EMPC); Real-Time Optimization (RTO); Modifier-adaptation; Moving Horizon Estimator (MHE); Systems identification.

1. INTRODUCTION

The typical hierarchical architecture for economic optimization and control in the process industries sees often two main layers based on optimization. As advanced controllers, model predictive control (MPC) systems, have become a standard in the process industries (Qin and Badgwell, 2003). The other optimization based layer is the one named real-time optimization (RTO), dedicated to the plant economic steady-state optimization. This feeds the calculated setpoints into the MPC algorithms that have the duty to guide a process there reliably, exploiting a (linear or nonlinear) dynamic model of the process and numerical optimization algorithms, dealing with constraints on outputs and inputs. It has been shown that many applications, nowadays, have not economical advantages following this separation (Engell, 2007).

Anyway both MPC algorithms and RTO suffers from plant-model mismatches. This can derive from model uncertainties and unmeasured disturbances so even model-free RTO algorithm (Guay and Peters, 2006) can suffer of offset problems and calculate non-economically optimum stationary points.

The offset correction in tracking MPC algorithms has been firstly systematized by Muske and Badgwell (2002) and Pannocchia and Rawlings (2003). Zero steady-state offset with respect to external setpoints is obtained by augmenting the nominal system with disturbances, i.e. building a disturbance model, and estimating both state and disturbance from output measurements. Pannocchia (2015) offers a comprehensive review about disturbance models and offset-free MPC design.

Also in the RTO literature plant-model mismatch issues have been analyzed. Recent proposals utilize a nominal fixed process model and appropriate measurements to guide the iterative scheme towards the optimum. These kind of methods that recursively adapt correction terms exploiting difference between actual and predicted functions or gradient are named "modifier-adaptation" (Chachuat et al., 2009). It has been demonstrated (Marchetti et al., 2009) that these modifiers guarantee to satisfy the necessary condition of optimality of the unknown plant. A strong drawback of this methodology is requiring information about the plant gradient. For this reason, gradient estimation is an active research area in the RTO literature (see e.g. (Costello et al., 2016; Marchetti et al., 2011) and references there in).

As above underlined, the RTO and MPC hierarchical division issues has led to the increased interest in merging the two layers. Several are the proposals to improve the effective use of dynamic and economic information throughout the hierarchy. Approaches that move the dynamic information into the RTO take the name of "D-RTO" (Würth et al., 2011). The D-RTO structure still requires the two layers existence. On the other hand, the interest here is to move economic information into the control layer. This approach requires the traditional tracking objective function substitution with the economic stage cost function used in the RTO layer. This formulation takes the name of economic MPC (EMPC) (Rawlings et al., 2012; Ellis and Christofides, 2014) and the RTO layer is completely eliminated. Being the stage cost in the finite horizon problem not strictly convex anymore, the traditional stability conditions for closed-loop system based on the Lyapunov function do not hold anymore. It has been shown that depending on certain systems and cost functions, oscillating solutions may be economically more profitable than steady-state ones (Angeli et al., 2009; Rawlings et al., 2012). Dissipativity (Angeli et al., 2012) and turnpike (Faulwasser and Bonvin, 2017) are the properties that play a role in the convergence of EMPC.

EMPC can suffer from converging to a non economically steady-state point when affected by plant-model mismatch. Some works in the literature indicate multi-model linear offset-free formulations as a solution to this problem (Alvarez and Odloak, 2012; Ferramosca et al., 2017). In (Vaccari and Pannocchia, 2016) it has been shown how even the traditional offset-free methodology can fail for the EMPC case. Another technique has been proposed, utilizing both offset-free MPC and modifier-adaptation idea in order to build a more reliable EMPC. In the present paper the work of Vaccari and Pannocchia (2016) is extended including a different kind of state estimation showing how the modifier-adaptation based technique achieves the ultimate optimal economic performance despite modeling errors and/or disturbances. Moreover, a new method based on system identification is here introduced in order to give a gradient estimation and overcome previous assumptions.

The rest of this paper is organized as follows. The problem definition and the current related works are presented in Section 2. The proposed method, with a detailed analysis and description of the new EMPC algorithm implementation is presented in Section 3. The presented algorithm and other variants are tested over a case study, and discussions about numerical results obtained are reported in Section 4. Section 5, finally, concludes the paper and presents possible future directions of this methodology.

2. PROBLEM DEFINITION

2.1 Plant and cost specification

In this paper we are concerned with the control of discrete time-invariant dynamical systems in the form:

$$\begin{aligned} x_p^+ &= F_p(x_p, u) \\ y &= H_p(x_p) \end{aligned} \tag{1}$$

in which $x_p \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$ are the plant state, control input and output at a given time, respectively, x_p^+ is the successor state. The plant output is measured at each time $k \in \mathbb{Z}$ and it is denoted by y_k. Functions $F_p : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ and $H_p : \mathbb{R}^n \to \mathbb{R}^p$ are not known precisely but are assumed to be differentiable.

Input and output are required to satisfy the following input and output constraints at all times:

$$u_{\min} \leq u \leq u_{\max}, \quad y_{\min} \leq y \leq y_{\max} \tag{2}$$

in which $u_{\min}, u_{\max} \in \mathbb{R}^m$ and $y_{\min}, y_{\max} \in \mathbb{R}^p$ are the bound vectors. We define the system (1) economically optimized, when a given cost function $\ell_e(y,u)$ is minimized, where $\ell_e : \mathbb{R}^p \times \mathbb{R}^m \to \mathbb{R}$.

Assumption 1. The cost function $\ell_e(y,u)$ is assumed continuously differentiable.

Given the system (1), an equilibrium point of this system is identified by the triple (x_s, u_s, y_s):

$$\begin{aligned} x_s &= F_p(x_s, u_s) \\ y_s &= H_p(x_s) \end{aligned} \tag{3}$$

and the related optimal one is then defined by:

$$(x_s^0, u_s^0, y_s^0) = \arg\min_{x,u,y} \ell_e(y,u) \tag{4a}$$

subject to

$$x = F_p(x,u) \tag{4b}$$

$$y = H_p(x) \tag{4c}$$

$$u_{\min} \leq u \leq u_{\max} \tag{4d}$$

$$y_{\min} \leq y \leq y_{\max} \tag{4e}$$

We assume (4) feasible and its solution (x_s^0, u_s^0, y_s^0) unique, albeit unknown.

Remark 2. The fact that (x_s^0, u_s^0, y_s^0) is unknown comes directly from the uncertainty in the description of (1).

2.2 Model and standard EMPC algorithm

In order to design an MPC algorithm, a process *model* has to be defined:

$$\begin{aligned} x^+ &= f(x,u) \\ y &= h(x) \end{aligned} \tag{5}$$

in which $x, x^+ \in \mathbb{R}^n$ denote the current and successor model states. The functions $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ and $h : \mathbb{R}^n \to \mathbb{R}^p$ are assumed to be differentiable.

Let $\boldsymbol{x} = \{x_0, x_1, \dots, x_N\}$ and $\boldsymbol{u} = \{u_0, u_1, \dots, u_{N-1}\}$ be, respectively, a state sequence and an input sequence. The Finite Horizon Optimal Control Problem (FHOCP) solved at each time is the following:

$$(\boldsymbol{x}^*, \boldsymbol{u}^*) = \arg\min_{\boldsymbol{x},\boldsymbol{u}} \sum_{i=0}^{N-1} \ell_e(y_i, u_i) \tag{6a}$$

subject to

$$x_0 = \hat{x}, \tag{6b}$$

$$x_{i+1} = f(x_i, u_i) \tag{6c}$$

$$u_{\min} \leq u_i \leq u_{\max} \tag{6d}$$

$$y_{\min} \leq h(x_i) \leq y_{\max} \tag{6e}$$

$$x_N = x_s^* \tag{6f}$$

in which $\hat{x}$ and x_s^* are the current estimate and steady-state value of the state model (5) respectively, and N is a positive integer representing the horizon length. Assuming problem (6) feasible, the associated receding horizon implementation is given by:

$$u = u_0^* \tag{7}$$

One of the main differences between tracking and economic MPC is the non strictly convexity of the cost function. As a matter of fact, since the EMPC utilizes directly the operating cost as stage cost, it may happen that $\ell_e(x_s, u_s) > \ell_e(x,u)$ for some feasible pair (x,u) that is not the steady state target (x_s, u_s). The concept of average asymptotic performance of economic MPC (Angeli et al., 2009; Rawlings et al., 2012) is based indeed on the fact that oscillating solutions may be economically more profitable than steady-state ones, depending on both systems and cost functions. Moreover, the standard Lyapunov function definition for closed-loop system cannot be used as a stability argument. Although, in the literature, a Lyapunov-based EMPC using an auxiliary MPC problem solution has been also formulated (Heidarinejad et al., 2012; Ellis and Christofides, 2014).

Assumption 3. In this work the steady-state operation is assumed to be more profitable than an oscillating behavior.

In this direction the concepts of dissipativity (Angeli et al., 2012; Rawlings et al., 2012) and turnpike (Faulwasser and Bonvin, 2017) has to be addressed. These properties play a key role in the analysis and design of schemes for EMPC. Faulwasser et al. (2015) have shown how in a continuous-time form, dissipativity of a system with respect to a steady state implies the existence of a turnpike and optimal stationary operation at the same steady state.

3. PROPOSED TECHNIQUE

Having revised the traditional economic MPC, we now illustrate the proposed method that aims to make the closed-loop system to converge to the true plant optimum defined in (4). Let us define the various constituents of the new algorithm.

3.1 Offset-free augmented system

Standard offset-free MPC algorithms are generally based on an augmented model (Muske and Badgwell, 2002; Pannocchia and Rawlings, 2003; Maeder et al., 2009; Morari and Maeder, 2012), whose general form can be written as:

$$\begin{aligned} x^+ &= F(x,u,d) \\ d^+ &= d \\ y &= H(x,d) \end{aligned} \tag{8}$$

in which $d \in \mathbb{R}^{n_d}$ is the so-called *disturbance*. The functions $F : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^{n_d} \to \mathbb{R}^n$ and $H : \mathbb{R}^n \times \mathbb{R}^{n_d} \to \mathbb{R}^p$ are assumed to be continuous and *consistent* with (5), i.e. $F(x,u,0) = f(x,u)$ and $H(x,0) = h(x)$. Since d is considered as other n_d states of the system, defining $\xi = [x^T, d^T]^T$ as the augmented state, we impose the following assumption regarding the non-linear system observability (Pannocchia et al., 2015).

Assumption 4. The augmented system (8) is observable.

3.2 State and disturbance estimation

Given the system (8) an observer is defined to estimate the augmented state. In this case, since the chosen model is non-linear and bound constraints are present on both outputs and inputs, the estimator selected is the so called Moving Horizon Estimation (Rawlings, 2013) (MHE). The new state estimate, at every sample time k, is the result of an optimization problem based on N_T past output data. This method has demonstrated good results against other state estimation techniques for non-linear systems such the one considered in this work.

Let us define $\boldsymbol{\xi} = \{\xi_0, \ldots, \xi_{N_T}\}$ as the augmented state sequence and $\mathbf{w} = \{w_0, \ldots, w_{N_T-1}\}$, $\mathbf{v} = \{v_0, \ldots, v_{N_T-1}\}$ for the augmented process noise and measurement noise sequences, where $w_j = [w_j^{x\,T}, w_j^{d\,T}]^T$. Hence the MHE problem to solve, at the generic time k, is the following:

$$\min_{\boldsymbol{\xi},\mathbf{w},\mathbf{v}} \quad \Gamma(\gamma) + \sum_{j=k-N_T+1}^{k} \ell_{MHE}(w_j, v_j) \tag{9a}$$

subject to:

$$\gamma = \hat{\xi}_{k-N_T+1|k-N_T+1} - \bar{\xi}_0 \tag{9b}$$

$$\hat{x}_{j+1} = F(\hat{x}_j, u_j, \hat{d}_j) + w_j^x \tag{9c}$$

$$\hat{d}_{j+1} = \hat{d}_j + w_j^d \tag{9d}$$

$$y_j = H(\hat{x}_j, \hat{d}_j) + v_j \tag{9e}$$

$$y_{\min} \leq H(\hat{x}_j, \hat{d}_j) \leq y_{\max} \tag{9f}$$

in which y_j represents the measurement at time j and N_T represents the horizon length of the MHE problem. The term $\Gamma(\gamma) = \frac{1}{2}\gamma' P_k \gamma + p_k$ approximates the arrival cost of the full estimation problem for times before $k - N_T + 1$. The weighting term P_k represents the inverse of the covariance of the *a priori* augmented state estimate $\bar{\xi}_0$. Until the data window is not filled, the optimization problem solved is the so called Full Information Estimation, i.e. the sum in (9a) becomes $\sum_{j=0}^{k}$ and the size of the problem increases at every iteration. Once N_T input and output data have been collected, the horizon moves one step forward and the optimization problem does not grow anymore, becoming the one in (9), i.e. the first data point is discarded as the new one is added. This shifting can be done in different ways and may cause the overlapping of two different data windows. The role of the term p_k is then to subtract double counted data points during this horizon movement. Its starting value is 0. Starting from chosen initial values, P_k and $\bar{\xi}_0$ are accordingly updated only when the window of N_T data has been collected, i.e. $k = N_T$, and their updating formula may differ depending on the arrival cost updating selected (Rao, 2000).

The estimation problem cost function $\ell_{MHE}(\cdot)$ in (9a) is usually quadratic and defined as follows:

$$\ell_{MHE}(w_j, v_j) = \|w_j^x\|^2_{Q_x^{-1}} + \|w_j^d\|^2_{Q_d^{-1}} + \|v_j\|^2_{R^{-1}} \tag{10}$$

in which $Q_x \in \mathbb{R}^{n \times n}$, $Q_d \in \mathbb{R}^{n_d \times n_d}$, $R \in \mathbb{R}^{p \times p}$ represent the covariance process and measurement noise respectively. Note how the process noise w_j for the augmented state ξ has different weights for the state x and disturbance d. This can help to address a more or less aggressive tuning depending on the system (8). The filtered estimate of $x(k)$ and $d(k)$ in (8), $\hat{x}_{k|k}$ and $\hat{d}_{k|k}$ are hence finally obtained from $\xi_{N_T-1|k}$, solution of problem (9).

3.3 Target calculation with modifier-adaptation technique

Given the current estimate of the augmented state $(\hat{x}_{k|k}, \hat{d}_{k|k})$, the offset-free economic MPC algorithm firstly computes the target problem modified in the following form:

$$(x^*_{s,k}, u^*_{s,k}, y^*_{s,k}) = \arg\min_{x,u,y} \ell_e(y,u) \tag{11a}$$

subject to

$$x = F(x, u, \hat{d}_{k|k}) \tag{11b}$$

$$y = H(x, \hat{d}_{k|k}) + (\lambda^G_{k-1})^T (u - u^*_{s,k-1}) \tag{11c}$$

$$u_{\min} \leq u \leq u_{\max} \tag{11d}$$

$$y_{\min} \leq y \leq y_{\max} \tag{11e}$$

in which $u^*_{s,k-1}$ is the steady-state input target found at the previous sampling time $k-1$, and λ^G_{k-1} is the modifier matrix calculated at the previous iteration with the following filtering relation:

$$\lambda^G_k = (1-\alpha_s)\lambda^G_{k-1} + \alpha_s \left(\nabla_u G_p(u^*_{s,k}) - \nabla_u G(u^*_{s,k}, \hat{d}_{k|k}) \right) \tag{12}$$

in which α_s is a scalar first-order filter constant $\in (0,1]$, $G : \mathbb{R}^{m+n_d} \to \mathbb{R}^p$ and $G_p : \mathbb{R}^m \to \mathbb{R}^p$ are the model and the plant steady-state input-to-output maps respectively, i.e.:

$$\begin{cases} x_s = F(x_s, \hat{d}_{k|k}, u_s) \\ y_s = H(x_s, \hat{d}_{k|k}) \end{cases} \Rightarrow y_s = G(u_s, \hat{d}_{k|k}) \tag{13}$$

The operator $\nabla_u(\cdot)$ is the gradient of the considered function respect to the variable u. The system is initially consistent with the traditional EMPC defined in Section 2.2, i.e. $\lambda^G_0 = 0$. The idea of building the problem (11) using the term λ^G_{k-1} is borrowed from the modifier-adaptation technique developed in the RTO literature (Marchetti et al., 2009). In (Vaccari and Pannocchia, 2016) it has been shown that iteratively, if a tracking cost function is substituted in (6) then the closed-loop system converges to the solution of (11) and this is also the optimal plant solution in (4), i.e.:

$$\lim_{k\to\infty} u_k = \lim_{k\to\infty} u^*_{s,k} = u^0_s \tag{14}$$

The proof involves a KKT matching of the two problems above mentioned and it is inspired by the work in (Marchetti et al., 2009).

3.4 Control optimization problem with modifier-adaptation technique

The FHOCP (6) is similarly modified. Hence the problem solved at each decision time is rewritten in the following form:

$$(\boldsymbol{x}_k^*, \boldsymbol{u}_k^*) = \arg\min_{\boldsymbol{x},\boldsymbol{u}} \sum_{i=0}^{N-1} \ell_e(y_i, u_i) \tag{15a}$$

subject to

$$x_0 = \hat{x}, \tag{15b}$$

$$x_{i+1} = F(x_i, u_i, \hat{d}_{k|k}) \tag{15c}$$

$$y_i = H(x_i, \hat{d}_{k|k}) + (\lambda_{k-1}^G)^T (u_i - u_{s,k}^*) \tag{15d}$$

$$u_{\min} \le u_i \le u_{\max} \tag{15e}$$

$$y_{\min} \le y_i \le y_{\max} \tag{15f}$$

$$x_N = x_{s,k}^* \tag{15g}$$

Analogously to the standard economic MPC, assuming problem (15) feasible, the associated receding horizon implementation is given by:

$$u_k = u_{0,k}^* \tag{16}$$

It has to be noted that the correction term is built analogously to the one in the target problem (11). Although, in this case, for convergence purposes, we use the current calculated target $u_{s,k}^*$.

3.5 Plant gradient estimation

As already underlined in the RTO literature (Marchetti et al., 2009) and also in (Vaccari and Pannocchia, 2016), the major drawback of the modifier-adaptation technique is that the modifier matrix λ_{k-1}^G construction and update in (12) requires the knowledge of the steady-state process gradient $\nabla_u G_p(\cdot)$, which obviously is unknown. In literature, many are the works focused on estimating these gradients involving a collection of past output data (see e.g. Gao et al. (2015); Costello et al. (2016)).

In this work a similar identification approach has been used in order to calculate an approximated value of the steady-state process gradient that we name as $\nabla_u \tilde{G}_p(\cdot)$. The gradient approximation routine is articulated in the following steps. Firstly, the closed-loop system is brought to a steady-state equilibrium until $k = k_{id}$. Then, for a determined amount of time that we define as identification horizon N_{id}, the result of the FHOCP (15) is excited by a random signal, i.e. the control law (16) is substituted by:

$$u_k = u_{0,k}^* + \sigma_k \tag{17}$$

in which $\sigma_k \in \mathbb{R}^m$ is a random vector. In order to avoid any feasibility problem, the *excited* input u_k is checked to be into its boundaries, otherwise it is saturated to nearest bound. For $k \in [k_{id}, k_{id} + N_{id}]$ we collect all the u_k calculated by (17) in a sequence named $U_{id} \in \mathbb{R}^{m \times N_{id}}$. Simultaneously, the corresponding y_k measurements are calculated evaluating (1) and, as well, stored into an output sequence Y_{id}, i.e. $Y_{id} \in \mathbb{R}^{p \times N_{id}}$. When $k = k_{id} + N_{id}$ a subspace identification is performed to obtain a local linearization of the plant system. The identification algorithm chosen to identify the process is the N4SID method (Overschee and Moor, 1994) using an open-source package [1]. In particular, we are interested in computing locally linearized state-space matrices A, B and C to compute the identified system gain $\nabla_u \tilde{G}_p(\cdot)^T = C(I-A)^{-1}B$, which is the best approximation of the steady-state gradient of the process. In order to avoid initial mismatch and numerical instability in the identification step, both the input and the output sequences are centered with respect to the model steady-state triple evaluated at $k = k_{id}$ with (11), i.e. $U_{id} - u^*{}_{s,k_{id}}$, $Y_{id} - y^*{}_{s,k_{id}}$. It is easy to prove that if the couple $(u^*{}_{s,k_{id}}, y^*{}_{s,k_{id}})$ is an equilibrium point of (1), the identified matrices A, B and C are the same to the one calculated identifying on the original sequences U_{id}, Y_{id}. The identification is then performed every iteration sliding the data windows forward. It has to be noted that for $k > k_{id} + N_{id}$ the original receding horizon law (16) is restored. To prevent numerical issues, identification is performed only if the norm of the current input sequence is greater than a threshold value τ_{id}, i.e. $\|U_{id}\| \ge \tau_{id}$.

It is worth noting that the whole gradient estimation procedure does not require any particular set of experiments on the plant. As a matter of fact, since the only plant measurements collected are transient ones, no additional sensors or controllers are needed.

[1] SIPPY (Systems Identification Package for PYthon) is available at GITHUB https://github.com/CPCLAB-UNIPI/SIPPY

Table 1. Actual reactor parameters.

Description	*Symbol*	*Value*	*Unit*
Kinetic constant 1	k_1	1.0	min^{-1}
Kinetic constant 2	k_2	0.05	min^{-1}
Reactor volume	V	1.0	m^3
A feed concentration	c_{A0}	1.0	$\frac{\text{kmol}}{\text{m}^3}$
B feed concentration	c_{B0}	0.0	$\frac{\text{kmol}}{\text{m}^3}$
A price	β_A	1.0	$\frac{€}{\text{kmol}}$
B price	β_B	4.0	$\frac{€}{\text{kmol}}$

4. CASE STUDY

4.1 Process and optimal economic performance

The EMPC application chosen is a Continuous Stirred Tank Reactor (CSTR), in which two consecutive reactions take place:

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C \tag{18}$$

The reactor is described by the following system of ordinary differential equations (ODE):

$$\dot{x}_1 = \frac{u}{V}(c_{A0} - x_1) - k_1 x_1 \tag{19}$$
$$\dot{x}_2 = \frac{u}{V}(c_{B0} - x_2) + k_1 x_1 - k_2 x_2$$

in which x_1 and x_2 are the molar concentrations of A and B in the reactor, c_{A0} and c_{B0} are the corresponding concentrations in the feed, u is the feed flow rate regulated through a valve, V is the constant reactor volume, k_1 and k_2 are the kinetic constants. For the sake of simplicity, the reactor is assumed to be isothermal and the state measurables. The parameters of the actual system are shown in Table 1. The process economics can be expressed by the running cost:

$$\ell(u, x_2) = \beta_A u c_{A0} - \beta_B u x_2. \tag{20}$$

Using the actual process parameters reported in Table 1, the process optimal steady-state is computed by solving the following optimization problem:

$$\min_u \beta_A u c_{A0} - \beta_B u x_2 \tag{21a}$$

subject to

$$\frac{u}{V}(c_{A0} - x_1) - k_1 x_1 = 0 \tag{21b}$$

$$\frac{u}{V}(c_{B0} - x_2) + k_1 x_1 - k_2 x_2 = 0 \tag{21c}$$

$$0 \le x_1 \le c_{A0} \tag{21d}$$

$$0 \le x_2 \le c_{A0} \tag{21e}$$

The calculated economic optimum is $u_s^0 = 1.043\,\text{m}^3/\text{min}$, $x_{1,s}^0 = 0.511\,\text{kmol}/\text{m}^3$ and $x_{2,s}^0 = 0.467\,\text{kmol}/\text{m}^3$. Although, we remark that this optimum point is supposed to be unknown since the controller have a wrong knowledge about the process parameters as explained below.

4.2 Model and controllers

For controller design, the two kinetic constants are supposed to be uncertain, i.e. the value known by the controller are $\bar{k}_1$ and $\bar{k}_2$, instead of k_1 and k_2. We compare the closed-loop behavior of three EMPC algorithms, all designed according to Section 3 using the same nominal model, cost function, and a sampling time of $\tau = 1.0\,\text{min}$. The economic cost function in (15) is:

$$\ell_e(y(t_i), u(t_i)) = \int_{t_i}^{t_i+\tau} \ell(u(t), y_2(t))dt = \int_{t_i}^{t_i+\tau} [\beta_A u(t) c_{A0} - \beta_B u(t) y_2(t)]\, dt \tag{22}$$

Remark 5. If the point-wise evaluation of $\ell(\cdot)$ was used as stage cost $\ell_e(\cdot)$, the system would not be dissipative (Angeli et al., 2012), i.e. the closed-loop system would not be stable.

The three controllers analyzed suffer all of the same plant-model mismatch; specifically $\bar{k}_1 = 1.2$ and $\bar{k}_2 = 0$, meaning that the first reaction is overestimated while the second one is ignored. Also the dynamic (19) is discretized using an implicit Euler method and the state disturbance model is used, i.e. the considered augmented system (8) can be rewritten as:

$$\begin{aligned} x^+ &= f(x,u) + d \\ d^+ &= d \\ y &= x \end{aligned} \tag{23}$$

The estimator tuning is $Q_x = 10^{-6}I$, $Q_d = I$, $R = 10^{-6}I$ and the chosen arrival cost updating is the one called *filtering*, i.e. the MHE problem is treated like a filter (Rao, 2000).

Hence the three controller formulations are defined as follows:

- EMPC0 is the standard economic MPC and uses only the state disturbance model in (23).
- EMPC1 uses the disturbance model in (23) and also the modified problems (11) and (15). Here we assume to know the plant gradient, i.e. $\nabla_u \tilde{G}_p(\cdot) = \nabla_u G_p(\cdot)$.
- EMPC2 is the EMPC1 where the gradient estimation technique described in Section 3.5 is applied. The identification threshold is $\tau_{id} = 0.1$, while the identification parameters are the following: $N_{id} = 60$, model order $n = 2$, future and past horizons $SS_f = SS_p = 10$.

4.3 Results

The closed-loop behavior of the above described controllers is depicted in Figure 1. We can immediately notice how both the EMPC1 and EMPC2 accomplish the goal to reach the economic optimum even under the model errors introduced before. The EMPC0, using only the disturbance (23), is not able to eliminate the offset reaching a different equilibrium point. The EMPC1 has a perfect knowledge of the plant gradients so its correction is virtually perfect. We can see that even under errors on k_1, the controller has no difficulty in reaching the optimal equilibrium point. This is an improved result that in (Vaccari and Pannocchia, 2016) could not be guaranteed. In the end we can see that before reaching $t = 20\,\text{min}$, EMPC2 behavior is identical to EMPC0 as expected. As a matter of fact, λ_k^G is updated for the first time from its original null value, only at time step $k = k_{id} + N_{id}$ (in this case $t = 80\,\text{min}$). After the identification has begun and the correction applied, the controller behavior changes and move towards the economic optimum. It has also to be noted that a second significant deviation reducing the offset happens around $t = 140\,\text{min}$. This corresponds to $k = k_{id} + 2N_{id}$, when the data window has shifted forward until the last random input calculated with (17) is discarded. Therefore we can conclude that EMPC1 gives the best performances but is not practically implementable, while EMPC2 is the only suitable for real application since uses only data collected from the process and no other major knowledge.

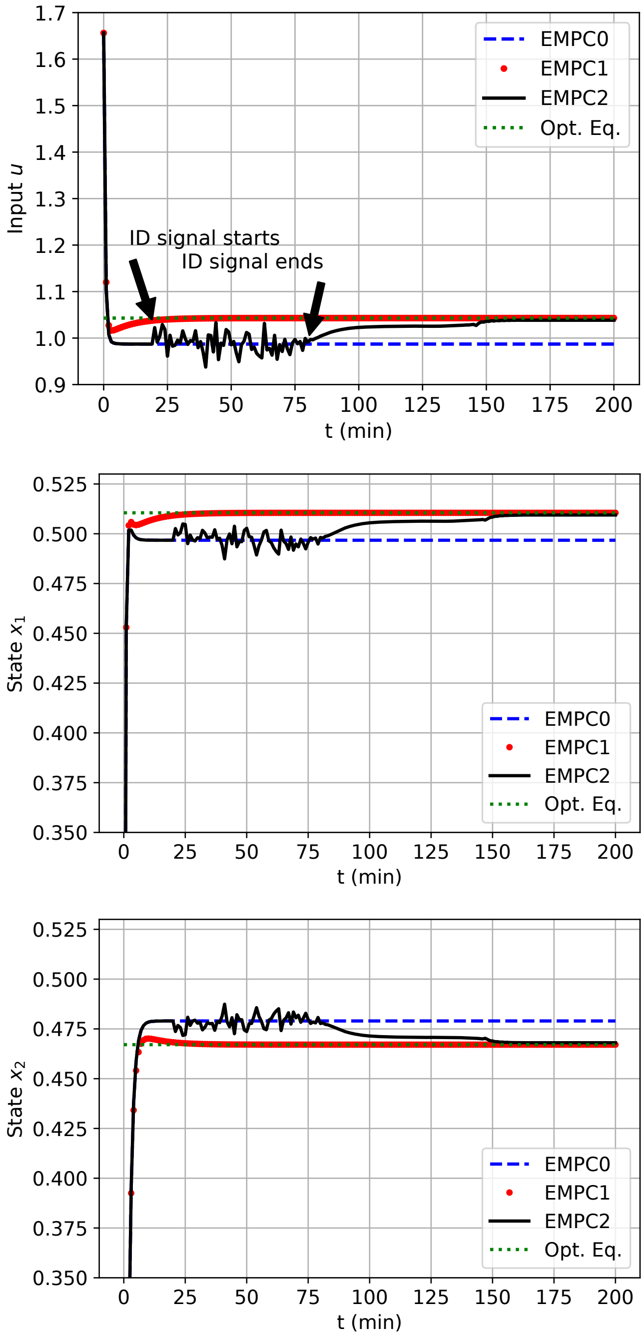

Fig. 1. Closed-loop results for the three selected controllers: input (top) and states (middle and bottom). Note that from 20 to 80 is evidenced the random signal used for identification purposes. The initial state values $x_1(0) = x_2(0) = 0$ are not displayed.

5. CONCLUSIONS

In this work we have proposed an economic model predictive control (EMPC) algorithm implementation with the goal to asymptotically achieves the optimal performance despite the presence of plant-model mismatch. The algorithm is based on a previously elaborated method (Vaccari and Pannocchia, 2016) that used a combination of the offset-free formulation and modifier-adaptation technique. We firstly introduced two new major differences between the algorithm proposed in this work and the one in literature: the Moving Horizon Estimation (MHE) technique has been selected as state and disturbance observer, and a plant gradient estimation methodology based on an identification algorithm has been implemented. We have applied our formulation on a CSTR example. Results show how the proposed algorithm can overcome plant-model mismatch and converge to the best economic equilibrium. It has been also proved how previous limitations on the mismatch structure have been overpassed. Furthermore, the gradient estimation methodology applied, using only transient measurements from the plants, opens up to possible applications to real processes.

Future research should focus on how a sustained economic performance can be maintained, and how one can understand or test if the reached equilibrium is the real economic optimum of the process.

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