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Shear modulus of masonry walls: a critical review

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Abstract

In the assessment of seismic performance of masonry buildings, the proper definition of mechanical parameters of masonry, the shear modulus in particular, is a critical issue. Moreover, considering that existing buildings are characterized by several masonry types, depending on the material as well as on the texture, mechanical parameters can vary in a very wide range, also because they depend on many other parameters and in particular on the integrity of the walls and on the stress level. Although the in situ or laboratory experimental evaluation of the *G* modulus has been the subject of a wide literature concerning flat jacks, diagonal and single compression and shear-compression test results, its outcomes are often contradictory. In effect, values given by different studies often differ significantly, even for the same class of masonry. Since the intrinsic scattering of the parameter is not sufficient by itself to justify the huge variability of the results, a critical discussion of the results as well as of the individual test arrangements is necessary to make the background more reliable, also in view of better addressing further studies-
A huge database has been setup combining masonry test results available in the relevant scientific literature with the test results obtained in the framework of the in situ experimental campaign carried out by the authors for the assessment of seismic vulnerability of masonry school buildings in the Municipality of Florence. The analysis of the database underlines that values of the shear modulus *G*, which is a fundamental parameter for the definition of capacity curve for walls commonly used in non-linear static analysis, are extremely scattered. Testing methodology and arrangement are discussed and a possible procedure is proposed to arrive to sounder estimations of relevant mechanical parameter of existing building masonry.

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Keywords: Seismic vulnerability; Masonry buildings; Seismic resistance; Seismic risk index; Pushover methods.

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1. Introduction

The proper definition of mechanical parameters of masonry is a critical issue in the assessment of the seismic performance of masonry buildings. The stiffness identification of masonry walls and therefore the correct definition of the shear modulus is a key step in classical pushover analysis for the evaluation of seismic performance (Croce et al., 2018).

Masonry is characterized by an inelastic, anisotropic and non-homogeneous material behavior; moreover, considering existing building, several masonry types, depending on the material as well as on the texture, can be detected in the built environment, characterized by mechanical parameters varying in a very wide range. Current code procedures in Italy (Italian Public Works Council, 2009) allow to derive mechanical parameters of existing masonry from the identification of the masonry type. Nevertheless, the identification of the class of masonry could be not sufficient to properly set the main mechanical parameters, especially the shear modulus, because they depend on many other parameters and in particular on the homogeneity and integrity of the walls.

Moreover, concerning the values of the shear modulus G to be assumed for masonry, the values stated in almost all National codes as well as in Eurocode EN1996-1-1 (G=0.4 E) seems to overestimate the shear modulus as reported in (Bosiljkov et al., 2005), (Tomažević., 2009) and (Zimmerman et al., 2011), due to the anisotropy of and the cracking of masonry. In effect, this value is simply obtained setting to $\nu = 0.25$ the Poisson modulus of the masonry, in the relationship linking the elastic constants in isotropic material,

$$G = \frac{E}{2(1+\nu)} \tag{1}$$

The in situ or laboratory experimental evaluation of the shear behavior of masonry walls is the subject of a wide literature concerning flat jack, diagonal compression and shear compression tests; but their outcomes are often contradictory and the values of shear modulus obtained according to different test procedures, can differ significantly for the same class of masonry or even for the same wall (Bosiljkov et al., 2005).

In the paper, a huge database concerning shear characteristics suitably collected is discussed. In the database, masonry test results available in the relevant literature have been supplemented with test results obtained by the authors in the framework of the in situ experimental campaign carried out for the assessment of seismic vulnerability of masonry school buildings in the Municipality of Florence.

Then, a critical discussion about the collected data is presented focusing the attention on the evaluation of the shear modulus G to be used for the definition of the capacity curve of masonry walls in non-linear static seismic analysis and in pushover analysis in particular.

Finally, testing methodology and arrangement are discussed and a possible procedure to be followed is proposed to arrive to the definition of masonry mechanical parameter to be used in the seismic assessment of existing masonry buildings.

2. Experimental procedures for the evaluation of masonry mechanical parameters

Several experimental procedures aiming to evaluate the main mechanical parameters of masonry walls, like shear modulus, elastic modulus as well as shear and compressive strength have been proposed in the literature, In this paragraph, the main testing procedures, flat jack compression tests, diagonal compression tests and shear compression tests (see Fig. 1), are shortly discussed.

2.1. Shear compression tests

This type of test derives from the Sheppard shearing test (Sheppard, 1985), in which the panel, separated from the wall by vertical cuts only, is subjected to the existing containment pressure and horizontal shear force. This test is carried out in situ, on masonry panels of 0,9 x 1,8 m around, to evaluate the shear strength.

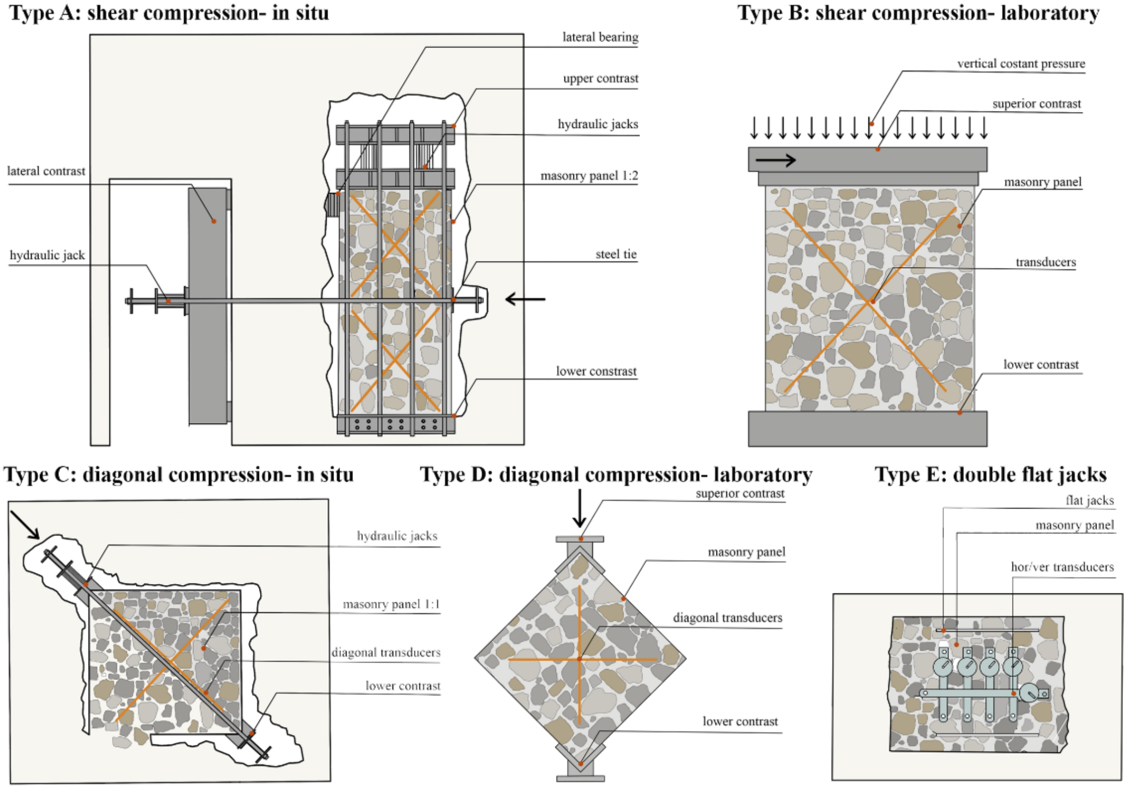

Fig. 1. Test types: (a) shear compression-in situ; (b) shear compression-laboratory; (c) diagonal compression in situ; (d) diagonal compression laboratory; (e) double flat jacks.

For the whole duration of the shear compression test, the panel is subjected to a vertical compressive stress of 0.3 MPa, that shall be kept constant throughout the test, by means of a suitable system of steel plates, steel rods and jacks. During the test, an increasing horizontal force is applied with a hydraulic jack at the level of the center of the panel by means of a metal profile connected to two steel bars, so introducing shear stresses over the entire thickness of the panel. At its lower and upper extreme, the panel is fixed to the steel structure by means of a suitable device and the two square panels, in which the whole panel can be subdivided (Fig. 1, Type A), are subject to different shear forces. In effect, tests have demonstrated that the highest quota of the applied horizontal load flows in the lower half-panel. In this test arrangement, displacements are measured placing 8 transducers on both side of the sample along the diagonals of the two square panels, while two additional transducers are placed on top, so that the horizontal displacements of the upper base and any possible rotations are measured. Six other transducers are positioned along each side of one vertical edge (at the base, at the center and on top). Transducers allow measurements in terms of time and pressure and the shear strength of the masonry is derived from

$$\tau_{\max} = \tau_k^{sc}\sqrt{1+\frac{\sigma_0}{1,5\tau_k^{sc}}} \tag{2}$$

as proposed by Turnšek and Ĉaĉoviĉ (1971). In eqn. (1) σ_0 is the vertical compression stress equal to 0.3 MPa, τ_k^{sc} is the shear strength of the masonry and $\tau_{\max}$ is the maximum shear stress defined as:

$$\tau_{\max} = \frac{T_{\max}}{A} \tag{3}$$

being T_{max} the maximum shear load in the lower half of the panel and A the horizontal cross-section of the panel.

This test can be also carried out in laboratory on masonry panels, fixed at the top and at the bottom and subjected to constant vertical load, applying increasing lateral forces on one of the two ends (Fig. 1, Type B).

2.2. Diagonal compression

The diagonal compression test is designed to evaluate the shear behavior of masonry panels. The laboratory test procedure is defined in the ASTM E519-81 specifications (ASTM, 1981) and it is generally performed on 1,20x1,20 m masonry panels of different thickness. During the in-situ test, the panel is insulated from the wall by means of 4 cuts made with diamond wire or circular saw. The lower part of the masonry remains attached to the wall wing, through a link 0,70 m wide at the lower horizontal edge. The test proceeds with the installation of the equipment on the corners of metal profiles; a jack placed on one of the two corners provide the compressive diagonal force. The panel is also equipped with 4 or more transducers, arranged along the diagonals on both sides, allowing to measure the deformations at each load step (Fig. 1, Type C). In the regular test procedure, couples of equal cycles of loading and unloading, with load increasing of 10 kN at each steps, are applied till to failure. Hence, the shear strength τ_k^{diag} is evaluated as:

$$\tau_k^{diag} = \frac{\sqrt{2}}{2}\frac{P_{max}}{A_n} \tag{4}$$

where P_{max} is the maximum load applied by the jack during the test and A_n is the net area of the panel a, according to the ASTM E519-81 specifications (ASTM, 1981).

Diagonal compression tests can also be carried out in laboratory, in this case the panel is isolated from the wall and rotated of 45° so that its diagonal is in vertical position. Compressive force along the diagonal is applied by means of a hydraulic jack through appropriate steel profiles applied to its corners (see Fig. 1, Type D).

2.3. Compression tests

In situ compression tests are usually carried out to determine the normal modulus of elasticity (E) and the compressive strength (f_m) of masonry to be used for the assessment of the performance of existing masonry building.

They are based on the use of two flat jacks and are described in ASTM C 1197-04 (ASTM International, 2004) and in RILEM MDT.D.5 (RILEM, 2004). Two horizontal cuts are executed on the wall delimiting a specimen with 0,40 m height, then the flat jacks are inserted in the cuts and centesimal deformometers are installed to measure horizontal and vertical displacement during the test, as it summarized in Fig. 1 (Type E). The maximum pressure of the test is used to estimate the compressive strength of the masonry, while the normal elastic modulus E and the Poisson ratio v, derived from the measured displacement, allow to derive the shear modulus through eqn. (1).

However, as it will be described later in the paper, the magnitude of horizontal displacements can be so small that it becomes hard to be appreciated correctly with common instruments thus leading to not reliable value of the apparent Poisson ratio and therefore of the apparent shear modulus.

3. Dataset definition

The first phase of the present study has been the definition of a database of test results, collected from relevant studies as well as from an in situ experimental campaign carried out by the authors for the assessment of seismic vulnerability of masonry school buildings in the Municipality of Florence. Only comprehensive and exhaustive papers, presenting full details of the tests results and clearly describing the test methodology and the characteristics of the masonry, have been considered in the database. Other data have been collected from the database of the Delta laboratory in Lucca, which kindly provided some tests reports carried out in masonry buildings situated in northern Tuscany and Emilia Romagna. The categories of masonry included in the research are irregular stone masonry, double-leaf stone masonry, with an inner core with poor mechanical characteristics, and single-leaf brick masonry.

These categories roughly correspond to the first, the second and the sixth type of masonry as defined in the Table C8A.2.2 in the annex A to the chapter C8 of the Guidelines for application of Italian Building Code (Italian Public Works Council, 2009). The collected data are resumed in the following Table 1-8.

Table 1. Shear-compression tests in brick masonry.

I.D.	τ_k (N/mm^2)	G (N/mm^2)	References
1-11	0,094;0,096;0,112;0,120;0,088;0,121 0,209;0,181;0,170;0,160;0,167	76;118;119;55;183;141 188;339;271;333;255	Zimmermann and Strauss, 2012
12	0,130	437	Borri et al., 2004
13-15	0,265;0,120;0,180	309;100;211	Corradi et al., 2008

Table 2. Diagonal compression tests in brick masonry.

I.D.	τ_k (N/mm^2)	E (N/mm^2)	G (N/mm^2)	References
1-4	0,053;0,068;0,072;0,069		355;506;30;131	Borri et al.,2000
5-6	0,054;0,066		23;71	DELTA Laboratory
7-8	0,090;0,115		600;856	Borri et al., 2008

Table 3. Double flat-jacks tests in brick masonry.

I.D.	E (N/mm^2)	G (N/mm^2)	References
1-19	1926;2235;1137;1195;1494;2709;1569;1187;1607;3284; 2517;1874;2287;2087;1863;2938;3532;2386;4891;	902;1005;521;597;655;672;784;356;516;1609; 1218;920;1084;819;921;1269;543;1146;2445;	DELTA Laboratory
20-22	3672;3107;1951;	977;1440;469;	Croce et al., 2017

Table 4. Shear-compression tests in irregular stone masonry.

I.D.	τ_k (N/mm^2)	E (N/mm^2)	G (N/mm^2)	References
1	0,040		75	Candela et al., 2011
2-9	0,121;0,095;0,118;0,189;0,094;0,097;0,211;0,072		200;116;274;240;173;326;333;99;	Borri et al., 2004
10	0,126		40	Sheppard, 1985
11-18	0,027;0,023;0,029;0,017;0,016;0,017;0,019;0,019		43;38;45;28;25;28;30;30	Modena and Bettio,1994
19-21	0,273;0,367;0,136		196;608;216	Borri et al., 2004
22	0,093		40	Tomaževic, 1992
23-24	0,048;0,039	793;379;	26;12	Angelini et al., 2007

Table 5. Diagonal compression tests in irregular stone masonry.

I.D.	τ_k (N/mm^2)	E (N/mm^2)	G (N/mm^2)	References
1-4	0,054;0,066;0,076;0,074;		23;7172;42	DELTA Laboratory
5-6	0,045;0,053;		19;26;	Borri et al., 2004
7-14	0,080;0,063;0,092;0,030;0,051;0,050;0,143		113;177;172;74;78;77;81	Brignola et al., 2006

Table 6. Double flat-jacks tests in irregular stone masonry.

I.D.	E (N/mm^2)	G (N/mm^2)	References
1-21	1562;1262;1134;2862;2864;915;1131;578;2087;1137;1137; 2938;2938;2254;1655;1528;2156;2961;1320;2627;2617;	498;419;407;1431;1358;442;549;286;448;556;531; 1241;1319;1084;576;669;1046;1306;637;1238;1081;	DELTA Laboratory
22-27	1166;2438;1622;1630;3972;2669;	546;1023;594;615;1010;885;	Croce et al., 2017

Table 7. Shear-compression tests in double-leaf stone masonry.

I.D.	τ_k (N/mm^2)	G (N/mm^2)	References
1-3	0,130;0,149;0,136;	328;308;216;	Borri et al., 2000
4-10	0,047;0,047;0,096;0,180;0,050;0,064;0,078;	38;65;101;77;201;154;133;	Corradi et al., 2008
11-12	0,013;0,067;	70;87;	Tomaževic, 1992
13-14	0,109;0,222;	290;249;	Borri et al., 2004

Table 8. Diagonal compression tests in double-leaf stone masonry.

I.D.	τ_k (N/mm^2)	G (N/mm^2)	References
1-19	0,072;0,047;0,072;0,068;0,053;0,059;0,307;0,249;0,339;0,278;	30;19;25;60;26;37;70;92;69;278;	Borri et al., 2000
	0,072;0,047;0,072;0,068;0,053;0,048;0,072;0,068;0,053;	30;19;25;60;26;19;25;60;26	
20-23	0,114;0,160;0,072;0,061;	285;105;36;74;	Modena et al., 1999
24-28	0,046;0,049;0,040;0,045;0,035;	105;55;165;80;132;	Corradi et al., 2008

In the Tables, available values of shear strength τ_k, elastic and shear moduli, E and G, are reported, in terms of the adopted test methodology, and masonry class. In particular, Tables 1-3 refer to brick masonry, in Tables 4-6 to irregular stone masonry and finally Tables 7-8 data to double layer stone masonry.

4. Analysis of the results

The collected data presented in the previous chapter, which include effects of inhomogeneity and cracking of the masonry panels, have been analyzed in order to evaluate the variability of the apparent shear modulus G for the same type of masonry according the different test arrangements. In Table 9 mean values and coefficient of variations for the shear modulus G are presented for the three different types of masonry.

Table 9. Results of different tests for shear modulus G.

Masonry type	G- double flat jacks		G- diagonal compression		G- shear compression	
	μ (N/mm^2)	COV	μ (N/mm^2)	COV	μ (N/mm^2)	COV
Brick Masonry	994	0.48	300	0.61	189	0.52
Stone Masonry	807	0.44	52	0.50	135	0.82
Double-leaf Stone Masonry	-	-	71	0.9	154	0.61

It is important to notice that values obtained from double flat jacks' tests according eqn. (1) are much higher than those obtained by the other test methods and may lead to unrealistic value of G/E ratio of around 0.4. Indeed, as showed in (Tomažević, 1999) and (Tomažević, 2009), experimental values of G/E ratio can vary between 0.06 to 0.25 and for these reasons the value most commonly assumed in the seismic assessment of masonry building is around 1/6. This difference can be motivated by the instrument resolution, which is too low in comparison with the actual value of horizontal displacements to be measured, so leading to inaccurate values of the apparent Poisson ratio and then of the apparent shear modulus. According to a previous works of Turnšek and Ĉaĉoviĉ (1971), where the relationship G=1100 τ_k is proposed, the possibility to derive a sound relationship between shear strength τ_k and shear modulus G has been also investigated.

Table 10. Results of shear compression tests (shear modulus).

Masonry type	G/τ_k	
	μ	COV
Brick Masonry	1421	0.50
Stone Masonry	1456	0.46
Double-leaf Stone Masonry	2020	0.67

The results are reported in Table 10, for the three examined masonry types, in terms of mean values and coefficient of variations. It is interesting to notice how a mean value of around 1400 is obtained for G/τ_k for single layer masonry, brick and stone categories, while higher values, around 2000, can be obtained for double leaf masonry, as shown in Figure 2 where quantile-quantile plots of the G/τ_k ratio are reported considering a normal distribution.

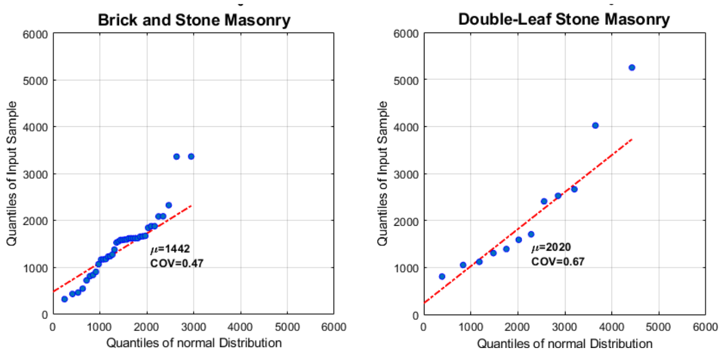

Fig. 2. QQ plot for G/τ_k for the three different types of masonry: (a) Brick Masonry.

5. Procedure for the definition of mechanical parameters of masonry

According to the results presented in the previous chapters, a procedure for the definition of the mechanical properties of masonry to be used in the assessment of seismic performance of masonry buildings is proposed. The procedure can be resumed in the following steps:

- evaluation of the elastic modulus of masonry E through in situ compressive tests by double flat jacks;
- definition of shear modulus G according to the experimental relationship G=0.15 E, according the average values resulting from the most reliable test arrangements;
- definition of the shear strength τ_k assuming τ_k=G/1500 for single leaf masonry and τ_k=G/2000 for double leaf masonry.

The rationale of the proposed approach is that the parameters should be derived from the elastic modulus, which is the parameter that can be measured more easily and precisely, using appropriate relationships to derive G and τ_k from it. It must be stressed that the shear modulus G is the most important parameter to be set in performing non-linear static analysis of masonry buildings as shown in (Croce et al., 2018). With the proposed approach the values of G and τ_k could be derived in a simplified and reliable way starting, for example, from in situ flat jack tests.

On these basis, further research works should aim to refine the ratios G/E and G/τ_k for different masonry types, so reducing the already underlined uncertainties in the evaluation of the seismic risk index.

Conclusion

In the paper, the critical issue regarding the proper definition of mechanical parameters of masonry to be used in the assessment of seismic performance of masonry buildings: shear modulus G and shear strength τ_k, has been investigated and discussed. First, combining data available in the relevant literature, with those obtained in the framework of the in situ experimental campaign carried out by the authors for the assessment of seismic vulnerability of masonry school buildings in the Municipality of Florence, a huge database of tests results has been collected concerning three different types of masonry, differentiating the data according the test method used: shear compression, diagonal compression or double flat jacks test.

The analysis of the database highlights the scattering of the value of the G modulus obtained according to different test arrangements. Since G plays a fundamental role in the evaluation of the capacity curve of masonry walls, commonly used in non-linear static seismic analysis, a more reliable procedure is proposed for the experimental evaluation of the shear modulus G and of the shear strength τ_k, focusing on the need to derive more precisely the ratios G/E and G/τ_k for different masonry types, so allowing the evaluation of the other relevant parameters from the modulus of elasticity E, which can be more easily derived from in situ tests.

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