Optimal Power Allocation for Full-Duplex Communications Over OFDMA Cellular Networks

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***Abstract*—We consider the optimal power allocation problem for an OFDMA small-cells network where the BSs operate in full-duplex mode. Although the problem formulation is not convex, we show that, leveraging on the intimate relationship between the minimum mean square error and the interference to noise ratio, it is possible to derive an iterative allocation scheme that provably converges to a local maximum of the sum rate. Moreover, owing to the convexity of the mean square error formulation, the proposed approach copes with unlimited number of interferers, and as such it is amenable for implementation on a generic multicell scenario. Despite we do not explicitly specify any sub-channel assignment constraint, the solution at convergence allows to fulfill the exclusivity assignment constraint in almost all cases , i.e., the constraint that each sub-channel can be assigned to at most two user pairs in the cell. The proposed allocation scheme allows to achieve fast convergence to a solution that approaches the performance of a power allocation scheme that includes an exhaustive search over all possible sub-channel assignments.**

I. Introduction

Next generation wireless communication systems are required to support growing demands for high data rates applications, which call for innovative technologies to efficiently exploit the available spectrum. Full-duplex (FD), which allows uplink and downlink transmission to occur simultaneously at the same frequency, represents a technology which has the potential to double the spectral efficiency of conventional half-duplex communication systems [1], [2]. However, the main limitation in FD operation is represented by self-interference (SI) caused by the signal leakage from the transceiver output to the input [3]. As a matter of fact, residual SI reduces the uplink coverage and precludes the use of FD technology in a large cell [4]. On the other hand, the increasing demand on high capacity systems can not be accommodated under the traditional infrastructure of cellular networks. Hence, to better utilize the available resources and to alleviate the huge infrastructure investment of operators, heterogeneous networks (HetNets) have been considered as a promising technique in LTE-advanced networks [2], [5]. In HetNets, low-power base stations (BSs), such as pico and femto-BSs, are intensively deployed to provide high data-rate services [6]. Thanks to the small transmitter-receiver distance and the reduction in the transmit power, pico and femto cells can facilitate the cancellation of SI leaking from a FD transmission to its reception. Therefore, it is natural and reasonable to integrate FD technology into femtocells [5] thus allowing a femto-base station (BS) to transmit and receive signals over the same spectrum. However, the actual gain that can be achieved by enabling FD at the pico or femto BS is still a matter of debate owing to the additional interference sources with respect to traditional HD systems. Indeed, during FD operation the downlink mobile equipment not only gets interference from other BSs, as in traditional HD systems, but also gets interference from uplink signals from both the same cell and the other cells. Similarly, the uplink suffers from additional interference coming from downlink transmissions of other cell and from residual SI of the same celle [7] [8]. Hence, to take the greatest possible advantage from the introduction of FD, the design of an optimized joint channel assignment and power allocation scheme that is able to cope with the increased interference scenario is needed. Such a problem is in general nonconvex, non linear, with mixed integer and continuous optimization variables even for single carrier systems [4]. In [9] a novel approach for joint subcarrier assignment and power allocation for FD-enabled OFDMA networks based on Lagrangian dual decomposition is proposed. The main limitation of this study is represented by the assumption of perfect SI cancellation, that cannot be achieved in practice. In [10] a different resource allocation approach based on Bender's decomposition is proposed that allows to take into account both self and inter-node interference. The proposed algorithm aims at minimizing the power consumption, and thus cannot be used for the throughput maximization in FD communications. In [11] a general optimization framework of sub-carrier assignment and transmission power control to maximize the overall system throughput is proposed. The algorithm considers a multi carrier scenario and takes the effects of self and inter-node interference into account. In this case, the interference due to the activities of neighboring cells is assumed constant and hence the proposed algorithm is appropriate for a single cell scenario.

In this paper we consider a distributed resource allocation problem for a typical OFDMA small-cells network where the BSs operate in FD mode. In this scenario, similarly to [11] and [4], we take into consideration all possible sources of interference in the network. The specificity of the approach proposed in this paper is that the allocation problem is formulated as a power-only allocation problem, rather than

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a joint sub-channel assignment and power allocation problem. Although this formulation is still non convex, we show that, leveraging on the intimate relationship between the minimum mean square error and the signal to interference ratio, it is possible to derive an iterative allocation scheme that provably converges to a local maximum of the sum rate. Despite the fact that we do not explicitly specify any sub-channel assignment constraint, the solution at convergence is shown to fulfill the exclusivity assignment constraint in almost all cases, i.e., the constraint that each sub-channel can be assigned at most to two users, one for the uplink and one for the downlink.

II. System Model and Problem Formulation

The accessible spectrum for the considered system consists of F orthogonal frequency division multiple (OFDM) sub-channels. Signal propagation in each sub-channel is assumed to experience large-scale and small-scale fading, which remains unchanged for the time horizon of radio resource allocation.

We consider a network with Q cells, where in each cell transmit a BS, M_q uplink users and N_q downlink users. Single-antenna uplink and downlink mobile users operate in *half-duplex*, while the BS, also equipped with a single antenna, is capable of *full-duplex* transmissions, i.e., it can transmit and receive simultaneously using the same frequency spectrum.

We denote by $\mathcal{U} = \cup_{q=1}^{Q} \mathcal{U}_q$ and $\mathcal{D} = \cup_{q=1}^{Q} \mathcal{D}_q$ the set of $M = |\mathcal{U}|$ uplink and $N = |\mathcal{D}|$ downlink users in the system, where $\mathcal{U}_q$ and $\mathcal{D}_q$ are the set of uplink and downlink users in cell q, respectively. The set of BSs is $\mathcal{Q} = \{1, 2, \ldots, Q\}$. We denote by $s_{i,f}$ the complex information symbol of i-th users on sub-channel f. The information symbols are assumed (as customary) zero-mean independent and identically distributed (i.i.d) random signals, i.e., $\mathbb{E}\left[s_{i,f} s_{j,g}^*\right] = 1$ if $i = j$ and $f = g$ and $\mathbb{E}\left[s_{i,f} s_{j,g}^*\right] = 0$, otherwise. The signal transmitted by each user is $x_{i,f} = \sqrt{P_{i,f}} s_{i,f}$, i.e., it is a scaled version of the information symbol with scaling factor $\sqrt{P_{i,f}}$ and transmission power $P_{i,f}$.

Under the assumption that all users can transmit on any subcarrier without any orthogonality requirement among users, let us first consider uplink connections for a generic cell q and subchannel f and express the signal received for user $i \in \mathcal{U}_q$ as

$$\begin{aligned} y_{i,f} = {} & \alpha_{q,i}(f)\sqrt{P_{i,f}} s_{i,f} + \underbrace{\sum_{\substack{j \in \mathcal{U}_q \\ j \neq i}} \alpha_{q,j}(f)\sqrt{P_{j,f}} s_{j,f}}_{\text{(a)}} \\ & + \underbrace{\sum_{\substack{m=1 \\ m \neq q}}^{Q} \sum_{j \in \mathcal{U}_m} \alpha_{q,j}(f)\sqrt{P_{j,f}} s_{j,f}}_{\text{(b)}} + \underbrace{\zeta_{q,q}(f) \sum_{j \in \mathcal{D}_q} \sqrt{P_{j,f}} s_{j,f}}_{\text{(c)}} \\ & + \underbrace{\sum_{\substack{m=1 \\ m \neq q}}^{Q} \beta_{q,m}(f) \sum_{j \in \mathcal{D}_m} \sqrt{P_{j,f}} s_{j,f}}_{\text{(d)}} + n_{q,f} \end{aligned} \tag{1}$$

where $n_{q,f}$ denotes the additive white Gaussian noise with distribution $\mathcal{CN}\left(0, \sigma_q^2\right)$, $\alpha_{q,i}(f)$ is the channel gain between transmitter $i \in \mathcal{U}$ and the BS $q \in \mathcal{Q}$, $\beta_{m,q}(f)$ is the channel gain between BSs $m, q \in \mathcal{Q}$ and $\zeta_{q,q}(f)$ is the residual gain relative to the self interference term at the BS q. Accordingly, (a) and (b) in (1) represent the intra-cell interference and the other-cells interference, respectively, while (c) is the residual SI due to non ideal SI cancellation at the BS and (d) is the other-cells interference due to concurrent downlink transmissions. Note that neither (c) nor (d) are present in classical cellular scenarios where uplink and downlink transmissions are separated in frequency or time domain.

As for the downlink connections, the signal received on sub-channel f at i-th receiver with $i \in \mathcal{D}_q$, can be expressed as:

$$\begin{aligned} y_{i,f} = {} & \delta_{i,q}(f)\sqrt{P_{i,f}} s_{i,f} + \underbrace{\delta_{i,q}(f) \sum_{\substack{j \in \mathcal{D}_q \\ j \neq i}} \sqrt{P_{j,f}} s_{j,f}}_{\text{(a)}} \\ & + \underbrace{\sum_{\substack{m=1 \\ m \neq q}}^{Q} \delta_{i,m}(f) \sum_{j \in \mathcal{D}_m} \sqrt{P_{j,f}} s_{j,f}}_{\text{(b)}} + \underbrace{\sum_{j \in \mathcal{U}_q} \eta_{i,j}(f)\sqrt{P_{j,f}} s_{j,f}}_{\text{(c)}} \\ & + \underbrace{\sum_{\substack{m=1 \\ m \neq q}}^{Q} \sum_{j \in \mathcal{U}_m} \eta_{i,j}(f)\sqrt{P_{j,f}} s_{j,f}}_{\text{(d)}} + n_{i,f} \end{aligned} \tag{2}$$

where $n_{i,f}$ is the additive white Gaussian noise with distribution $\mathcal{CN}\left(0, \sigma_i^2\right)$, $\delta_{i,q}(f)$ is the channel gain between the BS $q \in \mathcal{Q}$ and receiver $i \in \mathcal{U}$ and $\eta_{i,j}(f)$ is the channel gain between users $i, j \in \mathcal{U}$. As for the uplink case, (a) and (b) in (2) represent the intra-cell interference and the other-cells interference, respectively. Moreover, (c) in (2) is the intra-cell inter-node interference, while (d) is the other-cells inter-node interference. Note that, as for the uplink case, neither (c) nor (d) are present in classical cellular scenarios. Keeping in mind that the goal of this work is to derive an algorithm for optimally allocating the users' power, a close observation of both (1) and (2) shows that they can be rewritten in the form

$$\begin{aligned} y_{i,f} = {} & h_{i,i}(f)\sqrt{P_{i,f}} s_{i,f} + \sum_{j \in \mathcal{A}(i)} h_{i,j}(f)\sqrt{P_{j,f}} s_{j,f} \\ & + \sum_{j \in \mathcal{B}(i)} h_{i,j}(f)\sqrt{P_{j,f}} s_{j,f} + z_{i,f} \end{aligned} \tag{3}$$

where $\mathcal{A}(i) = \mathcal{U} \setminus i, \mathcal{B}(i) = \mathcal{D}, z_{i,f} = n_{q,f}$ if $i \in \mathcal{U}$, ie, i is an uplink user, and $\mathcal{A}(i) = \mathcal{D} \setminus i, \mathcal{B}(i) = \mathcal{U}, z_{i,f} = n_{i,f}$ if $i \in \mathcal{D}$. The correspondence between the coefficients $h_{i,j}$ and the various propagation gains can be inferred by confronting (1) and (2) with (3). For example, if i is an uplink user in the cell q, $h_{i,i} = \alpha_{q,i}$ and $h_{i,j}$ is either $\alpha_{q,j}$ if $j \in \mathcal{U}$, $\beta_{q,m}$ if $j \in \mathcal{D}_m$, or $\zeta_{q,q}$ if $j \in \mathcal{D}_q$; if i is a downlink user in cell q, $h_{i,i} = \delta_{i,q}$, $h_{i,j}$ is $\delta_{i,m}$ if $j \in \mathcal{D}_m$ $(m = 1, 2, \ldots, Q)$ and $h_{i,j}$ is $\eta_{i,j}$ if $j \in \mathcal{U}$. Assuming the interference is treated as additive

Gaussian noise, the achievable normalized rate (with respect to the bandwidth) in bps/Hz of a single-user decoder that decodes each transmitted symbol separately can be expressed for both uplink and downlink directions as

$$R_{i,f} = \log\left(1 + \gamma_{i,f}\right) \quad (4)$$

where, in accord with the notation in (3), the signal to interference-plus-noise ratio (SINR) $\gamma_{i,f}$ is computed as

$$\gamma_{i,f} = \frac{\left|h_{i,i}(f)\right|^2 P_{i,f}}{\sum\limits_{j\in\mathcal{A}(i)} \left|h_{i,j}(f)\right|^2 P_{j,f} + \sum\limits_{j\in\mathcal{B}(i)} \left|h_{i,j}(f)\right|^2 P_{j,f} + \sigma_{z,i}^2} \quad (5)$$

and $\sigma_{z,i} = \mathbb{E}\left\{|z_{i,f}|^2\right\}$.

Let's now assume that the received signal $y_{i,f}$ is multiplied by a scaling factor $g_{i,f}$ with the aim of minimizing the mean square error (MSE) $e_{i,f}$ defined as:

$$\begin{aligned} e_{i,f} &= \mathbb{E}_{\mathbf{s}_f,\, z_{i,f}} \left\{\left|g_{i,f} y_{i,f} - s_{i,f}\right|^2\right\} \\ &= \left|1 - g_{i,f} h_{i,i}(f)\sqrt{P_{i,f}}\right|^2 + \sum_{j\in\mathcal{A}(i)} \left|g_{i,f} h_{i,j}(f)\right|^2 P_{j,f} \\ &\quad + \sum_{j\in\mathcal{B}(i)} \left|g_{i,f} h_{i,j}(f)\right|^2 P_{j,f} + \left|g_{i,f}\right|^2 \sigma_{z,i}^2 \end{aligned} \quad (6)$$

By differentiating (6) with respect to $g_{i,f}$ and setting the derivative to zero, we can find the value of $g_{i,f}$ that minimizes the MSE as

$$g_{i,f}^* = \frac{\bar{h}_{i,i}(f)\sqrt{P_{i,f}}}{\sum\limits_{j\in\mathcal{U}} \left|h_{i,j}(f)\right|^2 P_{j,f} + \sum\limits_{j\in\mathcal{D}} \left|h_{i,j}(f)\right|^2 P_{j,f} + \sigma_{i,z}^2}. \quad (7)$$

The correspondent value for the minimum MSE (MMSE) is

$$e_{i,f} = \frac{1}{1 + \gamma_{i,f}} \implies w_{i,f}^* = 1 + \gamma_{i,f}. \quad (8)$$

A. Problem Formulation

Let us denote by $\mathbf{P}^{(U)} = \{P_{i,f}; f = 1, 2, \ldots, F; i \in \mathcal{U}\}$ and $\mathbf{P}^{(D)} = \{P_{i,f}; f = 1, 2, \ldots, F; i \in \mathcal{D}\}$ the vectors collecting all the the transmit powers for the uplink and downlink users, respectively. Accordingly, the fact that the achieved rate depends on all allocated powers, $R_{i,f}$ can be conveniently expressed as $R_{i,f}\left(\mathbf{P}^{(U)}, \mathbf{P}^{(D)}\right)$. We are now in the position of formulating the max sum rate allocation problem as:

$$\begin{aligned} \underset{\{\mathbf{P}^{(U)},\mathbf{P}^{(D)}\}}{\text{maximize}} \; & R_{tot} = \sum_{f=1}^{F} \sum_{i\in\{\mathcal{U},\mathcal{D}\}} R_{i,f}\left(\mathbf{P}^{(U)}, \mathbf{P}^{(D)}\right) \\ \text{subject to} \; & \\ & \sum_{f=1}^{F} P_{i,f} \le P_U \quad \forall i \in \mathcal{U}_q \quad q = 1, \ldots, Q \\ & \sum_{i\in\mathcal{D}_q} \sum_{f=1}^{F} P_{i,f} \le P_D \quad q = 1, \ldots, Q \end{aligned} \quad (9)$$

The constraints in (9) are designed to limit the power budget of uplink and downlink users. Note that problem (9) is not convex and hence hard to solve directly using standard optimization solvers.

III. Sub-optimal iterative allocation

Since centralized resource allocation, which requires inter-cell coordination, may not be feasible in practice due to random deployment and limited backhaul capacity [12], [13], to solve (9) we focus on a distributed approach performed independently at each BS. Inspired by the work in [14], we present an iterative algorithm that exploits the intimate relationship between the MMSE and the SINR and allows to provably achieve a local optimum of problem (9). Hence, we consider the following weighted MMSE problem:

$$\begin{aligned} \underset{\{\mathbf{P}^{(U)},\mathbf{P}^{(D)},\mathbf{w},\mathbf{g}\}}{\text{minimize}} \; & \sum_{f=1}^{F} \sum_{i\in\{\mathcal{U},\mathcal{D}\}} w_{i,f} e_{i,f} - \log(w_{i,f}) \\ \text{subject to} \; & \\ & \sum_{f=1}^{F} P_{i,f} \le P_U \quad \forall i \in \mathcal{U}_q \quad q = 1, \ldots, Q \\ & \sum_{i\in\mathcal{D}_q} \sum_{f=1}^{F} P_{i,f} \le P_D \quad q = 1, \ldots, Q \\ & \mathbf{w} \succeq 0 \end{aligned} \quad (10)$$

where $w_{i,f}$ are some positive weights and $\mathbf{w}$ and $\mathbf{g}$ are the vectors collecting all values of $w_{i,f}$ and $g_{i,f}$, respectively. Note that problem (10) is still not convex. Nevertheless, it can be shown that the procedure that iteratively optimizes one set of variables at the time converges to a local optimum of the original problem [15]. Moreover, once we fix the value of all sets of optimization variables except one and we solve (10) with respect to the remaining set of the variables, all the new problems are convex. Accordingly, the original problem (10) can be decomposed into four subproblems that can be iteratively solved one by one.

1) Optimizing with respect to $\mathbf{g}$*:* To further elaborate, let first assume to have some random initial power allocations $\mathbf{P}^{(U,0)}, \mathbf{P}^{(D,0)}$. Given the power allocations, employing (7) we are able to compute the optimal values of $g_{i,f}$ $\forall i \in \{\mathcal{U}, \mathcal{D}\}$ and $f = 1, 2, \ldots, F$.

2) Optimizing with respect to $\mathbf{w}$*:* Having fixed $\mathbf{P}^{(U,0)}, \mathbf{P}^{(D,0)}$ and $\mathbf{g}$ we can compute $e_{i,f}$ as in (6) and solve (10) in $\mathbf{w}$. In facts, differentiating the objective function in (10) and setting the result to zero yields

$$w_{i,f}^* = \frac{1}{e_{i,f}} \quad \forall i \in \{\mathcal{U}, \mathcal{D}\}, f = 1, 2, \ldots, F. \quad (11)$$

Since it is $0 < e_{i,f} \le 1$ the positive constraints on $\mathbf{w}$ are always met with (11).

3) Optimizing with respect to $\mathbf{P}^U$*:* Fixing the values of $\mathbf{P}^{(D,0)}$, $\mathbf{g}$ and $\mathbf{w}$, we are now able to consider the power allocation problem in the uplink. In this case, the optimization problem can be recast as a set of M convex quadratic subproblems (one for each user). In detail, we focus on the power allocation problem for user $i \in \mathcal{U}$. We can properly rearrange the terms in the summation (6) so to consider only those depending on the vector $\mathbf{P}_i$ that collects the power values of i on all subcarriers and solve

$$\begin{aligned} \underset{\{\mathbf{P}_i\}}{\text{minimize}} \quad & \sum_{f=1}^{F} w_{i,f}^* \left|1 - g_{i,f}^* h_{i,i}(f)\sqrt{P_{i,f}}\right|^2 \\ & + \sum_{\substack{j\in\mathcal{U} \\ j\neq i}} w_{j,f}^* \left|g_{j,f}^* h_{j,i}(f)\right|^2 P_{i,f} \\ & + \sum_{j\in\mathcal{D}} w_{j,f}^* \left|g_{j,f}^* h_{j,i}(f)\right|^2 P_{i,f} \\ & \text{subject to} \\ & \sum_{f=1}^{F} P_{i,f} \leq P_U \end{aligned} \tag{12}$$

Since (12) is convex and differentiable, the solution of (12) can be found using the KKT conditions. To elaborate, denoting by $I_{i,f}$ the term

$$I_{i,f} = \sum_{j\in\mathcal{U}} w_{j,f}^* \left|g_{j,f}^* h_{j,i}(f)\right|^2 + \sum_{j\in\mathcal{D}} w_{j,f}^* \left|g_{j,f}^* h_{j,i}(f)\right|^2, \tag{13}$$

the solution can be expressed as

$$P_{i,f} = \left[\frac{w_{i,f}^* g_{i,f}^* h_{i,i}(f)}{I_{i,f} + \mu_i}\right]^2 \quad f = 1, 2, \ldots, F, \tag{14}$$

where the Lagrange multiplier μ_i is chosen so that the power constraint for user i is met.

4) Optimizing with respect to $\mathbf{P}^{(D)}$*:* Fixing $\mathbf{g}, \mathbf{w}$ and $\mathbf{P}^{(U)}$, and by following the same approach for finding $\mathbf{P}^{(U)}$, the optimal solution of (10) for the downlink in cell q ($q = 1, 2, \ldots, Q$) is

$$P_{i,f} = \left[\frac{w_{i,f}^* g_{i,f}^* h_{i,i}(f)}{I_{i,f} + \mu_q}\right]^2 \ \forall i \in \mathcal{D}_q; f = 1, 2, \ldots, F, \tag{15}$$

where the Lagrange multiplier μ_q is chosen so that the power constraint for cell q is met.

A. Convergence behavior

Since our ultimate goal is to maximize the rate we want to show that the sum rate is increasing at each iteration. In order to do so, we need to add to our variables the apex (l) to indicate that they have been computed at the lth iteration. Moreover, we indicate with $e_{i,f}^{(l)} = \left(\mathbf{P}^{(l)}, g_{i,f}^{(l)}\right)$, ie the MSE computed with updated values of $\mathbf{P} = \{\mathbf{P}^{(U)}, \mathbf{P}^{(D)}\}$ and g. To elaborate, owing to the optimal construction of $\mathbf{P}^{(l+1)}$, we have

$$e_{i,f}\left(\mathbf{P}^{(l+1)}, g_{i,f}^{(l)}\right) \leq e_{i,f}^{(l)} \tag{16}$$

To further elaborate, since $g_{i,f}^{(l+1)}$ fulfills the MMSE criterion, $e_{i,f}^{(l+1)} \leq e_{i,f}\left(\mathbf{P}^{(l+1)}, g_{i,f}^{(l)}\right)$, so that it is

$$e_{i,f}^{(l+1)} \leq \sum_{i,f} e_{i,f}\left(\mathbf{P}^{(l+1)}, g_{i,f}^{(l)}\right) \leq e_{i,f}^{(l)} \tag{17}$$

Hence, since for any positive x it is $1 + \log(x) \leq x$, we have

$$1 + \log\left(w_{i,f}^{(l)} e_{i,f}^{(l+1)}\right) \leq w_{i,f}^{(l)} e_{i,f}^{(l+1)} \tag{18}$$

By summing (18) over $i \in \{\mathcal{U}, \mathcal{D}\}$ and $f = 1, 2, \ldots, F$ and combining the result with (17) yields:

$$\sum_{i,f} 1 + \log\left(w_{i,f}^{(l)}\right) \leq \sum_{i,f} w_{i,f}^{(l)} e_{i,f}^{(l)} - \log\left(e_{i,f}^{(l+1)}\right) \tag{19}$$

Since because of (11) it is $w_{i,f}^{(l)} = \left(e_{i,f}^{(l)}\right)^{-1}$ we get

$$\sum_{i,f} -\log\left(e_{i,f}^{(l)}\right) \leq \sum_{i,f} -\log\left(e_{i,f}^{(l+1)}\right) \tag{20}$$

or, equivalently, replacing (8) in (20)

$$R_{tot}\left(\mathbf{P}^{(l)}\right) \leq R_{tot}\left(\mathbf{P}^{(l+1)}\right). \tag{21}$$

We have shown that the the sum rate provably increases at each iteration, thus guaranteeing robust convergence to a local maximum solution. The proposed iterative allocation scheme is summarized in Algorithm 1. Notice that the considered approach can be seen as a strategic game with simultaneous updates among all users. In particular, similarly to the approach recently proposed in [16] for power allocation in device to device OFDMA communication scenarios, the proposed scheme allows to increase the sum rate at each move. Accordingly, the proposed algorithm could be modelled as a potential game with better response dynamics, where the potential function is represented by the system sum rate [17,18].

Algorithm 1: Iterative power allocation

- **Initialize:**
- **for** $i\,\{\mathcal{U}, \mathcal{D}\}$ **do**
  - Select an initial allocation $\mathbf{P}^{(U,0)}, \mathbf{P}^{(D,0)}$
  - Compute $g_{i,f}^{0}$ according to (7)
  - Compute $w_{i,f}^{(0)} = 1 + \gamma_{i,f}$
- $l \leftarrow 1$, $\Delta \leftarrow 1$;
- **while** $\Delta \neq 0$ **do**
  - **for** $i \in \{\mathcal{U}, \mathcal{D}\}$ **do**
    - Compute $P_{i,f}^{(l)}$ according to (14) and (15)
    - Compute $g_{i,f}^{(l)}$ according to (7)
    - Compute $w_{i,f}^{(l)} = 1 + \gamma_{i,f}$
  - $\Delta \leftarrow \left\|\mathbf{P}^{l} - \mathbf{P}^{(l-1)}\right\|$;
  - $l \leftarrow l + 1$

B. Implementation Issues

The proposed iterative algorithm is naturally amenable to a distributed implementation, where all BSs act independently. To this aim, at each iteration the qth BS needs to know $\gamma_{i,f}$ and $g_{i,f}^{(l)}$ for $i \in \mathcal{U}_q$, that can be evaluated through local estimation, on the basis of the received signals. Moreover, it is necessary to know $\gamma_{i,f}$ and $g_{i,f}^{(l)}$ for $i \in \mathcal{D}_q$, that can be estimated by downlink users and communicated back to the serving BS. Nevertheless, the implementation of the algorithm requires some global knowledge to evaluate $I_{i,f}$ reported in (13). Such terms can be estimated following a similar approach to the method described in [16] for a device to device scenario. In particular, we recognize that these terms represent the potential interference measured at the transmitters provided that the receivers send a properly weighted sounding reference signal, which spans all available sub-channels. Hence, the proposed allocation scheme can be implemented without requiring any explicit message passing neither among BSs, nor among users.

IV. Numerical Results

To examine the performance of the proposed allocation scheme, referred to as wMMSE-FD in the following, we consider a cellular system with cells of radius $R = 100$ m, each serving M_q uplink and N_q downlink mobile users. We perform Monte Carlo simulations, where at each simulation instance the positions of the mobile users are randomly generated in the cell with a minimum distance towards the serving BS of 10 m. The channel model used between each tx-rx pair is the same, with the justification that BSs do not have a significant height advantage in a typical pico and femto-cell deployment. Hence, we consider a channel attenuation due to path loss proportional to the distance between the transmitters and receivers, shadowing and fading. The path loss exponent is $\alpha = 4$, while shadowing is assumed log-normally distributed with standard deviation $\sigma_{SH} = 8$ dB. For each sub-channel, we assume an uncorrelated fading channel model with channel coefficients generated from the complex Gaussian distribution $\mathcal{CN}(0, 1)$. The maximum transmitting powers P_D and P_U are set to 0.1 and $0.1 \times M_d$ W, respectively, and the variances σ^2 of the additive zero-mean Gaussian noise are assumed to be the same for all receivers. The SI cancelation factor $\zeta_{q,q}(f)$ $(q = 1, 2, \ldots, Q; f = 1, 2, \ldots, F)$ at the BSs is set to a constant value of 110 dB, that is a reasonable value for the considered scenario [4].

In this setting, we first compare the performance of the proposed wMMSE-FD scheme with a joint sub-channel and power allocation scheme, denoted by REF. The REF scheme include an exhaustive search over all possible sub-channel assignments followed by power allocation. In particular, for each sub-channel assignment, power allocation is solved using the successive convex approximation approach proposed in [16]. The REF method incurs an exponential complexity in the number of sub-channels, and hence it is computational prohibitive especially when F is large. Hence, it is used as reference to compare the performance of the proposed scheme in a single cell scenario with limited number of sub-channels. The performance measure of interest is the achieved system sum rate representing the sum of the bits per channel use (bpcu) of each transmitter. To put in evidence the gain that can be obtained by FD operations, we also show the performance of an HD resource allocation algorithm, which applies the same wMMSE allocation approach proposed in this paper where half of the carriers are used for the uplink and the other half for the downlink so that neither SI nor inter-node interference is present. This scheme is denoted by wMMSE-HD.

Figures 1 shows the sum rate as a function of the average signal to noise ratio (SNR) for the single cell case, for $F = 4$, $M_q = 3$ and $N_q = 3$. The SNR for each

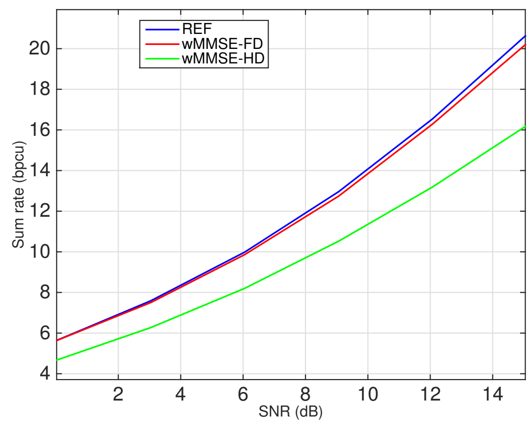

Fig. 1. Total sum rate for REF, wMMSE-FD, wMMSE-HD allocation schemes, for $Q = 1$, $N_q = 3$, $M_q = 3$ and $F = 4$.

simulation run is obtained by averaging over all nodes the received power divided by the noise variance in the case of uniform power allocation at the transmitters. Different SNR values are obtained by properly setting the noise variance. Notice that the proposed wMMSE-FD scheme yields similar performance as that of the REF scheme (with a somewhat higher rate achieved by REF), thus proving the validity of the proposed allocations scheme. Moreover, as expected both schemes clearly outperform wMMSE-HD.

Figures 2 shows the performance of the wMMSE-FD and wMMSE-HD schemes for $Q = 3$ (Figure 2 (a)) and $Q = 7$ (Figure 2 (b)), $F = 8$, $M_q = 6$ and $N_q = 6$. We report in the same figures both the aggregated (total) sum rate and the uplink (up) and downlink (down) sum rates. It is shown that all the sum rates scale with the number of cells even in the low SNR regime for both the HD and FD scenarios. This results demonstrates the adaptability of the proposed wMMSE approach to heterogeneous interference conditions typical of HetNets deployments. Moreover, the downlink connections outperform the uplink ones despite the total power budget is the same for both cases (i.e., 0.6 W per cell). This behavior is due to the higher downlink flexibility in assigning more power to those users that experience better channel conditions. Notice that operating in FD mode allows to outperform the classical HD cellular mode by nearly 50%, a result that is in line with what found in other works (e.g., see [11]). Furthermore, basing on all the results reported in Figures 2, we have verified that the proposed wMMSE-FD scheme allows to fulfill the exclusivity assignment constraint for 97% of all cases, i.e., a sub-channel is assigned to more than a two-user (one uplink and one downlink) pair in the cell in only 3% of cases. A closer inspection of such situations reveals that non exclusivity occurs when there are users that experience very high attenuations towards all the receivers and, as such, it does not affect the total sum rate. Finally, for 22% of all cases, sub-channels are assigned in HD mode, i.e., they are assigned exclusively to an uplink or a downlink user.

As for the convergence behavior of wMMSE-FD, in all the considered configurations reported in Figures 2 we have experienced a low number of iterations to convergence. In particular, the average number of iterations was approximately 10 for all cases, with a maximum that never exceeded 15. On the other hand, in the presence of channel variability, the training phase must be limited to the minimum necessary, and hence the fast convergence behavior of wMMSE-FD makes it particularly suitable for implementation over time varying wireless scenario.

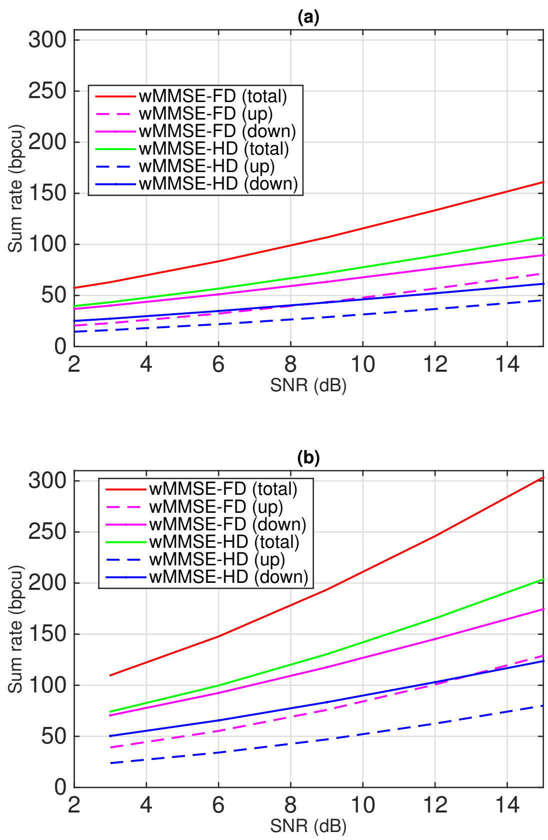

Fig. 2. Total sum rate, uplink sum rate and downlink sum rate for wMMSE-FD and wMMSE-HD allocation schemes, for $N_q = 6$, $M_q = 6$, $F = 8$, $Q = 3$ (a), and $Q = 7$ (b).

V. Conclusions

We have presented an iterative power allocation scheme for a typical OFDMA small-cells network where the BSs operate in FD mode. Hence, leveraging on the intimate relationship between the minimum mean square error and the signal to noise ratio, we have proved convergence to a local maximum of the sum rate. In a single cell scenario, the proposed allocation scheme allows to achieve fast convergence to a solution that approaches the performance of a power allocation scheme that includes an exhaustive search over all possible sub-channel assignments. Moreover, the proposed scheme can cope with an unlimited number of interferers, and as such it is amenable for implementation on a generic multicell scenario. As a matter of fact, the performance scales with the number of cells even in the low SNR regime. This results demonstrates the adaptability of the our solution to heterogeneous interference conditions typical of HetNets deployments.

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