Preliminary Stability Analysis Methods For PrandtlPlane Aircraft in Subsonic Conditions

Structured Abstract

Purpose: The present paper aims to assess the reliability and the limitations of analysing the flight stability of a box-wing aircraft configuration known as PrandtlPlane by means of methods conceived for conventional aircraft and well known in literature.

Design/methodology/approach: Results obtained by applying vortex lattice methods to PrandtlPlane configuration, validated previously with wind tunnel tests, are compared to the output of a "Roskam-like" method, here defined in order to model the PrandtlPlane features.

Findings: The comparisons have shown that the "Roskam-like" model gives accurate predictions for both the Longitudinal Stability Margin and Dihedral Effect, whereas the Directional Stability is always overestimated.

Research limitations/implications: The method here proposed and related achievements are valid only for subsonic conditions. The poor reliability related to lateral-directional derivatives estimations may be improved implementing different models known from literature.

Practical implications: The possibility to apply a faster method as the "Roskam-like" here presented has two main implications: 1) it allows to implement faster analyses in the conceptual and preliminary design of PrandtlPlane, providing also a tool for the definition of the design space in case of optimization approaches 2) it allows to implement scaling procedure, in order to study families of PrandtlPlanes or different aircraft categories.

Social implications: This paper is part of the activities carried out during the PARSIFAL project, which aims to demonstrate that the introduction of PrandtlPlane as air transport mean can fuel consumption and noise impact, providing a sustainable answer to the growing air passenger demand envisaged for the next decades.

Originality/value: The originality of this paper lies in the attempt of adopting analysis method conceived for conventional airplanes for the analysis of a novel configuration. The value of the work is represented by the knowledge concerning experimental results and design methods on the PrandtlPlane configuration, here made available in order to define a new analysis tool.

Keywords: PrandtlPlane; PARSIFAL; Flight Mechanics; Aircraft Design; VLM; Novel Configurations.

Paper type: Research paper

Abstract

The paper presents a method for the preliminary stability analysis of box-wing aircraft, called PrandtlPlane, proposed as a more efficient alternative to present commercial aircraft within the project "PARSIFAL" (Prandtlplane ARchitecture for the Sustainable Improvement of Future AirpLanes), funded by European Union under the Horizon2020 Program.

The presented method is derived from the well-known models proposed by Jan Roskam for conventional aircraft and focuses on the evaluation of α and β derivatives responsible for aircraft stability, with the aim of defining a preliminary design tool, useful to support the Vortex-Lattice Methods, commonly used for preliminary simulations and optimizations of PrandtlPlane aircraft.

Results obtained from both the vortex-lattice and the "Roskam-like" methods are compared for a reference PrandtlPlane configuration and a sensitivity analysis is performed on different configurations generated by varying a subset of design parameters Result of such comparison are discussed, analysing the accuracy of "Roskam-like" method and identifying the limitations, as well as the possible improvements.

Introduction

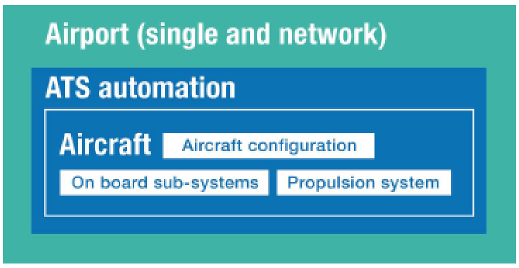
According to many studies carried out in the last years by European institutions and research bodies (ACARE, 2007; European Commission, 2011; EREA, 2012), the key requirements for the future worldwide development of the Air Transport System can be summarized as follows:

- to satisfy the increase of air traffic demand improving flight safety;
- to cut CO2 and NOx emissions and noise per unit of transport;
- to make travellers within Europe able to complete their journey within 4 hours (door-to-door).

Further requirements concern the recyclability of materials, the reduction of emissions during take-off operations and the capability of adopting alternative fuels and propulsion concepts.

In order to fulfil such requirements, the priority research areas indicated in Figure 1 have been defined.

Figure 1. Priority research areas (EREA, 2012)



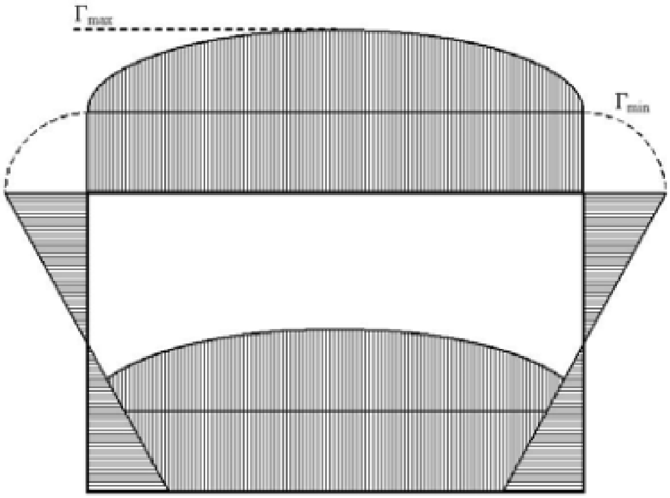
Among them, the *Aircraft Configuration* area plays a significant role in the challenge of improving aerodynamic efficiency and, therefore, reducing fuel consumption and pollution. Today, conventional tube-and-wing aircraft have reached a high level of maturity and further significant improvements will be difficult to achieve. In addition, changing the shape of aircraft to go far beyond current conventional configurations performance is becoming possible because of advances in materials, computational power and design techniques.

The novel aircraft configurations object of main studies in the last decades are the Blended Wing Body (BWB, Figure 2-left), the Truss Braced Wings (TBW, Figure 2-center) and the box-wing or PrandtlPlane (PrP, Figure 2-right).

Figure 2: Blended Wing Body (BWB), Truss Braced Wings (TBW) and PrandtlPlane (PrP) concepts

The PrP solution (Figure 2, right), presents the minimum induced drag among all the solutions available, since it represents the engineering application of the “best wing system” concept proposed by Prandtl (1924). According to Prandtl and as confirmed by further studies carried out at University of Pisa (Frediani and Montanari, 2009), the multiplane that provides the minimum induced drag for given lift and wingspan is a box-wing system, in which the induced velocities on the horizontal wings are constant along the wingspan and null on vertical wings (Figure 3).

Figure 3: Aerodynamic forces distribution of the “best wing system” (Frediani and Montanari, 2009)



Such important benefit makes the PrandtlPlane concept attractive for many applications in the commercial aircraft field (Frediani *et al.* 2012, Cavallaro and Demasi 2016). In addition,,previous studies on the PrandtlPlane concept applied to Light Sport Aircraft/Ultralight aircraft have shown further advantages concerning flight mechanics which confer to PrandtlPlane the capability of fulfilling the flight safety improvement requirements.

As described in details in (Frediani *et al.*, 2015), both numerical and experimental activities, including flight tests on scaled models, performed within the project *IDINTOS*, have shown the following advantages of the PrandtlPlane configuration:

- the wing loading is lower on rear wing and higher on front wing, which implies that stall occurs on the front wing first. When this happens, the rear wing introduces a significant negative pitching moment which increases longitudinal stability and makes the stall occurrence very smooth. In addition, when front wing is stalled the control surfaces on rear wing remain effective, allowing the pilot to control the aircraft even in post-stall conditions. Such "anti-stall" behaviour reduces the stall speed of the PrandtlPlane with reference to conventional airplanes, allowing safer take-off and landing operations and reducing the runway length necessary for such flight phases;
- pitch control can be performed by means of counter-rotating elevators, placed on front and rear wings, which introduce a pitching moment as a pure couple instead as the result of vertical force applied on the tail; this increases manoeuvring precision, improving safety in all the flight conditions in which the aircraft is close to the ground;
- since the two wings are placed at a significant distance from the centre of gravity, the pitch damping moment is higher than in a conventional case; as a consequence, the longitudinal stability is improved, with benefits on the safety side, as well as for the flight comfort.

The PARSIFAL Project

The present paper concerns part of the on-going research activities related to the project PARSIFAL ("Prandtlplane ARchitecture for the Sustainable Improvement of Future AirpLanes"), which has been funded by the European Union and is coordinated by the University of Pisa (Italy). The other partners are Delft University of Technology (Delft, Netherlands), ONERA (Meudon, France), ENSAM (Bordeaux, France), DLR (Hamburg, Germany) and SkyBox Engineering (Pisa, Italy). The research team is supported by an External Expert Advisory Board, made of representatives of aircraft manufacturers, airport management companies and airlines, which supports the consortium in order to maximize the impact of PARSIFAL outcomes.

PARSIFAL main objective is to design aircraft with the same wingspan of Airbus 320/Boeing 737 but with increased payload capabilities, similar to Airbus 330/Boeing 767 ones. Such improvement is possible thanks to the higher aerodynamic efficiency of the PrandtlPlane lifting system, which allows to reduce the wingspan without increasing fuel consumption (Jemitola and Fielding, 2012), and to new cabin layout options which can both increase the number of transported passengers and reduce the turn-around time (Frediani *et al.*, 2017; Abu Salem *et al.* 2018).

According to the aforementioned requirements of future aircraft, these features can provide a significant contribution for the improvement of the air transport system.

Scope of the work

The activities described in the present paper are part of the preliminary design phase of the project PARSIFAL, in which most of the efforts are focused on the definition of the design parameters of the PrP configuration as well as the boundary values to be adopted during the following optimization. More in details, the main drivers for the definitions of the optimization domain are related to flight mechanics requirements concerning first equilibrium and stability and then controllability (Schitanz and Scholz, 2011).

In past projects, such part of the design activity has been carried out performing a high number of simulations with Vortex-Lattice Methods (VLM), in order to evaluate correctly the box-wing aerodynamic behaviour.

Thanks to the experience achieved from previous research on PrP configurations, the evaluation of box-wing characteristics through VLM have been successfully validated by means of wind tunnel tests and, therefore, it is possible to consider VLM enough accurate for the preliminary design phases.

Such assessment provides the reason for the work here presented, in which VLM results are used as a mean of comparison to evaluate of the accuracy of a lower fidelity flight mechanics analysis tool, derived from the well-known methods used for conventional subsonic airplanes due to Roskam (1983). Such tool would speed-up and facilitate the definition of the design domain and, if properly validated, could be considered as an alternative to the VLM for optimisation purposes.

The present paper describes the first steps towards the definition of a method derived from the one proposed by Roskam and suitable for PrP configuration. The aim of this work is, in fact, focused on the analysis of part of the flight mechanic requirements, indicated here after:

- positive static stability margin:

$$h_n - h > 0 \tag{1}$$

where h and h_n are:

$$h_n = \frac{X_{NP}}{\bar{c}_a} \tag{2}$$

$$h = \frac{X_{CG}}{\bar{c}_a} \tag{3}$$

in which X_{CG} and X_{NP} are, respectively, the longitudinal positions of aircraft Centre of Gravity (*CG*) and Neutral Point (*NP*) referred to the leading edge of the mean aerodynamic chord ($\bar{c}_a$);

- directional stability, which depends on the sign of yaw moment (C_n) vs sideslip angle (β) derivative:

$$C_{n\beta} > 0 \qquad (4)$$

- dihedral effect, which depends on the sign of roll moment(C_l) vs β derivative:

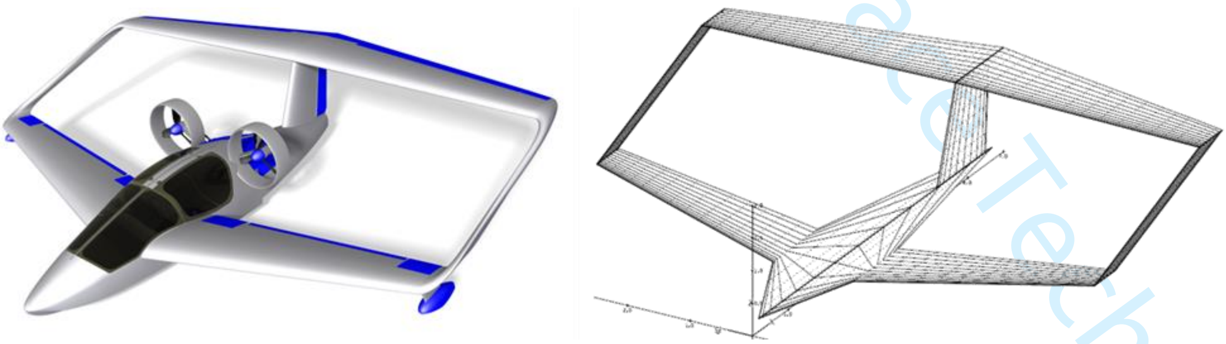
$$C_{l\beta} < 0 \qquad (5)$$

Next paragraphs provide some information on VLM implementation and validation for the PrP case and illustrate in details the model derived from Roskam. Finally a PrP reference configuration is defined and results of both methods are presented, including sensitivity analysis for a subset of design parameters.

Flight mechanic analysis through Vortex-Lattice Methods in subsonic range

According to previous research carried out at the University of Pisa (Frediani *et al.*, 2007; Frediani *et al.*, 2015), focused on the application of the PrandtlPlane configuration to light subsonic aircraft, the preliminary assessment of the aforementioned flight mechanic requirements can be performed by using Vortex-Lattice Methods. In particular, the use of the code AVL (Drela and Youngren, 2017) has been successfully implemented, both as evaluation or optimization tool (Rizzo, 2009), by means of a specific modelling strategy based on the replacement of the fuselage with a lifting surface with a similar planform (Figure 4). The choice of the “falt” model for the fuselage is the result of previous works, such as Zanetti (2015) for the “IDINTOS” case, in which the AVL accuracy has been investigated for several PrP configuration models (flat fuselage, fuselage as a distribution of doublets, fuselage removal, etc.). Whereas all the models provide a poor estimation of β-derivatives, the flat fuselage model is much more accurate in predicting α-derivatives, hence its choice represents the best compromise for the conceptual design phase.

Figure 4. AVL model of a light amphibious PrandtlPlane (Project “IDINTOS”, Frediani *et al.*, 2015)



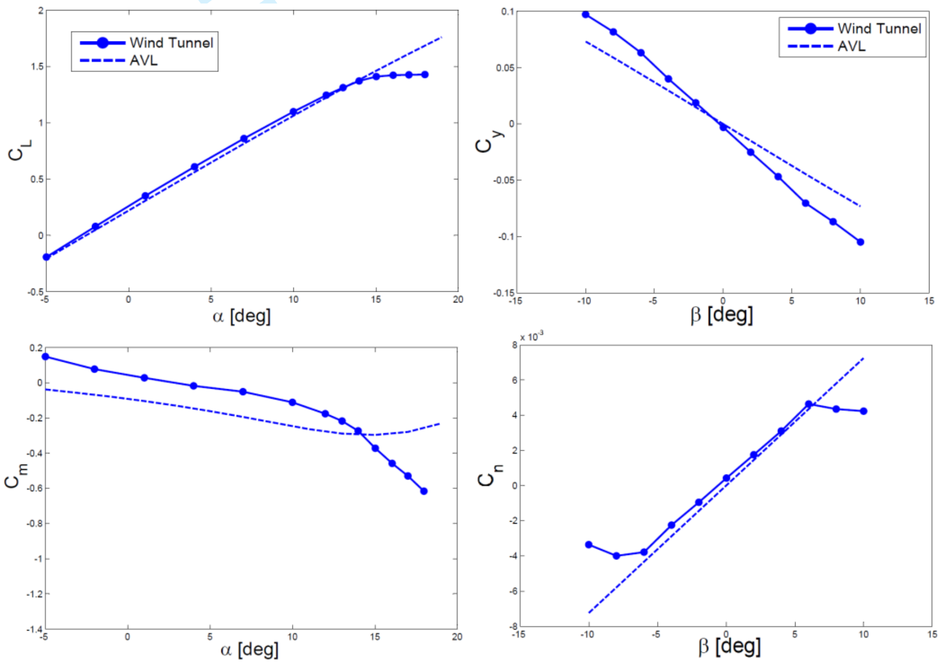
As detailed in Oliviero *et al.* (2016), the research activities carried out during the project “IDINTOS” have provided both numerical and experimental results concerning the aerodynamics and flight mechanics of the PrP configuration in subsonic flight (Cipolla *et al*., 2016). Given the data shown in Figure 5 (Oliviero *et al*., 2016), at trim condition (α=1°, β=0°), the following accuracy factors can be calculated:

- $k_{L\alpha} = C_{L\alpha}{}^{AVL} / C_{L\alpha}{}^{WT} = 0.99$
- $k_{m\alpha} = C_{m\alpha}{}^{AVL} / C_{m\alpha}{}^{WT} = 0.91$
- $k_{y\beta} = C_{y\beta}{}^{AVL} / C_{y\beta}{}^{WT} = 0.60$
- $k_{l\beta} = C_{l\beta}{}^{AVL} / C_{l\beta}{}^{WT} = 0.87$
- $k_{n\beta} = C_{n\beta}{}^{AVL} / C_{n\beta}{}^{WT} = 1.57$

In addition, the accuracy factor for the Static Stability Margin (SSM) can be calculated from previous data as follows:

- $k_{SSM} = (C_{m\alpha}{}^{AVL} / C_{L\alpha}{}^{WT}) / (C_{m\alpha}{}^{AVL} / C_{L\alpha}{}^{AVL}) = 0.92$

Figure 5. Comparison of results from wind tunnel tests and AVL analyses for the ultralight PrP "IDINTOS" (Oliviero *et al.*, 2016)



The accuracy factors presented above can be commented as follows:

- the sign of each derivative is predicted correctly;
- the AVL model of the PrP (Figure 4) allows a good evaluation of α-derivatives with error margins below 10% of wind tunnel values;
- the Static Stability Margin is predicted with a 10% error margin and in a conservative way;
- the flat fuselage model, chosen as compromise in terms of AVL accuracy, provides a poor representation of fuselage contribution to β-derivatives, hence the accuracy factors related to β-derivatives are much more distant from the value 1, with the only exception of $C_{l\beta}$ for which the error margin is below 15%.

Since the aim of this paper is to assess the accuracy of a "Roskam-like" approach for the evaluation of α and β derivatives for a generic PrP configuration in subsonic conditions, such accuracy factors can be helpful for the interpretation of the comparison between AVL and "Roskam-like" method results.

Application of conventional methods for aerodynamics derivatives evaluation to PrP configurations

As said, the method here presented is derived from Jan Roskam's "Methods for estimating stability and control derivatives of conventional subsonic airplanes" (Roskam, 1983), which is based on semi-empirical models for the evaluation of aerodynamic derivatives of conventional aircraft.

The method proposed by Roskam has been adapted to the PrP configuration in order to evaluate the aerodynamic derivatives in subsonic conditions. Therefore, as for the Roskam method, the hypothesis of absence of transonic effects, i.e. that Mach number (M) is much smaller than the critical value, is introduced:

$$M << Mcr \qquad (6)$$

Since the aim of PARSIFAL project is the study of box-wing aircraft shown, the Roskam method has been applied considering the main lifting surfaces of this configurations, illustrated in Figure 2: a front lower wing connected to the fuselage, a rear upper wing connected on top of the twin vertical tails, two vertical tip-wings, connecting the front wing to the rear one, and the twin vertical tails linked also to the fuselage.

This complex lifting system is geometrically described through the following main parameters:

- S: wing area of each lifting surface;
- b: wingspan of each lifting surface, assumed to be equal for front and rear wings;
- $\bar{c}_a$ or *mac*: mean aerodynamic chord of each lifting surface;
- $\bar{c}_g$: mean geometric chord of each lifting surface;
- AR: aspect ratio of each lifting surface, defined as $AR = b^2/S$;
- λ: taper ratio of each lifting surface, defined as $\lambda = c_t/c_r$, where c_t is the tip chord and c_r is the root chord;
- Λ: sweep angle of each lifting surface, which is typically defined at the leading edge (Λ_{LE}), at the quarter-chord point($\Lambda_{c/4}$,), or at the half-chord point ($\Lambda_{c/2}$). For constant λ wing portions, Equation (7) can be used to convert such quantities:

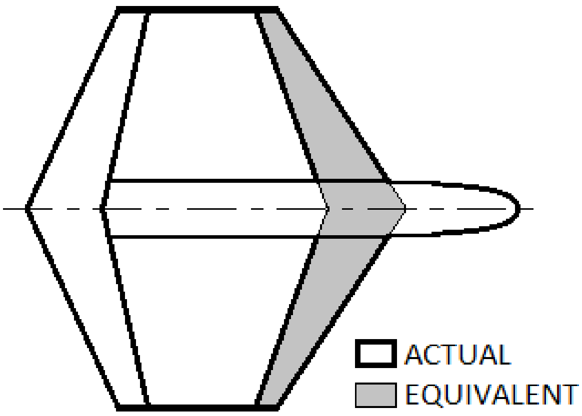
$$\tan(\Lambda_{nc}) = \tan(\Lambda_{mc}) - \frac{4}{AR}\left[(n-m)\left(\frac{1-\lambda}{1+\lambda}\right)\right] \qquad (7)$$

The following subscripts are introduced to identify the components of the box-wing architecture:

- *F*: front wing;
- *R*: rear wing;
- *B*: fuselage (or "body"), sometimes used in combination with *F* or *R* subscript to indicate wing-body systems;
- *T*: vertical tip-wings;
- *V*: twin vertical tails.

Since the PrP configurations here considered are characterized by the front wing intersecting the fuselage and the rear wing placed on top of the vertical tail, an equivalent area (S_F) has been defined only for the front wing by projecting the root section towards the longitudinal symmetry plane of the aircraft, as shown in Figure 6.

Figure 6. Equivalent wing area defined for the front wing



Longitudinal plane derivatives

<u>$C_{L\alpha}$ derivative</u>

In analogy with Roskam approach, the lift curve slope ($C_{L\alpha}$) of the PrP configuration is calculated trough Equation (8), in which the contributions of front wing-fuselage system and rear wing are divided:

$$C_{L\alpha} = C_{L\alpha_F} K_{FB} + C_{L\alpha_R} \frac{S_R}{S_F}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\eta_R \quad (8)$$

$C_{L\alpha F}$ and $C_{L\alpha R}$ can be calculated with the following expression, suitable for any lifting surface:

$$C_{L\alpha}\Big|_{3D} = \frac{2\pi AR}{2 + \sqrt{\frac{AR^2(1-M)^2}{C_{L\alpha}\big|_{2D}^{\,2}}\left(1 + \frac{\tan^2(\Lambda_{c/2})}{(1-M)^2}\right) + 4}} \quad (9)$$

in which $C_{l\alpha}$ is the airfoil lift curve slope.

Then, the other terms of Equation (8) are calculated as follows:

- The interference factor due to the fuselage K_{FB} is given by:

$$K_{FB} = 1 - 0.25\left(\frac{d_B}{b}\right)^2 + 0.025\left(\frac{d_B}{b}\right) \tag{10}$$

where d_B is the fuselage diameter;

- the downwash ratio $d\varepsilon/d\alpha$ is given by:

$$\frac{d\varepsilon}{d\alpha} = 4.44\left[K_{AR} K_{\lambda} K_R \sqrt{\cos(\Lambda_{c/4})}\right]^{1.19} \tag{11}$$

where, K_{AR}, K_{λ} and K_R are referred to the front wing and are calculate as follows:

$$K_{AR} = \frac{1}{AR} - \frac{1}{1 + AR^{1.7}} \tag{12}$$

$$K_{\lambda} = \frac{10 - 3\lambda}{7} \tag{13}$$

$$K_R = \frac{1 - h_R / b}{(2 l_R / b)^{1/3}} \tag{14}$$

The terms h_R and l_R in Equation (14) are defined as follows:

1. h_R is the vertical distance between front wing root chord and rear wing mean geometric chord;
2. l_R is the longitudinal distance between the points located at ¼ of the mean aerodynamic chords of the two wings .

- the dynamic pressure ratio at rear wing η_R is calibrated for the PrP configuration by using AVL, in order to introduce a correction with takes the particular downwash conditions on the rear wing into account. Given a PrP reference configuration, η_R is evaluated through the following steps:
  1. K_{FB} and $d\varepsilon/d\alpha$ and are obtained from Equation (10) and Equation (11);
  2. $C_{L\alpha F}$, $C_{L\alpha R}$ and $C_{L\alpha}$ are calculated using AVL for both the complete configuration and the isolated wings;
  3. η_R is obtained by reversing Equation (8).

$C_{m\alpha}$ derivative and Stability Margin

Once the model for $C_{L\alpha}$ has been calibrated for the PrP configuration, it can be used to evaluate the static Stability Margin (h_n) and, hence, $C_{m\alpha}$:

$$h_n = h_{n_{FB}} + \frac{\eta_R C_{L\alpha_R}}{C_{L\alpha}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\frac{S_R \bar{l}_R}{S_F \bar{c}_a} \tag{15}$$

where $h_{n\ FB}$ is the aerodynamic centre of the system composed of front wing and fuselage and $\bar{l}_R$ is the distance between this latter and rear wing aerodynamic centre.

In line with the scope of the model here presented, the h_n evaluation procedure is simplified by neglecting the interference effects due to the fuselage, hence approximating the term $h_{n\ FB}$ with the neutral point position of the isolated front wing ($h_{n\ F}$). Hence, Equation (15) has been modified as follows:

$$h_n = h_{n_F} + \frac{\eta_R C_{L\alpha_R}}{C_{L\alpha}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\frac{S_R l_R}{S_F \bar{c}_a} \tag{16}$$

where l_R is defined as for Equation (14).

According to Roskam, the longitudinal position of the aerodynamic centre of a generic lifting surface (X_{ac}) can be calculated through Equation (17):

$$X_{ac} = K_1\left(\frac{X'_{ac}}{c_R} - K_2\right)\bar{c}_g \tag{17}$$

where X_{ac} and X'_{ac} are referred to the leading edges of mean geometric chord and wing root chord, respectively. The terms in Equation (17) can be evaluated from charts (Roskam, 1983) and depend on wing geometry as follows:

- X'_{ac}/c_R depends on AR, M and Λ_{LE};
- K_1 depends on λ;
- K_2 depends on λ, AR and Λ_{LE}.

Once the positions of front and rear wing aerodynamic centres are known, it is possible to calculate l_R and hence h_n through Equation (16), in which all the other terms can be calculated as described previously.

Finally, given the CG position of the aircraft, it is possible to calculate pitch moment α-derivative referred to the *CG* as follows*:*

$$C_{m\alpha_{CG}} = C_{L\alpha}(h - h_n) \tag{18}$$

Sideslip angle derivatives

$C_{y\beta}$ derivative

According to approach proposed in Roskam (1983), the sideslip angle derivative of lateral force ($C_{y\beta\ V}$) can be calculated using Equation (19) for a single vertical surface on the plane of symmetry or Equation (20) for twin vertical surfaces.

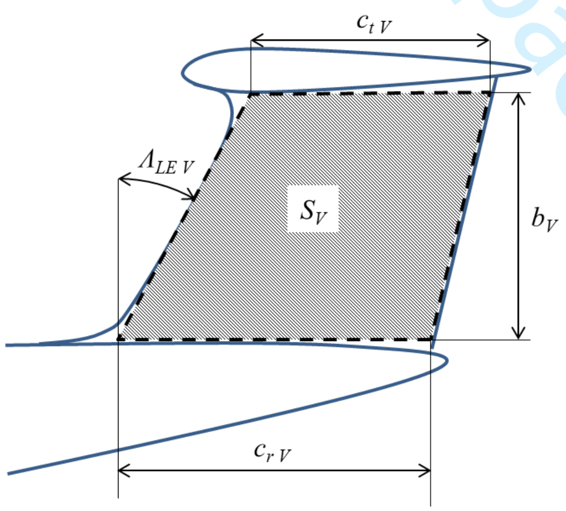
$$C_{y\beta_V} = -kC_{L\alpha_V}\left(1+\frac{d\sigma}{d\beta}\right)\eta_V \frac{S_V}{S} \qquad (19)$$

$$C_{y\beta_V} = -2\left(\frac{C_{y\beta_V(WB)}}{C_{y\beta_V(eff)}}\right)C_{y\beta_V(eff)}\frac{S_V}{S_F} \qquad (20)$$

Although the PrP configuration here considered has twin tails, in general the PrP architecture is compatible with both the solutions, therefore some details are hereafter provided for both the cases.

- In the single vertical surface model (Equation (19)):
    1. S_V is the area of the vertical surface (see Figure 7);
    2. *k* is an empirical factor depending on vertical surface span and fuselage geometry, which value can be found from charts (Roskam, 1983);
    3. $C_{L\alpha V}$ is the vertical surface lift coefficient, to be calculated according to Equation (8) ;
    4. the product $(1+d\sigma/d\beta)\cdot\eta_V$, i.e. the sidewash parameter by dynamic pressure ratio at the vertical surface, is calculated as indicated in Roskam (1983).
- In the twin vertical surfaces model (Equation (20)):
    1. S_V is the area of one of the vertical panels, calculated as shown in Figure 7;
    2. $C_{y\beta\,V(WB)}$ / $C_{y\beta\,Veff}$ depends on the span of vertical surfaces and on the distance between them, it can be found by means of charts (Roskam, 1983);
    3. $C_{y\beta\,Veff}$ depends on the span of vertical surfaces and can be found by means of charts (Roskam, 1983).

Figure 7. Definition of vertical panel effective geometry



In addition to twin vertical tails, Equation (20) has been used to calculate the $C_{y\beta}$ contribution given by the couple of vertical tip-wings.

$C_{l\beta}$ and $C_{n\beta}$ derivatives

As proposed by Roskam (1983), the β-derivatives of roll moment ($C_{l\beta}$) and yaw moment derivative ($C_{n\beta}$) can be calculated by summing the contributions given by all the aircraft components:

$$C_{l\beta} = C_{l\beta_{FB}} + C_{l\beta_R} + C_{l\beta_V} + C_{l\beta_T} \tag{21}$$

$$C_{n\beta} = C_{n\beta_F} + C_{n\beta_B} + C_{n\beta_R} + C_{n\beta_V} + C_{n\beta_T} \tag{22}$$

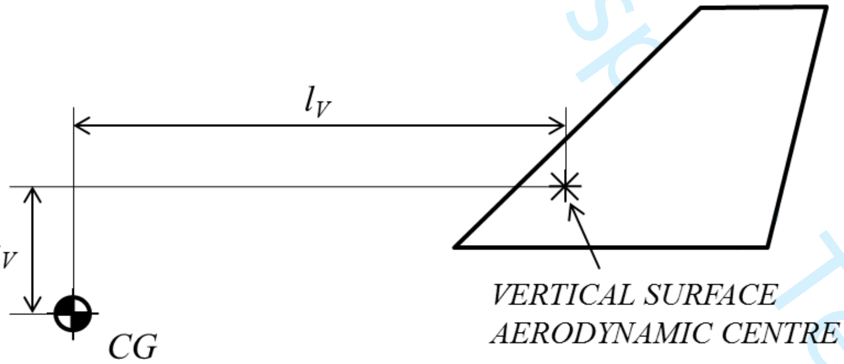
In particular, the moments generated by any vertical lifting surface (v) are calculated after evaluating $C_{y\beta}$ and the position of its aerodynamic centre:

$$C_{l\beta}\Big|_v = C_{y\beta}\Big|_v \left(\frac{z_v \cos\alpha - l_v \sin\alpha}{b} \right) \tag{23}$$

$$C_{n\beta}\Big|_v = -C_{y\beta}\Big|_v \left(\frac{z_v \sin\alpha + l_v \cos\alpha}{b} \right) \tag{24}$$

where z_v and l_v are defined as shown in Figure 8 for any vertical lifting surface. Since z_v and l_v identify the position of the aerodynamic centre, they can be calculated using the same approach proposed for the longitudinal plane. In particular, Equation (17) can be used to calculate l_v, whereas z_v depends on the location of the mean aerodynamic chord of the vertical surface.

Figure 8. Definition of vertical and horizontal distance between generic vertical surface aerodynamic centre and aircraft CG



In Equation (21), $C_{l\beta\,FB}$ can be calculated through the following equation:

$$C_{l\beta_{FB}} = 57.3\left\{ C_L \left[\left(\frac{C_{l_\beta}}{C_L} \right)_{\Lambda_{c/2}} K_{M_\Lambda} K_B + \left(\frac{C_{l_\beta}}{C_L} \right)_{AR} \right] + \Gamma \left(\frac{C_{l_\beta}}{\Gamma} K_{M_\Gamma} + \frac{\Delta C_{l_\beta}}{\Gamma} \right) + \left(\Delta C_{l_\beta} \right)_{Z_F} + \theta \tan\Lambda_{c/4} \left(\frac{\Delta C_{l_\beta}}{\theta \tan\Lambda_{c/4}} \right) \right\} \tag{25}$$

where Γ is the wing geometric dihedral angle, θ is the wing twist between root and tip sections of the wing and, according to Roskam:

- C_L is the airplane lift coefficient;
- $(C_{l\beta}/C_L)_{\Lambda_{c/2}}$ is the wing sweep contribution depending on λ, AR, $\Lambda_{c/2}$;
- K_{M_Λ} is the compressibility correction sweep depending on M and $\Lambda_{c/2}$;
- K_B is a fuselage correction factor depending on AR and $\Lambda_{c/2}$;
- $(C_{l\beta}/C_L)_{AR}$ is the aspect ratio contribution depending on λ and AR;
- $C_{l\beta}/\Gamma$ is the wing dihedral effect depending on λ, AR and $\Lambda_{c/2}$;
- K_{M_Γ} is the compressibility correction to dihedral depending on AR, M and $\Lambda_{c/2}$;
- $\Delta C_{l\beta}/\Gamma$ and $(\Delta C_{l\beta})_{Z_F}$ are body induced effects on the wing height depending on fuselage geometry;
- $\Delta C_{l\beta}/(\theta \cdot \tan\Lambda_{c/4})$ is a wing twist correction factor depending on AR.

Then, the term $C_{l\beta\,R}$ of Equation (21) is given by:

$$C_{l\beta_R} = C_{l\beta_{RB}} \frac{S_R}{S_F} \qquad (26)$$

where $C_{l\beta\,RB}$ can be obtained from Equation (25) by applying to rear wing the same procedure previously detailed for front wing-fuselage system.

Concerning the evaluation of the directional stability derivative $C_{n\beta}$, as said, the contribution of vertical tip-wings and twin tails can be calculated through Equation (24), wings' contributions ($C_{n\beta\,F}$ and $C_{n\beta\,R}$) can be neglected at low angles of attack and, hence, it is necessary only to evaluate the fuselage contribution $C_{n\beta\,B}$.

As proposed by Roskam (1983), this term can be calculated by using Equation (27), where:

- K_N is an empirical factor for body and body + wing effects, depending on fuselage geometry and provided through on charts;
- K_{Rl} is a correction factor depending on the Reynolds number of the fuselage, which can be obtained from charts;
- S_{BS} is the side area of the fuselage;
- l_B is the fuselage length.

$$C_{n\beta_B} = -57.3 K_N K_{R_l} \frac{S_{B_S}}{S} \frac{l_B}{b} \qquad (27)$$

PrP configuration tests cases and results

Analysis on reference PrP configuration

Figure 9. Dimensions of the PrP reference configuration used in the Roskam method (half-model)

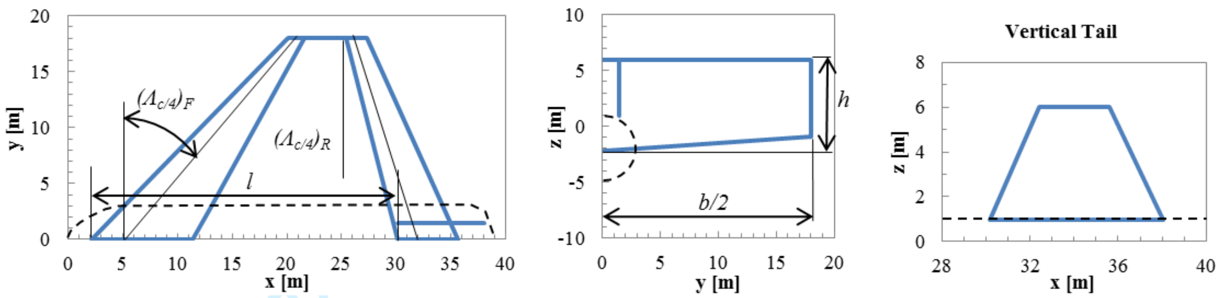
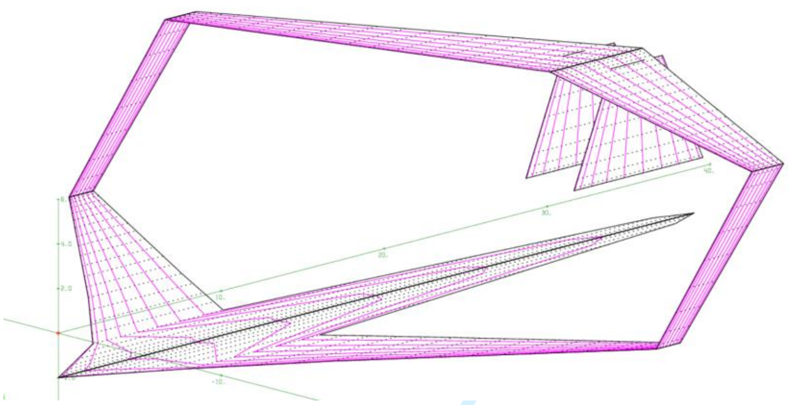


Figure 10. AVL model of the PrP reference configuration



The PrP reference configuration has the following main characteristics:

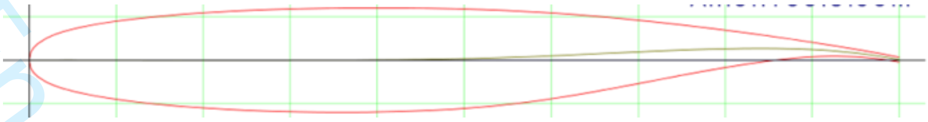
- the main dimensions of the lifting system and vertical tails are shown in Figure 9 and listed in Table 1, shows the same configuration as modelled in AVL;

Table 1. Main dimensions of PrP reference configuration

Reference dimensions		Configuration parameters	
$\bar{c}_a$ [m]	5.46	Front Wing $\Lambda_{c/4}$ [deg]	38
S [m^2]	194	Rear Wing $\Lambda_{c/4}$ [deg]	-20
b [m]	36	h/b	0.22
x_{CG} [m]	17.9	l/b	0.70
Front wing		**Rear wing**	
C_r [m] (projected)	9.27	C_r [m]	5.60
C_t [m]	1.50	C_t [m]	1.90
b [m]	36	b [m]	36
S [m^2]	194	S [m^2]	135
$\Lambda_{c/4}$ [deg]	45	$\Lambda_{c/4}$ [deg]	-20
Γ	4	Γ	0
AR	6.7	AR	9.6
λ	0.16	λ	0.34
Twin Tails		**Fuselage**	
C_r [m]	7.91	d_B [m]	5.90
C_t [m]	3.16	l_B [m]	39
b [m]	5.05	S_{BS} [m^2]	215

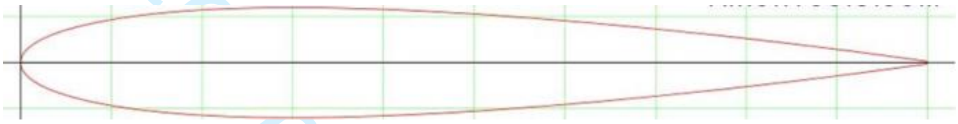
- front and rear wing sections have the shape of the supercritical airfoil NACA SC20-410 (shown in Figure 11), chosen during the preliminary studies of PARSIFAL project (Cipolla *et al.*, 2018);

Figure 11. Airfoil NACA SC20-410



- vertical tails are made with a NACA 0012 airfoil (Figure 12);

Figure 12. Airfoil NACA 0012



As indicated in Table 1, the configuration parameters are the sweep angles of both wings, the ratio *h/b*, and the ratio *l/b*, in which *l* is defined as the distance between the leading edges of wings' root chords.

By applying the calibration procedure explained in Paragraph 0 to the PrP reference configuration the dynamic pressure ratio at the rear wing has been calculated, obtaining a value of η_R equal to 0.98.

It is worth to note that the reference dimensions with which the aerodynamic coefficients have been calculated in AVL have been chosen in order to operate in analogy with the method proposed by Roskam for conventional airplanes. Therefore, the reference values for wing area, mean aerodynamic chord and wingspan are those related to the front wing equivalent area, defined according to Figure 6.

The aerodynamic derivatives calculated with AVL and "Roskam-like" method, properly implemented in a Matlab program, are summarized in Table 2. Results show a good accordance for $C_{L\alpha}$ and $C_{l\beta}$, a fair accordance for $C_{m\alpha}$ and $C_{y\beta}$, whereas the error is more significant in $C_{n\beta}$ estimation. The error concerning Stability Margin evaluation is presented in percentage of reference chord in order to have a direct understanding of the implications of such differences.

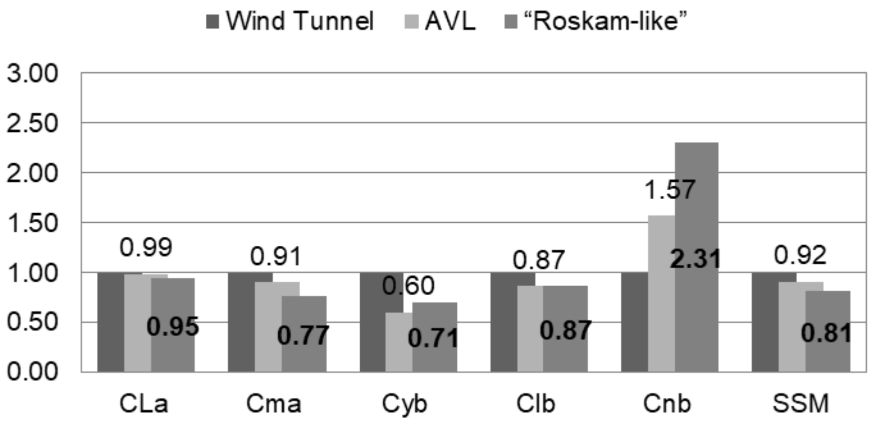
From the standpoint of the assessment of flight mechanic requirements, in this case Dihedral Effect and Stability Margin are well estimated; in addition, being the error on this latter negative, the "Roskam-like" evaluation is conservative. On the other hand, the Directional Stability is significantly overestimated, making the "Roskam-like" method not conservative.

Table 2. Results of comparison for the reference configuration

Derivatives	"Roskam"	AVL	Error (ref. AVL)
$C_{L\alpha}$ [1/rad]	6.28	6.56	-4%
$C_{m\alpha}$ [1/rad]	-0.23	-0.27	-15%
$C_{y\alpha}$ [1/rad]	-0.84	-0.72	17%
$C_{l\beta}$ [1/rad]: Dihedral Effect	-0.13	-0.13	0%
$C_{n\beta}$ [1/rad]: Directional Stability	0.22	0.15	47%
h_n - h: Static Stability Margin (% $\bar{c}_a$)	3.7%	4.1%	Diff.: -0.5%

In terms of accuracy with regards to experimental data, Figure 13 shows how the "Roskam-like" method provides a lower accuracy for all the derivatives with the only exception of $C_{l\beta}$, thus confirming a good estimation of Dihedral Effect. In addition, the prediction of Stability Margin is fair conservative, whereas Directional Stability is overestimated.

Figure 13. Accuracy factors for the reference configuration



Since such results may be positively affected by the η_R calibration, carried out on this configuration, a sensitivity analysis is presented in next paragraph in order to define if the errors between the two evaluation tools are anyway related to the configuration.

Sensitivity to design parameters

In this section some configuration parameters have been varied, to perform sensitivity tests:

- **Test Case 1**: the absolute values of wings' sweep angles (***$\Lambda_{c/4}$***) have been reduced and tip-wings have been modified accordingly;
- **Test Case 2**: the ratio between vertical distance between wings and wingspan (***h/b***) has been increased, changing the tip-wings length accordingly and moving the double fin vertically without changing its shape;
- **Test Case 3**: the ratio between horizontal distance between wings and wingspan (***l/b***) has been increased, changing the tip-wings length accordingly and moving the double fin horizontally without changing its shape.

As summarized in Table 3, for each design parameter two variations have been introduced: the first one (a) of 10% and the second one (b) of 30%.

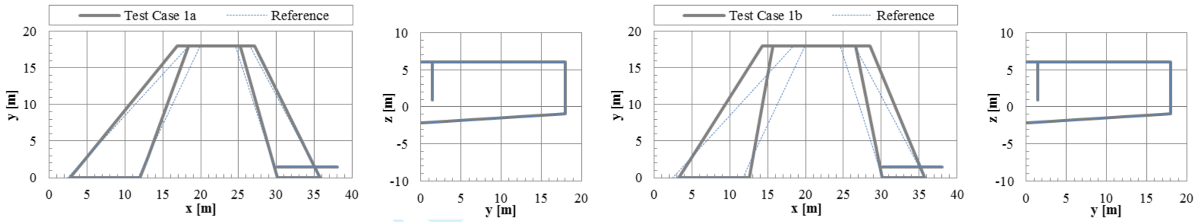
Table 3. Design parameters variations for test cases

Configuration Parameter	Reference	Test Case 1a	Test Case 2a	Test Case 3a	Test Case 1b	Test Case 2b	Test Case 3b
Front wing $\Lambda_{c/4}$ [deg]	38	34 *(-10%)*	38 *(0%)*	38 *(0%)*	27 *(-30%)*	38 *(0%)*	38 *(0%)*
Rear wing $\Lambda_{c/4}$ [deg]	-20	-18 *(-10%)*	-20 *(0%)*	-20 *(0%)*	-14 *(-30%)*	-20 *(0%)*	-20 *(0%)*
h/b	0.22	0.22 *(0%)*	0.24 *(10%)*	0.22 *(0%)*	0.22 *(0%)*	0.29 *(30%)*	0.22 *(0%)*
l/b	0.70	0.70 *(0%)*	0.70 *(0%)*	0.77 *(10%)*	0.70 *(0%)*	0.70 *(0%)*	0.91 *(30%)*

Test cases 1a and 1b: $\Lambda_{c/4}$ variation

The PrP configurations generated by reducing the $\Lambda_{c/4}$ absolute value of both wings are shown in Figure 14. Fuselage and the vertical tails dimension and positions have not changed.

Figure 14. Configurations of test cases 1a and 1b (half-model, fuselage and vertical tail dimensions as in reference configuration)



Results reported in Table 4 and Table 5 show that, with the exception of $C_{L\alpha}$ and $C_{l\beta}$, the errors have increased, although the increase does not seem directly related to the variation of design parameters. As for the reference configuration, the Dihedral Effect is well estimated, whereas the Stability Margin evaluation is less accurate as much as the design parameters is far from the reference value. Also in this case, the Stability Margin evaluation is conservative, whereas Directional Stability is overestimated.

Table 4. Results of comparison for Test case 1a (-10% $\Lambda_{c/4}$)

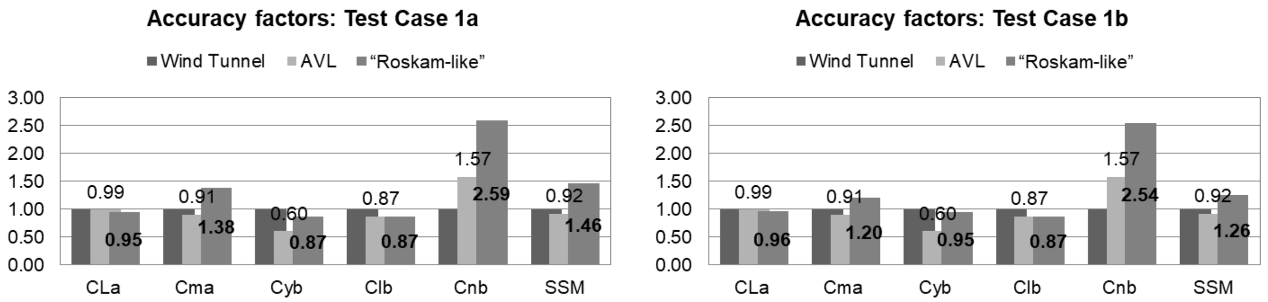
Derivatives	"Roskam"	AVL	Error (ref. AVL)
$C_{L\alpha}$ [1/rad]	6.44	6.71	-4%
$C_{m\alpha}$ [1/rad]	0.29	0.19	53%
$C_{y\alpha}$ [1/rad]	-0.96	-0.67	43%
$C_{l\beta}$ [1/rad]: Dihedral Effect	-0.13	-0.13	0%
$C_{n\beta}$ [1/rad]: Directional Stability	0.23	0.14	64%
$h_n - h$: Static Stability Margin (% $\bar{c}_a$)	-4.5%	-2.8%	Diff.: -1.7%

Table 5. Results of comparison for Test case 1b (-30% $\Lambda_{c/4}$)

Derivatives	"Roskam"	AVL	Error (ref. AVL)
$C_{L\alpha}$ [1/rad]	6.67	6.90	-3%
$C_{m\alpha}$ [1/rad]	0.93	0.70	33%
$C_{y\alpha}$ [1/rad]	-0.96	-0.61	57%
$C_{l\beta}$ [1/rad]: Dihedral Effect	-0.12	-0.12	0%
$C_{n\beta}$ [1/rad]: Directional Stability	0.21	0.13	62%
$h_n - h$: Static Stability Margin (% $\bar{c}_a$)	-13.9%	-10.1%	Diff.: -3.8%

In terms of accuracy compared to wind tunnel data, Figure 15 shows how the “Roskam-like” method provides again a good estimation of the Dihedral Effect and a conservative prediction of Stability Margin (which is negative in test cases 1a and 1b), whereas Directional Stability is highly overestimated.

Figure 15. Accuracy factors for Test Cases 1a and 1b



Test cases 2a and 2b: h/b variation

The PrP configurations generated by increasing *h/b* are shown in Figure 16; fuselage and the vertical tails dimension have not changed, although these latter have been moved upwards together with the rear wing.

Figure 16. Configurations of test cases 2a and 2b (half-model, fuselage and vertical tail dimensions as in reference configuration)

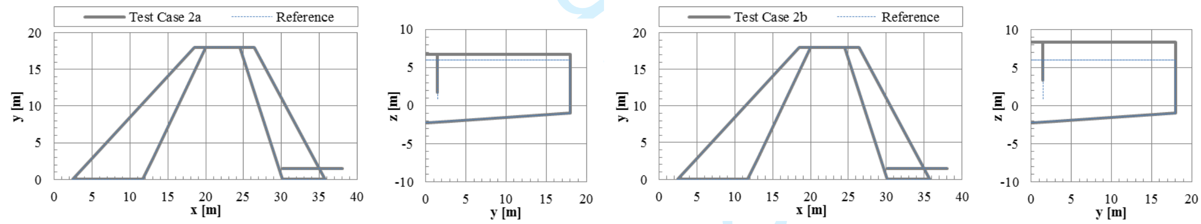


Table 6. Results of comparison for Test case 2a (+10% *h/b*)

Derivatives	“Roskam”	AVL	Error (ref. AVL)
$C_{L\alpha}$ [1/rad]	6.31	6.60	-4%
$C_{m\alpha}$ [1/rad]	-0.28	-0.36	-22%
$C_{y\alpha}$ [1/rad]	-1.07	-0.80	34%
$C_{l\beta}$ [1/rad]: Dihedral Effect	-0.15	-0.15	0%
$C_{n\beta}$ [1/rad]: Directional Stability	0.26	0.16	63%
$h_n - h$: Static Stability Margin (% $\bar{c}_a$)	4.4%	5.5%	Diff.: -1.0%

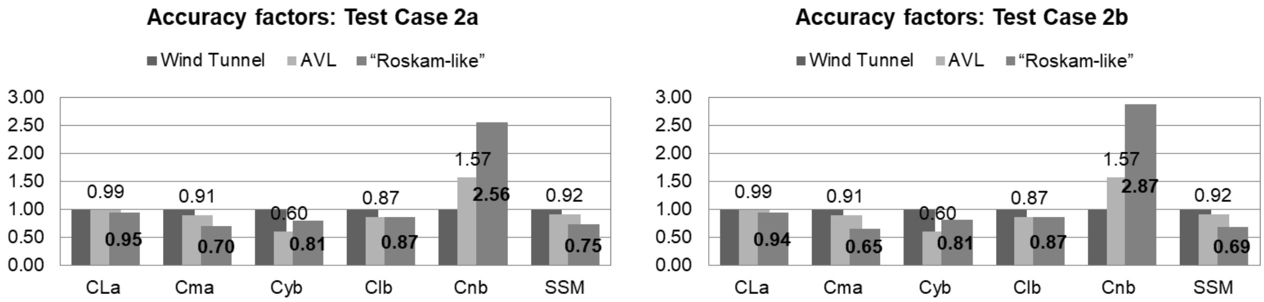
Table 7. Results of comparison for Test case 2b (+30% *h/b*)

Derivatives	“Roskam”	AVL	Error (ref. AVL)
$C_{L\alpha}$ [1/rad]	6.37	6.67	-4%
$C_{m\alpha}$ [1/rad]	-0.39	-0.54	-28%
$C_{y\alpha}$ [1/rad]	-1.28	-0.95	35%
$C_{l\beta}$ [1/rad]: Dihedral Effect	-0.19	-0.19	0%
$C_{n\beta}$ [1/rad]: Directional Stability	0.31	0.17	82%
$h_n - h$: Static Stability Margin (% $\bar{c}_a$)	6.1%	8.1%	Diff.: -2.0%

Results in Table 6 and Table 7 show that, with the exception of $C_{L\alpha}$ and $C_{l\beta}$, the errors increase as much as *h/b* is increased, which can be explained with the correlation between the dynamic pressure acting on the rear wing, hence

η_R, and the vertical distance between this latter and front wing wake, which is related to *h/b*. Nevertheless, the Stability Margin is fairly and conservatively estimated, even for the higher *h/b* value. Dihedral Effect is well predicted, whereas Directional stability is again overestimated. As Figure 17 shows, the same conclusions of previous test cases can be drawn concerning accuracy errors with respect to experimental data.

Figure 17. Accuracy factors for Test Cases 2a and 2b



Test cases 3a and 3b: l/b variation

The PrP configurations generated by increasing l/b are shown in Figure 18; fuselage and the vertical tails dimension have not changed, although these latter have been moved rearwards together with the rear wing.

Figure 18. Configurations of test cases 3a and 3b (half-model, fuselage and vertical tail dimensions as in reference configuration)

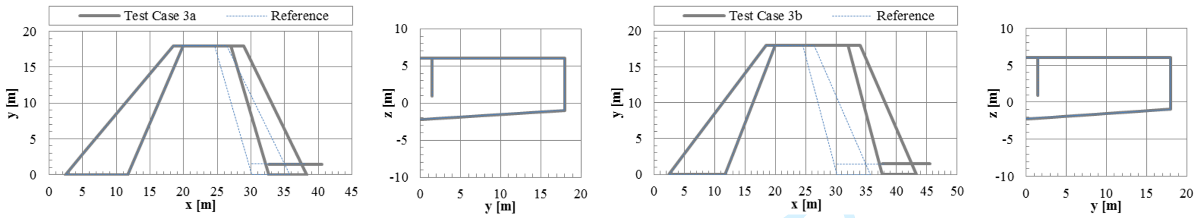


Table 8. Results of comparison for Test case 3a (+10% *l/b*)

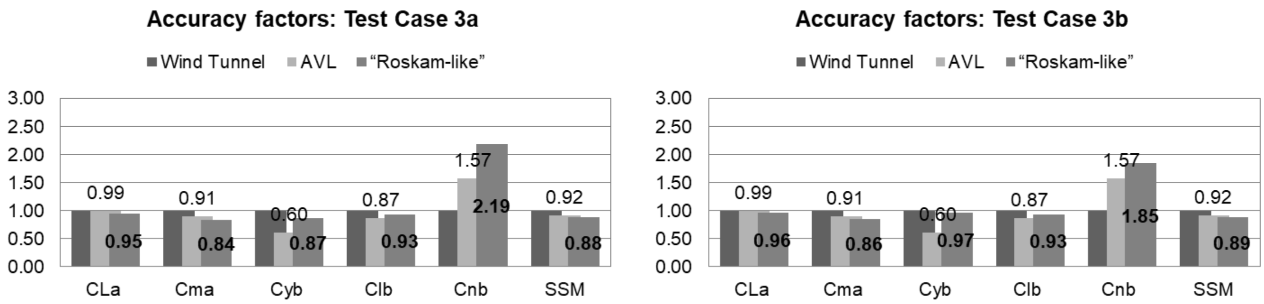
Derivatives	"Roskam"	AVL	Error (ref. AVL)
$C_{L\alpha}$ [1/rad]	6.32	6.57	-4%
$C_{m\alpha}$ [1/rad]	-1.27	-1.37	-7%
$C_{y\alpha}$ [1/rad]	-0.96	-0.67	43%
$C_{l\beta}$ [1/rad]: Dihedral Effect	-0.14	-0.13	8%
$C_{n\beta}$ [1/rad]: Directional Stability	0.25	0.18	39%
$h_n - h$: Static Stability Margin (% $\bar{c}_a$)	20.1%	20.9%	Diff.: -0.8%

Table 9. Results of comparison for Test case 3b (+30% *l/b*)

Derivatives	"Roskam"	AVL	Error (ref. AVL)
$C_{L\alpha}$ [1/rad]	6.40	6.57	-3%
$C_{m\alpha}$ [1/rad]	-3.40	-3.59	-5%
$C_{y\alpha}$ [1/rad]	-0.96	-0.60	60%
$C_{l\beta}$ [1/rad]: Dihedral Effect	-0.14	-0.13	8%
$C_{n\beta}$ [1/rad]: Directional Stability	0.27	0.23	17%
$h_n - h$: Static Stability Margin (% $\bar{c}_a$)	53.1%	54.6%	Diff.: -1.5%

Results in Table 8 and Table 9 do not allow to find a correlation between the error of “Roskam-like” model and *l/b* variation. Contrary to *h/b* and $\Lambda_{c/4}$, *l/b* affects the Dihedral Effect, although the error is quite small. The Stability Margin is well and conservatively estimated, whereas the Directional Stability prediction remain not conservative and affected by significant errors (although reduced for higher *l/b* value).

Figure 19. Accuracy factors for Test Cases 3a and 3b



Also in these test cases, the Roskam-like method accuracy is fairly good for Dihedral Effect and Stability Margin evaluation, whereas the Directional Stability is too much overestimated (Figure 19).

Conclusions and further developments

In the present paper, a method for the evaluation of α and β derivatives of subsonic airplanes characterized by a PrandtlPlane (PrP) architecture is proposed with the aim of introducing a fast preliminary design tools, useful to support, and sometimes replace, the Vortex-Lattice Methods (VLM) commonly used for preliminary simulations and optimizations.

In particular, the attention is focused on those derivatives which influence the longitudinal and lateral-directional stability of the aircraft, i.e.: $C_{L\alpha}$ and $C_{m\alpha}$, whose ratio defines the Static Margin of Stability, $C_{l\beta}$, i.e. the Dihedral Effect, $C_{l\beta}$, and the Directional Stability $C_{n\beta}$. The good accuracy of evaluating such derivatives with the VLM code AVL is presented as well, discussing the comparison with wind tunnel data from previous research on PrP aircraft.

The method here proposed is derived from the well-known models proposed by Roskam (1983) for conventional aircraft and it is presented in details in this paper, in order to show which way Roskam models have been adapted to the box-wing lifting system of the PrP.

Results obtained with both the VLM and the “Roskam-like” method have been compared for a reference PrP configuration, the same used to calibrate the method itself by means of AVL, and for a series of different configurations, generated through the variations of the following design parameters: the front and rear wing sweep angles ($\Lambda_{c/4}$), the ratio between vertical distance between wings and wingspan (*h/b*) and the ratio between horizontal distance between wings and wingspan (*l/b*).

The comparisons have shown that the "Roskam-like" model gives accurate predictions for both the Stability Margin and Dihedral Effect, whereas the Directional Stability is always overestimated. More in details:

- the error on Stability Margin evaluation is lower than 5% of the reference chord for all the configurations generated, the prediction is always conservative and the accuracy of $C_{L\alpha}$ evaluation is good as well;
- the error on Dihedral Effect, hence $C_{l\beta}$, evaluation is below 1% with the only exception of configurations generated by varying *l/b*, for which the error increases up to 8% making the prediction not conservative;
- the error on Directional Stability is for most of the cases between 40% and 80% and the prediction is not conservative for all of them.

Concerning the dependence of such results on the variations of design parameters from the reference configuration values, used to calibrate the variable dynamic pressure ratio η_R in "Roskam-like" method, for most of the cases a correlation is not evident. The main exception is the case of *h/b* whose increase generates bigger errors, since it affects directly the dynamic pressure on the rear wing, hence η_R.

In addition, for each derivative an "accuracy factor" has been introduced in order to assess of the "Roskam-like" method accuracy on a scale derived from results of a VLM with "flat" fuselage, previously validated by means of experimental data.

Both in terms of derivatives and accuracy factors, the comparison between AVL and the "Roskam-like" shows that this latter can be used in conceptual and preliminary design phases to evaluate both the Stability Margin and the Dihedral Effect of PrP configurations, since the errors introduced in derivatives evaluation are acceptably small and, even considering the variation of main design parameters, results are conservative.

This does not happen for the Directional Stability, for which neither the VLM with "flat" fuselage nor the "Roskam-like" method are accurate. Moreover all the data here presented show that the "Roskam-like" method produce not conservative results, hence more investigations on $C_{n\beta}$ evaluation are required.

Further development may be focused on refining the models adopted for vertical twin tails, e.g. introducing the effects of rear wing, and vertical tip wings, whose spanwise side force distribution is different than conventional vertical surfaces.

Acknowledgments

The present paper presents part of the activities carried out within the research project PARSIFAL ("Prandtlplane ARchitecture for the Sustainable Improvement of Future AirpLanes"), which has been funded by the European Union under the Horizon 2020 Research and Innovation Program (Grant Agreement n.723149).

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