Chapter 23

Affect and aesthetics in mathematical problem solving: New trends, methodologies, results and critical aspects

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Abstract. Since the end of the Eighties, the researchers in mathematics education stressed the need to go beyond a purely cognitive approach for the analysis of the students' performance in problem solving activities. The main reason for the emersion of this need was the difficulty in the interpretation of the failure of students who seemed to have the required cognitive resources. In this frame, the role of affective constructs (emotions, beliefs, attitudes and values) and aesthetic considerations in students' decisions during problem solving was analysed, described and stressed. In a certain sense, we can affirm that: on the one hand, the studies about problem solving are at the origins of the appearance of the specific field of research on affect; on the other hand, the development of the studies about affective constructs and aesthetics *competencies* have shed light on several phenomenon emerged in problem solving activities. So there is an indissoluble link between affect, aesthetic and problem solving. This book offers several and interesting new reflections about this close relationship.

Key words: aesthetics, affect, problem solving, mathematics education.

Research on mathematics-related affect has grown exponentially in the last decades, as showed by recently published overviews (Hannula et al., 2016; Hannula, Pantziara & Di Martino, 2018).

The beginning of the *modern* interest toward affect in mathematics education research is around the Eighties when, exactly in order to understand and interpret the students' failures in problem solving activities, different researchers underlined the need to go beyond a purely cognitive approach. The main focus is the

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failure in problem solving activities of students who seem to have the required cognitive resources.

In the volume Mathematical Problem Solving: multiple research perspectives (Silver, 1985a), Silver included affect within the issues he considered need to be addressed in the development of the research on mathematical problem solving:

> The first issue may appear to be out of place in the current climate of cognitive research on human problem solving [...] However, it is precisely because of the current climate that the issue is so important [...] Anyone who has worked with students in mathematics classes knows that affective considerations have a substantial face validity and such weak research support deserves to be studied in more detail. The time has to come to renew our efforts to uncover the affective/cognitive links in problem solving. (Silver, 1985b, p. 253-354)

In the same volume, McLeod affirmed:

> Limiting one's research perspective to the purely cognitive seems acceptable for those interested mainly in the performance of machines; however, researchers who are interested in human performance need to go beyond the purely cognitive if their theories and investigations are to be important for problem solving in mathematics classrooms. (McLeod, 1985, p. 268)

Four years later, the volume Affect and Mathematical Problem Solving (McLeod & Adams, 1989) was published. In the volume, for the first time in the field of mathematics education, the affective factors were taken into account to analyse students' behaviour and difficulties *internal* to a specific mathematical activity: problem solving. In some sense, the publication of Affect and Mathematical Problem Solving represented the beginning of a new era in mathematics education, despite that – as Mayer (1990, p. 36) claims – it "raises interesting questions but provides few answers".

Our community has come a long way since the end of the Eighties: different answers have been found to the questions posed in Affect and Mathematical Problem Solving and new challenges in the field of affect are emerged. The present volume is proof of that: new relevant issues, approaches, methodologies and results concerning affect and problem solving are presented and described (see Table 23.1).

Table 23.1 Synthesis of chapters in section 3: affect and aesthetics in mathematical problem solving

Chapter	Focuses	Sample	Methodology
17	Attitude, mathematical competition, problem solving	Grade 5, 6, 7, 8 students	Mixed: interviews, questionnaires, e-mails,
18	Aesthetic, affect, problem solving	Graduate students	Qualitative: interviews
19	Aesthetic, problem solving	Primary pre-service teachers	Qualitative: journals
20	Emotions, mathematical competition, prob-	Students of the third cycle of Basic Edu-	Mixed: students' mathematical products, journals, Facebook posts, online

	lem solving	cation	survey
21	Engagement, problem solving	Grade 11 and Grade 12 students	Qualitative: direct observation
22	Emotions, technology, problem solving	Grade 9 students	Qualitative: video recorded data, interviews

I will try to discuss briefly and schematically all these aspects, also highlighting some theoretical and methodological critical issues that arise.

Two of the six chapters of the section affect and aesthetics in mathematical problem solving concern aesthetic issues. They offer a really interesting picture of the related research, discussing the role of the aesthetic in mathematics education and the relationship between aesthetic and affect.

Concerning this latter issue, essentially we can recognize two theoretical approaches:

1. Aesthetic is seen as a part of affect. According to Goldin (2000), affect has a tetrahedral nature, it includes the three dimensions (emotions, beliefs and attitudes) considered by McLeod in the very famous 1992 Handbook (McLeod, 1992) and a fourth dimension related to values. In this theoretical model, aesthetic falls in this fourth dimension;
2. Aesthetic and affect are closely intertwined but different domains with different functions. According to Sinclair (2008), the aesthetics functions as a non-logical form of knowing, drawing the attention of the perceiver to a phenomenon, while the affective can active the awareness of these perceptions.

Sinclair and Rouleau stress another dichotomy in the research on aesthetic: aesthetic can be seen as an objective and absolute judgment reflecting the view of a cultural elite, or researchers can assume a contextual and *democratic* view of the aesthetic. This latter approach appears to be more significant for mathematics education: the classroom has to be the environment where the judgment (also the aesthetic one) are negotiated and shared.

The two chapters discuss the role of aesthetic in the problem solving process, starting from the role of aesthetic in the work of research mathematicians. This approach is complex because – as Sinclair and Rouleau underline – few mathematicians write about their problem solving processes, and still fewer explicit aesthetic (or affective) considerations.

This interesting approach is based on an *epistemological* assumption: if something is relevant in the work of professional mathematicians then it can (have to?) be relevant in mathematics education. This assumption has also driven relevant research in the field of affect: for example Liljedahl research about the Aha! experience (Liljedahl, 2008).

Anyway, the researcher needs to explicit why (and subsequently how) they suppose to use their knowledge of the mathematicians' problem solving experiences for promoting aesthetic and affective responses in the mathematics classroom. Concerning aesthetic, the mathematicians' experience reveals the connection between aesthetic considerations and decisions or evaluations. Since decisions and evaluations are crucial in the problem solving process, and the development of the problem solving competence is a crucial goal of mathematics instruction, it appears relevant to promote aesthetic responses in the teaching of mathematics. Moreover, to promote aesthetic experiences can be a teaching strategy in repairing a negative relationship with mathematics.

The attention on aesthetic issues in mathematics education is therefore fully justified. How to promote aesthetic responses in the students' mathematical experiences is still an outstanding issue.

Presmeg and Sinclair & Rouleau discuss some possibilities to promote aesthetic responses in problem solving activities (both with students and with pre-service teachers). All the authors stress how the level of difficulty of the problem is a key-factor: it is crucial that the problem appears neither too easy nor too hard.

This observation appears to be strictly related to the idea of flow introduced by Liljedahl in his chapter. According to the theory developed by Csíkszentmihályi (1990), flow is a state in which people are so involved in an activity that nothing else seems to matter. Using Peter's words, flow is one of the only ways for mathematics education researchers to talk productively about engagement.

Flow is possible only when there is a balance between challenge and individual's ability. Therefore, the theory of flow is useful for interpreting student reactions when faced with an imbalance between challenge and their skill.

Liljedahl's approach is particularly interesting because it is not only theoretical but also directly linked to practice. In particular, he discusses the flow is facilitated by specific problem solving settings, for example when a classroom is conducted according to the Building Thinking Classrooms framework of teaching (Liljedahl, 2016).

The theory of flow is particularly stimulating because it describes the balance between challenge and ability as a dynamic process where teachers play a crucial role. As a matter of fact, through appropriate feedbacks, teachers have to manage two different *dangerous* situations: when challenge exceeds skills (frustration) and when skills exceed challenge (boredom).

In this volume, also the chapter developed by Daher, Swidan and Masarwa is focused on the dynamics aspects of affect. In particular, scholars compare positioning and emotions in learning algebra with technology of high and middle achieving students.

This kind of studies is particularly relevant but also complex from a methodological point of view.

They are relevant because on the one hand studies that focus on the dynamics of affective states in classroom are still rare (Hannula, 2012); on the other hand, further evidence of the dynamic progression of motivational and emotional states through the problem solving process is needed (Lewis, 2017).

They are complex because the data are collected through direct observation of students during problem solving activities, and direct observation is confronted with the *issue of circularity*. Lester was the first to discuss this issue in relation to the research on beliefs, but his considerations are easily generalizable to other affective constructs:

> For researchers to claim that students behave in a particular manner because of their beliefs and then infer the students' beliefs from how they behave involves circular reasoning. *The reasoning goes something like this: Question: How do you know that students' beliefs infuence how they do mathematics? Answer: Because in our study students did mathematics in a certain way. Question: But how do you know that the students'beliefs contributed to this behavior? Answer: Because they would not have behaved this way if they did not hold these beliefs*. (Lester, 2002, p. 348)

We have discussed the issue of circularity related to methodologies based on direct observation, but more in general, for their nature, it is difficult to infer affective states or traits. If it is true that affective constructs are not directly observable, it is also true that individuals themselves are often not conscious of these processes (Hannula, Pantziara & Di Martino, 2018): studies based on self-report are not however without their methodological critical aspects.

Chapter 17 and chapter 20 of this book describe studies based on different kinds of self-report: some of these self-report (for example the students' posts on Facebook) are particularly original and I believe they deserve a special attention for the future of the interpretative research about affect.

The two studies are focused on the potential of mathematical problem solving competitions for promoting positive emotions and attitudes towards mathematics. The main goal of these competitions is affective: they are seen as way of motivating participants to engage with mathematics and therefore of developing appreciation for mathematics.

As already mentioned, these studies are based on the collection of self-reports before, during and after the competition.

On the one hand, these studies are interesting because they monitor the students' evolution in the period. On the other hand, the comparison between initial and final self-report – particularly relevant in order to monitor the evolution of attitude towards mathematics – has a clear problematic: typically the final self-report is filled by a minority of participants and this minority is likely representative of those for whom the competition was a good experience.

Methodological issues a part, these competitions certainly have interesting features. They are inclusive: the students' participation is voluntary and the problems have to be intended for all. But they are also detached by the official curriculum: there is more choice for the selection of the problems. In this frame, it is easier to develop realistic mathematically rich problems having more connections with other areas.

These features certainly can contribute to develop a positive attitude towards this kind of mathematical competition. It remains unexplored and problematic if and how these experiences (out of school) can have effects on the students' attitude towards mathematics at school.

In conclusion, the six chapters of the section affect and aesthetics in mathematical problem solving of the present volume offer an interesting and diverse contribute. They deal with significant issues, offering new theoretical approaches and new methodologies, but also highlighting new critical aspects in the research about affect, aesthetic and problem solving.

I want to replicate the hope that Silver stated in his introduction to the volume *Teaching and Learning Mathematical Problem Solving: multiple research perspectives* (Silver, 1985a, p. ix): "If this volume leads to an increased understanding of the nature of past problem-solving research, or an increased appreciation of the potential benefits of cross-fertilization of ideas among workers in different fields, or a renewed enthusiasm for attacking some of the underrepresented themes and issues from previous research, then it will have served its purpose".

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