Approximate Solutions to Circle-to-Circle Solar Sail Orbit Transfer

Alessandro A. Quarta* and Giovanni Mengali†

University of Pisa, I-56122 Pisa, Italy

Introduction

Solar sailing is known to be a feasible solution for deep space missions requiring extremely high values of Δv. As such, it represents a valid option for heliocentric trajectories involving significant variations of orbital inclination, in particular for missions towards the inner region of the Solar System [1, 2]. Among the scenarios envisaging a substantial plane change, those involving a heliocentric transfer between circular orbits of different radii have stimulated different research studies [3, 4].

In principle, the mission analysis does not constitute a challenge in this case, as it may be reduced to a classical trajectory optimization problem [5, 6]. However, especially for solar sails of current or next generation with low-medium performance [7–9], the optimal trajectory design often requires a significant amount of simulation time. This is mainly due to the long transfer times, on the order of some years, and to the trajectory complexity, which is characterized by a number of revolutions around the Sun and by a continuous variation of the sail control parameters [10].

For these reasons, different mathematical models [4, 11, 12] have been developed to give, with a reduced amount of simulation time, not only an estimate of the main mission performances, but also accurate information about the structure of the optimal trajectory in a

*Research Assistant, Department of Aerospace Engineering, a.quarta@ing.unipi.it. Senior Member AIAA.

†Associate Professor, Department of Aerospace Engineering, g.mengali@ing.unipi.it. Senior Member AIAA.

three-dimensional circle-to-circle (direct) orbit transfer. This is exactly the context within which the contribution of this Note is inserted. More precisely, the aim of this work is to reappraise the analytical model originally proposed by Wiesel and Alfano [13] for a spacecraft equipped with a low-performance solar electric propulsion system, and to apply it, in a similar mission scenario, to a heliocentric transfer trajectory tracked by a low-performance, ideal, flat solar sail. In this sense, the obtained results extend to a three-dimensional case the semi-analytical model recently discussed in Ref. [14].

Simplified Dynamical Model

Consider a flat, ideal (that is, perfectly reflecting) solar sail that, at the initial time $t_0 \triangleq 0$, covers a circular heliocentric orbit of semimajor axis a_0 and inclination i_0 with respect to the ecliptic plane. The mission aim is to reach a circular heliocentric orbit with prescribed values of semimajor axis a_f and inclination i_f.

Assume that, during the whole transfer, the eccentricity e of the osculating orbit is sufficiently small, such that its value may be neglected within the spacecraft's equations of motions. This situation is representative, for example, of a solar sail of first generation [7, 15, 16], whose performance in terms of characteristic acceleration a_c is sufficiently small. Recall that a_c is defined [17] as the maximum solar sail propulsive acceleration when the Sun-spacecraft distance is $r_\oplus \triangleq 1\,\mathrm{AU}$. Note that the assumption of negligible eccentricity along the whole transfer is in accordance with the model proposed by Wiesel and Alfano [13].

The variations of a and i with the mean anomaly M are described by the following simplified Lagrange's variational equations [17]

$$\frac{\mathrm{d}a}{\mathrm{d}M} = 2\,\beta\,a\,\cos^2\alpha\,\sin\alpha\,\cos\delta \tag{1}$$

$$\frac{\mathrm{d}i}{\mathrm{d}M} = \beta\,\cos^2\alpha\,\sin\alpha\,\cos M\,\sin\delta \tag{2}$$

where $\beta \triangleq a_c/(\mu_\odot/r_\oplus^2)$ is the sail lightness number [17], which coincides with the ratio of the maximum propulsive acceleration to the (local) solar gravitational acceleration. Because the parking orbit is circular, the mean anomaly M is measured counterclockwise from the Sun-spacecraft line at the beginning of the transfer, that is, $M(t_0) = 0$.

In Eqs. (1)-(2), the term $\alpha \in [0, \pi/2]$ is the sail cone angle, that is, the angle between the Sun-spacecraft line and the direction of the propulsive acceleration vector, whereas $\delta \in [0, 2\pi]$ is the sail clock angle, see Fig. 1. The latter is measured counterclockwise from the transverse direction of a heliocentric Radial-Transverse-Normal $\mathcal{T}_{\text{RTN}}$ reference frame [18], whose unit vectors are $\hat{\boldsymbol{i}}_{\text{R}}$, $\hat{\boldsymbol{i}}_{\text{T}}$, and $\hat{\boldsymbol{i}}_{\text{N}}$. In particular, a clock angle $\delta = \{0, \pi\}$ corresponds to a propulsive acceleration vector belonging to the spacecraft's osculating orbital plane and, as such, it is unable to vary the orbital inclination.

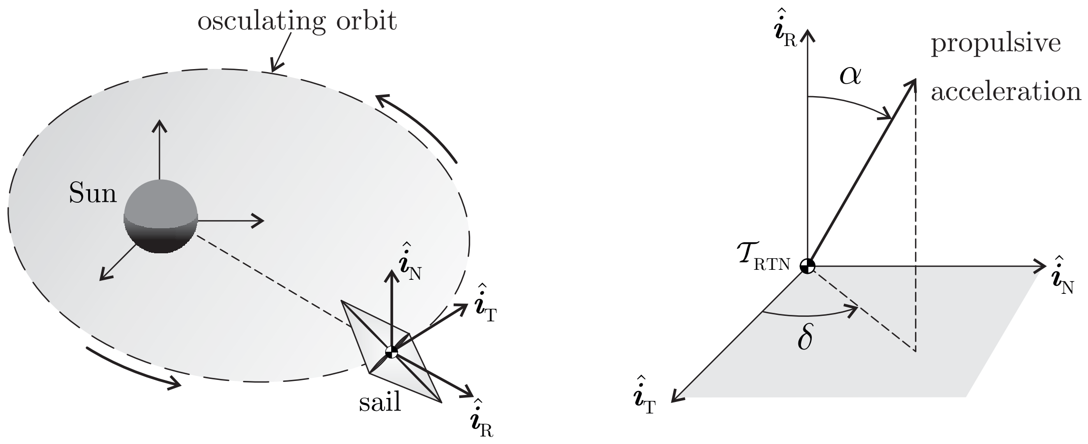

Figure 1: Reference frame and solar sail control angles (α, δ)**.**

The spacecraft motion is studied within an optimal framework, that is, by minimizing the mean anomaly $M_f \triangleq M(t_f)$ at the final (given) transfer time t_f. This corresponds to maximizing the performance index $J \triangleq -M_f$. As will be discussed next, under some suitable assumptions the trajectory that minimizes M_f (or maximizes J) is a good approximation of the minimum-time transfer trajectory.

Using an indirect approach, consider the variables adjoint to the semimajor axis λ_a, and to the inclination λ_i, and introduce the Hamiltonian function of the problem

$$\mathcal{H} \triangleq 2\,\lambda_a\,\beta\,a\,\cos^2\alpha\,\sin\alpha\,\cos\delta + \lambda_i\,\beta\,\cos^2\alpha\,\sin\alpha\,\cos M\,\sin\delta \tag{3}$$

which defines the two Euler-Lagrange equations [19]

$$\frac{\mathrm{d}\lambda_i}{\mathrm{d}M} \triangleq -\frac{\partial\mathcal{H}}{\partial i} = 0 \qquad , \qquad \frac{\mathrm{d}\lambda_a}{\mathrm{d}M} \triangleq -\frac{\partial\mathcal{H}}{\partial a} = -2\,\lambda_a\,\beta\,\cos^2\alpha\,\sin\alpha\,\cos\delta \tag{4}$$

Because $\mathcal{H}$ is independent of the orbital inclination, the adjoint variable λ_i is a constant of motion. A second integral of motion is obtained when Eq. (1) is substituted into the second of (4). Indeed, with simple calculations, it may be verified that

$$a\,\lambda_a = a_0\,\lambda_{a0} \tag{5}$$

where λ_{a0} is the (unknown) initial value of λ_a. According to Eq. (5), the adjoint variable λ_a is obtained as a function of a provided that its initial value λ_{a0} is given. This result avoids the need of a numerical integration of the second of the Euler-Lagrange equations (4). Moreover, because $a_0 > 0$, the sign of the product $a\,\lambda_a$ is a function of the sign of λ_{a0} only, whose value is an output of the boundary value problem associated to the optimal problem. Note that the special situation in which $\lambda_{a0} = 0$ corresponds to having $\lambda_a = 0$ along the whole trajectory, see Eq. (5).

According to Pontryagin's maximum principle, the optimal sail cone angle α is given by the constant value $\alpha = \alpha^\star \triangleq \arctan(1/\sqrt{2})$. The latter coincides with the well known value of α that maximizes the transverse component of the local propulsive acceleration. The value $\alpha = \alpha^\star$ is often used for local optimization trajectories [4, 11, 17]. As far as the optimal sail clock angle is concerned, the maximization of $\mathcal{H}$ with respect to δ provides $\delta = \delta^\star$, with

$$\sin\delta^\star \triangleq \frac{\operatorname{sign}(\lambda_i)\,\cos M}{\sqrt{k^2+\cos^2 M}} \qquad , \qquad \cos\delta^\star \triangleq \frac{\operatorname{sign}(\lambda_{a_0})\,|k|}{\sqrt{k^2+\cos^2 M}} \tag{6}$$

where $\operatorname{sign}(\cdot)$ is the signum function (with $\operatorname{sign}(0) \triangleq 0$) and k is a dimensionless constant, whose value depends on the parking orbit characteristics and the initial value of adjoint variables according to the relationship

$$k \triangleq \frac{2\,a_0\,\lambda_{a_0}}{\lambda_i} \tag{7}$$

Equation (7) states that k takes a finite value provided that $\lambda_i \neq 0$. The noteworthy case in which $\lambda_i = 0$ will be discussed next, along with the other special case $\lambda_{a_0} = 0$.

Substituting the optimal values $\alpha = \alpha^\star$ and $\delta = \delta^\star$ into Eqs. (1)-(2) and (3), the following three first order differential equations are obtained:

$$\frac{\mathrm{d}}{\mathrm{d}M}\left(\frac{\ln a}{\widetilde{\beta}}\right) = \frac{2\,|k|\operatorname{sign}(\lambda_{a_0})}{\sqrt{k^2+\cos^2 M}} \tag{8}$$

$$\frac{\mathrm{d}}{\mathrm{d}M}\left(\frac{i}{\widetilde{\beta}}\right) = \frac{\operatorname{sign}(\lambda_i)\,\cos^2 M}{\sqrt{k^2+\cos^2 M}} \tag{9}$$

$$\mathcal{H} = \widetilde{\beta}\,|\lambda_i|\,\sqrt{k^2+\cos^2 M} \tag{10}$$

where k is given by Eq. (7), whereas $\widetilde{\beta} \triangleq 2\,\beta/(3\sqrt{3})$ represents a sort of modified sail lightness number that contains the optimal value of the sail cone angle (namely $\alpha = \alpha^\star$).

Because $\widetilde{\beta} > 0$, from Eq. (8) it is found that a positive (negative) value of λ_{a_0} corresponds to an increase (decrease) of the semimajor axis a of the osculating orbit as a function of the mean anomaly M, which, in turn, implies an orbit raising (lowering). On the other hand, Eq. (9) shows that a positive (negative) value of λ_i corresponds to an increase (decrease) of the orbital inclination when M is increased. Therefore, when the characteristics of both the parking and the final (target) orbit are given, that is, $(a_0,\, i_0)$ and $(a_f,\, i_f)$ are known, the sign of the two adjoint variables is simply given by

$$\operatorname{sign}(\lambda_{a_0}) = \operatorname{sign}(a_f - a_0) \qquad , \qquad \operatorname{sign}(\lambda_i) = \operatorname{sign}(i_f - i_0) \tag{11}$$

The previous equations may be substituted into Eqs. (8)-(9). An interesting result is that the optimal transfer trajectory, in the space of the two dependent variables $\ln a/\widetilde{\beta}$ and $i/\widetilde{\beta}$, is described by two uncoupled differential equations in the independent variable M. More precisely, for a given sail lightness number β and a quadruple (a_0, a_f, i_0, i_f), Eqs. (8)-(9) and (11) provide the two following integral relationships

$$\frac{|\ln(a_f/a_0)|}{\widetilde{\beta}} = f(k, M_f) \triangleq 2\,|k| \int_0^{M_f} \frac{\mathrm{d}M}{\sqrt{k^2 + \cos^2 M}} \tag{12}$$

$$\frac{|i_f - i_0|}{\widetilde{\beta}} = g(k, M_f) \triangleq \int_0^{M_f} \frac{\cos^2 M}{\sqrt{k^2 + \cos^2 M}}\,\mathrm{d}M \tag{13}$$

where it is implicitly assumed that $a_f \neq a_0$ and $i_f \neq i_0$. The special cases in which either $a_f = a_0$ or $i_f = i_0$, are strictly related to the situations $\lambda_{a_0} = 0$ or $\lambda_i = 0$. However these special cases cannot take place simultaneously otherwise the problem solution would be trivially $M_f = 0$, which means that the parking orbit coincides with the target orbit.

Note that the integrals in the right sides of Eqs. (12) and (13) may be rewritten as a function of elliptic integrals of first and second kind. However, from a practical viewpoint such a possibility does not change the fact that a numerical scheme is anyway necessary to calculate their value. For this reason those integrals are left in their original form, on the understanding that, for a given pair (k, M_f), the nonnegative functions f and g are calculated with one of the quadrature algorithms available in the literature. For example, assuming

$M_f \in [1, 25]$ revs and $|k| \in [0.01, 10]$ and using an adaptive Gauss/Lobatto quadrature method [20] with an absolute error tolerance of 10^{-8}, the isocontour lines for f and g are shown in Fig. 2.

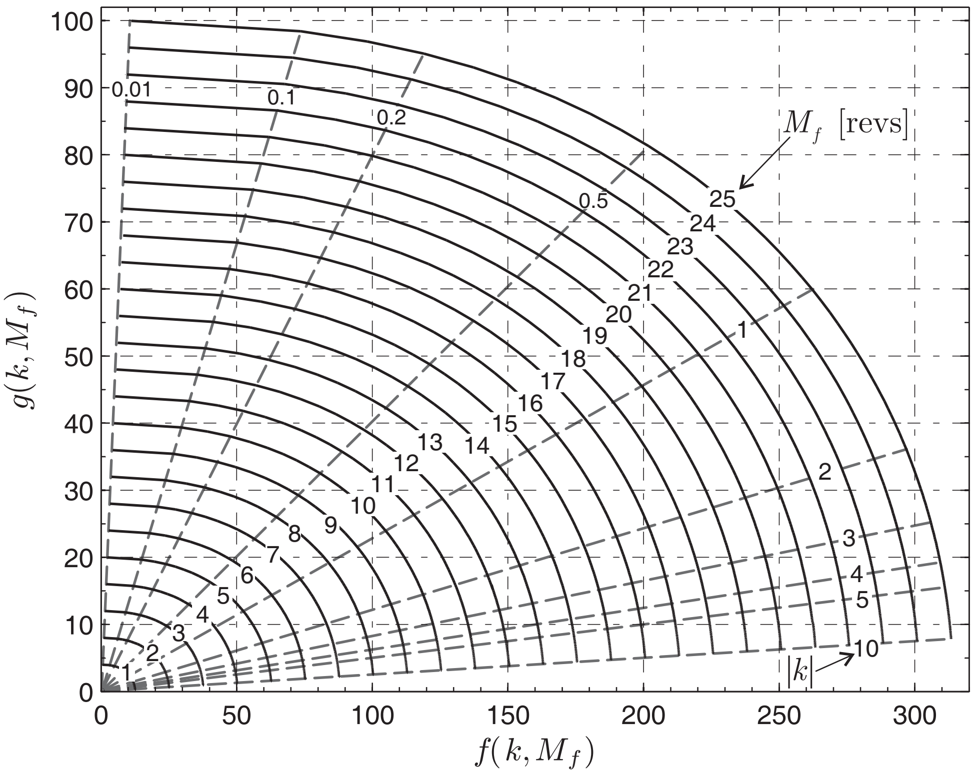

Figure 2: Contour line of f and g as a function of $|k|$ and M_f, see Eqs. (8)-(9).

The minimum value of M_f, and the corresponding value of k, are obtained by solving a nonlinear algebraic system constituted by Eqs. (12)-(13), whose numerical solution is straightforward. A first estimate of the unknowns k and M_f, useful for initializing the numerical procedure, is provided by the data summarized in Fig. 2. When the pair (k, M_f) has been found, the adjoint variable λ_i is obtained by introducing the transversality condition $\mathcal{H}_f = 1$ into Eq. (10). Finally, Eqs. (7) and (11) provide the value of λ_{a_0} that completes the solution of the optimal problem.

The semimajor axis of the osculating orbit varies with the mean anomaly according to

Eqs. (8) and (11), viz.

$$a = a_0 \exp\left[\widetilde{\beta}\,\mathrm{sign}\,(a_f - a_0)\; f(k, M)\right] \qquad \text{with} \qquad M \in [0,\, M_f] \tag{14}$$

Using this last equation and the fact that $\mathrm{d}M/\mathrm{d}t = \sqrt{\mu_\odot/a^3}$, where $\mu_\odot$ is the Sun's gravitational parameter, the total flight time t_f is calculated through a quadrature algorithms [20] using the equation:

$$t_f = \sqrt{\frac{a_0^3}{\mu_\odot}} \int_0^{M_f} \exp\left[\frac{3\,\widetilde{\beta}\,\mathrm{sign}\,(a_f - a_0)\; f(k, M)}{2}\right] \mathrm{d}M \tag{15}$$

If $a_f = a_0$ (which corresponds to a simple orbital cranking) the previous relationship for the flight time t_f must be changed, as is now discussed.

The special cases $\lambda_{a_0} = 0$ and $\lambda_i = 0$ deserve a specific discussion. Assume first that $\lambda_{a_0} = 0$ and $\lambda_i \neq 0$, and consider the spacecraft motion in the plane (a, i). Because $\widetilde{\beta}$ is constant and different from zero, Eq. (8) states that a remains constant independent of M. Observing that in general $\mathrm{d}i/\mathrm{d}M \neq 0$, the case $\lambda_{a_0} = 0$ corresponds to a pure cranking maneuver in which the solar sail only changes its orbital inclination starting from a circular orbit of radius a_0. The sail moves along a range of circular orbits [1,3] whose inclination varies progressively until the final inclination i_f is reached. The optimal control law for the sail clock angle (6) states that δ is piecewise constant with M, and it changes its value twice per full revolution around the Sun. Indeed, if $k = 0$, Eq. (6) states that $\delta^\star = \pi/2$ (or $\delta^\star = 3\pi/2$) when $\mathrm{sign}\,(\lambda_i \cos M) = 1$ (or $\mathrm{sign}\,(\lambda_i \cos M) = -1$). Such an optimal control law coincides with the law that locally optimizes the variation of orbital inclination, in accordance with Refs. [11, 17]. In this case, from Eq. (7) it is found that $k = 0$, while Eq. (13) reduces to

$$\frac{i_f - i_0}{\widetilde{\beta}\,\mathrm{sign}\,(i_f - i_0)} = \int_0^{M_f} |\cos M|\, \mathrm{d}M \tag{16}$$

whose integration is a simple matter. Once the value of M_f that solves Eq. (16) is found, the flight time may be expressed as

$$t_f = \sqrt{\frac{a_0^3}{\mu_\odot}}\, M_f \tag{17}$$

which replaces Eq. (15) when $a_f = a_0$.

The second special case is $\lambda_i = 0$ and $\lambda_{a_0} \neq 0$, that is, according to Eq. (7), when $|k| \to \infty$. In this case Eq. (9) states that the orbital inclination i is a constant of motion and, therefore, $i_f = i_0$. Indeed, the optimal control law shows that $\sin\delta^\star = 0$, see Eq. (6), which defines a two-dimensional transfer trajectory. In the latter case the right hand side of Eq. (1) becomes $2\,\mathrm{sign}\,(\lambda_{a_0}) \equiv 2\,\mathrm{sign}\,(a_f - a_0)$, and the semimajor axis of the osculating orbit varies according the law

$$a = a_0\, \exp\left[2\,\widetilde{\beta}\,\mathrm{sign}\,(a_f - a_0)\; M\right] \tag{18}$$

from which the minimum value of the final mean anomaly is found to be

$$M_f = \frac{\ln\,(a_f/a_0)}{2\,\widetilde{\beta}\,\mathrm{sign}\,(a_f - a_0)} \tag{19}$$

while Eq. (15) provides the total flight time as

$$t_f = \sqrt{\frac{a_0^3}{\mu_\odot}}\,\frac{(a_f/a_0)^{3/2} - 1}{3\,\widetilde{\beta}\,\mathrm{sign}\,(a_f - a_0)} \tag{20}$$

Equation (20) coincides with the relationship found in Ref. [14] using a different approach and, more important, by minimizing the total flight time t_f.

Results and Discussion

It might be argued that the problem of minimizing M_f is of limited practical importance because the typical performance index for a solar sail based mission is the total flight time [17]. However, under some additional assumptions (besides the zero eccentricity of the osculating orbit) the transfer trajectories that minimize the final mean anomaly M_f are nearly coincident with the minimum-time transfer trajectories. As stated in the previous section this is true, for example, when the spacecraft performs a pure cranking maneuver ($\lambda_{a_0} = 0$), or when it executes a circle-to-circle two-dimensional transfer ($\lambda_i = 0$). Indeed, for the pure cranking maneuver it may be verified that Eqs. (16)-(17) give the same results of those discussed by McInnes [17] for locally optimal trajectories. On the other hand, as far as the circle-to-circle two-dimensional transfer is concerned, it has been already noted that Eq. (20) coincides with the approximate expression for the minimum flight time discussed in Ref. [14].

More involved is the analysis of a situation in which the spacecraft must vary both the semimajor axis and the orbital inclination. In this case the effectiveness of estimating the minimum flight time using the flight time corresponding to the minimum value of the mean anomaly must be validated by numerical simulations. A comparison between the two flight times has been performed using an optimization program, based on modified equinoctial parameters, whose mathematical model is described in Ref. [21]. Extensive simulations have shown that for characteristic accelerations $a_c \leq 0.5\,\mathrm{mm/s^2}$ and variations of orbital inclination $\Delta i \triangleq |i_f - i_0| \leq 25\,\mathrm{deg}$, the difference in terms of flight time between the optimal value and the approximate value obtained using the simplified procedure discussed in this Note is less than 5%.

For example, consider a parking orbit along the ecliptic plane ($i_0 = 0$) with a radius $a_0 = 1\,\mathrm{AU}$, and a target orbit with inclination $i_f = 15\,\mathrm{deg}$ and radius $a_f = 0.48\,\mathrm{AU}$. Assuming an ideal solar sail with a characteristic acceleration $a_c = 0.5\,\mathrm{mm/s^2}$ (that is, $\widetilde{\beta} = 3.24531 \times 10^{-2}$) and using the previous simplified model, the flight time obtained by means of the simplified model is $t_f \simeq 571.2$ days. The same mission, when analyzed using an optimization software that considers the actual spacecraft's equations of motion, may be completed with a minimum flight time of about 577.5 days and 2.64 revolutions around the Sun. The difference between the two transfer times is therefore about 1% only.

For this example the variation of i is illustrated in Fig. 3 as a function of a. According to the simplified model, the orbital inclination varies nearly linearly with the semimajor axis, while the (truly) optimal variation shows a marked increase of inclination when the spacecraft reaches its minimum distance from the Sun (in this example equal to 0.48 AU).

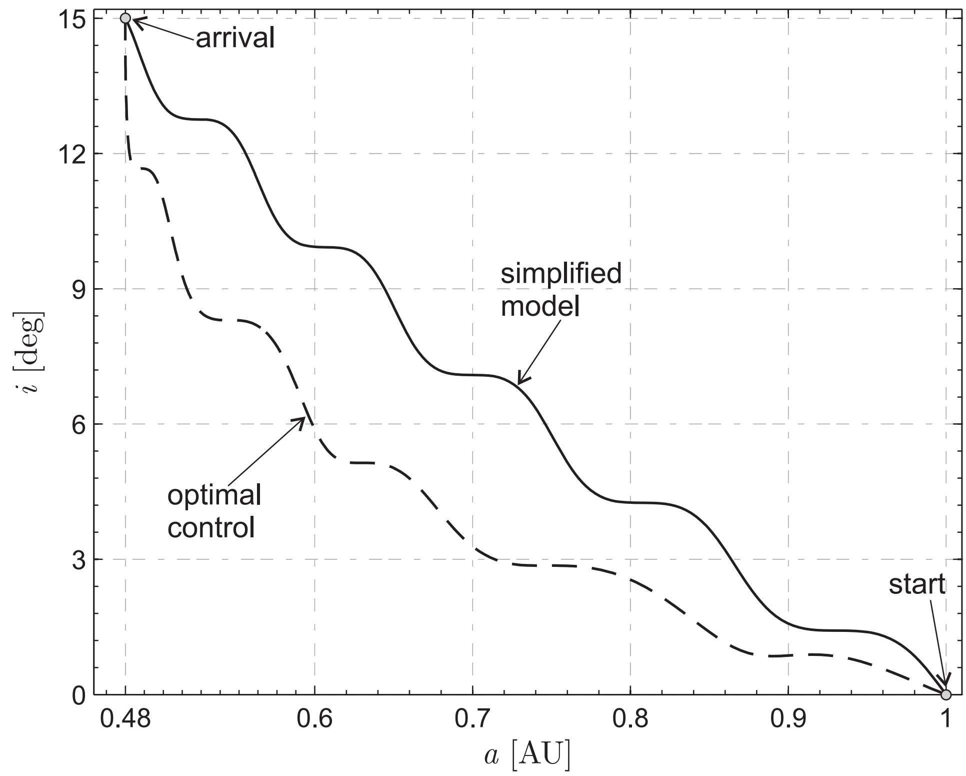

Figure 3: Orbital inclination vs. semimajor axis when $a_c = 0.5\,\mathrm{mm/s^2}$.

To summarize, while in the simplified mathematical model the mean rate of change of

i with respect to a is roughly constant during the transfer, in a truly optimal trajectory a marked variation in orbital inclination takes place in the closeness of the transfer orbit perihelion. Such a peculiarity of the truly optimal trajectory answers for the differences between the simplified model and the optimal model when the mission requires a substantial variation of both orbital inclination and semimajor axis.

An exemplary case is constituted by the Solar Polar mission, whose primary goal is a scientific analysis of the solar polar regions [2] using a circular target orbit with radius 0.48 AU and an inclination greater than 75 deg with respect to the ecliptic plane. The lack of accuracy provided by the proposed simplified model is related to the particular structure of the optimal transfer orbit. Indeed, as shown by Sauer [1], assuming an orbit lowering with $a_f = 0.48\,\mathrm{AU}$ and a variation of orbital inclination of about ninety degrees, the optimal trajectory for an ideal solar sail with a characteristic acceleration $a_c = 0.5\,\mathrm{mm/s^2}$ may be virtually divided into two phases.

A first phase, whose length is about 578 days, in which the spacecraft uses the propulsive thrust mainly for reducing the semimajor axis until a nearly circular orbit is reached with a radius of 0.48 AU and an inclination of about 15 deg with respect to the parking orbit. In a second phase, with a length of about 1212 days, the orbital inclination is varied by maintaining a nearly constant Sun-spacecraft distance at about 0.48 AU from the star.

When the approximate model developed in this Note is applied to the case of Solar Polar mission (assume $a_0 = 1\,\mathrm{AU}$, $a_f = 0.48\,\mathrm{AU}$, $i_0 = 0$, $i_f = 90\,\mathrm{deg}$, and $a_c = 0.5\,\mathrm{mm/s^2}$ to obtain results comparable with those by Sauer [1]), it is found that the total flight time is $t_f \simeq 2676$ days. However, according to Ref. [1], the same mission may be fulfilled with a minimum total flight time of 1790 days, and the corresponding error of the simplified model exceeds therefore 50%.

Nevertheless, even within such an unfavorable mission scenario, it will be shown now that it is still possible to use the previous simplified method by introducing suitable corrections suggested by the particular structure of the optimal solution. Actually if the mission is virtually divided into two parts, that is, an orbit lowering with a variation of inclination of 15 deg followed by a pure orbit cranking of 75 deg, the simplified model can be successfully employed. Indeed, as far as the orbit lowering phase is concerned, the flight time estimated with the simplified model is about 577.5 days (a value nearly coincident with that obtained in Ref. [1]). For the cranking phase, assuming that it is performed at a distance of 0.48 AU from the Sun and using Eq. (16), the final mean anomaly is $M_f = 3619.5$ deg (about 10.05 revs). This value, when substituted into Eq. (17), estimates a cranking time of about 1221.3 days. With this simplified approach the total mission time is therefore 1798.8 days, which is nearly coincident with that found by Sauer [1].

The latter result implies an a priori knowledge of the structure of the optimal transfer trajectory, which in turn corresponds to the knowledge of the variation of orbital inclination when the spacecraft approaches the perihelion distance. In the previous example the optimal variation $\widetilde{\Delta i} = 15$ deg was known in advance due to the results of Ref. [1]. However, even if the optimal value of $\widetilde{\Delta i}$ were not known, the previous approximate model could be still employed through a parametric approach in which the total flight time vs $\widetilde{\Delta i}$ is calculated by points. To better illustrate this algorithm, consider again the previous Solar Polar mission assuming now that the optimal value of $\widetilde{\Delta i}$ is not a priori known. Because $\widetilde{\Delta i} \in [0,\, 90]$ deg, consider the set of equispaced values $\mathcal{S} = \{0, 90/n, 2 \cdot 90/n, 3 \cdot 90/n, \ldots, 90\}$ deg, where n is a given integer number. For each value of $\widetilde{\Delta i} \in \mathcal{S}$, the total flight time may be calculated as the sum of the time necessary to move the spacecraft from $\{a = a_0,\, i = 0\}$ to $\{a = a_f,\, i = \widetilde{\Delta i}\}$, and the time required to perform the (pure) cranking phase from $\{a = a_f,\, i = \widetilde{\Delta i}\}$ to $\{a = a_f,\, i = i_f\}$. Repeating the same procedure for all values of the set $\mathcal{S}$ results in the curve illustrated in Fig. 4, which confirms that the minimum total mission time corresponds to about $\widetilde{\Delta i} \simeq 15\,\text{deg}$. More precisely, the value obtained through the simplified model is $\widetilde{\Delta i} \simeq 11\,\text{deg}$, with a flight time of 1787 days. Notably, Fig. 4 is generated with a minimum simulation effort, much less than that required to solve the full optimal control problem.

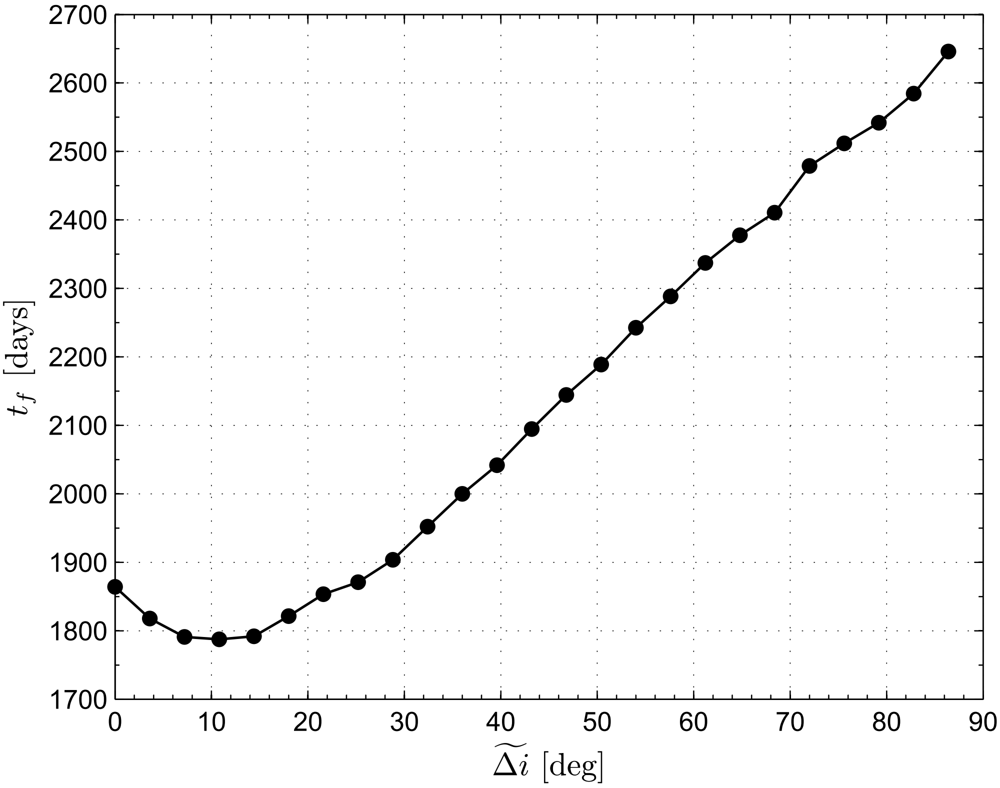

Figure 4: Total flight time as a function of $\widetilde{\Delta i}$ for the Solar Polar mission ($a_f = 0.48\,\text{AU}$, $i_f = 90\,\text{deg}$ and $a_c = 0.5\,\text{mm/s}^2$).

Conclusions

The problem of trajectory optimization for a circle-to-circle orbit transfer with plane change using a low-performance solar sail is a rather complex task, which usually requires a significant amount of simulation time. The methodology developed in this Note guarantees a quick estimate, with a precision level consistent with a preliminary mission analysis, of the optimal performance of an ideal solar sail.

In particular, the proposed mathematical model provides a good approximation of the minimum flight time when the variation of orbital inclination is on the order of a few ten degrees. In addition, the same mathematical model may also be adapted, with minor changes, to study mission scenarios involving high variations of orbital inclination. In this sense, the proposed model is therefore useful also when high energy missions are considered.

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