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## **“THINK ABOUT YOUR MATH TEACHERS”**

### **A NARRATIVE BRIDGE BETWEEN FUTURE PRIMARY TEACHERS’ IDENTITY AND THEIR SCHOOL EXPERIENCE**

Pietro Di Martino\* & Silvia Funghi\*\*

\* Pisa University \*\* Modena e Reggio Emilia University

*Pre-service teachers approach their professional learning in mathematics with a complex set of needs and wants. These needs and wants are strongly affected by the tension deriving from the realisation of the gap between what an individual wants to become as a mathematics teacher (his/her ideal of mathematics teacher) and what he/she believes to be at present. Professional identity as a mathematics teacher can be seen as a continuous development arising from this gap. For these reason, both as researchers and as teacher educators, it appears significant to study what ideals of positive and negative mathematics teachers the future teachers have.*

#### **INTRODUCTION AND THEORETICAL FRAMEWORK**

Pre-service teachers approach their professional learning in mathematics with a complex set of needs and wants. In particular, they approach their path trying to satisfy their own wants and needs in the light of their previous experiences as math students (Liljedahl, 2014).

These previous experiences largely determine future teachers’ mathematical identity (Kaasila, 2007). According to Kaasila, we define mathematical identity as the set of narratives that pre-service teachers create to describe themselves as mathematics learners and teachers. In particular, identity as a mathematics teacher can be seen as a continuous development arising from the gap between the ideal of good mathematics teacher that a pre-service teacher has into his/her mind and the teacher the individual thinks to be at the present moment of his/her formation (Sfard & Prusak, 2005).

In this framework the case of future primary teachers appears particularly interesting: several researches highlight that many of them lived hard experiences with mathematics, developing negative emotions towards mathematics and towards the fact that they will have to teach mathematics (Coppola et al., 2013). Therefore, there is often a strong tension between what the individual is and what he/she wants to become as a mathematics teacher (Krzywacki & Hannula, 2010). As underlined by Liljedahl et al. (2014), the management of tensions defines pre and in-service teachers’ wants and needs and affect their decisions (respectively in their approach to professional learning and in their school practice).

The study of future primary teachers’ mathematical identity as mathematics teachers appears particularly significant to understand how they approach to opportunities offered them during the professional development (Lutovac & Kaasila, 2014). As Krzywacki and Hannula (ibidem) underline, pre-service teachers’ identity as

mathematics teachers is strongly influenced by the real mathematics teachers met during the school period.

For these reasons, as researchers in mathematics education, we believe that it is interesting to compare future primary teachers' viewpoints about what school teachers need to teach mathematics effectively with the existing great amount of literature on that issue (Ball & Bass, 2000; Mason, 2008; Oliveira & Hannula, 2008). On the other hand, as teacher educators, it is crucial to offer to future teachers the opportunities for reflection on: their own learning, their experiences with understanding of mathematics, as well as on the approaches used by their teachers to introduce and discuss topics (William, 2001).

Therefore, within a wider study about teachers' mathematical identity, beliefs and emotions towards mathematics, we have developed a narrative study aimed at identifying which traits future primary teachers consider distinctive of effective mathematics teachers and which traits they consider distinctive of ineffective mathematics teachers.

## METHODOLOGY

**Procedure and population.** The study developed through two different phases. In this paper we will focus on the second one, but we believe that, in this section, it is important to briefly sketch the study in its wholeness.

The first phase involved 212 future primary teachers enrolled in the first year of the university degree for primary school teachers of six different Italian universities. They were asked to answer in anonymous way to a questionnaire composed by 7 open questions about their past experiences, beliefs and emotion towards mathematics, within 1 hour. In particular, Q2 was: "*What has been your past experience with mathematics during the school period? Can you describe an episode occurred during your school period that you consider crucial in the development of your current relationship with mathematics?*". By the analysis of the collected data, it emerges as 'mathematics teacher' is the most recurrent factor: we found 131 occurrences of teachers on 212 answers to Q2 (the 62% of the collected narrations). There are other recurrent aspects in the episodes narrated (such as successes or failures in math, specific topics, the transition from a school level to another one), but mathematics teacher results by far the most recurrent factor both in *negative* episodes and in *positive* one. This result affected the development of the second phase of our study, that it is the focus of this paper. We decided to investigate about which traits future primary teachers consider distinctive of effective (ineffective) mathematics teachers stimulating a reflection about the approaches used by their mathematics teachers, possibly recognizing both *positive* and *negative* model of mathematics teacher in their school experiences.

The second phase involved 59 future teachers enrolled in the first year of the university degree for primary school teachers of seven different Italian universities: 44 of them (75%) answered to an online questionnaire composed by 11 questions (9 open-ended

and 2 close-ended questions) and 15 of them (25%) answered to a semi-structured interview concerning the same topics of the online questionnaire. The interviews had not a settled time: it varied in a range from 25 to 65 minutes. The interviews were audio-recorded and then fully transcribed.

In this paper, we focus on the future teachers' answers to the questions Q6: "*Think about your math teachers. Is there one you would like to become like? In what way? Why?*" and Q7: "*Think about your math teachers. Have you ever thought 'I never should act with my students like s/he did with me'? Why?*". Questions Q6 and Q7 were included both in the questionnaire and in the oral interviews.

**Rationale.** The choice of the research instruments is never neutral. In our case, the narrative approach is not only coherent with the definition of mathematical identity assumed: as Kaasila (2007) underlines, through this methodological approach, what pre-service teachers consider really important in their experiences comes to the fore. Individuals develop their sense of identity by describing themselves as protagonists of different stories: what creates the identity of the individual is the identity of the story, not the other way around. We chose to use both an open-ended questionnaire and interviews because we believe that the two instruments complement each other. As a matter of fact, the use of questionnaire permits to collect a wider range of answers and, according to Cohen et al. (2007), an open-ended question can catch the authenticity, richness, depth of response, honesty and candor which are the hallmarks of qualitative data. On the other hand, questionnaires have their limitations: they are still one-way, when compared with interviews. Moreover, Kaasila (ibidem) has highlighted the potential of narrative interviews for the study of pre-service teachers' mathematical identity.

Regarding the analysis of the narrative data collected, we refer to the work of Lieblich et al. (1998). They recognize two main choices related to two independent dichotomies. The first choice concerns the narrative unit of analysis: holistic (the narrative is analyzed as a whole) vs categorical (specific utterances are singled out from the complete narrative) analysis. The second choice concerns the traditional dichotomy between the attention to the content or the attention to the form of a narrative. Our analysis approach was mainly content-categorical oriented, being considered particularly suitable to study a phenomenon common to a group of people (Kaasila, ibidem).

## RESULTS AND DISCUSSION

Not all respondents answer affirmatively to Q6 or Q7: some of them explicit that they did not recognize a *positive* or *negative* model of mathematics teacher in their school memories (FTQ19<sup>4</sup>: "*Actually I have never met great mathematics teachers: for one reason or the other, they have never fully satisfied me*"). Within our sample, it emerges

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<sup>4</sup> Here, as well as in the next excerpts, the letter Q refers to Questionnaire, the letter I refers to Interview and the number indicates the progressive numbering of the respondents.

a gap between the percentage of respondents that does not recognize positive model in their mathematics teachers (32%) and the percentage (only the 12,5%) of those who declared they have not memories of negative model of mathematics teachers (in these percentages, we had not considered positive or negative references to academics). In the light of these data, it seems that, reflecting on their experiences, future primary teachers have greater ease in recognizing the negative traits in the teaching styles their mathematics teachers used, rather than the positive ones.

The analysis of the data collected through the questionnaire and interviews permits to describe a long list of traits that future teachers associate to their mathematics teachers (see table 1 below). The answers to Q6 and to Q7 permit to identify the traits associated respectively to positive models of mathematics teacher (positive traits) and to negative models of mathematics teachers (negative traits).

<b>Positive traits</b>	<b>Negative traits</b>
Competence in math	Incompetence in math
Competence in teaching math	Incompetence in teaching math
Clarity in explanation	Ambiguity in explanation
Interactive teaching methods	Frontal teaching method
Ability to show the link between math and real life	Inability or disregard in going beyond the content included in the syllabus
Relational Approach	Instrumental Approach
Passion for math	Coolness for math
Passion for teaching math	Coolness for teaching math
Serenity	Aggression
Severity	Severity
Attention to students' needs and difficulties	Indifference for students' needs and difficulties
Confidence in students' capability	Doubts about students' capability
Ability to develop a good relationship with students	Inability to develop a good relationship with students

Table 1: Duality between positive and negative teachers' traits.

It clearly emerges a duality between positive and negative traits. There is a unique *anomaly*: severity. Some respondents consider severity as a negative trait that can contribute to create a bad climate in the classroom (FTQ9: “*I don't want to be like my primary teacher: she is rude and severe. I was intimidated by her, therefore I was stuck, I went into a panic*”). Other respondents whereas underline their conviction that a

certain level of severity is needed to be a respected and effective teacher (FTQ15: “*She was an excellent teacher: she was severe and very good in teaching*”).

Analysing traits in table 1, we can recognize some aspects included in the Mathematical Knowledge for Teaching model (Ball & Bass, 2000). We observe that the references to the common content knowledge are mainly stressed in the answers to Q7 rather than in the answers to Q6, and they are almost always combined to pedagogical aspects (FTQ37: “*My secondary teacher was incompetent and unable to interact with teenagers*”). Future primary teachers seem to be aware that having a solid content knowledge is a necessary but not a sufficient condition to be an effective teacher, in particular at primary school level. This is also evidenced by the greater number of recurrent traits related to pedagogical content knowledge or to affective aspects.

A significant outcome of our survey is the attention given by future teachers to the view of mathematics their teachers offered. In his famous paper, Skemp (1976, p.6) stated: “*I now believe that there are two effectively different subjects being taught under the same name, ‘mathematics’*”, introducing the concepts of relational and instrumental mathematics. According to this classification, within our sample, we found a general appreciation for mathematics teachers that have proposed a relational approach to mathematics (FTQ42: “*She always got in-depth when explaining. The first question was always “Why?” and never “How we have to solve it?”*”, FTQ22: “*I have appreciated my mathematics teacher from the beginning because she tried to teach us to look beyond memorization of formulas*”), and conversely a widespread criticism toward teachers with an instrumental approach to math (FTQ27: “*I have had teachers that forced me to memorize formulas and to recite rules*”).

On the other hand, discussing why an instrumental approach to mathematics appears to be often so appealing for teachers and students, Skemp describes some apparent *advantages* of this approach to mathematics. In particular he underlines that within its own context, instrumental mathematics is usually easier to understand and the rewards are more immediate. The analysis of the interviews shows as the appreciation for “relational mathematics teachers” is often the result of a posteriori reflections, based on a greater awareness (FTI7: “*For a long time I thought that mathematics was characterized by memorization. Probably I was focused on memorization rather than understanding (...) Now I’m understand that I have never found teachers that try to explain me the reasons beyond mathematical facts (...) many facts were simply assumed (...) but I was not able to understand: perhaps I needed further and different explanations*”). Sometimes the awareness of the weakness of an instrumental approach to mathematics emerges from an a posteriori comparison with the educational results of a relational approach (FTI1: “*My mathematical experience has been linear and planned (...) I was very relaxed and satisfied. My sister had a more troubled path, but this path permits to her and many of her classmates to develop the flexibility that I haven’t (...) Probably I have a more complete preparation in mathematics (...) but, when I have to manage with a new situation, a problem that goes beyond the*

*application of memorized schemas, I am disoriented, whereas she is more ready, prepared and reactive because of her flexibility”).*

As we anticipated, a great emphasis to affective aspects emerges in future teachers' answers. In particular, *passion* and *calm* are considered crucial quality for an effective mathematics teacher. Passion for mathematics and passion for teaching mathematics are considered both essential to convey passion for mathematics to the students. However, these two passions are not always coincident (FTQ4's answer to Q7: “*my teacher surely loved mathematics, but he was not interested in its teaching at all*”). A calm teacher's presence is considered a key element in order to have an appropriate classroom climate and to promote passion for mathematics (FTQ26: “*She had charisma and she was so calm that I was enchanted during her math explanation*”). Particularly interesting that, in the answers to Q7, all the future teachers' memories involving an aggressive teacher are related to experiences at the primary level (FT15: “*I've always promised myself that I won't behave like my primary teacher. I would avoid to result aggressive and intimidate pupils*”).

On the other hand, the most stressed traits in the affective side concern two aspects of the teacher's *attention* to the students.

The first one is the teacher's confidence in students' mathematical potential (FTQ37: “*my teacher have always really appreciated my math ability. Therefore I would like to have this talent in showing the appreciation for student's ability*”). In particular, the richness of the data collected through interviews permitted to highlight the strong emotions elicited in students when they reach the awareness that teacher (and adults in general) has low confidence in their mathematical abilities. As underlined by FTI6, this awareness can persuade the student to be not able in math: “*I was dealing with people who had no confidence in me. This fact demoralized me a lot. Judgments like ‘Okay, after all she is not able to understand’ or ‘Okay, after all she is not able to do the appropriate reasoning’ convinced me that I was not able to do math (...)* When I met someone that believes in me, I gained confidence in my abilities (...) I believe that it is important to interact with a teacher that believes in you, in particular this support is fundamental at the primary school”.

The second one is teacher's attention to students' mathematical difficulties (FTI31: “*She had a positive attitude towards students' difficulties, she was inclusive: when we had difficulties in assimilating some topics, she tried new teaching methods*”; FTQ17: “*She wasn't interested in the development of our cognitive processes but she only focused in finishing the curriculum. She just wrote on the blackboard and she did not consider our difficulties. When we asked her something, she always answered that she hadn't got time to reply*”).

To conclude, we want to underline that in the interviews future teachers find the time to describe the evolution of their convictions about positive and negative ways of teaching mathematics. Particularly interesting in this sense is the episode narrated by FTI13: “*It happened that during my practicum I was paired with the primary teacher*

*that I have had when I was student. At that time, I liked very much her teaching methods and I liked the activities that she proposed to us. But in my practicum I have seen...I had a flashback: she proposed the same experiences that she used with us 15 years earlier. Identical! Identical! I have thought: 'No, I don't want to do this...ever!'"*

## CONCLUSION

There are a lot of papers in the field of mathematics education focused on what characteristics primary teachers should have in order to teach mathematics effectively. These studies have the ambition to affect the way teachers' education programs are developed. We strongly believe that it is highly relevant to listen the voice of the future teachers about this issue. "Teachers do not approach their professional learning as blank slates" (Liljedahl et al., 2015, p. 193): their beliefs and opinions about their experiences affect their wants and needs in the professional development setting. Knowing these wants and needs and their relationship with future primary teachers' experiences with mathematics is significant both as researchers and math educators.

In our study, the request to produce a mathematical autobiography forced the respondents to re-enact and re-consider their own past experiences, in order to develop a new awareness about their own wants and needs. In particular, it emerges that future primary teachers go beyond the boundary delimited by mathematical knowledge for teaching in their reflections, placing a strong emphasis to affective aspects in their judgments about mathematics teachers.

The goal is not to draw a more complete list of what primary teachers need to teach mathematics effectively. According to Mason (2008, p. 317), future teachers' attention needs to be focused not on a list of prescriptions, but on noticing: "The aim of teacher education is to prepare the ground so that novice teachers will find themselves increasingly sensitised to noticing possibilities for initiating, sustaining or completing actions which they might not previously have had come to mind". On the other hand, as Liljedahl (2014) underlines, the recognition of future teachers' wants and needs should have an impact on how we view our role as facilitators.

In particular, it appears fundamental to create bridges between the research results and the action for improving practices. Our study underlines the need to incorporate in a systematic way *affect* in the education program for future primary teachers. Some steps in this direction are being taken (Gómez-Chacón, 2008), but we believe that many more will need to be taken soon.

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