Inside teachers' affect:

teaching as an occasion for math-redemption

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Abstract

Mathematics education is strongly interested in defining "what is necessary for teaching mathematics effectively". The main directions of research emphasize the cognitive side of the answer to this question, trying to describe what kind of knowledge is needed in order to teach mathematics effectively. Starting from the point that teachers' affect plays a crucial role in determining the quality of teaching, we discuss this issue from a theoretical point of view, describing our perspective in detail and introducing the construct of "attitude towards mathematics teaching". We discuss some of the results of a study we conducted in order to investigate the attitude towards mathematics and its teaching of 189 primary school pre-service teachers. In particular, we focus on the emotional component, describing what we call the "math-redemption" phenomenon.

Key words

Attitude towards mathematics, attitude towards mathematics teaching, teachers' training, math-redemption

Introduction

> *Historians predict that unless we examine and learn from the past we are condemned to relive it. It appears that their prediction may be in the process of fulfilment in the area of mathematics education* (Mihalko 1978, p.35).

This was the incipit of the work of Mihalko about mathematics teacher education more than thirty years ago. He described a cycle that it is not difficult to

recognize also nowadays in many countries: students' performance in mathematics below public expectations, cries of indignation, rethinking of the way to introduce mathematics in classroom (with an abuse of the label "new math"), implementation of the "new math" in the curriculum, poor results obtained and so on. Mihalko underlined that the introduction of "new math" in school curriculum is usually more virtual than real because it is not accompanied by an adequate teacher preparation. Consequently, he began to discuss the meaning of "adequate teacher preparation" recognizing cognitive and affective goals:

> *In the cognitive area we need a teacher education curriculum which assures knowledge and competency in mathematics as well as a knowledge of the philosophical, historical, psychological, and sociological aspects of education. In the affective area we need a stimulus for the growth of teachers' ability and desire of knowledge* (Mihalko ibidem, p.36).

In some sense he can be seen as a precursor in the field about teachers' professional development in mathematics education: for the first time the awareness of the need of a deep reflection about "what is necessary for teaching mathematics effectively" was made explicit.

About ten years later, Shulman (1986) developed his very famous perspective that recognized three different components of knowledge necessary for teaching: Curricular Knowledge (CK), Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK). PCK represented the real innovation: a knowledge "which goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching" (Shulman ibidem, p.9).

Shulman's perspective has had a great impact on the research about teachers in mathematics education, having inspired many important studies in the field: Ball and Bass (2003) explicitly referred to Shulman's work in the development of their theory of Mathematical Knowledge for Teaching (MKT). The work of Ball and Bass has also many links with the pioneering work of Mihalko, since they share the starting educational problem, the working assumption and the reformulation of the problem with a shift of attention from students to teachers:

> *We seek in the end to improve students' learning of mathematics (...) We focus on teacher knowledge based on the working assumption that (...) the goal of improving students' learning depends on improving teachers' knowledge (...) The problem: what mathematics do teachers need to know to teach effectively?* (Ball & Bass ibidem, p.3).

Based on the shared assumption that the quality of students' learning is related to the teacher knowledge, Ball and Bass's perspective about mathematics teacher development refers exclusively to the cognitive aspect. But, according to Zembylas (2005, p.467, emphasis as in original):

> *Teacher knowledge is located in 'the lived lives of teachers, in the values, beliefs, and deep convictions enacted in practice, in the social context that encloses such practices, and in the social relationship that enliven the teaching and learning encounter'. These values, beliefs and* emotions *come into play as teachers make decisions, act and reflect on the different purposes, methods and meanings of teaching.*

This view supports our strong conviction that the answer to the question "what is necessary for teaching mathematics effectively" cannot be limited to what teachers know and that it must include considerations about what teachers believe and feel. As underlined in what may be considered the initial manifesto of the modern research on affective factors in mathematics education: "All research in mathematics education can be strengthened if researchers will integrate affective issues into studies of cognition and instruction" (McLeod 1992, p.575).

Teachers' affect: theoretical framework

Since the early research in the field of affect, the interest about teachers' beliefs, emotions and attitudes in mathematics is mainly motivated by the conviction that these factors influence teachers' practice and then strongly affect the quality of students' learning in mathematics: "the teacher's attitude is a potent force in the classroom" (Burton 1979, p.131).

Initially the focus of the research was placed on finding – through quantitative studies – cause-effect relationships between affective factors hold by a teacher and his/her classroom practice. This approach is problematic and lead to inconsistent or even contradictory results: for example the problem of the incon-

sistency between beliefs professed by teachers and their practice is well-known (Di Martino & Sabena 2010).

In the nineties, the research on affect in mathematics education developed through a shift from a normative-positivistic paradigm, to an interpretative one (Zan et al. 2006). A gradual affirmation of the interpretative paradigm in social sciences, related to a greater attention towards aspects of the complexity of human behavior, has led researchers in mathematics education to abandon the attempt of explaining behavior through measurements or general rules based on a cause–effect scheme, and to search for new interpretations. After this change of paradigm, there was a growing awareness among mathematics educators of the central role of affect in mathematics learning and teaching (Tsamir & Tirosh 2009). But, as Philipp (2007, p.309) underlines, there is a great imbalance between research on teachers' beliefs, and research on teachers' emotions:

> *One noteworthy difference between research on teachers' beliefs and affect is that whereas research on teachers' beliefs has been extensive and subsumed into almost all areas of research on mathematics teaching and learning, the study of teachers' affect has not.*

Actually, this imbalance is not peculiar only to mathematics education:

> *Despite the enormous blossoming of psychological research on emotions since the early 1980s, little of this work has informed current research on teachers (...) Researchers also know little about how teachers regulate their emotions, the relationship between teachers' emotions and motivation, and how integral emotional experiences are in teacher development* (Sutton & Wheatley 2003, p.328).

The research about teachers' emotions in mathematics education has mainly focused on primary pre-service teachers, in particular studying and well-documenting the problem of primary teachers' negative emotions (anxiety, fear, etc.) towards mathematics (Wood 1987; Hannula et al. 2007; Di Martino & Sabena 2011). Many researchers stress the importance of preventing or overcoming these negative emotions as a necessary condition to improve the quality of mathematical learning:

> [Mathematics teachers] *cannot be expected to generate enthusiasm and excitement for a subject for which they have fear and anxiety. If the cycle of mathophobia is to be broken, it must be broken in the teacher education institution* (Mihalko ibidem, p.36).

Our conviction is that interpreting (and counteracting) the phenomenon called mathophobia needs to consider teachers' affect in its entirety. From this point of view, if Philipp (ibidem) stresses the complete lack of integration between the research on teachers' emotions and the research on teachers' beliefs, many scholars give theoretical emphasis to the strong relationship between beliefs and emotions (Hannula 2009). In particular, Di Martino and Zan (2010; 2011) consider this relationship at the basis of their three-dimensional model of attitude (TMA model, Figure 1). In their view, attitude towards mathematics is characterized by three strictly interrelated dimensions: emotional disposition towards mathematics, view of mathematics and perceived competence in mathematics:

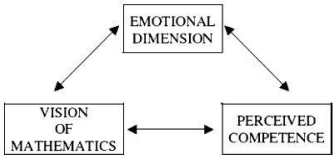

Figure 1:The Three-dimensional Model of Attitude (Di Martino & Zan, 2010).

Within the interpretative paradigm, Di Martino and Zan's theoretical research fits with the strong incentive "to develop constructs that might be applied to help make sense of teaching and learning environments" (Philipp ibidem, p.264). In this paradigm the single affective construct is no longer a trait of the observed subject, predictive for his/her behaviors, but instead it is a model of the observer, useful to interpret and understand processes of teaching and learning (Ruffel et al. 1998).

In this framework, research in mathematics education has underlined that to analyze teachers' affect it is necessary to consider not only their attitude towards mathematics but also towards its teaching (Relich, Way & Martin 1994, p.56).

According to this view, we have extended the model of attitude, considering also teacher's emotional disposition, view and perceived competence towards mathematics teaching. We conducted a study focused on primary pre-service teachers' attitude towards mathematics and its teaching. The study has a two-fold goal: on the one hand – as teachers' educators – to help future teachers becoming aware of the attitudes that they hold (it is the first step towards an eventual change), on the other hand – as researchers – to investigate the six dimensions involved in attitude towards mathematics and attitude towards its teaching and their mutual relationships.

In this paper, we focus and discuss in particular about emotional component: in the Italian context, as discussed in previous study (Di Martino & Sabena 2011), primary pre-service teachers have often very negative emotions towards mathematics.

We think that it is important to investigate the relationship between pre-service teachers' emotions towards mathematics and two different aspects: their past experiences as math-students and the emotions elicited in knowing that they have to teach mathematics in future.

Methodology

The sample of the study is represented by 189 future primary school teachers of two different Italian Universities: a small University in the South and a bigger one in the North. The subjects were enrolled in the courses on Mathematics and its Teaching that take place during the first year of the University degree for primary school teachers.

We developed an open questionnaire on the basis of the evaluation of the results gained with questionnaires used in previous researches with primary pre-service teachers (Di Martino & Sabena, 2010; 2011). The questionnaire was administered in the very first lesson of the course at both the Universities in the a.y. 2011-2012. Respondents were asked to answer anonymously, providing a nickname. The used questionnaire is composed by 12 questions focused on the three components of attitude (according to the TMA model), declined along the two dimensions of mathematics and its teaching.

The questions can be organized into the six resulting factors as showed in Table 1:

	Mathematics	*Mathematics teaching*
Emotional disposition	4. Write three emotions you associate to the word "mathematics". 5. How was your relationship with mathematics as a student? □Positive □Negative □Indifferent □Ups and downs Explain why you think that your relationship was so.	10. Which emotions do you feel in knowing that you will have to teach mathematics? Why?
View	1. Write three adjectives you associate to the word "mathematics". 2. What is, in your opinion, a positive feature of mathematics? Why do you think so? 3. What is, in your opinion, a negative feature of mathematics? Why do you think so? 6. Indicate three qualities you consider necessary in order to succeed in mathematics. 8. For which reasons, in your opinion, can students have bad results in mathematics? 9. In your opinion, why is it important that mathematics is taught at school?	12. Which characteristics should have in your opinion, a "good" mathematics teacher?
Perceived competence	7. In which measure do you think to have the qualities written in the previous answer?	11. Try to describe some difficulties you expect to meet in teaching mathematics.

Table 1. The questionnaire questions' categorization according to the developed model.

The questionnaire is oriented to capture relationships and dynamics developing over time. In particular, we were interested in seizing links between the past experience as students (e.g. Question 5) and the perspective towards the future teaching (questions 10-12). Present is of course pervasive, since every answer is filtered by the subjects' present views and emotions.

In this paper, we analyze the answers to Questions 4,5 and 10, related to the emotional dispositions towards mathematics and towards its teaching.

For what concern the methodology of analysis, descriptive statistics was used as an analytical tool to gain insights into the data. Dubar & Demaziere (1998) proposed an approach, called analytical, in order to systematically produce

sense from people's words. Final outcome of this analytical process is the construction of a set of categories, properties, and relationships.

Results

We based our analysis of the answers to the question related to the emotional disposition, on the work of Ortony, Clore and Collins (1988) about the cognitive origin of the emotions. They describe emotions as "valenced reactions" to consequences of events, action of agents, or aspects of objects. They classify the first class of reactions (to events) in being pleased and displeased, the second class of reactions (to agents) in approving and disapproving, and the latter (to objects) in liking and disliking.

We use these dichotomies for a first rough classification into positive/negative emotions coming from the answers to Question 4 and Question 10, but in our analysis we consider Ortony, Clore and Collins' s view that:

> *An analysis of emotion must go beyond differentiating positive from negative emotions to give a systematic account of the qualitative differences among individual emotions such as fear, envy, anger, pride, relief, and admiration* (Ortony et. al ibidem, p. 12).

The analysis of Question 4 (Q4) reveals a predominance of negative emotions (*anxiety, fear, panic*) elicited by mathematics with respect to the positive ones (*satisfaction, enjoyment, curiosity*). This predominance emerges both in terms of numerical occurrence (the 28% of the sample expresses only negative emotions in his/her answers, while the 20% expresses only positive emotions towards mathematics), and in the intensity (for example *panic* appears to be hottest than *curiosity* or *satisfaction*).

Q5 asks future teachers to give a judgment on their personal relationship with mathematics, and to provide an explanation. Respondents can be divided into the four groups provided by the questionnaire: **P**ositive **R**elationship (**PR**: 23%), **N**egative **R**elationship (**NR**: 16%), **I**ndifferent **R**elationship (**IR**: 1%), and **F**luctuating **R**elationship (**FR**: 60%). In this case, no questionnaire is left unanswered; moreover almost all the respondents provide detailed descriptions of their relationship with maths: the past relationship with mathematics appears something on which future teachers have much to tell. By a qualitative analysis of these rich answers we can categorize the perceived causes of

such relationship (in some case the recognized causes are more than one). In particular, comparing the answers in the *extreme* groups (i.e. the **PR** and the **NR**) we obtain the data summarized in Table 2:

Declared causes for the relationship with maths at school	
Positive Relationship Group	*Negative Relationship Group*
Teacher (60%)	Teacher (52%)
Innate characteristics (5%)	Innate characteristics (40%)
Success and its emotional consequences (23%)	Failure and its emotional consequences (36 %)
Interest in the discipline (17%)	Disinterest in the discipline (16%)

Table 2. Perceived causes for the relationship with maths.

Focusing on similarities and differences between the two columns of Table 2 we observe that:

- in both groups the majority of respondents recognizes in one of their school teacher the main factor in the determination of their own relationship with maths at school;
- in the **NR**-group a great relevance is given also to attributed innate "limiting" characteristics (for example, Elilee writes "*Surely this is because I am limited and more inclined to the humanities*") whereas this aspect is little mentioned in the PR-group, roughly 5%;
- the relationship with mathematics is often identified also with the success or failure experiences and their emotional burden. This aspect appears strong both in the positive (for example, Minu writes: "*I always liked the sense of satisfaction felt when I solve a problem*") and in the negative cases (as an example June answers: "*Besides my difficulty in following and understanding maths, a strong sense of anxiety has accompanied me every time there was a maths test in classroom*"), confirming that the emotional disposition and the perceived competence dimensions are deeply intertwined.

As this brief discussion shows, the analysis of Q5 provides insights on another dimension of the TMA model: respondent's view of mathematics teaching. In fact, their narrative accounts shed light on what they consider good and bad qualities of mathematics teachers. With this regard, two main features emerge as crucial and pervasive in both the **PR** and the **NR** groups: to be able/unable

in helping the students to understand mathematics (cognitive dimension); to be able/unable in transmitting the love for mathematics (affective dimension).

Analyzing the answers to Q10 we observe that those respondents that are in **PR**-group and declare positive emotions towards maths declare also positive feelings towards the idea of having to teach maths.

In the case of negative relationship or negative emotional disposition towards mathematics, the correlation with emotional disposition towards its teaching is not unidirectional. As a matter of fact many respondents that declare negative emotions towards mathematics (Q4) or a negative relationship in the past with it (Q5), display positive feelings related to the perspective of having to teach it. Indeed the 40% of the sample declare a positive emotional disposition towards the idea having to teach mathematics compared with the 30% that is scared by the same perspective.

In our opinion, it is very significant that the most used word in the answers to this question is: responsibility. In many cases, the difficulties met in the personal school experience are considered as the main stimulus for the future work (Nadi, for example, writes: "*It is exciting to think that I might give children what had not been given to me*"). However, in other cases, the negative past experience appears to be the origin of rooted and precise beliefs about mathematics teaching that influence negatively the perspective of teaching mathematics. These beliefs appear to be closely related with the view of mathematics teaching emerging from answers to Q5, highlighting the fear to be unable in helping the students at a cognitive level (as an example Cielo affirms: "*I feel anxiety because I might not be able to transmit the love for the subject*") and in conveying the pleasure to do maths on an affective level (in the protocol of Camilla 89 we can read: "*I am discouraged because I do not feel able to explain to a child topics as multiplication tables, division that I now consider routine*").

Moreover, some people with negative relationship with mathematics see in the perspective of teaching a possibility for *redeeming* themselves in their relationship with mathematics, whereas others, on the contrary, declare to feel insecure in accomplish a work that they consider important but difficult.

As a matter of fact, almost all the respondents consider teaching of mathematics a very difficult challenge, but there is a clear distinction between those that see it as a stimulating challenge, and those that see it as an insurmountable obstacle. In the latter case, strong negative emotions are elicited by the idea of having to teach mathematics. This remark highlights that negative feelings towards the perspective of having to teach maths are strongly influenced by a low perceived competence towards mathematics as, for example, it emerges from Nello's protocol: "*Fear, because I have not the necessary basis of mathematics for teaching it*".

But the same negative feelings are also associated to a low perceived competence towards mathematics teaching, as for example in RedQueen's protocol: "*Fear, because I would like to transmit my passion to other people, but I fear not to find suitable methods to be effective*".

Conclusions

As teacher educators we have a dual goal: a research-goal that is to understand and recognize the most significant variables involved in the process of teachers' development, and an educational-goal that is to promote teachers' growth.

These goals are clearly related and linked to the answer to the question "what do maths teachers need to teach effectively?".

Starting by the perspective that knowledge is only one side of the coin, and by the documented and alarming phenomenon of negative feelings towards mathematics (and sometimes towards its teaching) of primary pre and in-service teachers, we are interested in conducting theoretical and empirical studies to interpret this phenomenon and to recognize its causes. It is important to know the phenomenon, to study its effect on teachers practice and, on the basis of this, to develop strategies to overcome difficult experiences.

Some interesting outcomes emerge by the preliminary analysis of pre-service teachers' answers to our questionnaire. First, the awareness of the role of the school-teacher in their relationship with mathematics comes out as a fundamental topic. Another very significant issue coming from our analysis is the desire for math-redemption expressed by many respondents among those who declare negative past relationship with mathematics. This is a central point, because as teacher educators we have the chance of leveraging this desire to

break the chain connecting the negative past school experiences with the negative feelings towards mathematics of many primary pre-service teachers. Moreover, we can observe, from our analysis, that the degree of confidence about the possibility of math-redemption is strictly linked to both cognitive aspects (PCK, SMK) and affective ones.

Mathematics teacher education is a quite recent field of research in mathematics education: the first Handbook of Mathematics Teacher Education was published in 2008 by Sullivan and Wood. As we read in the preface of this handbook, "most research papers in mathematics teacher education put a major focus on the content dimension" and there is very little literature about teachers affect.

Thirty years after the famous paper by Schoenfeld (1983), we are convinced that there is the need to go beyond the purely cognitive also in the research about mathematics teacher education, and to explore teachers' affect in its wholeness.

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