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Gianpaolo Rossini and Cecilia Vergari

Abstract

In many industries, it is quite common to observe firms delegating the production of essential inputs to independent ventures jointly established with competing rivals. The diffusion of this arrangement and the favourable stance of competition authorities call for the assessment of the social and private desirability of Input Production Joint Ventures (IPJV), which represent a form of input production cooperation, scantly investigated so far. IPJV can be seen as an intermediate organizational setting lying between the two extremes of vertical integration and vertical separation, with a major difference, due to partial collusion. Our investigation is based on an oligopoly model with horizontally differentiated goods. We characterize the conditions under which IPJV is privately optimal finding that firms' incentives may be welfare detrimental. We also provide a rationale for the empirical relevance of IPJV both in terms of its ability to survive and in terms of disengagement incentives which account for the large number of divorces among members of joint ventures. The stance of the paper as to IPJV is more cautious with respect to the received wisdom of competition authorities and in favour of the wide application of the rule of reason.

KEYWORDS: Input Production Joint Venture, horizontal mergers, oligopoly

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1 Introduction

In many industries firms delegate the production of an essential input to an independent venture carried out in cooperation with one (or more) firm(s) competing in the downstream market for the final good. Examples may be found in many sectors.

In the automotive industry, for instance, Ford and PSA produce and design diesel engines in a specific joint venture; Ford and Fiat produce in a jointly owned plant the “KA” and the “500” on the basis of several common inputs.

In the electronic industry Sony jointly manufactures with rival Sharp liquid crystal displays.

In the media production, newspapers process raw news obtained from press agencies they jointly own, like Associated Press in the U.S.A. and ANSA in Italy.

In the chemicals, giant companies jointly own plants where ethylene and other basic components for plastics are manufactured, as in the recent agreement between Dow and Kuwait Petroleum Corporation (Hewitt, 2008).

Many other industries display instances of joint ventures in upstream sections of production. A great deal of them may be visible even to the accidental observer.

Most joint ventures devoted to the manufacturing of an essential input are autonomous companies owned and governed on an equal foot by delegates of firms operating and competing among each other in the downstream section of the vertical chain of production.[1]

This arrangement, we term *Input Production Joint Venture* (IPJV), may be regarded as an intermediate organizational setting lying between vertical integration, where the essential input is entirely manufactured in-house, and vertical separation, where the intermediate good is bought from external, independent firms operating in the upstream section of the vertical chain of production. A major difference with respect to both vertical integration and vertical separation arises from the fact that IPJV implies a kind of collusion, owing to the union and concentration in a single company of input production facilities, otherwise allocated in competing firms.[2]

[1] There are also joint ventures which are owned and governed on the basis of asymmetric shares. For instance, Alenia Aeronautica and Sukhoi Holding have given rise to a joint venture with shares respectively 51% and 49% for the marketing of a commercial aircraft jointly produced with shares respectively of 25% and 75%. We stick to the most common and simple case of symmetric partcipation.

[2] The *joint venture* we are going to investigate differs from the merger of existing activities between rival firms, as investigated, for instance, in Milliou and Petrakis (2007) and in Inderst and Wey (2003). The *joint venture* is a fresh production activity with a governance shared by the founding firms. A major effect of this difference is that *mergers* usually undergo a distinct probationary

Our interest in IPJV is generated by four facts. *First*, the diffusion of IPJV as a common practice in input manufacturing. *Second*, the benign treatment of antitrust authorities towards IPJV. *Third*, the high rate of divorces among member firms. *Fourth,* the small number of contributions on IPJV in the Industrial Organization (IO) literature, mostly focussed on vertical separation, vertical integration[3] and partial vertical separation.[4] Instances of IPJV have been investigated, in a different perspective, in the management literature.[5]

In the ensuing pages we wish to discover whether the fairly positive stance of antitrust authorities towards IPJV is well grounded or poorly justified and why divorces are so frequent in such a popular vertical venture. In this sense, we try to fill some gaps in the received analysis of IPJV, examining feasibility, private and/or social desirability and stability of IPJV in scenarios with static uncertainty and distinct strategic modes.

The closest case so far analyzed in IO is Research Joint Venture (RJV) with a large and consolidated literature.[6] With RJV firms share information and choose R&D investment levels maximizing joint profit. Unlike RJV, an IPJV requires an independent input producer owned in equal stakes by downstream firms. The profit of the venture accrues *ultimately* to the downstream firms, which own the IPJV, making for consolidated profits. In this sense an IPJV is a particular case of an Equity Joint Venture, as defined by Hewitt (2008, p. 96):

> "An Equity Joint Venture ... is a joint venture or alliance which has the following characteristics, namely where (i) each party has an ownership interest in a jointly owned business, (ii) the jointly owned business has a distinct management structure in which each party directly participates and (iii) the parties share the profits (or losses) of the jointly owned business".

Even though this arrangement is a kind of partial collusion, so far it has not given rise to much antitrust complain and suit. Perhaps that is due to the similarities with RJV where sharing and coordination of R&D decisions among firms

inquiry by antitrust authorities while most *joint ventures* do not or, when probed, enjoy by and large a more benign treatment.

[3]For a recent survey of theoretical and empirical issues on vertical integration, see Lafontaine and Slade (2007).

[4]Analyses of partial outsourcing can be found in Alvarez and Stenbacka (2007), Shy and Stenbacka (2005), Moretto and Rossini (2008).

[5]See, for example, Hewitt (2008) and the rich management literature surveyed. As for the IO literature there are some notable exceptions that we review at the end of this Section.

[6]The seminal paper on R&D cooperation is d'Aspremont and Jacquemin (1988), extended by, e.g., Kamien *et al.* (1992), generalized by Amir *et al.* (2003) and Lambertini and Rossini (2009).

competing in the product market has been proved to be socially beneficial.[7] The theoretical and empirical underpinnings of RJV have generated a favorable stance by the U.S. Department of Justice, the U.S. Federal Trade Commission and several other antitrust authorities.[8]

> "RJVs often provide procompetitive benefits, such as sharing the substantial economic risks involved in R&D, increasing economies of scale in R&D beyond what individual firms could realize....The antitrust enforcement agencies also view most RJVs as procompetitive and typically analyze them under the rule of reason because of their potential to enable participants to develop more quickly or efficiently new or improved goods, services, or production processes. Under the *Competitors Collaboration Guidelines*, the agencies will not ordinarily challenge a RJV when there are three or more other independently controlled firms with comparable research capabilities and incentives".[9]

However, an IPJV cannot a priori be thought to provide the same private and social benefits of a RJV.

The IPJV could be regarded as a subset of Production Joint Venture and/or Equity Joint Venture. Towards these arrangements the stance of market authorities has been mostly benign[10] due to supposed procompetitive effects and the related benefits consumers pick up:[11]

> "Courts typically have analyzed true production joint ventures under the rule of reason and generally have upheld them".[12]

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[7]The pionereeing contribution comes from Kamien *et al.* (1992) who found that joint process research and development is welfare maximizing when firms compete à la Cournot in the product market and in most cases of Bertand competition. The existence of RJV spillovers are crucial to the result.

[8]Shapiro and Willig (1990) examine the benefits and dangers of production joint ventures. They point out that the free riding and scale economies arguments associated with R&D activities are less pronounced for production activities and they raise doubts about special antitrust treatment of production joint ventures. Despite of that the "Antitrust Guidelines for Collaborations Among Competitors" jointly issued by the U.S. Federal Trade Commission and the U.S. Department of Justice (2000) exhibit a fairly mild stance. The new 2010 U.S. Guidelines (see: Shapiro, 2010) are concerned only with horizontal mergers, while for vertical mergers and ventures the above mentioned 2000 Guidelines are still the basic reference rule.

[9]Jacobson (2007) p. 447.

[10]See, Federal Trade Commission and U.S. Department of Justice (2000), p.1.

[11]We mostly refer to the U.S.A.. EU and U.S. rules and jurisprudence are not much distant on this issue.

[12]Jacobson (2007), p. 450.

The "benign neglect" of antitrust authorities may come not only from likely spillovers but also from the high volatility of IPJV. As reported in the management literature, almost one half of joint ventures end in a divorce (Hewitt, 2008, p.12). We do not know exactly the causes of IPJV fragility. Therefore, we deem worthwhile to go through this unsolved issue in order to discover whether a fairly positive stance of antitrust agencies towards IPJV is partly the result of the high volatility of this kind of venture. To this aim we develop an oligopoly model with horizontally differentiated goods in the downstream market and the production of a homogeneous essential input in the upstream market. Enterprises may choose either to manufacture an essential input in-house (vertical integration) or to construct an IPJV by delegating to an (independent) upstream company the input production. Strategic interactions are assessed in a two stage game framework to which a third stage is added where vertical arrangements are chosen.

We resolve IPJV mostly vis à vis vertical integration with a few extensions to vertical separation cases. We trace how private and social incentives to form an IPJV change with the intensity of competition in the downstream market. The degree of competition in the downstream market turns out to be the main determinant of the incentive to join efforts upstream.

Some canonical results on the social superiority of vertical integration under linear pricing will be confirmed also in the presence of positive fixed cost in the input production. IPJV (partial or complete, according to whether only some or all firms participate in the joint venture) is privately preferred to vertical integration for high levels of competition in the downstream market, even in the most unfavorable scenarios for IPJV, i.e., with zero fixed costs. *A fortiori*, with positive fixed costs, since IPJV eliminates wasteful duplication.

Private profitability of IPJV goes up as we move to more competitive downstream market structures, i.e., as the number of firms increases, as products become closer substitutes, and as we go from Cournot to Bertrand competition modes. The advantage of IPJV, when competition gets tougher, is due to the fact that downstream firms are able to reap profits secured by the upstream firm (they jointly own) so as to compensate for the reduced returns in the fiercely contested downstream market. With IPJV firms are less afraid of downstream competition since they are sheltered by their upstream profit "reservoir". A far-reaching message follows: the more intensively firms fight in a market the more likely they may look for collaborations in a closely related market.

Like in any cartel or merger agreement, in an IPJV, there are incentives to leave the alliance. When companies compete with vertically integrated rivals and product market competition is fairly mild, there arises a lure to waive the IPJV either to build fresh autonomous vertically integrated firms (disengagement) or to set up smaller IPJVs (splitting). We shall detect levels of product market substitutability where the lure is quite catching. Nonetheless, we shall find circumstances in which incumbent vertically integrated enterprises may stand ready to compensate disengaging IPJV members to let them stay in, making IPJV stable. This result is in tune with the "facilitating collusion" argument according to which the presence of vertically integrated firms makes upstream collusion easier (Riordan, 2008).

As anticipated above, the IO literature on input joint venture is not abundant since most papers focus on research joint ventures rather than on collaborations aimed at input production. Nonetheless, a few notable contributions provide brilliant insights. Spencer and Raubitschek (1996) analyze Production Joint Ventures (PJVs) as a defence strategy against expensive and sometimes restricted supply of foreign intermediate goods in technologically advanced industries, such as semiconductors during the 1980s and 1990s in the U.S.. They show that, under imperfect competition, domestic high cost PJVs can be profitable for member firms as they reduce foreign monopoly power and import prices. The analysis is cast in a homogeneous goods framework with Cournot competition with firms signing exclusive contracts with the PJV. Morasch (2000a) and Morasch (2000b) provide another rationale for firms' incentives to form input production joint ventures in both a closed market and in a trade setting. Strategic contracts between alliance members regarding transfer prices and upstream cooperation are a way to affect downstream competition. In our framework downstream competition has a different effect since it determines the incentives to join efforts upstream. In Morasch's work (Morasch, 2000b), collaborations in the upstream market represent a commitment device to overcome the negative strategic effect arising in horizontal mergers under Cournot competition.[13] Finally, in Chen and Ross (2003), two firms compete in the downstream market and cooperate upstream. The input joint venture yields the same profit of a horizontal merger. Formally, this paper is quite close to ours. However, Chen and Ross (2003) consider general demand functions, while we consider a linear demand model; moreover, they limit their scope to two perfectly symmetric firms. As they acknowledge in their conclusion, the coordination between the IPJV and the owners is hard to reach in an asymmetric framework. We provide a different IPJV setting to bypass the hurdles associated to differences across firms. We extend the analysis to more than two firms, so as to encompass also mixed vertical arrangements. We consider both splitting and quitting the IPJV and the effects of product market competition on the stability of the IPJV.

The outline of the paper is as follows. In Section 2 we outline the framework of the investigation. In Sections 3 and 4 we compare vertical integration with IPJV in duopoly and oligopoly settings with Cournot competition. We examine the role of product differentiation, fixed cost and competition in the downstream product

[13] See Salant *et al.* (1983).

market in determining private and social desirability of IPJV. In Section 5 we consider the incentives to break the IPJV, a question analyzed mainly in the managerial literature on joint ventures. Finally, in Section 6 we extend our model to Bertrand competition and to static uncertainty. The conclusion may be found in Section 7.

2 The framework

We consider a Cournot oligopoly with n firms producing differentiated goods.[14] Each enterprise i produces the quantity q_i sold at price p_i and costs are $c_i q_i$. The demand system is given by linear inverse schedules $p_i = a - q_i - b\sum_{j\neq i} q_j$ in the region of quantities where prices are positive. The parameter $a > 0$ represents market size; $b \in [0,1]$ measures the degree of substitutability between the final products (if $b = 1$, products are perfect substitutes; if $b = 0$, products are independent).

Manufacturing a final good requires an essential input produced either by the firm itself (vertical integration - VI) or by an (independent) upstream (U) enterprise. An IPJV implies that downstream (D) firms set up a U venture owned in equal stakes by the D firms. More precisely, for the input production some (or all) D firms create an Equity Joint Venture (Hewitt, 2008) whose profits accrue ultimately to the D firms themselves, making for their *consolidated profits*.

As it is customary in the literature on vertical relationships we assume that one unit of input is embodied in each unit of output (perfect vertical complementarity). Input production requires a fixed commitment equal to $f \geq 0$. The marginal cost is constant and equal to $z < a$; without loss of generality we set $z = 0$.

Given the above described inverse demand system for the final goods, Cournot competition leads to different equilibria according to the vertical arrangement and the resulting U market structure. The vertical interaction between U and D is modelled as a two stage-game solved backwards:[15] the first stage covers the setting of the input price, w, the second stage comprises competition in the output (downstream) market. As such the solution concept we use is the subgame perfect Nash equilibrium (SPNE).

In what follows we distinguish the duopoly, $n = 2$, from the oligopoly case, $n \geq 3$. In a duopoly, we figure two scenarios: either the two firms are VI or they jointly own, on an equal foot, the independent U producer of the essential input, while competing among themselves in the D section. In contrast in an oligopoly with $n \geq 3$, we shall be able to extend the analysis to three distinct scenarios: in the first, all firms are VI, in the second, dubbed partial IPJV, some enterprises are VI and others participate in the IPJV, in the third all companies join and make for

[14] We consider Bertrand competition in Section 6.

[15] Recent examples may be found in Sappington (2005) and Arya *et al.* (2008).

one or more IPJVs. However, these scenarios are not exhaustive. Other vertical arrangements could arise: all firms may operate independently and vertically separated (VS); some firms may be VS and the rest VI. In our analysis we take VI as a benchmark to be matched with IPJV, because VI is socially superior to VS, as extensively proved in the literature.[16] For this reason we concentrate on VI versus IPJV.[17]

We examine duopoly and oligopoly in turn. The former simpler framework allows us to point out the trade off between VI and IPJV focusing on fixed cost savings and the degree of competition in the D market. We complete the investigation by extending the analysis to arrangements where VI firms compete with non-integrated rivals acquiring the essential input from an independent producer serving all non integrated companies.

3 Duopoly

Before going through the comparison of different arrangements, we define the vertical governance and the firms' objective functions in the case of IPJV. We then proceed with the proper duopoly analysis.

3.1 Governance of IPJV

The governance of the IPJV can take different shapes. Yet, not all of them may be feasible from an operative standpoint. In order to limit the investigation only to acceptable arrangements, we survey three governance structures.

1. First, each D parent firm i chooses both the input price and the output quantity maximizing *consolidated* profit, π^i_{cons} with:

   $$\pi^i_{cons} = \pi_{iD} + \pi_U/2$$

   i.e., the sum of the *operative* profit raised in the D market, π_{iD}, and the share of profit obtained by the U firm, $\pi_U/2$. In this case U is a passive subject.
2. Secondly, the IPJV acts independently of the owners maximizing its own profit, π_U while the D parent firms maximize *consolidated* profits.
3. Finally, the IPJV's objective is π_U and the D parent firms maximize their own *operative profit*, π_{iD}.

[16] See: Sappington (2005), Arya *et al.* (2008).

[17] To make our choice more robust, we go through a formal comparison with VS in the duopoly case and we show that VI is a dominant strategy for each firm.

We analyze these three cases in turn.[18]

Governance 1. The D parent firms, which are the owners of the U enterprise on an equal foot, provide guidelines to U as to the input price they wish to be charged. This strategy is pursued by the D firms maximizing an objective function made up of their respective operative profit augmented by one half the U profit. Therefore, each D firm i maximizes its *consolidated* profit π^i_{cons} rather than its *operative* profit, π_{iD}. This governance arrangement suffers a coordination impasse since there is no common input price U may charge, unless the two D firms are equal in all respects (symmetry). Formally, D firm i's objective function reads as follows:

$$\pi^i_{cons}\left(q_i, q_j\right) = \left(p_i - c_i\right) q_i + (1/2)\left[w_i(q_i + q_j)\right] \text{ for } i, j = 1,2 \text{ and } i \neq j.$$

The first part, $(p_i q_i - c_i q_i)$, is the *operative* profit of D firm i, π_{iD}. The second part is one half the *operative* profit of the U firm, since U is equally owned by the two D firms. Within the square brackets the aggregate quantity produced is multiplied by the input price w_i. As it can be seen, we are bound to consider two distinct prices since each D firm has to optimally set its own preferred input price dictated by its objective function. More specifically, this governance is modeled as a two stage game entirely played by the D rivals-coowners of the passively behaving U company. In the first stage the D firms set their preferred input price, while in the second stage they set the output quantity. Maximization of *consolidated* profits by the D firms gives rise to the following second stage equilibrium quantities:

$$q_i = \frac{2a(2-b) + 2\left(bc_j - 2c_i\right) - 2w_i + bw_j}{2(b+2)(2-b)} \text{ for } i, j = 1,2 \text{ and } i \neq j.$$

Each D firm sets an optimal input price w_i as a result of the first stage of the game:

$$w_i = \frac{ba\left(b - b^2 + 4\right) + c_j\left(2b - b^2 + 4\right) + c_i\left(b^3 - 6b - 4\right)}{(b - b^2 + 4)(b+1)} \text{ for } i, j = 1,2 \text{ and } i \neq j.$$

The two input prices are not equal. Each D firm would like to charge a distinct input price. Their difference is:

$$w_i - w_j = \frac{\left(8 - b^2\right)\left(c_j - c_i\right)}{4 + (1-b)b} \geq 0 \iff c_j \geq c_i. \tag{1}$$

From (1) we see that the most efficient firm would like to set a higher input price. An impasse arises if D firms wish to dictate the input price to U since no unique

[18] In this section without loss of generality we abstract from the fixed cost of production, i.e., we set $f = 0$.

equilibrium exists unless firms are perfectly symmetric, i.e., $c_i = c_j$. This scenario has been analyzed by Chen and Ross (2003) under the case of perfect symmetry, showing that the market equilibrium is equivalent to a horizontal merger in D (cartel outcome). The drawback of this governance arrangement for U, and the reason why we exclude it, is that an agreement is reached only in the particular case $w_i = w_j = w$.[19]

Assuming hereafter $c_i = c_j = 0$, we can easily derive the equilibrium price, quantity, operative profit in D, operative profit in U and industry profit:

$$\begin{aligned} p_m &= \frac{a}{2}, \\ q_m &= \frac{a}{2(b+1)}, \\ \pi_{mD} &= \frac{a^2(1-b)}{4(b+1)^2} \\ \pi_{mU} &= \frac{a^2 b}{(b+1)^2} \\ \Pi_m &= \frac{a^2}{2(b+1)} \end{aligned} \tag{2}$$

This equilibrium corresponds to a cartel solution, i.e., a *monopoly* in the D market.

Governance 2. Alternatively, the U producer acts independently of the D firms. Its objective function is π_U, whereas the parents D maximize their consolidated profits, π^i_{cons}. Then, from the second (output) stage we get:

$$\max_{q_i} \pi^i_{cons}(q_i, q_j)$$
$$\iff q_i = \frac{2a - w}{2(b+2)} \text{ for } i, j = 1, 2 \text{ and } i \neq j,$$

and from the first (input) stage we have:

$$\begin{aligned} w &= \arg\max_{w} \pi_U(w) \\ &\iff w_c = a \end{aligned}$$

[19] One way out of this kind of impasse could be a bargaining on the input price between the two D firms. However, asymmetric bargainings are often unable to lead to an equilibrium when the D firms own the IPJV both in equal shares or in asymmetric shares (see Kalai and Smorodinsky (1975) for asymmetric bargainings; see Rossini and Vergari (2010) for a formal proof of the above statement).

Equilibrium price, quantity, operative profit in D, operative profit in U and industry profit are:

$$p_i = \frac{a(3+b)}{2(2+b)}, \tag{3}$$

$$q_i = \frac{a}{2(b+2)}, \tag{4}$$

$$\pi_{iD} = -\frac{a^2(1+b)}{4(b+2)^2} < 0 \tag{5}$$

$$\pi_U = \frac{a^2}{2+b}$$

$$\pi^i_{cons} = \frac{a^2(b+3)}{4(b+2)^2} \tag{6}$$

$$\Pi = \frac{a^2(3+b)}{2(b+2)^2} \tag{7}$$

Note that the *operative profit* in the D market (5) is negative.

Governance 3. U is an independent producer and the Ds maximize their *operative* (rather than *consolidated*) profit. In particular, while prices, quantities and industry profit are the same as in (3), (4) and (7), respectively, the distribution of profit along the vertical chain differs; equilibrium profits in U and D are:

$$\pi_U = \frac{a^2}{2(2+b)} < \frac{a^2}{2+b}$$

$$\pi_{iD} = \frac{a^2}{4(2+b)^2} > 0,$$

while firm i consolidated profit is:

$$\pi^i_{cons} = \frac{a^2(b+3)}{4(b+2)^2}. \tag{8}$$

As a result of this short survey of governances, we exclude **Governance 1** due to coordination problems on the input price; we also drop **Governance 2** since it leads to the same consolidated profit of case 3 but it implies negative *operative* profits in D. The simplest feasible governance of IPJV turns out to be **Governance 3**: the U firm is an autonomous entity owned by D parent companies, which completely delegate the IPJV, making the U firm the pivot actor as regards market price.[20] This governance solves any potential coordination problem (it is consistent with both symmetric and asymmetric costs) and it ensures positive *operative* profits in each stage of the production chain without reducing the amount of the owners' *consolidated* profits. Therefore, *we assume* ***Governance 3*** *and adopt it in the case of IPJV*.[21]

3.2 IPJV versus VI

We now proceed by comparing vertical integration (VI) with IPJV scenarios.

Beginning with VI, we consider two (symmetric) VI rival firms, each comprising a U and a D activity. Firm i's profit is:

$$\pi_i = q_i p_i - f, \; i = 1, 2.$$

Quantity competition yields the customary symmetric equilibrium with the following price, quantity, individual and industry profits (superscript C stands for Cournot, subscript VI for vertical integration):

$$\begin{aligned} p_{VI}^C &= \frac{a}{2+b} \\ q_{VI}^C &= \frac{a}{2+b} \\ \pi_{VI}^C &= \left(\frac{a}{2+b}\right)^2 - f \\ \Pi_{VI}^C &= 2\left(\frac{a}{2+b}\right)^2 - 2f > 0 \\ \iff s = f/a^2 &< \frac{1}{(2+b)^2} \equiv s_{VI}^C(b), \end{aligned} \tag{9}$$

where $s = f/a^2$ is a measure of fixed cost vis à vis market size.

The *second* scenario is based on *IPJV*. D firms jointly own, on an equal foot, the independent U producer of the essential input, while competing among themselves in the D section. Both D firms get the input at the linear price w selected at the

[20] With linear pricing the U decision produces a negative externality for the D firms which is internalized as long as the profits in D are taken into account. This is **Governance 1** which yields a cartel outcome in the particular case of perfect symmetry. In **Governance 2** and **Governance 3**, U is an independent producer and does not take into account the negative externality. Therefore it sets a higher input price than the cartel. This leads to an equilibrium price (expression 3) which is higher than the monopoly price (expression 2).

[21] This assumption holds for the oligopoly case as well.

input stage by the (single) U producer that maximize profit, $\pi_U = w\left(q_i + q_j\right) - f$. As seen above, the input price is set at the monopoly level, $w_M = a/2$ and IPJV symmetric equilibrium magnitudes are given by (3) and (4) (labeled with subscript J):

$$\pi_J^C = \frac{a^2}{4(b+2)^2},$$
$$\pi_U^C = \frac{a^2}{2(b+2)} - f,$$

where π_J^C denotes the operative profit of each D firm. Industry profits are:

$$\begin{aligned} \Pi_J^C &= \frac{(b+3)a^2}{2(b+2)^2} - f > 0 \\ \iff & f/a^2 < \frac{(b+3)}{2(b+2)^2} \equiv s_J^C(b). \end{aligned} \tag{10}$$

There are two further possible scenarios. Both firms are vertically separated (VS). An enterprise is VI and the other is VS. The adoption of VI by both firms is a dominant strategy with respect to VS. To prove it, consider the game in normal form in Table 1 below.

	VI	VS
VI	$\frac{a^2}{(2+b)^2} - f, \frac{a^2}{(2+b)^2} - f$	$\frac{a^2(4+b)^2}{16(2+b)^2} - f, \frac{a^2}{4(2+b)^2}$
VS	$\frac{a^2}{4(2+b)^2}, \frac{a^2(4+b)^2}{16(2+b)^2} - f$	$\frac{4a^2}{(b-4)^2(2+b)^2}, \frac{4a^2}{(b-4)^2(2+b)^2}$

(Table 1)

Firm 1 is the row player and firm 2 is the column player. The matrix contains the equilibrium profits for the two firms associated to each possible outcome. Cell (VI,VI) represents the first scenario analyzed in this Section; (VS,VS) corresponds to a market configuration with two enterprises in U and two in D; (VI, VS) is a market with one VI firm and one VS. For each couple of strategies to be feasible we need a constraint on the fixed cost:[22]

$$f < \frac{a^2(2-b)}{8(2+b)}.$$

Given this restriction, (VI, VI) is the unique Nash equilibrium of this game. We thus exclude VS from the comparison with IPJV as it is always privately and socially dominated.

[22] This constraint comes from the non-negativity condition of profit for the U producer when one firm is VI and the other is VS.

Comparing the equilibrium values of VI and IPJV we write the following Proposition.

Proposition 1 Private and social efficiency of IPJV and VI in a Cournot duopoly.

a) Assume that input production does not require any fixed cost, i.e., $f = 0$*. In a Cournot duopoly, a producer of a final good is indifferent between vertically integrating and participating to an IPJV as long as the final goods are homogeneous. In contrast, when the final goods are horizontally differentiated, downstream firms strictly prefer VI. Consumers are always better off under VI, which turns out to be Pareto superior.*

b) Assume that the fixed cost in U is $f > 0$ *and let* $s = f/a^2$ *be a measure of fixed cost vis à vis market size. IPJV turns out to be privately preferred for large* s *(high relative fixed cost) and for high* b *(low differentiation). Consumers' preferences do not change with respect to point a). Social welfare is superior with VI for relatively low fixed costs, while, for high fixed commitment, IPJV is the only feasible arrangement and becomes socially superior by default.*

Proof. See the Appendix (8.1). ■

Discussion. Whenever the fixed cost of producing the essential input is sufficiently high, IPJV is privately more efficient. This becomes more likely as differentiation decreases. While the first effect is fairly obvious, the second is less clear-cut and points to the influence of differentiation on D competition. As $b \to 1$ industry profits "migrate" to U since the D section becomes more competitive driving down profits. The opposite occurs for VI which suffers from a tougher competition in D and does not benefit from any U profit buffer since it internally transfers the input at the marginal cost. In our duopoly scheme IPJV is socially efficient only when fixed costs are reasonably high even if it leads to a sort of U collusion coupled to double marginalization. This negative effect has to be contrasted with the wasteful duplication of fixed costs associated to VI.

The large area of private superiority of IPJV, even in the duopoly case, accounts for the observed diffusion of Equity Joint Ventures along the vertical chain of production and for the fact that firms fiercely competing in one market may show a tendency to collaborate in a related market.

4 The oligopoly setting

Now, we generalize the investigation conducted in the above section. The extension allows for a richer analysis with the introduction of mixed cases, not contemplated in the duopoly framework, with VI firms competing with rivals adopting

IPJV. These cases may shed further light on the stance of the U.S. Department of Justice and the U.S. Federal Trade Commission quoted in the introduction (Jacobson, 2007; p. 447) and contained in both the Guidelines on joint ventures (2000) and the new Guidelines on Horizontal Mergers (2010).

We survey three distinct scenarios.[23] In the *first*, all firms adopt *VI*, in the *second*, dubbed *partial IPJV*, some firms adopt VI, while others take part in IPJV, in the *third* all companies participate in IPJV making for *complete IPJV*.

First, complete VI. We have $n \geq 3$ (symmetric) firms each comprising the U and the D sections of the vertical production process. Equilibrium individual and aggregate profits, $\forall i = 1,...,n$, are:[24]

$$\pi_{iVI} = \left(\frac{a}{2+b(n-1)}\right)^2 - f,$$

$$\Pi_{VI} = n\left(\frac{a^2}{(2+b(n-1))^2} - f\right) > 0 \iff s = \frac{f}{a^2} < \frac{1}{(2+b(n-1))^2} \equiv s^{VI}(n,b). \tag{11}$$

Second, partial IPJV. There are $(n-k)$ D firms competing with k VI firms, while jointly owning the independent input producer which sets price w. The D firms' operative profits are:

$$\pi_{iD} = p_i q_i - w q_i, \quad i = 1,...,n-k$$

while the VI firms' profits are:

$$\pi_{jVI} = p_j q_j - f, \quad j = n-k+1,...,n$$

with $k \geq 1$ and $n > k$. Cournot competition leads to the following input price equilibrium:

$$\begin{aligned} w &= \arg\max_w \left(w \sum_{i=1}^{n-k} q_i - f\right) \\ &\iff w_P = \frac{a(2-b)}{4-2b(1-k)}. \end{aligned} \tag{12}$$

Remark *The price set by the partial IPJV, w_P, is independent of the number of firms in the industry, n, it is decreasing in the number of VI firms, k, and in the*

[23]These scenarios do not exhaust the set of possibile combinations of vertical arrangements. We discuss this issue in the next Section.

[24]For the sake of clearness, equilibrium magnitudes, which are not in the text, can be found in Appendix (8.2) for all vertical arrangements.

degree of product differentiation, b (the tougher the competition in D, the lower the input price).

Discussion. The effect on the input price of the increase in the number of VI firms runs contrary to the so called "raising rivals cost effect" due to Ordover et al. (1990). According to this result, the U division of the new vertical merger (the VI firm) may foreclose D rivals. A less competitive U market obtains with a consequent increase in the input price. In our framework, as the number of VI competitors goes up the input price set by the U division of the IPJV decreases. The intuition is that the IPJV faces a more competitive market due to the increase in the number of "efficient" VI firms, which get the input at its marginal cost. Then, the IPJV is bound to set a lower input price.

Equilibrium profits are, with $j = n-k+1,...,n$ and $i = 1,...,n-k$:

$$\pi_{jVI}(k) = \frac{a^2 (b(k+n-2)+4)^2}{4(b(k-1)+2)^2 (b(n-1)+2)^2} - f, \quad (13)$$

$$\pi_{iD} = \frac{a^2}{4(b(n-1)+2)^2},$$

$$\pi_U = \frac{(2-b)(n-k)a^2}{4(b(k-1)+2)(b(n-1)+2)} - f,$$

$$\pi_{cons}^{pJ} = \pi_{iD} + \frac{1}{n-k}\pi_U, \quad (14)$$

where π_{jVI} and π_{iD} are the profits of VI and D firms, respectively; π_{cons}^{pJ} is the consolidated profit of a firm participating in the (partial) IPJV. Industry profit is:

$$\Pi_{PJ}(k) = \frac{a^2\left(k^2b^2(b(n-1)+n+2)-k\left(b^3\left(n^2-1\right)-3b^2\left(n^2-2n-1\right)-12bn-4\right)\right)}{4(b(n-1)+2)^2(b(k-1)+2)^2} + \\ + \frac{a^2\left(n(b-2)^2(b(n-1)+3)\right)}{4(b(n-1)+2)^2(b(k-1)+2)^2} - f(k+1).$$

For future reference, industry profit with $k = 1$ is:

$$\Pi_{PJ}(1) = \frac{\left(-4bn+12n-b^2+4bn^2+2b^2n-b^2n^2+4\right)a^2}{16(bn-b+2)^2} - 2f > 0 \quad (15)$$

$$\iff s < \frac{\left(-4bn+12n-b^2+4bn^2+2b^2n-b^2n^2+4\right)}{32(bn-b+2)^2} \equiv s^{PJ}(n,b). \quad (16)$$

Third, complete IPJV. The n D companies build an Equity Joint Venture for the IPJV. The Equity Joint Venture is thus the unique input producer and sets the monopoly input price $w_M = a/2$. The equilibrium individual and industry profits, $\forall i = 1,...,n$, are:

$$\pi_{iD} = \frac{a^2}{4(bn-b+2)^2},$$
$$\pi_U = \frac{na^2}{4(bn-b+2)} - f,$$
$$\pi_{cons}^J = \pi_{iD} + \frac{1}{n}\pi_U, \tag{17}$$
$$\Pi_J = \frac{(b(n-1)+3)a^2n}{4(b(n-1)+2)^2} - f > 0 \iff s < \frac{(b(n-1)+3)n}{4(b(n-1)+2)^2} \equiv s^J(n,b). \tag{18}$$

Let us compare the distinct market and vertical arrangements seen above. We classify them according to the degree of downstream market competition measured by b and n, since, as b and n increase, competition in D becomes tougher.[25] To perform the comparison we split the feasible set of the differentiation parameter b into distinct areas which depend on n.[26] In what follows we abstract from fixed cost assuming $f = 0$ and we confine to a partial IPJV where a single VI firm, i.e., $k = 1$, competes with $(n-1)$ D companies which possess the IPJV. These two assumptions simplify the analysis without spoiling the results and basic intuitions. Later on, we will discuss extensions to $f > 0$ and $k > 1$.

By comparing industry profits in the three analyzed vertical arrangements, we get the following thresholds:

$$\Pi_{VI} - \Pi_J = \frac{(-b(n-1)+1)a^2n}{4(bn-b+2)^2} > 0 \iff b < \frac{1}{n-1} \equiv b^{PJ}(n) \tag{19}$$

$$\Pi_{VI} - \Pi_{PJ}(1) = \frac{\left(b^2(n-1)-4bn+4\right)(n-1)a^2}{16(bn-b+2)^2} > 0 \iff b < \frac{2\left(n-\sqrt{n^2-n+1}\right)}{n-1} \equiv b^{VI}(n) \tag{20}$$

$$\Pi_J - \Pi_{PJ}(1) = \frac{(bn-b-2)a^2}{16(bn-b+2)} > 0 \iff b > \frac{2}{(n-1)} \equiv b^J(n). \tag{21}$$

Simple algebra shows that $b^J(n) > b^{PJ}(n) > b^{VI}(n)$. From these comparisons we can derive the following:

[25] As b increases, products become closer substitutes and the market size (the total quantity) decreases (See Singh and Vives, 1984). As for the number of firms, an increase in n, which also defines the number of varieties, determines an increase of the market size (because of consumers' love for variety); however as firms' profits decrease with n, we can take n as another measure of competition.

[26] As standard in the oligopoly literature, we treat n as a real number. Clearly, we will take into account the integer problem when it is necessary.

Proposition 2 Private and social efficiency of complete IPJV, partial IPJV and VI.

Assume that the fixed cost of production is zero, i.e., $f = 0$ and that the partial IPJV is such that there is a single VI firm which competes with $(n-1)$ D companies, i.e., $k = 1$.

a *For sufficiently high levels of product differentiation, i.e., $b \in [0, b^{VI}(n))$, the private ranking is: VI $\succ$ partial IPJV $\succ$ complete IPJV;*

b *For upper intermediate levels of product differentiation, i.e.,$b \in \left(b^{VI}(n), b^{PJ}(n)\right)$, the private ranking is: partial IPJV $\succ$ VI $\succ$ complete IPJV;*

c *For lower intermediate levels of product differentiation, i.e., $b \in \left(b^{PJ}(n), b^{J}(n)\right)$, the private ranking is: partial IPJV $\succ$ complete IPJV $\succ$ VI;*

d *For low levels of product differentiation, i.e., $b \in (b^{J}(n), 1]$, the private ranking is: complete IPJV $\succ$ partial IPJV $\succ$ VI.*

e) As for the social welfare (SW) we have the following ranking: $SW_{VI} > SW_{PJ} > SW_{J}$, independently of b and n. However, for reasonably high fixed costs the social desirability of VI vanishes and (partial or complete) IPJV becomes the most desirable setting.

Proof. See Appendix (8.3). ■

Discussion. The above results emphasize the effect of competition, measured by b and n, on private (industry) preferences concerning vertical arrangements. As the degree of product differentiation decreases firms prefer to switch from VI to (partial or complete) IPJV. This result somewhat replicates and extends the duopoly outcome seen above. However, with more than two firms we are able to analyze the effect of n as well as the interaction between n and b. Since D competition gets fiercer as the number of firms goes up, only high levels of differentiation are able to preserve the private advantage of VI. On the contrary, under IPJV the D firms are able to “shift” to U the profit swept away by tougher downstream competition. With IPJV in U the D firms are able to compensate the lost profit in D with the monopoly profit obtained by the single independent U producer. The U section becomes a profit “reservoir” for firms bound to compete fiercely in D. If we turn to a different market structure with one VI firm, we have partial IPJV. In this case, the U market becomes more contestable since the VI firm makes its own input in-house and drives down the input demand faced by the incumbent IPJV enterprise. This translates into a lower w. Therefore, a VI firm selling in D provides an automatic policing of the U market. This is an external effect and it occurs even if the VI company does not sell any input to the rival D firms, which keep on buying the input exclusively from the IPJV.

The limitation of the analysis to zero fixed costs is adopted for a neater investigation of the effects of D market structure (n, b) on the private ranking of the three distinct vertical arrangements. With positive fixed cost the qualitative results in oligopoly do not change as far the effects of b and n are concerned. Nonetheless, a positive f increases the likelihood of the adoption of IPJV, making this arrangement privately more desirable, since average fixed costs for individual firms go down with respect to the VI case. This effect has been properly investigated in Proposition **1**. As far as social welfare is concerned, positive fixed costs affect the ranking and make IPJV the most desirable arrangement.[27]

Consider now the case of $k \in \{2,3,...,n\}$. As $k \to n$, partial IPJV tends to disappear as the market is going to be made entirely by VI firms. Notice that the thresholds $b^{VI}(n)$ and $b^{J}(n)$, which define respectively the lower and the upper limit of the area where partial IPJV is the preferred setting, now depend also on k. Moreover, $\partial b^{J}/\partial k \leq 0$, as it can be easily verified.[28] Further, as k increases, industry profits of partial IPJV decrease, whereas those of VI do not change, making for an indirect proof that $\partial b^{VI}/\partial k > 0$. Therefore, in the limit, $k \to n$, the interval $\left(b^{VI}(n), b^{J}(n)\right)$ will vanish: as the number of VI firms (k) in the partial IPJV configuration increases, the likelihood that partial IPJV is the most preferred setting tends to zero.

5 Incentives to break the IPJV

In Proposition 2 we have analyzed industry preferences for the three vertical arrangements. However, a frequently observed weakness of a joint venture (JV) is the inability to last due to incentives to walk away. Some JVs survive, some JVs suffer disengagement of one or more members, some JVs may split. The issue of disengagement and/or falling apart of JV is widely analyzed in the managerial literature on JV (Hewitt, 2008).

Starting from the complete IPJV scenario, we evaluate the incentives to quit an IPJV. The outside options for firms willing to leave the joint venture are the following.

- *Disengaging:* some firms, once acquired the proper know-how, decide to disengage and produce the essential input in-house (VI).
- *Splitting:* the unique IPJV splits into many IPJVs.

[27] In most received literature RJV is deemed superior because of internalization of spillovers, i.e., externalities. Here, we do not introduce any external effect in the input production. Were there technology external effects, they would add to the private benefits of IPJV and have an impact similar to the saving of fixed costs that IPJV brings about. In all these cases we may see areas of social preference.

[28] The threshold would be: $b^{J}(n,k) = \frac{4}{n-1+\sqrt{(n-1)(n+8k-9)}}$ whose limit for $k \to n$ is $\frac{1}{n-1} = b^{PJ}(n)$.

- *Vertical separation:* some firms decide to quit and, simultaneously, new upstream firms enter the market for the input.

As far as disengagement is concerned, we have seen the equilibrium outcome in the above Section. It coincides either with the partial IPJV scenario, when only some firms leave the IPJV and vertically integrate, or with the complete VI scenario, when all firms turn to VI. As for vertical separation (VS), it does not seem a genuine option for the D firms, since VS implies the simultaneous occurrence of two events: the entry of a new U firm which is not under the control of the D firm and the quitting of the IPJV by one D firm. In that rare case the D firm would buy the essential input at a high price (because of market power in U) without enjoying part of the U operative profits.[29] As for splitting, we analyze in the following subsection the equilibrium outcome with a general number of IPJVs. We conclude the Section making the proper comparisons of the incentives to break the IPJV either by disengaging or by splitting.

5.1 Splitting

Consider a general number of IPJVs. Assume there are $v \in [2,n]$ firms in each IPJV so that the number of IPJV in the market is n/v. Notice that for at least two IPJVs to exist we need $n \geq 4$. The equilibrium input price is then:

$$w_J(b,n,v) = \frac{a(2-b)}{4+b(n-2-v)} > 0,$$

decreasing in b and n and increasing in v (as $v \to n$, we get the monopoly price). Equilibrium consolidated and industry profits are (index S stands for splitting):

$$\pi^s_{cons}(v) = \frac{\left(5b-3bn+bv-b^2+b^2n-6\right)(b-bn+bv-2)a^2}{(2b-bn+bv-4)^2(bn-b+2)^2} - \frac{f}{v} \tag{22}$$

$$\Pi_S(v) = n\frac{\left(5b-3bn+bv-b^2+b^2n-6\right)(b-bn+bv-2)a^2}{(2b-bn+bv-4)^2(bn-b+2)^2} - \frac{n}{v}f. \tag{23}$$

Industry profits are n times the consolidated profit accruing to each firm. Consolidated profits change with the number of IPJV members as follows:

$$\frac{\partial}{\partial v}\pi^s_{cons} = (b-2)\,a^2 b\frac{\left(b^2(n-1)(v-n)-2b+4\right)}{(b(n-v-2)+4)^3(bn-b+2)^2} > 0$$

$$\iff b > b_2(v) \equiv \frac{\left(-1+\sqrt{(4(n-1)(n-v)+1)}\right)}{(n-v)(n-1)} \in (0,1).$$

[29] It is easily proved that there is no incentive to leave the IPJV and vertically separate (alone or with other firms). If a deviation occurs from the complete IPJV, it is either by disangaging or by splitting. In Appendix (8.5), we formally prove this statement for the four-firm oligopoly case.

These considerations prove the following.

Lemma 3 *Consolidated profits are U shaped with respect to the number of members: they are decreasing when goods are sufficiently differentiated (competition in D is mild), they are increasing when goods are close substitutes.*

Then, it appears that for b low (highly differentiated goods) the lower the number of IPJV members, the better: the extreme scenario is with $v = 2$. In contrast, for b high, when goods are close substitutes, complete IPJV ($v = n$) is preferable with respect to *splitting* into more than one IPJV.

5.2 Splitting or disengaging? Some comparisons

We now investigate the incentives to leave the unique IPJV, abstracting from the fixed cost.

First, consider the individual profitability from leaving the IPJV and vertically integrate when all firms disengage. Formally, compare the extreme cases of VI and complete IPJV. As they represent symmetric market structures, individual and industry preferences coincide. Therefore, we may write:

$$\pi_{iVI} - \pi^{J}_{cons} > 0 \iff b < b^{PJ}(n) \tag{24}$$

where $b^{PJ}(n)$ is defined in (19) and is decreasing in n. From (24) we see that there is an incentive for all firms involved in IPJV to leave the joint venture and become VI, provided b is sufficiently low. Divorce becomes more likely the lower is the number of firms in the market (as b^{PJ} is decreasing in n). In other words, the incentive to quit the IPJV plot and become VI is higher when the market is made by few firms and/or the degree of differentiation is high (low b). In these circumstances IPJV turns out to be quite fragile and bound to fall apart due to pressing private incentive to disengage.[30]

Next, consider the individual profitability from splitting the IPJV (again symmetric framework). Therefore, we compare consolidated profit under complete IPJV (defined in 17) with consolidated profit under splitting (defined in 22):

$$\pi^{s}_{cons}(v) - \pi^{J}_{cons} > 0 \iff b < b_S(v) \equiv \frac{\left(n-v-4+\sqrt{(v-33n+40)(v-n)+16}\right)}{2(n-v)(n-1)} \tag{25}$$

A unique IPJV is more profitable than many IPJVs as long as goods are close substitutes. We see that for b sufficiently high, there is no incentive for splitting. In line

[30]These conclusions hold for zero fixed costs. Strictly positive fixed costs erode the incentives to disengage from IPJV.

with the previous comparison of IPJV vs. VI, we find that the tougher competition in D, the higher the incentive to form and preserve an IPJV as large as possible (complete IPJV). There is a trade-off between a lower input price under splitting (where there are n/v U producers) and a higher U-profit cushion under complete IPJV. The balance of this trade-off depends on b. Notice that this result holds as long as fixed cost are very low ($f \to 0$), which is the worst scenario for complete IPJV since an advantage of this vertical arrangement is fixed costs saving. The profitability of complete IPJV clearly increases in the presence of positive fixed cost.

Now, consider firms' preferences over complete VI and splitting:

$$\pi_{iVI} - \pi_{cons}^{s}(v) > 0 \iff b < b_{VIS}(v) \equiv \frac{\left(-n+\frac{1}{2}+\frac{1}{2}\sqrt{4(n(3n-5)-2v(n-1))}\right)}{(n-1)(n-v-1)} \tag{26}$$

Complete VI dominates splitting for b sufficiently low (from both the individual and the industry point of view).

The *intermediate situation* of partial IPJV features two types of actors, the VI firm and the D firms, owners of the independent IPJV. Simple computations show that the VI firm is worse off than the $(n-1)$ D firms if fixed costs are high enough. If we abstract from fixed costs, the VI firm enjoys a variable cost advantage (input price) with respect to the D firms. Therefore, it holds a larger market share allowing for higher profits. However, if each company in the IPJV adopts the VI arrangement, the advantage of the incumbent VI firm (the n^{th} enterprise in the market) fades away as the equation below shows:

$$\pi_{nVI}(1) - \pi_{iVI} = \frac{(bn-b+8)(n-1)a^2 b}{16(bn-b+2)^2} > 0,$$

where $\pi_{nVI}(1)$ is defined in (13) with $j=n$ and $k=1$. The above positive difference defines the loss of profit of the incumbent VI firm when remaining firms turn to VI. As for the D firms, they gain from disengaging only in some areas of parameters b and n. To verify this we compute the difference representing the incentive to disengage of the D firms belonging to the (partial) IPJV plot. This difference is given by the following:

$$\pi_{iVI} - \pi_{cons}^{pJ} > 0 \iff b < \frac{(n-2)-\sqrt{\left(n^2-6n+6\right)}}{(n-1)} \equiv b_1(n), \tag{27}$$

which is a real number only for $n > 4$.

Finally, compare splitting with disengaging (partial IPJV). From the viewpoint of the unique VI firm, we compare the profit from splitting (defined in 22) with the profit from leaving the complete IPJV to go VI (defined in 13), i.e.,

$$\pi_{nVI}(1) - \pi^{s}_{cons}(v).$$

This difference, function of $v = \frac{n}{i}$ for $i = 2, 3, 4, ..,$ is always positive. Then, *a firm leaving the unique IPJV always prefers vertical integration with respect to splitting*. From the viewpoint of the IPJV members, we compare the profit from splitting (defined in 22) with the profit from remaining in the partial IPJV when one firm disengages (defined in 14), i.e.,

$$\pi^{s}_{cons}(v) - \pi^{pJ}_{cons}.$$

Simulations show that, *for IPJV members, splitting is preferred to disengaging for b low as well as for b high and n low enough. In contrast for b and n high, disengaging is more profitable than splitting also for the IPJV members*.[31] Finally, we compare industry profits under partial IPJV (defined in 15) with those under splitting (defined in 23), i.e.,

$$\Pi_{PJ} - \Pi_{S}(v).$$

Simulations show that *industry profits are higher under disengagement (partial IPJV with only one VI firm) than under splitting for n not too low*. This holds for any level of b (the higher b the lower the threshold for n, the minimal threshold is $n = 5$ for $b = 1$).[32] This means that disengagement dominates splitting as long as n is sufficiently high. This preference increases with b. In contrast splitting is more likely to dominate for b (and n) low. The partial dominance of disengagement over splitting (i.e., multiple IPJV) could make for an investigation limited to disengagement. Even if confined to a limited parameter set, this outcome reflects the popularity of disengagement in the management literature (Hewitt, 2008) which, on its turn, is the sign of much more frequent occurrence of it in everyday industry life.

From the above comparison of individual incentives we can summarize firms' preferences over distinct vertical arrangements. Considering $v = \frac{n}{i}$ for $i = 2, 3, 4, ..$ the ranking for the thresholds of b is:

$$b_{VIS}(v) < b^{VI}(n) < b_1(n) < b^{pJ}(n) < b^{J}(n) < b_S(v) < 1.$$

Then, we can write the following Lemma:

[31] See Appendix (8.4).

[32] See Appendix (8.4).

Lemma 4 *Assume that the fixed cost of production is zero, i.e., $f = 0$ and that partial IPJV is such that there is a single VI firm which competes with $(n-1)$ D companies, i.e., $k = 1$.*

i) For $n = 3,4$ the D firms have a positive incentive to leave the IPJV plot since they reap larger profits if they disengage, $\forall b$, and disengagement always dominates splitting (for $n = 4$). For sufficiently low levels of product substitutability, i.e., $0 < b < b^{VI}(n)$, the VI firm has no incentive to stop disengaging. For higher levels of product substitutability, i.e., $b > b^{VI}(n)$, complete disengagement may not occur since the existing unique VI firm can prevent disengagement by compensating D rivals. This is feasible since the loss of the VI firm would be higher than the gain the D firms would get if they turn to VI.

ii) For $n = 5,6...$ firms may have an incentive to leave the IPJV, either vertically integrating (disengagement) or splitting. For sufficiently low levels of product substitutability, i.e., $0 < b < b_{VIS}(v)$, all D firms disengage. For $b_{VIS}(v) < b < b^{VI}(n)$, splitting takes place. For $b^{VI}(n) < b < b_S(v)$, firms' preferences over splitting versus disengagement depend on v and n. Finally for $b_S(v) < b < 1$ there is no incentive to leave the unique IPJV.

iii) For n high enough, disengagement always dominates splitting which never takes place. For sufficiently low levels of product substitutability, i.e., $0 < b < b^{VI}(n)$, the D firms disengage. For $b^{VI}(n) < b < b_1(n)$, the VI company can halt disengagement since industry profits of partial IPJV are larger than those of complete VI. Finally for $b_1(n) < b < 1$ there is no lure to disengage.

Proof. See Appendix (8.4). ■

From the considerations above and from Lemma 4, we can derive the following proposition which provides a taxonomy of subgame perfect Nash equilibria (SPNE) related to different vertical arrangements.

Proposition 5 *Assume that the fixed cost of production is zero, i.e., $f = 0$, and that partial IPJV is such that there is a single VI firm which competes with $(n-1)$ D companies, i.e., $k = 1$.*

For low product market substitutability, i.e., $b < b_{VIS}(v)$, the adoption of VI by all firms is a SPNE. For high product market substitutability, i.e., $b > b_S(v)$, the adoption of IPJV by all firms is a SPNE. For intermediate levels of product substitutability deviations from complete IPJV occur either by disengaging or by splitting. Whenever partial IPJV dominates splitting (multiple IPJVs) from the industry point of view, partial IPJV turns out to be a SPNE if a mechanism is set up whereby the VI firm(s) compensates the D firms lest they quit the IPJV.

Proof. See Appendix (8.6). ■

Discussion. From the analysis conducted above, we see that, for extreme values of product differentiation, firms' preferences are clear. All firms disengage and vertically integrate under mild competition (b low enough) as they get high profit and enjoy an input cost as low as possible (because they produce the input in-house). On the contrary, all firms join the unique IPJV under tough competition (b high enough) as they benefit from the U-profit cushion coming from the U monopolist which compensates for the low D profit. For intermediate levels of product differentiation, different combinations of splitting and disengagement can arise in equilibrium. Indeed, we do not cover the entire set of infinite possibilities arising from all the combinations of these vertical arrangements.[33] Nevertheless, our analysis makes the point. IPJV (complete or partial) allows firms to recoup in the U section of production the profits lost in D. On the other hand, splitting into more than one IPJV and VI allow firms to get the essential input at a lower price. The balance between these opposite forces depends on the intensity of competition in D.

Notice that, whenever disengagement dominates splitting, for intermediate levels of product substitutability, partial IPJV persists since the VI firm has an incentive to stop disengagement by other firms since the industry profits of partial IPJV are larger than those of complete VI. Thus, our setting is consistent with what maintained by the so called ”facilitating collusion argument” according to which the presence of VI firms makes upstream collusion easier (Riordan, 2008).

The existence of a large set of Nash equilibria witnesses both the private efficiency of IPJV and its ability to survive. However, large areas of incentives to quit and a simple incentive compatible mechanism explains the high frequency of divorces recorded in the management literature, in joint ventures of all kinds, IPJV included and, may be, also the justification of the mild stance of antitrust agencies.

6 Extensions

We extend the analysis on IPJV to Bertrand competition and market demand uncertainty.

6.1 Bertrand versus Cournot competition

We confine to the duopoly case and investigate the profitability of VI vs. IPJV when firms compete à la Bertrand. Then, we compare the results with Cournot.

[33]As an illustrative example, we go through all vertical arrangements in the particular case of four-firm oligopoly (Appendix 8.5).

Considering the same linear inverse demand system, $p_i = a - q_i - bq_j$, the demand schedule, for $b \neq 1$, becomes $q_i = \alpha - \beta p_i + \delta p_j$, with

$$\alpha = \frac{a}{1+b}, \beta = \frac{1}{(1-b)(1+b)}, \delta = \frac{b}{(1-b)(1+b)}.$$

Under VI, price competition yields the following symmetric equilibrium (B superscript stands for Bertrand):

$$p_{VI}^{B} = \frac{a(1-b)}{(2-b)}$$
$$\pi_{VI}^{B} = \frac{a^2(1-b)}{(b-2)^2(b+1)} - f$$
$$q_{VI}^{B} = \frac{a}{(2-b)(b+1)},$$

so that industry profits are $\Pi_{VI}^{B} = 2\frac{a^2(1-b)}{(b-2)^2(b+1)} - 2f$, with:

$$\Pi_{VI}^{B} > 0 \iff \frac{f}{a^2} < \frac{(1-b)}{(b-2)^2(b+1)} \equiv s_{VI}^{B}(b). \quad (28)$$

Under IPJV, the equilibrium magnitudes are:

$$p_{J}^{B} = \frac{a(3-2b)}{2(2-b)}$$
$$\pi_{J}^{B} = \frac{a^2(1-b)}{4(b-2)^2(b+1)}$$
$$\pi_{U}^{B} = \frac{a^2}{2(2-b)(b+1)} - f$$
$$q_{J}^{B} = \frac{a}{2(2-b)(b+1)},$$

so that aggregate profits are $\Pi_{J}^{B} = \frac{(3-2b)a^2}{2(b-2)^2(b+1)} - f$, with:

$$\Pi_{J}^{B} > 0 \iff \frac{f}{a^2} < \frac{(3-2b)}{2(b-2)^2(b+1)} \equiv s_{J}^{B}(b). \quad (29)$$

Comparing the equilibrium values of the two vertical arrangements with distinct competition modes, we obtain the following result:

Proposition 6 Bertrand duopoly, IPJV comparison, social desirability of competition mode

a) (Bertrand) With Bertrand competition in the D market, if $f = 0$, *IPJV is privately preferred to VI as long as* $b \in [\frac{1}{2}, 1)$. *Social welfare is superior with VI, for relatively low fixed cost, whereas for relatively high fixed cost, VI is no longer feasible and IPJV becomes socially superior by default.*

b) (IPJV comparison) IPJV under Bertrand competition yields larger quantities, lower D prices, higher U profits and lower D profits than under Cournot competition, i.e., $q_J^B > q_J^C$, $p_J^B < p_J^C$, $\pi_U^B > \pi_U^C$ *and* $\pi_J^B < \pi_J^C$*; industry profits and consumer surplus are higher under Bertrand.*

c) (Social desirability of competition mode) For low product substitutability, $b \leq 1/2$*, under Bertrand, VI is adopted and is socially preferred to any Cournot competition vertical arrangement (VI or IPJV). In contrast, for high product substitutability,* $b > 1/2$*, and low relative fixed cost, IPJV is chosen with Bertrand, while VI prevails with Cournot competition which achieves social superiority.*

Proof. See Appendix (8.7). ■

Discussion. The relative profitability of IPJV vis à vis VI increases when we go from Cournot to Bertrand competition mode, regardless of fixed cost. Under Bertrand competition, IPJV is strictly preferred to VI when goods are sufficiently substitutable even with zero fixed cost. Positive fixed cost clearly reinforces the private advantage of IPJV.

With Bertrand competition the D externality reshapes the distribution of profits along the vertical chain.[34] The U section becomes more profitable and the D section less profitable vis à vis Cournot. Overall Bertrand competition yields bigger industry profits, due to the sale of a larger quantity that drives up U profits, overcompensating for the squeeze in D. As a result, Bertrand competition is socially preferred to Cournot under IPJV.

More precisely, Bertrand always ensures a higher welfare than Cournot with both vertical arrangements, i.e., $SW_{VI}^B > SW_{VI}^C$ and $SW_J^B > SW_J^C$. Nevertheless, while in Cournot private and social preferences coincide for any b, (VI is always better than IPJV), in Bertrand they diverge for $b \in (1/2, 1)$. In this range firms strictly prefer to coordinate on IPJV which is the outcome under Bertrand competition. As a consequence, Bertrand is socially desirable for $b < 1/2$, while Cournot for $b > 1/2$, (as $SW_{VI}^C > SW_J^B$). In this static setting traditional conclusions about price and quantity competition (Singh and Vives,1984) may be reversed. This is in line with Arya, *et al*. (2008) who maintain that only if we allow for the selling of input

[34] We refer to the externality associated to linear pricing, whereby a lower D price increases U profit.

by a VI firm to D competitors we may have the same social ranking of Cournot vs. Bertrand.

6.1.1 Private and social choices

In some industries firms have the option of selecting the competition mode. In other sectors this opportunity is absent, for instance, because supply is capacity constrained in the short - medium run. In regulated sectors, market authorities may resolve to push firms to a specific mode of competition, excluding the option of adopting the privately preferred control. In other surveilled sectors, like private and/or public utilities, market authorities may ask for a specific vertical arrangement, requiring either dismissals or serving specific firms and customers in the intermediate section of the vertical chain of production.

On the trace of these considerations, it may be worth to list private and public incentives when firms and/or market authorities choose both competition mode and vertical arrangement. To this purpose, we broaden the investigation on the private and social preferences for competition mode and vertical arrangements by comparing, in the next proposition, industry profit and social welfare for the four combinations, VI and IPJV under Cournot and under Bertrand competition. To this purpose we define:

$$\begin{aligned} s^C(b) &\equiv \frac{\left(2-2b^2+b\right)(2-3b)}{2(b+2)^2(b-2)^2(b+1)}, \\ s_1^W(b) &\equiv \frac{(5-4b)}{4(b-2)^2(b+1)}, \\ s_2^W(b) &\equiv \frac{\left(10-3b-2b^2\right)\left(2-2b^2+b\right)}{4(b+2)^2(b-2)^2(b+1)}. \end{aligned} \tag{30}$$

Proposition 7 Private and social desirability of competition modes and vertical arrangements.

a) Assume $f=0$. For high levels of product differentiation, $b\in(0,1/2)$, industry profit ranking is the following: $\Pi^C_{VI}>\Pi^B_{VI}>\Pi^B_J>\Pi^C_J$. For upper intermediate product differentiation, $b\in(1/2,0.56)$, the ranking becomes $\Pi^C_{VI}>\Pi^B_J>\Pi^B_{VI}>\Pi^C_J$. For lower intermediate product differentiation, $b\in(0.56,2/3)$, the ranking becomes $\Pi^C_{VI}>\Pi^B_J>\Pi^C_J>\Pi^B_{VI}$. Finally, for low product differentiation, $b>2/3$, we have $\Pi^B_J>\Pi^C_{VI}>\Pi^B_{VI}>\Pi^C_J$. On the contrary, the social ranking is $SW^B_{VI}>SW^C_{VI}>SW^B_J>SW^C_J$ independently of b. Then, social and private desirability of the combined competition and vertical choice never match.

b) Assume $f > 0$ and define $s = f/a^2$ as a measure of relative fixed cost. For $s < s^C(b)$ the private ranking for high and intermediate differentiation $(b < 2/3)$ assigns the top position to Π^C_{VI}. For $s \in \left(s^C(b), s^B_{VI}(b)\right)$ the top position in the private ranking is taken by Π^B_J. If we move to $b > 2/3$ the top position is taken by Π^B_J for any feasible s. The social ranking does not change with respect to a) as long as $s < s^B_{VI}(b)$. However, for $s > s^B_{VI}(b)$, VI under Bertrand competition is no longer feasible and the social welfare ranking is as follows: $SW^C_{VI} > SW^B_J > SW^C_J$ for $s \in \left(s^B_{VI}(b), \min\{s^C_{VI}(b), s^W_2(b)\}\right)$; for $s > \min\{s^C_{VI}(b), s^W_2(b)\}$, $SW^B_J > SW^C_{VI} > SW^C_J$ as long as VI under Cournot competition is feasible, i.e., for $s < s^C_{VI}(b)$, otherwise $SW^B_J > SW^C_J$.

Proof. See Appendix (8.8). ■

Discussion. In the proposition above we privately and socially compare the joint choice of market competition mode and vertical setting in a duopoly. The social planner always prefers Bertrand competition and VI for any b and s. On the contrary, firms prefer Cournot and VI if $b < 2/3$ and low s, while for $b > 2/3$ they rather choose IPJV and Bertrand. The implication is that private and social preferences never match as long as all the four vertical arrangements are feasible. Only for relatively high fixed costs, i.e., $s > s^B_{VI}(b)$, private and social incentives coincide.

6.2 Profit volatility of VI and IPJV

Since the rationale behind many joint ventures is risk sharing, a further intriguing question may regard the relative preference for distinct vertical arrangements under uncertainty. Then, we inquire into the effect of uncertain market demand for final goods on IPJV desirability. The answer comes from a simple extension to a triopoly framework with one VI firm competing with two D firms which jointly own, on an equal foot, an independent input producer (partial IPJV). We enrich the model considering both Cournot and Bertrand competition.

We adopt the following inverse demand functions:[35]

$$\begin{aligned} p_1 &= a - q_1 - b(q_2 + q_3) + e \\ p_2 &= a - q_2 - b(q_1 + q_3) + e \\ p_3 &= a - q_3 - b(q_1 + q_2) + e \end{aligned} \tag{31}$$

where e is the additive random shocks with $E(e) = 0$, and $E\left(e^2\right) = \sigma^2$.[36]

[35] As in Klemperer and Mayer (1986).

[36] A deeper investigation of risk sharing requires the introduction of idyosyncratic shocks. We leave this research avenue for future work.

We begin with Cournot competition. D firms maximize expected profits selecting a quantity to which they will stick regardless of the realization of the stochastic shock. This quantity is anticipated by U, which chooses the profit maximizing input price w. This price is deterministic, i.e., independent of e. Given the optimal w, parents D realized profits and prices depend on the stochastic shock. As usual, the VI firm maximizes expected profit of the entire vertical chain of production. The set of equilibrium realized profits is:

$$\pi_D^C = \frac{a(a+4(1+b)e)}{16(1+b)^2}$$

$$\pi_U^C = \frac{a^2\,(2-b)}{8\,(1+b)}$$

$$\pi_{cons}^C = \pi_D^C + \frac{\pi_U^C}{2}$$

$$\pi_{VI}^C = \frac{a(b+2)(2a+ab+4e(1+b))}{16(1+b)^2}$$

As it can be seen, only π_U^C is certain, while π_D^C, π_{cons}^C and π_{VI}^C are affected by uncertainty. If we take expectations we see that all expected profits are equal to certainty profits of the corresponding triopoly case. Therefore, the private rankings do not change with respect to what seen above in the certainty case. However, volatility is not irrelevant. If we compare the variance of π_{VI}^C with that of π_{cons}^C, we discover that the former is more volatile than the latter:

$$var\left(\pi_{VI}^C\right) = \frac{a^2(2+b)^2}{16\,(1+b)^2}\sigma^2 > var\left(\pi_{cons}^C\right) = \frac{a^2}{16\,(1+b)^2}\sigma^2.$$

This means that profit volatility of the IPJV is lower than that of VI. The U joint venture is not affected by uncertainty and therefore its profit provides a sort of cushion against risk for D firms which own U. On the contrary the VI firm does not enjoy this pillow of sure profit and therefore shows higher volatility. IPJV does not generate any direct risk sharing along the vertical chain, since the U joint venture is somehow isolated. Then, parent D firms bear all risk while enjoying the safety cushion of U sure profits.

A different story can be told when firms compete à la Bertrand, while facing the same kind of demand uncertainty. In that case the equilibrium profits are:

$$\pi_D^B = \frac{a(1-b)(a+ab+4e)}{16(2b+1)}$$
$$\pi_U^B = \frac{a(1-b)(2+3b)(a+ab+4e)}{8(1+b(3+b-2b^2))}$$
$$\pi_{cons}^B = \pi_D^B + \frac{\pi_U^B}{2}$$
$$\pi_{VI}^B = \frac{a(1-b)\left(2-b^2+3b\right)\left(a(b+1)\left(2-b^2+3b\right)+4e\left(1-b^2+b\right)\right)}{16\left(b^2-b-1\right)^2(2b+1)}$$

As far as the different levels of volatility of the two arrangements are concerned, we can write the following variances:

$$var\left(\pi_{VI}^B\right) = \frac{a^2(1-b)^2((b-3)b-2)^2}{16(1+b(3+b-2b^2))^2}\sigma^2 < var\left(\pi_{cons}^B\right) = \frac{a^2(1-b)^2((b-4)b-3)^2}{16(1+b(3+b-2b^2))^2}\sigma^2.$$

It appears that the ranking of volatility under Bertrand competition is reversed. Risk sharing along the vertical chain in the IPJV is quite balanced since both U and D profits are affected by uncertainty and, therefore, they share the burden of volatility of IPJV.

On the basis of these considerations, we may write the following result.

Proposition 8 Effect of demand uncertainty on IPJV

Additive demand uncertainty does not change the ranking of private and social preference of IPJV in both Cournot and Bertrand competition with respect to certainty. However, risk affects the relative volatility of the two vertical arrangements and risk sharing along the vertical chain: under Cournot competition IPJV is less volatile than VI and the entire market risk is born by D firms, while under Bertrand competition the IPJV is more volatile and risk is shared between U and D firms.

Discussion. The above result underlines the main goals of IPJV, i.e., risk sharing along the vertical chain of production. IPJV changes the distribution of risk between U and D according to whether Bertrand or Cournot is adopted. This may make a competition mode the preferred one thanks to the more comfortable risk allocation it provides. With Cournot the IPJV provides a safe heaven for D consolidated profits, while with Bertrand risk is spread along the vertical chain between U and Ds.

7 Conclusion

In these pages we have conducted an inquiry into the social and the private desirability of the Input Production Joint Venture (IPJV), an intermediate semi-collusive organizational setting lying between the two extremes of vertical integration and vertical separation. As narrated in the management literature, this kind of joint venture is widely adopted in many industries and seen with favour by most competition authorities and, in particular, by the U.S. Department of Justice and the Federal Trade Commission, as it appears in the Antitrust Guidelines for Collaborations among Competitors (2000) and the recent Guidelines on Horizontal Mergers (2010). IPJVs do not raise competitive concerns from antitrust authorities thanks to economies of scale and synergies they may bring about, which enjoy an assessment mostly based on the *rule of reason*. Yet, IPJV is definitely a form of partial collusion and, as far as we know, there are not many robust results to sustain the benign stance of antimonopoly institutions unless fixed costs saving are quite high and there is a clear benefit to consumers.

When compared to vertical integration, in several circumstances, IPJV turns out to be privately desirable but inefficient from a social point of view, even if it avoids wasteful duplication of fixed cost. Even though our approach is confined to linear pricing, it seems that IPJV should be considered somewhat more cautiously since it represents a kind of partial collusion in the U section and its social desirability cannot be taken for granted, unless society places a heavy emphasis on secondary effects of fixed costs, for instance, for environmental reasons. We have shown that firms' incentives to form an IPJV increase with the degree of downstream market competition. The intuition is that with an IPJV the U section of production may provide a kind of profit "reservoir" for firms. In this sense a more general message may come from our analysis maintaining that fierce competition in a market induces a genuine lure to look for collaborations in a closely related market.

A partial justification of the mild stance of antitrust agencies towards IPJVs may spring from the short life expectancy of these collaborations, since a high rate of early divorces occurs among firms engaging in IPJV (Hewitt, 2008). We have tried to interpret this widely observed stylized fact and we have gone through the issues of disengagement, i.e., quitting, and of splitting the IPJV. By investigating market structures where firms doing IPJV compete with vertically integrated rivals, we find that, if product market substitutability is low, there is an incentive for firms doing IPJV to walk away and turn to vertical integration. However, this incentive could be (privately) efficiently neutralized via a transfer mechanism whereby incumbent vertically integrated firms convince IPJV members to stay in. A strong incentive for IPJV to last exists when downstream market competition is fierce since, in that case, the only shelter for low differentiated firms is to collaborate in the upstream section. A further privately desirable feature arises under uncertainty since IPJV may provide different hedging opportunities both with Cournot and Bertrand competition.

All these arguments may explain why the antitrust agencies have adopted a stance which looks quite temperate at first sight, mostly based on the *rule of reason* and quite immune from *per se* evaluations.

8 Appendix

8.1 Proof of Proposition 1

a) Assume first that $f = 0$. Profits' comparison gives:

$$\Pi_J^C - \Pi_{VI}^C = (b-1)\frac{a^2}{2(b+2)^2} \leq 0.$$

As for the consumer surplus, it is higher under VI, where the equilibrium price is lower, i.e., $p_J^C - p_{VI}^C = \frac{a(b+1)}{2(b+2)} > 0$. Therefore, VI is privately and socially preferred.

b) Suppose now $f > 0$, the previous comparison becomes:

$$\begin{aligned} \Pi_J^C - \Pi_{VI}^C &= \frac{f\left(8b+2b^2+8\right) - a^2(1-b)}{2(b+2)^2} \geq 0 \\ \iff& \frac{f}{a^2} \geq \frac{(1-b)}{2(b+2)^2} \equiv \bar{s}(b) \end{aligned} \tag{32}$$

where $s \equiv f/a^2$ is a relative measure of fixed cost with respect to market size. As for social welfare (SW) we have to compare the sums of consumer surplus and industry profits in the two cases (VI and IPJV). Straightforward calculations lead to the definition of the following threshold:

$$\widetilde{s}(b) = \frac{(5+b)}{4(b+2)^2}, \tag{33}$$

below which the SW of VI is larger than the SW of IPJV. Given that

$$\widetilde{s}(b) - s_{VI}^C(b) = \frac{(b+1)}{4(b+2)^2} > 0$$

where $s_{VI}^C(b)$ is defined by (9). We conclude that, in the feasible set of parameters, VI is always socially preferred.

For the sake of easier understanding, we plot (9), (10), (32) and (33) in the plane (b,s) in Figure 1 below.[37] The upper solid line defines $s_J^C(b)$ above which neither vertical arrangement is feasible. The intermediate solid line defines $s_{VI}^C(b)$ below which VI is feasible. The lower solid line defines $\bar{s}(b)$ above which IPJV is privately preferred to VI, while below this line VI is preferred. The dashed line represents the SW frontier, $\widetilde{s}(b)$ below which VI is socially preferred to IPJV.

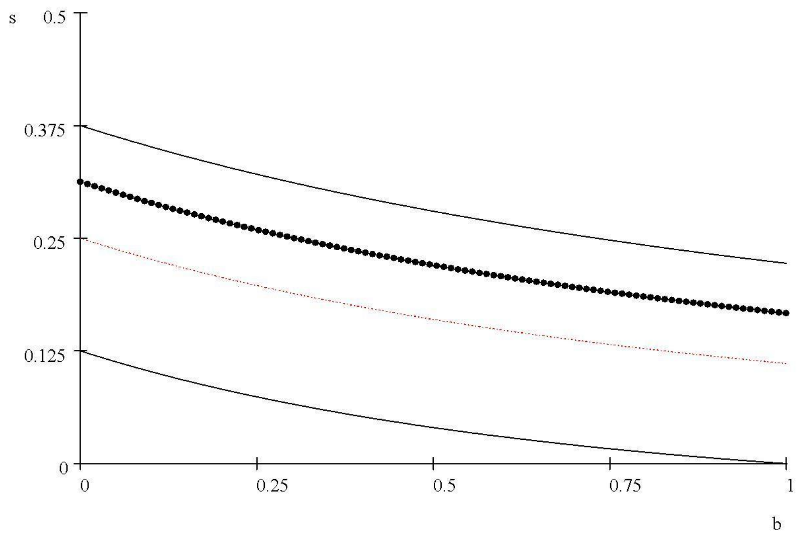

Figure 1: Threshold lines for private and social ranking of IPJV vs VI.

8.2 Equilibrium magnitudes

We report here equilibrium magnitudes that are not in text.

Equilibrium quantity and price under VI, $\forall i = 1, ..., n$, are:

$$q_{iVI} = \frac{a}{2+b(n-1)},$$
$$p_{iVI} = p_{VI} = \frac{a}{2+b(n-1)},$$

[37] Diagrams are provided in the proofs for the sake of easier understanding as complimentary exposition.

Equilibrium prices under partial IPJV are, with $j = n-k+1,\dots,n$ and $i = 1,\dots,n-k$:

$$p_{jVI} = \frac{a\left(b\left(k+n-2\right)+4\right)}{2\left(b\left(n-1\right)+2\right)\left(b\left(k-1\right)+2\right)},$$
$$p_i = \frac{a\left(6-b^2\left(n-1\right)-b\left(5-2n-k\right)\right)}{2\left(b\left(n-1\right)+2\right)\left(b\left(k-1\right)+2\right)},$$

where p_{jVI}, and p_i are the prices of VI and D firms, respectively.

Equilibrium quantity and price under complete IPJV, $\forall i = 1,\dots,n$, are:

$$q_{iJ} = \frac{a}{2\left(bn-b+2\right)},$$
$$p_{iJ} = \frac{\left(bn-b+3\right)a}{2\left(bn-b+2\right)}.$$

Equilibrium quantity and individual profits under splitting are:

$$q_s = \frac{(2-b+b(n-v))a}{(4-2b+b(n-v))(bn-b+2)}$$
$$\pi_{sD} = \frac{(2-b+b(n-v))^2a^2}{(4-2b+b(n-v))^2(bn-b+2)^2}$$
$$\pi_{sU} = \frac{(b-bn+bv-2)(b-2)a^2v}{(2b-bn+bv-4)^2(bn-b+2)} - f$$

where index S stands for splitting.

8.3 Proof of Proposition 2

Simple algebra shows that $b^J(n) > b^{PJ}(n) > b^{VI}(n)$ (defined in 19, 20 and 21). For the sake of simplicity, we plot (19), (20) and (21) in the plane (n,b) in Figure 2 below. The upper solid line defines $b^J(n)$, above which complete IPJV is the preferred vertical arrangement. Notice that this threshold is meaningful, i.e., lower than 1, only for $n \geq 4$. The intermediate solid line defines $b^{PJ}(n)$, above which partial IPJV is preferred to complete IPJV, which is better than VI. Between $b^{PJ}(n)$ and the lower solid line which defines $b^{VI}(n)$, partial IPJV is preferred to VI, which is better than complete IPJV. Finally, between the horizontal axis and the lower solid line, VI is preferred to partial IPJV which is better than complete IPJV.

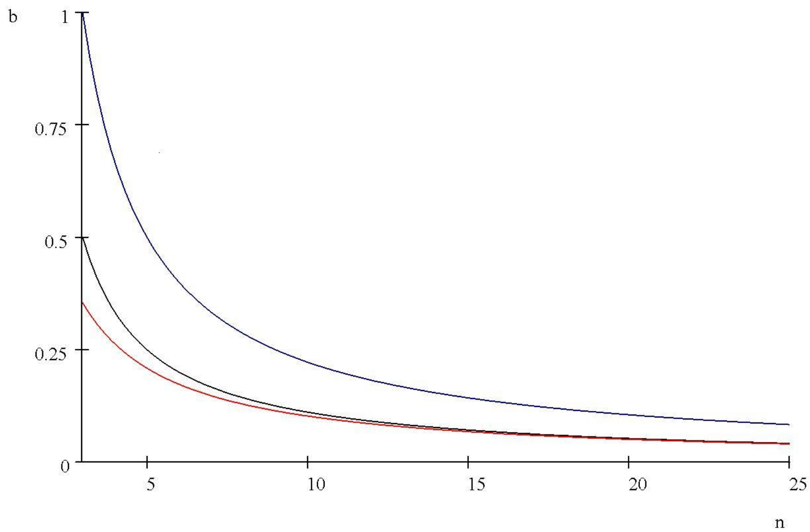

Figure 2: Industry surpluses with VI, partial, complete IPJV.

As for social welfare ranking, we have:

$$SW_{VI} = \frac{a^2(3+b(n-1))n}{2(2+b(n-1))^2}$$

$$SW_{PJ} = \frac{a^2(4(5+7n)+b(n-1)(16-3b+3(4+b)n))}{32(2+b(n-1))^2}$$

$$SW_J = \frac{a^2(7+3b(n-1))n}{8(2+b(n-1))^2}.$$

whose ranking, $SW_{VI} > SW_{PJ} > SW_J$ is independent of the values of n and b. Assume next $f > 0$. Social welfare comparisons then depend on the relative fixed cost, $s = f/a^2$ as follows:

$$SW_{VI} - SW_J = \frac{(bn-b+5)a^2n}{8(bn-b+2)^2} - f(n-1) > 0$$

$$\iff s < \frac{(bn-b+5)n}{8(n-1)(bn-b+2)^2} \equiv \widetilde{s}_1(n,b)$$

$$SW_{PJ} - SW_J = \frac{(3bn-3b+10)a^2}{32(bn-b+2)} - f > 0 \iff s < \frac{3b(n-3)+10}{32(bn-b+2)} \equiv \widetilde{s}_3(n,b)$$

$$SW_{VI} - SW_{PJ} = \frac{\left(-16b+4bn+3b^2-3b^2n+20\right)(n-1)a^2}{32(bn-b+2)^2} - f(n-2) > 0$$

$$\iff s < \frac{\left(-16b+4bn+3b^2-3b^2n+20\right)(n-1)}{32(bn-b+2)^2(n-2)} \equiv \widetilde{s}_2(n,b)$$

Some form of joint venture in the input production becomes socially preferred to VI for sufficiently high levels of s. For the social ranking we also need the thresholds of s which define the feasibility of the three vertical arrangements, $s^{VI}(n,b) < s^{PJ}(n,b) < s^{J}(n,b)$ defined in (11), (16) and (18), respectively. It is easily shown that the ranking of these thresholds depend on b and n. In particular, comparing complete IPJV and VI, we find that if $s > \widetilde{s}_1(n,b)$, IPJV is socially preferred to VI. We now check whether beyond this threshold, VI is feasible. For this to be possible we need:

$$s^{VI}(n,b) > \widetilde{s}_1(n,b) > 0 \iff b < \frac{3n-8}{n(n-1)} \equiv \widetilde{b}(n).$$

As an instructive example, we consider two extreme cases for b.

- First, set $b = 0.1$. VI is socially superior for $s < \min\{\widetilde{s}_1, \widetilde{s}_2\}$, partial IPJV is superior for $s \in (\widetilde{s}_2, \widetilde{s}_3)$ and complete IPJV is superior for $s > \max\{\widetilde{s}_1, \widetilde{s}_3\}$. Notice that for $s > s^{VI}$ VI is not feasible anymore.
- Second, set $b = 0.9$. In this case the ranking of the s thresholds is clear-cut: $\widetilde{s}_2 < s^{VI} < \widetilde{s}_1 < \widetilde{s}_3$. VI is socially superior for $s < \widetilde{s}_2$, partial IPJV is superior for $s \in (\widetilde{s}_2, \widetilde{s}_3)$ and complete IPJV is superior for $s > \widetilde{s}_3$. Notice that for $s > s^{VI}$ VI is not feasible anymore.

8.4 Proof of Lemma 4

We begin stating that

$$\pi_{iVI} - \pi_{cons}^{pJ} > 0 \iff b^2(n-1) + b(4-2n) + 2 > 0.$$

This polynomial has two roots: $b_1 = \frac{(n-2)-\sqrt{(n^2-6n+6)}}{(n-1)}, b_2 = \frac{(n-2)+\sqrt{(n^2-6n+6)}}{(n-1)}$. They are real numbers only for $n > 4$. Note that for $n = 3$ splitting (S) is not a feasible option; for $n = 4$, disengaging always dominates splitting (this is proved in the following Section "Example: four-firm oligopoly"). Therefore, for $n = 3,4$, we can drop S from the comparison.

Then, i) for $n = 3,4$ the difference $\pi_{iVI} - \pi_{cons}^{pJ}$ is strictly positive for all feasible b. In other words, each D firm obtains a positive surplus by disengaging from partial IPJV. This occurs in both cases, i.e., when each firm leaves the IPJV plot on an individual basis and when all IPJV firms leave as a group.

ii) For $n \geq 5$, the two roots, b_1 and b_2 are real. In particular $b_1 \in (0,1)$, while $b_2 \geq 1$, and is not acceptable. Therefore, $\pi_{iVI} - \pi_{cons}^{pJ} > 0 \iff b < b_1$. Firms may have the incentive for splitting which in some cases dominates disengagement. In particular $\pi_{iVI} - \pi_{cons}^{s}(v) > 0 \iff b < b_{VIS}(v)$. For $b \in \left(b_{VIS}(v), b^{VI}(n)\right)$, from individual profits' comparisons, we obtain the following unambiguous firms' preferences: $S \succ VI \succ$ partial $IPJV \succ$ complete $IPJV$, therefore splitting takes place. For $b \in \left(b^{VI}(n), b_S(v)\right)$, from individual profits' comparisons, we obtain the following firms' (inconclusive) preference orderings: $S \succ VI \succ J$ and $PJ \succ VI \succ J$.

The comparison of splitting (S) with disengagement (PJ) depends on the parameters. From the viewpoint of the unique VI firm: $\pi_{nVI}(1) - \pi^s_{cons}(v) > 0$ computed in $v = \frac{n}{i}$ for $i = 2, 3, 4, ...$ From the viewpoint of the IPJV members the sign of the difference, $\pi^i_{cons}(v) - \pi^{pJ}_{cons}$ is ambiguous. It is positive for $b \to 0$ and it is positive for $b \to 1$ and n high enough. For example, $\pi^i_{cons}\left(\frac{n}{2}\right) - \pi^{pJ}_{cons} > 0 \iff 32n + 128 - 2b^2\left(48n - 10n^2 + n^3 - 48\right) + 2b\left(40n + n^2 - 96\right) + b^3(n-1)(n-4)^2 > 0$. The only negative term is $-2b^2\left(48n - 10n^2 + n^3 - 48\right)$, consider this polynomial in $b = 1$. It is positive if $n < 15$ and negative for $n > 16$; $\pi^i_{cons}\left(\frac{n}{3}\right) - \pi^{pJ}_{cons} > 0 \iff 24n + 72 + b\left(48n + 2n^2 - 108\right) + b^3\left(15n - 7n^2 + n^3 - 9\right) + b^2(16n^2 - 60n - 2n^3 + 54) > 0$, the only negative term is $b^2\left(16n^2 - 60n - 2n^3 + 54\right)$. In $b = 1$ it is positive if $n < 12$ and negative for $n > 13$; $\pi^i_{cons}\left(\frac{n}{4}\right) - \pi^{pJ}_{cons} > 0 \iff 192n + 512 + b(352n + 18n^2 - 768) + b^3\left(112n - 57n^2 + 9n^3 - 64\right) + b^2\left(132n^2 - 448n - 18n^3 + 384\right) > 0$, the only negative term is $b^2\left(132n^2 - 448n - 18n^3 + 384\right)$. In $b = 1$ it is positive if $n < 12$ and negative for $n > 13$. From the industry point of view, the comparison is: $\Pi_{PJ} - \Pi_S(v) > 0 \iff -32b\left((n-1)(v-n) - 2\right) + 4b^2(6nv - 4v - 4n - v^2 + nv^2 - 2n^2v - n^2 + n^3) + b^4(n-1)^2(v-n+2)^2 - 4b^3(n-1)(nv^2 - 2n^2v + 6n + 2nv - 2v + n^3 - 2n^2 - 4) - 64 < 0$. In $b = 0$, $\Pi_{PJ} - \Pi_S(v) > 0$; in $b = 1$, $\Pi_{PJ} - \Pi_S(v) > 0$ for n not too low: $\Pi_{PJ} - \Pi_S\left(\frac{n}{2}\right) < 0 \iff n \geq 6$; $\Pi_{PJ} - \Pi_S\left(\frac{n}{3}\right) < 0 \iff n \geq 5$; $\Pi_{PJ} - \Pi_S\left(\frac{n}{4}\right) < 0 \iff n \geq 5$. In $b = \frac{1}{2}$, $\Pi_{PJ} - \Pi_S(v) > 0$ for n not too low: $\Pi_{PJ} - \Pi_S\left(\frac{n}{2}\right) < 0 \iff n \geq 12$, $\Pi_{PJ} - \Pi_S\left(\frac{n}{3}\right) < 0 \iff n \geq 10$, $\Pi_{PJ} - \Pi_S\left(\frac{n}{4}\right) < 0 \iff n \geq 9$. In $b = \frac{1}{3}$, $\Pi_{PJ} - \Pi_S(v) > 0$ for n not too low: $\Pi_{PJ} - \Pi_S\left(\frac{n}{2}\right) < 0 \iff n \geq 17$, $\Pi_{PJ} - \Pi_S\left(\frac{n}{3}\right) < 0 \iff n \geq 14$, $\Pi_{PJ} - \Pi_S\left(\frac{n}{4}\right) < 0 \iff n \geq 13$. Figure 3 depicts the thresholds defined by $b_{VIS}(v)$ (expression 26) computed in $v = \frac{n}{2}$ as an example, $b^{VI}(n)$ (20) and $b_S(v)$ (25) computed in $v = \frac{n}{2}$, in the plane (n, b). The solid upper line represents the frontier defined by $b_S\left(\frac{n}{2}\right)$. Above the line there is no incentive to leave the IPJV, while below the incentive is nonnegative. The solid lower line defines the frontier $b_{VIS}\left(\frac{n}{2}\right)$. Below it, complete disengagement is profitable and feasible and preferred to splitting. Above it and below $b^{VI}(n)$, splitting is more profitable than complete disengagement.

iii) Looking at the comparison $\Pi_{PJ} - \Pi_S(v)$, analyzed above, we conclude that for n high enough industry profits are higher under partial IPJV than under splitting. Therefore disengagement dominates splitting and it never takes place since the unique VI firm, under partial IPJV, can prevent splitting via a compensation mechanism. Figure 4 depicts the thresholds defined by (20) and (27) in the plane (n, b).

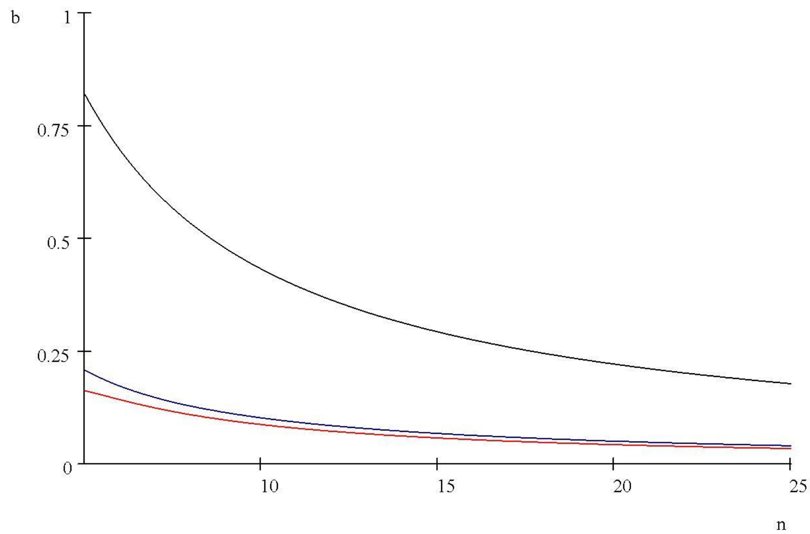

Figure 3: Ranges of b for incentives to leave the IPJV.

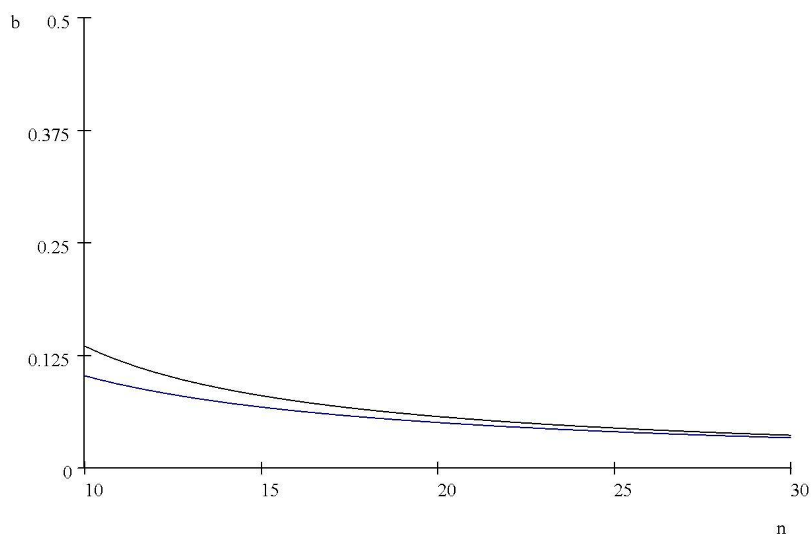

Figure 4: Ranges of b for incentives to disengage.

The solid upper line represents the frontier defined by b_1. Above the line there is no incentive to disengage from IPJV, while below the line the incentive is nonnegative. The solid lower line defines the frontier $b^{VI}(n)$. Below it, disengagement is profitable and feasible. Above it and below b_1, the VI firm could compensate the IPJV firms if they agree not to move, since the loss the VI would face in the case of disengagement would be larger than the gain IPJV firms could secure.

8.5 Example: a four-firm oligopoly

We present an example with four firms and analyze all possible vertical arrangements.

Suppose there are four D firms in the market forming an IPJV. The possible deviations from this starting situation are:

1. one firm leaves the IPJV and vertically integrate (in the market we are left with one VI firm and one IPJV);
2. two firms leave the IPJV and vertically integrate (we have two VI firms and one IPJV);
3. all firms quit and vertically integrate (complete disengagement: we are left with four VI firms);
4. two firms leave the IPJV and form a new one (splitting: we have two IPJVs).
5. one firm leaves the IPJV and vertically separates
6. two firms leave the IPJV and vertically separate
7. all firms leave the IPJV and vertically separate
8. one firm leaves the IPJV and vertically integrates, another leaves the IPJV and vertically separates.

In the initial situation, each firm obtains:

$$\pi^J_{cons} = \frac{3(b+1)a^2}{4(3b+2)^2} - \frac{f}{4}$$

and industry profits are $\Pi_J = 4\pi^J_{cons}$. In case 1, equilibrium profits for each firm of the IPJV and for the VI firm are, respectively:

$$\begin{aligned}
\pi^1_{cons} &= \frac{a^2\left(4b-3b^2+6\right)}{8(2+3b)^2} - \frac{f}{3} \\
\pi^1_{VI} &= \frac{(3b+4)^2 a^2}{16(3b+2)^2} - f \\
\pi^1_{cons} - \pi^1_{VI} &= -\frac{(5b+2)a^2}{16(3b+2)} + \frac{2}{3}f < 0 \text{ for } f \text{ low}
\end{aligned}$$

and industry profits are: $\Pi^1_{PJ} = 3\pi^1_{cons} + \pi^1_{VI} = \frac{(48b-9b^2+52)a^2}{16(3b+2)^2} - 2f$. In case 2, equilibrium profits for each firm of the IPJV and for each VI firm are, respectively:

$$\begin{aligned}
\pi^2_{cons} &= \frac{a^2\left(5b-3b^2+6\right)}{4(2+b)(2+3b)^2} - \frac{f}{2} \\
\pi^2_{VI} &= \frac{4a^2(1+b)^2}{(b+2)^2(3b+2)^2} - f \\
\pi^2_{cons} - \pi^2_{VI} &= -\frac{\left(5b+b^2+2\right)a^2}{4(b+2)^2(3b+2)} + \frac{f}{2} < 0 \text{ for } f \text{ low}
\end{aligned}$$

and industry profits are: $\Pi^2_{PJ} = 2\pi^2_{cons} + 2\pi^2_{VI} = \frac{(28-3b^3+15b^2+48b)a^2}{2(3b+2)^2(b+2)^2} - 3f$. In case 3, equilibrium profits for each VI firm are:

$$\pi^3_{VI} = \frac{a^2}{(3b+2)^2} - f$$

and industry profits are: $\Pi^3_{VI} = 4\pi^3_{VI}$. In case 4, equilibrium profits for each firm of each IPJV are:

$$\pi^4_{cons} = \frac{a^2(2+b)\left(6-3b^2+5b\right)}{16(2+3b)^2} - \frac{f}{2}$$

and industry profits are: $\Pi^4_S = 4\pi^4_{cons}$. In case 5, equilibrium profits for each firm of the IPJV and for the VS firm are, respectively:

$$\begin{aligned}
\pi^5_{cons} &= \frac{2a^2(4+5b)^2\left(6-3b^2+4b\right)}{(2+3b)^2\left(3b^2-16b-16\right)^2} - \frac{f}{3} \\
\pi^5_{VS} &= \frac{4a^2(1+b)^2(4+3b)^2}{(3b+2)^2\left(16b-3b^2+16\right)^2} \\
\pi^5_{cons} - \pi^5_{VS} &= 2\frac{\left(32-14b^2+48b-31b^3\right)a^2}{\left(3b^2-16b-16\right)^2(3b+2)} - \frac{f}{3} > 0 \text{ for } f \text{ low}
\end{aligned}$$

and industry profits are: $\Pi^5_{VS} = 3\pi^5_{cons} + \pi^5_{VS}$. In case 6, equilibrium profits for each firm of the IPJV and for each VS firm are, respectively:

$$\begin{aligned}
\pi^6_{cons} &= \frac{a^2(2+b)(4+5b)^2\left(6-3b^2+5b\right)}{4(2+3b)^2\left(10b+b^2+8\right)^2} - \frac{f}{2} \\
\pi^6_{VS} &= \frac{16a^2(1+b)^4}{(2+3b)^2\left(10b+b^2+8\right)^2} \\
\pi^6_{cons} - \pi^6_{VS} &= \frac{\left(64-53b^3+54b^2+144b-25b^4\right)a^2}{4\left(10b+b^2+8\right)^2(3b+2)} - \frac{f}{2} > 0 \text{ for } f \text{ low}
\end{aligned}$$

and industry profits are: $\Pi_{VS}^6 = 2\pi_{cons}^6 + 2\pi_{VS}^6$. In case 7, equilibrium profits for each VS firm are:

$$\pi_{VS}^7 = \frac{4a^2(1+b)^2}{(4+b)^2(2+3b)^2}$$

and industry profits are: $\Pi_{VS}^7 = 4\pi_{VS}^7$. In case 8, equilibrium profits for each firm of the IPJV, for the VI firm and for the VS firm are, respectively:

$$\begin{aligned}
\pi_{cons}^8 &= \frac{a^2(2+b)(4+5b)^2\left(6-3b^2+5b\right)}{4(2+3b)^2\left(12b+3b^2+8\right)^2} - \frac{f}{2} \\
\pi_{VI}^8 &= \frac{4a^2(1+b)^2(4+5b)^2}{(2+3b)^2\left(12b+3b^2+8\right)^2} - f \\
\pi_{VS}^8 &= \frac{16a^2(1+b)^4}{(2+3b)^2\left(12b+3b^2+8\right)^2} \\
\pi_{cons}^8 - \pi_{VI}^8 &= -\frac{(5b+4)^2\left(5b+b^2+2\right)a^2}{4\left(12b+3b^2+8\right)^2(3b+2)} + \frac{f}{2} < 0 \text{ for } f \text{ low} \\
\pi_{cons}^8 - \pi_{VS}^8 &= \frac{\left(64-53b^3+54b^2+144b-25b^4\right)a^2}{4\left(12b+3b^2+8\right)^2(3b+2)} - \frac{f}{2} > 0 \text{ for } f \text{ low} \\
&\Rightarrow \pi_{VI}^8 > \pi_{cons}^8 > \pi_{VS}^8
\end{aligned}$$

and industry profits are: $\Pi_{VS}^8 = 2\pi_{cons}^8 + \pi_{VI}^8 + \pi_{VS}^5$.

Comparing these profits we obtain the following.

$$\begin{aligned}
\pi_{cons}^J - \pi_{cons}^1 &= \frac{ba^2}{8(3b+2)} + \frac{1}{12}f > 0 \\
\pi_{cons}^J - \pi_{VI}^1 &= -\frac{a^2}{16} + \frac{3}{4}f < 0 \text{ for } f \text{ low} \\
\Pi_J - \Pi_{PJ}^1 &= \frac{(3b-2)a^2}{16(3b+2)} + f
\end{aligned}$$

For $f \to 0$, $\Pi_J - \Pi_{PJ}^1 > 0 \iff b > \frac{2}{3}$.[38] From the second inequality we see that a firm may have the incentive to leave the IPJV and vertically integrate. From the third, we see that for b high enough disengagement can be stopped.

[38] This is $b^{VI}(4)$ defined in 21.

$$\begin{aligned}
\pi^1_{cons} - \pi^2_{VI} &= -\frac{\left(6b+2b^2+b^3+4\right)a^2}{8\left(b+2\right)^2\left(3b+2\right)} + \frac{2}{3}f < 0 \text{ for } f \text{ low} \\
\pi^1_{cons} - \pi^2_{cons} &= \frac{\left(2-b\right)a^2 b}{8\left(3b+2\right)\left(b+2\right)} + \frac{1}{6}f > 0 \\
\pi^1_{VI} - \pi^2_{VI} &= \frac{\left(384b+712b^2+564b^3+198b^4+27b^5+64\right)a^2}{16\left(3b+2\right)^2\left(b+2\right)^2} > 0 \\
\Pi^1_{PJ} - \Pi^2_{PJ} &= \frac{\left(368b+716b^2+540b^3+180b^4+27b^5+48\right)a^2}{16\left(3b+2\right)^2\left(b+2\right)^2} + f > 0
\end{aligned}$$

From the first inequality we see that also a second firm may have the incentive to disengage and VI, however going from case 1 to case 2, both the unique VI firm and the IPJV are worst off: the VI firm would suffer from tougher competition for the presence of a second VI firm, the D firms participating to the IPJV would lose for a lower number of participants. Thus there is a two-side incentive to prevent other firms from disengaging. Thus case 1 dominates case 2.

$$\begin{aligned}
\pi^1_{VI} - \pi^3_{VI} &= \frac{3\left(3b+8\right)a^2 b}{16\left(3b+2\right)^2} > 0 \\
\pi^1_{cons} - \pi^3_{VI} &= \frac{\left(4b-3b^2-2\right)a^2}{8\left(3b+2\right)^2} + \frac{2}{3}f < 0 \text{ for } f \text{ low} \\
\Pi^3_{VI} - \Pi^1_{PJ} &= \frac{3\left(3b^2-16b+4\right)a^2}{16\left(3b+2\right)^2} - 2f
\end{aligned}$$

For $f \to 0$, $\Pi^3_{VI} - \Pi^1_{PJ} > 0 \iff b < 0.263$.[39] From these comparisons we see that case 1 dominates case 3 for b sufficiently high: for $b < 0.263$ all firms prefer VI, otherwise case 1 takes place as the unique VI firm can prevent the others from disengaging.

$$\begin{aligned}
\pi^4_{cons} - \pi^J_{cons} &= \frac{\left(1-b\right)\left(3b+4\right)a^2 b}{16\left(3b+2\right)^2} - \frac{f}{4} > 0 \text{ for } f \text{ low} \\
\pi^4_{cons} - \pi^2_{VI} &= -\frac{\left(16b-8b^2+13b^4+3b^5+16\right)a^2}{16\left(3b+2\right)^2\left(b+2\right)^2} + \frac{f}{2} < 0 \text{ for } f \text{ low}
\end{aligned}$$

From the first inequality we conclude that there is an incentive for splitting. From the second inequality we see that, if a deviation occurs from the complete IPJV,

[39] This is $b^{VI}(4)$ defined in 20.

it is by disengaging rather than by splitting. Case 2 dominates case 4. Compare industry profits under case 1 versus case 4 to see whether the unique VI firm can prevent splitting via a compensation mechanism:

$$\Pi^1_{PJ} - \Pi^4_S = \frac{\left(56b + 76b^2 + 39b^3 + 20\right)a^2}{16\left(3b+2\right)^2} > 0$$

For any b, the unique VI firm has the incentive to stop splitting and so disengagement dominates splitting.

$$\begin{aligned}
\pi^J_{cons} - \pi^5_{VS} &= \frac{\left(896b - 48b^2 - 432b^3 + 27b^4 + 512\right)(b+1)a^2}{4\left(3b^2 - 16b - 16\right)^2 (3b+2)^2} - \frac{f}{4} > 0 \\
&\quad \text{for } f \text{ low}, \\
\pi^J_{cons} - \pi^6_{VS} &= \frac{\left(288b + 156b^2 - 4b^3 + 3b^4 + 128\right)(b+1)a^2}{4\left(10b + b^2 + 8\right)^2 (3b+2)^2} - \frac{f}{4} > 0 \text{ for } f \text{ low}, \\
\pi^J_{cons} - \pi^7_{VS} &= \frac{\left(8b + 3b^2 + 32\right)(b+1)a^2}{4(3b+2)^2(b+4)^2} - \frac{f}{4} > 0 \text{ for } f \text{ low}.
\end{aligned}$$

From these comparisons we conclude that leaving the IPJV and vertically separate, alone or with other firms is never optimal.

$$\pi^8_{VI} - \pi^1_{VI} = -\frac{\left(7680b + 22\,592b^2 + 33\,984b^3 + 28\,064b^4 + 12\,528b^5 + 2754b^6 + 243b^7 + 1024\right)a^2}{16\left(12b + 3b^2 + 8\right)^2 (3b+2)^2} < 0$$

From this inequality we see that case 1 also dominates case 8.

Summing up, assume that the fixed cost is sufficiently low. If one firm disengages and vertically integrates, it is better off because it is the most efficient firm in the market (it now gets the input at its marginal cost) and competes with the IPJV which benefits from sharing the (low enough) fixed cost but suffers from double marginalization. If two firms disengage and vertically integrate, the VI firm faces tougher competition for the presence of a second VI firm and the D firms participating to the IPJV lose owing to a lower number of members. Thus there is a two-sided incentive to prevent other firms from disengaging. One VI firm competing with an IPJV dominates splitting, that is two IPJVs. However complete disengagement (case 3) takes place for $b < 0.263$ as the unique VI firm cannot stop the other firms. Partial IPJV (case 1) takes place for $b \in \left(0.263, \frac{2}{3}\right)$ and complete IPJV (the starting point) occurs for $b > \frac{2}{3}$.

8.6 Proof of Proposition 5

Looking at the industry profits, we see that for $b < b_{VIS}(v)$, (lower solid line in Figure 3), VI is strictly better than both splitting and partial or complete IPJV, so that the adoption of VI by all firms is a NE. For $b > b_S(v)$, (upper solid line in Figure 3), aggregate profits are higher with complete IPJV: the D firms have the incentive and the possibility to persuade the single VI firm to join the venture. For intermediate values of b, there is an incentive to leave the IPJV. However, the preferences over disengagement and splitting are ambiguous, as we can see from the industry profits' comparison.

Whenever partial IPJV dominates splitting, we have that a compensation mechanism allows partial IPJV to survive. In fact for $b^{VI}(n) < b < b_1$ the D firms in the partial IPJV would gain from the switch. However, the difference between the profit of the VI firm in the presence of disengagement and the VI profit without disengagement is larger than the difference between the profits of the Ds with disengagement and those without disengagement. For $b_1 < b < b^J$ the D firms participating to partial IPJV have no incentive to disengage as $\pi_{iVI} < \pi^{pJ}_{cons}$: in this area partial IPJV is a NE. Finally, for $b > b^J > b_1$, aggregate profits are higher with complete IPJV.

8.7 Proof of Proposition 6

a) Comparing the joint profits under the two scenarios we find that:

$$\Pi^B_J - \Pi^B_{VI} = \frac{(2b-1)\,a^2}{2\,(b-2)^2\,(b+1)} + f.$$

Assume $f = 0$, $\Pi^B_J - \Pi^B_{VI} > 0 \iff b > 1/2$. For positive fixed cost we get a picture which is qualitatively the same as for Cournot duopoly (Figure 1): the private profitability of IPJV increases as goods become closer substitutes. In the range of f/a^2, where both vertical arrangements are feasible, social welfare is always higher with VI. For relatively high fixed cost, i.e., $s \in \left(s^B_{VI}(b), s^B_J(b)\right)$, VI is no longer feasible and IPJV becomes socially superior by default.

b) The IPJV scenario, under price and quantity competition, yields the following equilibrium comparisons:

$$q_J^B - q_J^C = \frac{b^2 a}{2(b+1)(b+2)(2-b)} > 0,$$
$$p_J^B - p_J^C = \frac{-b^2 a}{2(b+2)(2-b)} < 0,$$
$$\pi_U^B - \pi_U^C = \frac{b^2 a^2}{2(b+1)(b+2)(2-b)} > 0,$$
$$\pi_J^B - \pi_J^C = \frac{-b^3 a^2}{2(b+2)^2(b-2)^2(b+1)} < 0,$$
$$\Pi_J^B - \Pi_J^C = \frac{\left(4-2b-b^2\right)a^2 b^2}{2(b+2)^2(b-2)^2(b+1)} > 0.$$

c) Let us go through the efficiency of the two modes of competition. Comparing social welfare of Cournot and Bertrand outcomes for any level of b, we obtain:

$$SW^B - SW^C = \begin{cases} \text{for } b \in \left[0, \frac{1}{2}\right], SW_{VI}^B - SW_{VI}^C = \frac{\left(4-b^2-2b\right)a^2 b^2}{(b+2)^2(b-2)^2(b+1)} > 0 \\ \text{for } b \in \left(\frac{1}{2}, 1\right), SW_J^B - SW_{VI}^C < 0 \iff \frac{f}{a^2} < \frac{\left(10-3b-2b^2\right)\left(2-2b^2+b\right)}{4(b+2)^2(b-2)^2(b+1)} \end{cases}.$$

8.8 Proof of Proposition 7

By comparing industry profits for the four combinations, we get the following:

$$\Pi_{VI}^C - \Pi_J^B > 0 \iff s < s^C(b),$$
$$\Pi_{VI}^B - \Pi_{VI}^C = \frac{-4b^3 a^2}{(b+2)^2(b-2)^2(b+1)} < 0.$$
$$\Pi_J^B - \Pi_{VI}^B > 0 \iff s > \frac{(1-2b)}{2(b-2)^2(b+1)} \equiv s^B(b) \tag{34}$$
$$\Pi_{VI}^B - \Pi_J^C > 0 \iff s < \frac{\left(b+b^2+2\right)\left(2-3b-b^2\right)}{2(b+2)^2(b-2)^2(b+1)} \equiv s_1(b). \tag{35}$$

If $f = 0$, $\Pi_{VI}^C > \Pi_J^B \iff b < 2/3$, $\Pi_J^B > \Pi_{VI}^B \iff b > 1/2$ and $\Pi_{VI}^B - \Pi_J^C > 0 \iff b < 0.561$. If $f > 0$, $\Pi_{VI}^C > \Pi_J^B \iff s = f/a^2 < s^C(b)$, $\Pi_J^B > \Pi_{VI}^B \iff s > s^B(b)$, which is positive only for $b < 0.5$; and $\Pi_{VI}^B - \Pi_J^C > 0 \iff s < s_1(b)$, which is positive only for $b < 0.561$. As for social welfare we have the following comparisons:

$$SW_{VI}^{B} - SW_{J}^{B} > 0 \iff s < \frac{(5-4b)}{4(b-2)^2(b+1)} \equiv s_1^W(b) \tag{36}$$

$$SW_{VI}^{B} - SW_{VI}^{C} = \frac{\left(4-b^2-2b\right)a^2b^2}{(b+2)^2(b-2)^2(b+1)} > 0$$

$$SW_{J}^{B} - SW_{J}^{C} = \frac{\left(12-2b-3b^2\right)a^2b^2}{4(b+2)^2(b-2)^2(b+1)} > 0$$

$$SW_{J}^{B} - SW_{VI}^{C} < 0 \iff s < \frac{\left(10-3b-2b^2\right)\left(2-2b^2+b\right)}{4(b+2)^2(b-2)^2(b+1)} \equiv s_2^W(b). \tag{37}$$

a) Assume $f = 0$. From the above inequalities we can write the following industry profit ranking: for $b < 1/2$, $\Pi_{VI}^{C} > \Pi_{VI}^{B} > \Pi_{J}^{B} > \Pi_{J}^{C}$; for $b \in (1/2, 0.56)$, $\Pi_{VI}^{C} > \Pi_{J}^{B} > \Pi_{VI}^{B} > \Pi_{J}^{C}$; for $b \in (0.56, 2/3)$, $\Pi_{VI}^{C} > \Pi_{J}^{B} > \Pi_{J}^{C} > \Pi_{VI}^{B}$; finally, for $b > 2/3$, $\Pi_{J}^{B} > \Pi_{VI}^{C} > \Pi_{J}^{C} > \Pi_{VI}^{B}$. As for social welfare we have the following ranking: $SW_{VI}^{B} > SW_{VI}^{C} > SW_{J}^{B} > SW_{J}^{C}$.

b) Assume $f > 0$. The thresholds for the relative fixed cost, $s = f/a^2$, defined in (28), (30), (34), and (35) are decreasing in b such that:

$$s^B(b) < s_1(b) < s^C(b) < s_{VI}^B(b).$$

For $s > s_{VI}^B(b)$, VI under Bertrand competition is not feasible. Therefore we focus on the area $s < s_{VI}^B(b)$. $s^B(b)$, $s_1(b)$ and $s^C(b)$ are non-negative for $b < 1/2$. In this range of product differentiation, for $s < s^B(b)$, the industry profit ranking is $\Pi_{VI}^{C} > \Pi_{VI}^{B} > \Pi_{J}^{B} > \Pi_{J}^{C}$; for $s \in \left[s^B(b), s_1(b)\right]$, it is $\Pi_{VI}^{C} > \Pi_{J}^{B} > \Pi_{VI}^{B} > \Pi_{J}^{C}$; for $s \in \left[s_1(b), s^C(b)\right]$, it is $\Pi_{VI}^{C} > \Pi_{J}^{B} > \Pi_{J}^{C} > \Pi_{VI}^{B}$; for $s \in \left[s^C(b), s_{VI}^B(b)\right]$, it is $\Pi_{J}^{B} > \Pi_{VI}^{C} > \Pi_{J}^{C} > \Pi_{VI}^{B}$. For $b \in (0.5, 0.56)$, $s^B(b)$ is negative, so that the first range of s vanishes. For $b \in (0.56, 2/3)$, also $s_1(b)$ becomes negative, so that there are only two ranges $s \in \left[0, s^C(b)\right]$ and $s \in \left[s^C(b), s_{VI}^B(b)\right]$. Finally, for $b > 2/3$, only $s_{VI}^B(b)$ is positive, so that in the whole range of feasible s, the ranking is $\Pi_{J}^{B} > \Pi_{VI}^{C} > \Pi_{J}^{C} > \Pi_{VI}^{B}$. As for social welfare, the thresholds defined by (33), (36) and (37) are not binding because they are larger than the feasible fixed cost. Therefore, in the area of feasible relative fixed cost s, the social welfare ranking is: $SW_{VI}^{B} > SW_{VI}^{C} > SW_{J}^{B} > SW_{J}^{C}$. However, for $s > s_{VI}^B(b)$, VI under Bertrand competition is no longer feasible and the social welfare ranking depends on $s(b)$ as follows: $SW_{VI}^{C} > SW_{J}^{B} > SW_{J}^{C}$ for $s \in \left(s_{VI}^B(b), \min\{s_{VI}^C, s_2^W\}\right)$; for $s > \min\{s_{VI}^C, s_2^W\}$, $SW_{J}^{B} > SW_{VI}^{C} > SW_{J}^{C}$ as long as VI under Cournot competition is feasible, i.e., for $s < s_{VI}^C$. This result is proved by comparing the relevant thresholds for the relative fixed cost. In particular, making the proper differences we obtain the following ranking:

$$s_{VI}^{B} < s_{2}^{W} < s_{1}^{W} < s_{J}^{B}.$$

As for $s_{VI}^{C}(b)$, it is bigger than s_{VI}^{B} and lower than s_{1}^{W}, however its ranking with respect to s_{2}^{W} depends on b.

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