The construct of attitude in mathematics education

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Abstract This chapter addresses some crucial theoretical issues about research on attitude towards mathematics, a field that has a very long tradition in mathematics education, since early studies on attitude already appeared more than 60 years ago. Up to this point research on attitude in mathematics education has passed through a shift of perspectives, methodologies used and types of issues treated: the investigation of the relationship between attitude and achievement, the discussion and development of tools for measuring / assessing /observing, the critique for the lack of a theoretical framework, the analysis of the relationship between attitude and other affective constructs, and between attitude and cognition. Through the theoretical debate about these issues, the aim of the chapter is to highlight new directions for research on attitude. But tracing the 'story' of the construct attitude is also significant to understand – through a theoretical lens - the mosaic of relationships and interactions within the affect field.

Introduction

In a certain sense, research on attitudes towards mathematics can be viewed as paradigmatic of research within mathematics education.
This research field lies at the intellectual crossroads of many different domains (e.g. mathematics, psychology, cognitive science, epistemology, semiotics, anthropology), and often deals with constructs which have been developed in those domains, to face with (new) issues emerging in mathematics education (Sierpinska et al., 1993).
The construct of *attitude* was introduced in the first decades of the 19th century in the context of social psychology in order to foresee

individual's choices in contexts such as voting, or buying goods. *Attitude* is seen as a trait of an individual that has a direct influence upon his/her behaviour:

> An attitude is a mental and neural state of readiness, organized through experience, exerting a directive and dynamic influence upon the individual's response to all objects and situations with which it is related. (Allport, 1935, p. 810)

In mathematics education, early studies about attitude already appeared in the middle of the 20th century. These pioneering studies were deeply affected by the field (social psychology) in which the construct was born, both as regard the characterization of attitude, seen as an individual's trait capable of influencing his/her own behaviour (Aiken, 1970), and the methods used to assess and measure it.

In this context, the search for a measurement of attitude was the main goal: Dutton (1951), in one of the first studies concerning attitude and mathematics, explicits the aim of measuring pupils' and teachers' attitude towards arithmetic using Thurstone scales. As a matter of fact, following the trend in social psychology, the measurement of attitude was mainly carried out by the means of unidimensional scaling methods, ad hoc constructed, like Thurstone and Likert scales.

From early studies on attitude to mathematics up to now many things have changed in research on attitude, some of them deeply influenced by a change of perspectives in mathematics education. At this time, attitude is considered (together with beliefs, emotions and values) one of the constructs that characterize a new field of research: that of affect.

Research on attitude, as often happens, has not been characterized by a linear path. Over the years, the researchers' position, about basic issues like the definition itself of attitude and the instruments used to assess the construct, has dramatically changed, and new issues and goals have been identified.

This feature of research on attitude increases the general need emerged with strength in the last two decades, for a clear theoretical systematization of the research's results in mathematics education. As a matter of fact, this need in mathematics education has become even indispensable, due to the massive development of the research field and, in particular, with the identification of its *cumulative* and

universal characters (Boero & Szendrei, 1998). This view of the field is strictly linked with the characterization of the nature of the research findings:

> Researchers in education have an intellectual obligation to push for greater clarity and specificity (...) [in mathematics education] findings are rarely definitive; they are usually suggestive. Evidence is not on the order of proof, but is cumulative. (Schoenfeld, 2000, p. 647-648)

Therefore, coherently with the *cumulative* characterization of research in mathematics education, our view is that tracing, with critical eyes, the story of the research on attitude allows to understand – through a theoretical lens – the mosaic of the relationships and interactions between attitude's definitions and assessing instruments, and the influence on both these issues of the shift from a normative paradigm to an interpretive one.
Moreover, this systematization is necessary to trace the future of the research on attitude: identifying new issues, developing suitable methods, and warning against the replacement of the old same mistakes.

Early studies about attitude in mathematics education: the problematic relationship between attitude and achievement

During its early period (ranging from the first half of the 20th Century to the end of the Eighties), research in mathematics education on attitude follows the trend of research in social psychology. The definition of attitude is rarely made explicit, and implicitly seems to refer to the orientation of behaving in a certain way. A central point of research is the development or the refinement of measuring instruments and sampling methods:

> The search for more adequate questionnaire and sampling techniques and factors underlying attitudes toward these subjects [arithmetic and mathematics] continues to be an important area for research. (Dutton, 1951, p. 418)

In this period, the predominant methodology is quantitative and statistical: as a matter of fact the quantitative and statistical approach is considered a sort of warranty of the scientific nature of the discipline.

The research on attitude reflects the evolution of the field of mathematics education: an in-depth discussion about the very *nature* of this emerging field has not been developed yet. According to Kilpatrick (1992, p. 15), in that period, "*the measurement movement begins*". The quantitative primacy in the methods used has its roots in the search for scientific-acceptance of a young discipline that begins to move its first steps:

> From the beginnings of the century through its three-quarter point, such inquiry [inquiry in math education] become increasingly "scientific", that is, ostensibly objective and rigorously quantified. (Schoenfeld, 1994, p. 698)

On the other hand, the attention paid to the measurement instruments is also linked to the main goals of early studies on attitude: the identification of causal correlations between attitude and other significant factors.
Feierabend (1960), in the first review related to attitude within mathematics education, highlights two main reasons for the academic interest towards this construct. The former, according to the development of the construct in social psychology, is related to the view of attitude as a *selective factor* because of its correlation with the choice of enrolling/not enrolling in advanced mathematical courses:

> Mathematics, geometry, and algebra are the courses which, when disliked in high school, have the highest percentage of students who never take a course in this area again. This implies the operation of such a strong selective factor that by the time students reach college, only the students with a strong positive attitudes will still be taking mathematics; the rest have negative attitudes which may increase in strength with the operation of time and the lack of counteracting influences. (Feierabend, 1960, p. 19)

The latter concerns the relationship between attitude and mathematical achievement:

> A series of recent investigations have attempted to explain differences in school performance among students of equal abilities on the basis of their attitudes. (Feierabend, ibidem, p. 11)

This second reason also involves taking into account gender differences in mathematics achievement and in problem solving-ability:

> There are sex differences in problem-solving ability unrelated to general mental ability, special abilities, or specific knowledge (...) [he] attempted to show that the differential performance of the two sexes was due to a difference in attitude toward problem-solving. (Feierabend, ibidem, p. 17)

In his review Feierabend advances some criticisms of the research on attitude, but his criticism is limited to some aspects related to the development of instruments or to the statistical analysis carried out. No reference is mentioned about the lack of theoretical clarity, in particular no explicit definition of attitude is provided: a naïve view of the construct emerges. The term 'attitude' is used to address different constructs, such as preference, interest, motivation.
Ten years later Aiken (1970) summarizes in the following way early research on attitude:

> The major topics covered were: methods of measuring attitudes towards arithmetic and mathematics; the distribution and stability of mathematics attitudes; the effects of attitudes on achievement in mathematics; the relationship of mathematics attitudes to ability and personal factors. (Aiken, 1970, p. 592)

It is relevant that even Aiken's list does not include any reference to the topic 'nature of the construct *attitude*' (that will become a major topic in the research on attitude in the early nineties).
From the reviews carried out by Feuerabend and Aiken and from the exam of the literature of that period (Reyes, 1984), clearly emerges that the efforts of most studies are focused on the search for evidence of a causal relationship between "something called *attitude*" (Neale, 1969, p.631) and others variables, in particular mathematical achievement. This causal relationship is seen even as a hypothesis of the etiology of attitude towards maths (Aiken & Drager, 1961).
The search for a causal relationship reveals a normative approach, that strongly drives research on attitude and seems to justify, and in a certain sense to reinforce, the great attention paid to the measurement instruments, rather than to the theoretical clarification of the construct.
Despite its theoretical limits, this first-phase of research on attitude has been fruitful, producing significant results that, coherently with a *cumulative view* of research, have given some decisive contributions for the new-era of research.
The most significant can be considered the initial assumption of this kind of research: non-cognitive factors strictly interact with cognitive factors, and have a crucial role in the learning of mathematics. This assumption is a sort of *break in the wall* of the purely cognitive approach to mathematics education, and will be decisive in the development of the specific field of affect in mathematics education:

not purely cognitive factors – and in particular attitude – assume a relevance in the study of the mathematical learning.

> The attitudes of students toward mathematics play a vital part in their learning (...) Important for the study of attitudes toward mathematics is the idea that an attitude involves both cognitive and non-cognitive aspects. (Corcoran & Gibb, 1961, p. 105)

In addition to this, the great emphasis placed on methods resulted in a refinement of many observational instruments.
This makes it possible to highlight important issues related to the observation of attitude (and more in general of affective constructs), such as the one related to the tendency of individuals to reply to the questionnaires according to what is socially accepted and valued, rather than expressing their own thoughts, i.e. the so called social desiderability phenomenon (Kloosterman and Stages, 1992).
Moreover, in this early period, research on attitude consolidated two significant findings.
A first result – confirmed by many studies – is related to the relationship between attitude towards mathematics and the choice of mathematics courses. For instance, Aiken (1970), in his literature review of the research on attitude, states that there is a good deal of evidences that the choice of enrolling in advanced mathematics courses is significantly affected by attitude.
A second important finding refers to gender differences in mathematical achievements. In particular, the valuable work of Elizabeth Fennema and Julia Sherman (1977) highlights the differences in attitude towards mathematics between males and females, offering a new and significant key of interpretation for gender differences in mathematics achievements:

> Since the study of mathematics appears not to be sex-neutral, attitudes toward mathematics may reflect cultural proscriptions and prescriptions (...) These data certainly indicate that many females have as much mathematical potential as do many males. The generalized belief that females cannot do well in mathematics is not supported. (Fennema & Sherman, 1977, p. 69)

This result, that nowadays can appear unquestionable, was not so obvious before the work of Fennema and Sherman.
Even if, as we have underlined, the first period of research on attitude has surely provided several important findings and suggested research hypotheses, it has also shown its strong limits from the very

beginning. The analysis of these limits has surely been crucial for the development of the research on attitude.

According to Bishop (1992), carrying out a research study in mathematics education requires taking into account three components: enquiry (which concerns the reason for the research activity), evidence and theory. The initial studies on attitude seem to be mainly centred on search for evidence, identifying in the belief on a causal relationship between attitude and achievement in mathematics the main reason for the research activity, and neither developing any theoretical framework nor clarifying the nature of the construct. But in spite of the efforts devoted to developing measuring instruments, research fails in showing not only a causal relationship in the direction attitude – achievement, but also a clear correlation between them.

Aiken (1970) reports the results of several studies in which this correlation is far from being clear, highlighting the need for clarifying its very nature. Almost thirty years later, Ma and Kishor (1997) analysing the results from 113 different studies, conclude that this correlation is not statistically significant. Assuming that this correlation does exist, they identify the cause of the failure in proving it in the inappropriateness of the observing instruments used in research on attitude towards mathematics up to that point. Actually, the instruments used to measure attitude towards mathematics have been criticized by many researchers, because their nature "*exceptionally primitive*" (Leder, 1985).

However, starting from the Eighties, the belief that the major weakness of research lays in the lack of clarity at the theoretical level, in the definition of the construct itself, is becoming even more acknowledged. Kulm (1980) underlines that a trend exists to avoid an explicit definition of attitude towards mathematics, and to adopt instead operational definitions implied by the kinds of instruments used to measure attitude. This lack of interest in characterizing the construct produces a gap between the definition of attitude and its measurement (Leder, 1985), and results in the lack of reliability of the observational instruments.

Using Germann's words to summarize criticism towards the first phase of research about attitude:

> First, the construct of attitude has been vague, inconsistent, and ambiguous. Second, research has often been conducted without a theoretical model of the relationship of attitude with other variables. Third, the attitude instruments themselves are judged to be immature and inadequate. (Germann, 1988, p. 689)

In other words, the naive-theoretical approach that characterizes early studies on attitude appears to be inadequate to the normative-positivistic paradigm in which those studies were conducted. As a matter of fact, this paradigm demands isolating and clearly identifying the variables in order to interpret the statistical results and to compare them:

> Sometimes no description or definition of what is meant by a particular variable is even included in the research report. This makes interpretation of results difficult and detracts from efforts to compare results across studies. (Hart, 1984, p. 573)

For this reason, a re-thinking of research on attitude is a process which already began at the end of the Eighties, involving many aspects of the research in the field: the paradigm in which is framed, the goals that it pursues, the construct's definition, the relationship between the construct and other (affective and cognitive) factors, the development of observational tools and the discussion about the methods for analysing data.

The theoretical debate about *attitude* in mathematics education

In 1992, within the well-known "*Handbook of research on mathematics teaching and learning*", McLeod traces the way for a reconceptualization of research on affect in mathematics education. He identifies three different constructs – beliefs, attitudes and emotions[1] – that, in his view, vary in stability and differ in the degree of the role played by cognition.
McLeod's work starts by a crucial premise:

> Affective issues play a central role in mathematics learning and instruction (...) If research on learning and instruction is to maximize its impact on students and teachers, affective issues need to occupy a more central position in the minds of researchers. (McLeod, 1992, p. 575)

[1] Later, De Bellis and Goldin (1999) propose 'values' as fourth construct of the affective domain.

He underlines the relationship between the role acknowledged to affective factors and the constructivist view of mathematics learning:

> If we believe that the learner is someone who only receives knowledge rather than someone who is actively involved in constructing knowledge, our research program could be entirely different in terms of both the affective and the cognitive domain (McLeod, ibidem, p. 576)

The need for a reconceptualization is strictly connected with the criticism of the previous research on attitude:

> Research on affect has been voluminous, but not particularly powerful in influencing the field of mathematics education. It seems that research on instruction in most cases goes on without any particular attention to the affective issues (...) A major difficulty is that research on affect has not usually been grounded in a strong theoretical foundation. (McLeod, ibidem, p. 590)

Therefore, McLeod underlines as research on affect has to pay particular attention to three aspects strictly intertwined: the discussion of theoretical issues, the development of a wider variety of methods, the analysis of the relationships among affective constructs and between affect and cognition.
Once again, the development of research on attitude is deeply influenced by the simultaneous development of the field of mathematics education by the end of the Eighties. In this period, many scholars debate on the nature of mathematics education and on the criteria for quality of the research in the field. In particular, coherently with the goal of universalization of the research results, the request for a theoretical clarification of the constructs used in the research is emphasized:

> A community of scholars engaged in the research of common areas with common themes, however, has responsibility to communicate ideas and results as clearly as possible using common terms. For these reasons, it is important to use the terms consistently, accurately, and appropriately once their definitions have been agreed on. (Pajares, 1992, p. 315)

What is attitude towards mathematics?

The discussion on the theoretical aspect starts with the 'definition problem': what is attitude towards mathematics?
A first critical issue refers to the object itself of attitude: mathematics. As a matter of fact, if some researchers refer to a 'unique' attitude toward mathematics (Haladyna, Shaughnessy and Shaughnessy,

1983), other claim that many different attitudes exist, according to the different topics and activities that are considered (Tirosh, 1993); still others propose to distinguish between attitude to mathematics seen as a branch of scientific knowledge, and mathematics as school subject (Schoenfeld, 1989), or even that attitude can refer to different objects and situations, such as the mathematical content, the characteristics of mathematics, the kind of teaching, the mathematical activities in class and the teacher of mathematics (Kulm, 1980).
Moreover, this complexity increases when we consider in addition to the variety of objects of attitude, also the variety of subjects: whose attitude? Research on attitude has dealt with a large variety of individuals: students, perspective and in-service teachers, students' parents, and, more in general, adults.
But the major aspect of complexity regarding the definition problem is that it involves not only the characterization of the construct 'attitude', but also that of positive / negative attitude, a dichotomy that pervades research, both implicitly and explicitly.
Classic studies regarding the relationship between attitude and achievement in fact investigate the correlation between *positive* attitude and success. In the same way, studies aiming to change attitude actually end up in setting the objective of transforming a *negative* attitude into a *positive* one.
As already mentioned, a large portion of studies shows a lack of a clear definition of the construct: attitude tends to be defined implicitly and a posteriori through the instruments used to measure it (Kulm, 1980; Leder, 1985; Daskalogianni & Simpson, 2000).
In social psychology the most recent theories agree on the multidimensionality of the construct, and make reference to a *tripartite model*, according to which attitude has a cognitive, an affective, and a behavioural component (Eagly & Chaiken, 1998).
Within the field of mathematics education many explicit definitions of attitude refer to this tripartite model, describing it by means of three components: the emotional disposition towards mathematics, the set of beliefs regarding mathematics, and the behaviour related to mathematics (Hart, 1989; Leder, 1992; Ruffell et al., 1998). However some studies – generally in the earliest period of research – assume a 'simple' characterization, seeing attitude as a general emotional disposition (Haladyna et al., 1983).

Both the two definitions show their theoretical but also operational and didactical limits (Di Martino & Zan, 2001).
The *simple* definition does not make explicit reference to cognitive aspects, although many researchers that assume this definition use models (see Mandler, 1984; Ortony, Clore & Collins, 1988) that emphasize the relationship between emotion and cognition, describing emotional experience as the result of a combination of cognitive analyses and physiological responses. In this framework, it is the interpretation given by an individual to an experience that elicits the emotion, and not the experience itself:

> First, the meaning comes out of the cognitive interpretation of the arousal. This meaning will be dependent on what the individual knows or assumes to be true. In other words, the individual's knowledge and beliefs play a significant role in the interpretation of the interruption. (McLeod, 1992, p. 578)

As regard the characterization of positive / negative attitude, according to the simple definition, it is clear what a positive (negative) attitude is: a positive (negative) emotional disposition toward the subject.
This characterization can be useful to deal with issues such as the choice of mathematics courses or the comparison between different groups of individuals, but it seems inadequate to deal with complex issues such as success in mathematics. In this context, the idea of positive attitude that emerges from the simple definition is not considered very significant by many mathematics education researchers, who underline the importance of linking a positive emotional disposition with an epistemologically correct view of the discipline (Ernest, 1988). In the same vein, the crucial issue of promoting a positive attitude risks to lose its significance, if the goal of developing a positive emotional disposition toward mathematics is not associated to the goal of promoting a *positive* view of the discipline. Considering only the emotional aspects has even a great didactical risk, since it can lead teachers to avoid complex tasks in order to avoid negative emotions.
Discussing similar issues about the attitude definition in the early research leads Kulm (1980) to conclude:

> It is probably not possible to offer a definition of attitude towards mathematics that would be suitable for all situations, and even if one were agreed on, it would probably too general to be useful. (Kulm, 1980, p. 358)

The awareness that the *suitableness* of the construct depends by the issues studied, will lead up to the idea of '*working definition*' (Daskalogianni & Simpson, 2000).
As regard the *tripartite* model, the main critical aspect is that the implicit assumption of a link between attitude and behaviour becomes part of the definition itself of the construct. This theoretical choice exposes research to the risk of circular reasoning, as well described by Lester (2002) for the belief-construct:

> A central difficulty is that the fundamental assumption undergirding much of this research rests on a shaky logical foundation. Specifically, a basic assumption is the beliefs influence peoples' thinking and actions. However, it is also often assumed that beliefs lie hidden and so can be studied only by inferring them from how people think and act. For researchers to claim that students behave in a particular manner because of their beliefs and then infer the students' beliefs from how they behave involves circular reasoning. (Lester, 2002, p. 346)

In the light of these critical aspects, a third definition of attitude emerges in which behaviours are not explicitly mentioned: attitude towards mathematics is described as the pattern of beliefs and emotions associated with mathematics (Daskalogianni & Simpson, 2000).
This choice overcomes the risk of circularity, but it still remains the theoretical problem of identifying a positive/negative attitude according a multidimensional dimension (Di Martino & Zan, 2003). This identification not only requires a characterization of the positive/negative dichotomy for every dimension (emotions, beliefs, possibly behaviour), but also to identify if and how the dichotomies referred to the single components can result in a unique characterization of positive/negative attitude. This issue is strictly connected with the choice of the instruments used to measure attitude.

Instruments used to measure attitude

As Leder (1985) claims, the lack of interest in characterizing the construct produces a gap between the definition of attitude and its measurement: as a matter of fact the instruments traditionally used in order to assess and measure attitudes do not vary according to the different definitions and to the fact that an explicit definition of attitude is given or not.

The instruments used are almost exclusively self-report scales (Kulm, 1980; Leder, 1985; Mc Leod, 1987) such as Thurstone or Likert scales. They propose items that take into consideration beliefs and behaviour as well as emotions: for example 'Mathematics is useful', 'I think about arithmetic problems outside school', 'I like problem solving'. Therefore, these instruments make implicit reference to the tripartite model, regardless of the explicit choice of this kind of definition.

Even if the instruments used appear increasingly sophisticated, the measurement generally results in a reduction to the positive / negative bipolarity, obtained by summing points relating to the three dimensions: cognitive, affective and behavioural.

While some scholars play down this operation, observing that '*the correlation among measures of the three components, although leaving room for some unique variance, are typically of considerable magnitude*' (Ajzen, 1988, p.22), others consider this reduction as contradicting the recognised complexity of the tripartite model (Eagly & Chaiken, 1998). Reducing to a single score the description of attitude is also in contrast with the original idea of Thurstone and Chave (1929) who claim that attitude is a complex construct that cannot be measured with a single score, but it requires several indices. They underline that the choice of the characteristics to be measured – as it happens in measuring a physical object like a table, when one can decide to evaluate the length, the width, the height – depends on the context.

But the theoretical debate about research on attitude highlights other critical issues in the *measurement process*.

First of all, the assessment of each component opens significant problems, due to the limits of questionnaires. As regards beliefs, the mismatch between beliefs expoused and beliefs in action is well known (Schoenfeld, 1989), just like the already mentioned *social desiderability* phenomenon (Kloosterman and Stages, 1992). Regarding emotions, researchers have underlined the difference between the *opinion* given about an emotion and the *emotion* itself (Ruffell et al., 1998), and the limits of instruments such as questionnaires and interviews in capturing emotional reactions that are not conscious (Schlöglmann, 2002).

A second critique concerns the choice of the items that, in the case of questionnaires, is fully demanded to the researchers. As a matter of fact, the respondents have only to express his/her agreement/disagreement about some arguments chosen by the researcher: how can we be sure that the object of the items is relevant for the respondent? In other words, using the terminology introduced by Green (1971) how can be we sure that the corresponding beliefs are psychologically central in the respondent's belief system?
A third critical aspect concerns the choice of the score attributed to each of the items, that requires to identify what is a *positive* emotion / belief / behaviour (this shows the strong relationship between the definition and the measurement problem).
Essentially:
- When *positive* refers to an emotion, normally means 'perceived as pleasurable'. So anxiety when confronting a problem is seen as negative, while the pleasure in doing mathematics is evaluated as positive.
- When *positive* refers to beliefs, is generally used with the meaning 'shared by the experts'. This approach points out its first limit when some studies highlight that a single pattern of beliefs shared by the experts in mathematics does not exist (Mura, 1993, 1995; Grigutsch & Törner, 1998). In the light of this, identifying several different typical patterns of beliefs towards mathematics shared by the experts becomes necessary. Still now, it remains an open issue that could lead to define some different patterns to be taken as models.
- When it refers to a specific behaviour, *positive* generally means 'successful'. In the school context, a successful behaviour is generally identified with high achievement. This characterization leads to the problem of how to assess achievement (Middleton & Spanias, 1999).
A further problem is that generally the difference between the various meanings is rarely made explicit. If the researcher does not declare his/her choices, it becomes problematic to interpret results obtained within a study, and to perform comparisons with different studies.
But even if this ambiguity is overcome by making explicit the choices made, in our opinion other problems remain.

Some studies overlap the three meanings of 'positive' (related to emotion, belief and behaviour) through implicit assumptions: for example, that a 'positive' belief (i.e. shared by the experts) is associated with a successful behavior and elicits a pleasurable emotion; or that a pleasurable emotion is necessarily associated with a positive behaviour in mathematics, and vice versa for negative emotion.
In evaluating a belief (or an emotion) as 'positive' or 'negative' according to the emotion and behaviour related to it, a cause/effect model is assumed, according to which the same belief *causes* the same emotion or the same behaviour in all individuals. Moreover, not only it is assumed that a certain belief has an emotional component, but also the significance of that emotional component; not just that it is linked to behaviour, but also the type of behaviour.
In this case, the cure seems worse than the disease, since this approach does not take into account the very complex nature of the relationship between beliefs, emotions and behaviour.
As a matter of fact, various studies about emotions (Evans, 2000) suggest the possibility that for certain subjects, an optimal level of anxiety exists, above which, but also below which, performance is reduced.
Concerning the relationship between beliefs and emotions, in a study carried out with 211 high school students aged between 14 and 18 years (Di Martino & Zan, 2002) we investigated this relation in the case of the belief "*In mathematics there is a reason for every thing*", also used in many scales for assessing attitude towards mathematics. Students were asked to fill in a questionnaire including the following item:

Choose the option you most agree with: ☐ In mathematics there is always a reason for every thing (B) ☐ It is not true that in mathematics there is always a reason for every thing (not B) And: ☐ I like ☐ I don't like ☐ I find indifferent this characteristic of mathematics

It resulted that only the 51,7% of the sample fell in the two *expected* groups (i.e. B-I like and not B-I don't like). But overall there was not distinction in the percentage of belief B-holder between the

groups of mathematical high achievers and low achievers. The distinction between the two cohorts of students resulted in the emotion associated with this beliefs: the 76% of the high-achievers that are belief B-holder likes this characteristic of math, while this percentage dramatically decreased to 28% within the low-achievers group.
About the combination 'epistemological correct belief – negative emotion', we suggest two possible interpretations. There is the possibility that the negative emotion is *directly* related to the belief. On the other hand, also the possibility that the emotional disposition is not directly linked to the single belief, but to the interaction of it with other ones, has to be taken into account.
This remark questions the possibility to characterize a single belief as positive or negative, without considering the connection with other beliefs an individual may have (the belief system):

> Because they [single beliefs] offer a limited glimpse into a much broader system and because understanding their connections and centrality is essential to understanding the nature of their effect, researchers must study the context-specific effects of beliefs in terms of these connections. (Pajares, 1992, p. 326)

More specifically, Rokeach (1968), describing belief systems, recognizes the dimension of *centrality* underlining that not all the beliefs have the same importance for the individual. Central beliefs play a prominent role in people's belief system, and consequently in influencing their behaviour. As Eagly (1967) observes, beliefs about self are generally considered more central that other ones.
Consider for example the interaction between the belief B used in our study ("*In mathematics there is a reason for every thing*") and the (likely) central self-belief "*I am not able to understand these reasons*": it can result in unproductive behaviours such as avoiding answering or giving random answers (Di Martino, 2004).
These reflections highlight the inadequacy of the assumption of a cause/effect relationship between a specific belief and emotions or behaviours. The interaction is more complex, since it involves the belief systems (and not only the single belief) and strongly depends on the individual.
In order to grasp this complexity, and following the results of this debate on attitude, a movement towards the overcoming of the normative approach and the use of an interpretative approach for research on attitude emerges.

The attitude construct in the reconceptualization of the affective domain in mathematics education

Once again, the history of research on attitude reflects the evolution of the field of mathematics education: the theoretical debate about attitude develops in parallel to a new interpretive perspective that begins to emerge within the field of mathematics education. This perspective, in contrast with the normative – positivistic one, significantly affects the discussion about the theoretical characterization of the constructs.

The gradual affirmation of the interpretive paradigm in social sciences, due to a greater attention paid to the complexity of human behaviour, led researchers in mathematics education to abandon the attempt of explaining behaviour through measurements or general rules based on a cause–effect scheme, and to search instead for new interpretive tools (once again drawing on other domains):

> The purpose of doing interpretivist research (...) is to provide information that will allow the investigator to "make sense" of the world from the perspective of participants. (Eisenhart, 1988, p. 103)

It is a significant shift in focus, with an emerging attention to the understanding of phenomena ("making sense of the world") rather than a description of the phenomena itself: in a certain sense, it is a shift from product to process (Schoenfeld, 1994).

The interpretive approach directly influences also re-thinking methods since the limits of the statistic methods become evident:

> Through the 1980s and into the 1990s (…) with a shift in focus there was a concomitant shift in methods (including the reporting of clinical interviews, process and simulation models, field observations and participant observations), because a new class of phenomena required a new set of explanations a new set of tools to uncover them. (Schoenfeld, 1994, p. 703)

This shift of perspectives gives new strength to research on attitude that was stuck in the causal-relationship paradigm. In particular, attitude gains renewed popularity in the studies aimed at interpreting the failure in problem solving activities of students who seem to have the required cognitive resources.

In 1989, the book "*Affect and mathematical problem solving*" (McLeod & Adams, 1989) – that collects different contributions by several authors – represents the turning point of research on affec-

tive constructs, and in particular on attitude. In fact, for the first time the affective constructs are used not only to prove a numerical correlation with an outcome (the mathematical achievement), but also to interpret a process (the interactions between affective and cognitive aspects in problem solving activity).
Therefore, the need for theoretical clarification in mathematics education (that is also related to the possibility and the will of a cumulative development in the field) appears to be a fundamental issue in the specific research on affect:

> There was a lack of definition, lack of clarity, and lack of connections to mathematics. It is possible to avoid making the same mistakes again as new ideas and research methodologies are employed. It is hoped that new researchers on affect will be clear about what is being studied, precise in definition, and respectful of what has been learned previously. (Fennema, 1989, p. 209)

The double occurrence of the adjective 'new' in Fennema's words it is not casual: it is shows the awareness that new perspectives and new more complex issues force to rethinking the affective constructs. In particular, the shift from a normative paradigm to an interpretive one also results in a discussion (re-definition) about goals, definitions and methods.
It grows the belief that the study on attitude towards mathematics could offer interpretive instruments to understand the reasons of an individual's intentional actions in the mathematical context (Zan et al., 2006). This *belief* is supported by the initial evidence coming from related research in the field of neuroscience:

> There is apparently some neurological basis for asserting a link between affective and cognitive aspects of human functioning. (Silver, 1985, p. 253)

More recently, Damasio (1996) highlights the close relationship between affect and decision-making processes.
The theoretical construct of 'attitude towards mathematics' is not anymore a construct finalised to explain causes of behaviour, thus enabling researchers to predict it, but instead as a flexible and multidimensional interpretative tool, aimed at describing the interactions between affective and cognitive aspects in mathematical activity. In particular, attitude should allow interpreting people's decisions in mathematical activities, and, if necessary, suggesting strategies to modify them.

In this context, particularly significant is the position about the definition itself of the construct attitude of Ruffel, Mason and Allen:

> Reflecting on them [some previous studies about attitude] led us to challenge the very construct of attitude. We are also led to challenge the cause-and-effect model underlying much attitudinal research. We now see *attitude* as at best a complex notion, and we conjecture that perhaps it is not a quality of an individual but rather a construct of an observer's desire to formulate a story to account for observation. (Ruffel et al., 1998, p. 1)

It could be argued that the same thing can be said about every theoretical construct, and not only in mathematics education. In fact, in our opinion, this position expresses the awareness that any phenomenon is observed by a particular point of view, thus highlighting the role of the researcher/observer that is not a mere measurer. It represents the overcoming of a naive approach, in which attitude is seen as a quality of an individual objectively measurable, and the transition to a theoretical approach.

In line with this perspective, Daskalogianni and Simpson (2000) assume that the definition of attitude becomes a *working definition*, functional to the problems that the researchers pose themself. Therefore, having different definitions of the construct appears natural, and a single definition is not any more valued in terms of *correctness* (is it the *right* definition?) but instead in terms of *suitability* for addressing a specific research problem in mathematics education (Di Martino & Zan, 2010). According to the classification of the research proposed by Bishop (1992), this kind of approach characterizes the new trend of research on attitude as *problem-led*.

The theoretical re-thinking of the research on attitude brings to the exploration of new methods of inquiry in the field. Coherently with their positions, Ruffel, Mason and Allen (1998) underline the inadequacy of the *measurement approach*, giving up to use the verb 'measuring' and substitute it with the verb 'probing'.

At the beginning of the new millennium, a criticism towards the use of quantitative methods in the research on attitude clearly emerges, and a movement towards the use of a qualitative approach begins.

It emerges the awareness that qualitative methods, and in particular the use of narratives, makes it possible to take into account those beliefs and emotions which are psychologically central for the respondents. Some studies using essays, diaries, interviews and also the observation of behaviour in natural settings or in structured sit-

uations appear (Karsenty & Vinner, 2000; Hannula, 2002; Zan & Di Martino, 2007; Kaasila, 2007).
Differently from what happens with the traditional attitude scales, the respondents are not requested to express an agreement / disagreement with respect to items chosen by others, but telling their mathematical stories, they can make explicit the aspects that they consider relevant for their relationship with mathematics. As a matter of fact, the pivotal motivation for using narrative in educational research is the following:

> Humans are storytelling organisms who, individually and socially, lead storied lives. The study of narrative, therefore, is the study of the ways humans experience the world. (Connelly & Clandinin, 1990, p. 2)

As a consequence, after almost ten years after the *manifesto* of McLeod, the shift in focus in research on attitude provokes a shift in perspectives and methods: a real *revolution*.

The TMA model: a definition of attitude grounded on students' narratives

In the described framework, within an Italian National Project, we investigated about the use of the diagnosis '*this student has a negative attitude*' by mathematics teachers to interpret mathematical difficulties.
The results of the study (Polo & Zan, 2006) show not only that the diagnosis is frequently used (in the different school levels), but, above all, that, in the majority of cases, it represents a claim of surrender rather than an interpretive step capable of steering future action: a sort of black box.
These results persuaded us that, in order to turn the 'negative attitude diagnosis' into a useful instrument for both practitioners and researchers, it was necessary to link the theoretical construct of attitude to the practice. This fitted with the strong incentive "*to develop constructs that might be applied to help make sense of teaching and learning environments*" (Philipp, 2007, p.264).
Therefore we carried out a study, based on the collection and analysis of students' autobiographical narratives, aimed at constructing a

characterisation of students' attitude towards mathematics strictly linked to students' experience (Di Martino & Zan, 2010).
Our hypotheses in choosing autobiographical essays is that pupils tend to explicitly evoke those events and remarks about their past that they deem important, and they also tend to paste fragments, introducing some causal links, not in a logical perspective but rather in a social, ethical and psychological one (Bruner 1990).
We assume that in order to describe the kind of relationship an individual has with mathematics, and consequently to suggest a characterisation of attitude towards mathematics strictly linked to the experience, this pasting process is more important than an *objective* report of one's experience with the discipline at school.
As Bruner claims:

> It does not matter whether the account conforms to what others might say who were witnesses, nor are we in pursuit of such ontologically obscure issues as whether the account is 'self-deceptive' or 'true'. Our interest, rather, is only in what the person thought he did, what he thought he was in, and so on. (Bruner, 1990, pp. 119–120)

In doing so, a theoretical model for attitude emerges from the data collected through a cyclical analytical process, what Glaser and Strauss (1967) call a *grounded theory*.
In this kind of process, the autobiographical texts are analysed in order to systematically produce sense starting from the individuals' narrations: the final outcome is the identification of a set of categories and relationships aimed at understanding and interpreting different behaviours (Demazière & Dubar, 1997).
We collected and analysed 1662 anonymous essays entitled "Maths and me: my relationship with maths up to now", written by students whose school levels ranged from grade 1 to grade 13[2].
From our study, it emerges that when students describe their relationship with mathematics, almost all of them refer to one (or more) of the following three dimensions:

- the emotional disposition towards mathematics,
- the vision of mathematics,
- the perceived competence in mathematics.

[2] The sample of the study was not chosen on a statistical basis, but through the collaboration of teachers who voluntarily accepted to participate in our research.

This result suggests the Three-dimensional Model for Attitude (TMA) represented in Fig. 1.

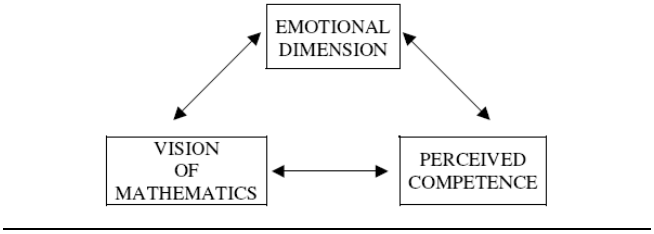

Figure 1: The Three-dimensional Model for Attitude

TMA takes explicitly into account the close relationship among the three dimensions. The research study also highlighted the subjectivity of these interactions, confirming the need for observational tools suitable for tracking it:

> The proposed model of attitude acts as a *bridge* between beliefs and emotions, in that it explicitly takes into account beliefs (about self and mathematics) and emotions, and also the interplay between them. However, in order for it to become effective theoretical and didactical instruments, the construction and use of consistent instruments for observation, capable of taking into account its complexity, is needed. (Di Martino & Zan, 2011, p. 479)

The analysis of the students' autobiographies also suggests the development of a new approach to the positive/negative characterization of attitude, confirming how the reduction of the dichotomy positive/negative attitude to the mere emotional dimension can be questionable. As a matter of fact, we found that negative emotional dispositions towards mathematics may be associated with different patterns of attitude, depending on the student's perceived competence and vision of mathematics as well as on the relationships amongst the three dimensions. Coherently with this observation and with the multidimensional characterization of the construct in TMA, we assume a definition of 'negative attitude' that explicitly makes reference to the negativity of at least one of the three dimensions:

> The multidimensionality of the model underlines the inadequacy of the positive/negative dichotomy for attitude referred only to the emotional dimension (like/dislike), and rather suggests considering an attitude as *negative*, when at least one of the dimensions is *negative*. In this way, we can outline *profiles* of negative attitude, depending on the dimension that appears to be *negative*. (Di Martino & Zan, 2010, p. 44)

We identify two polarities for each dimension, and assume as negative: an emotional disposition resulting in disliking mathematics, a low perceived competence, and – according to the characterization of Skemp (1976) – an instrumental vision of mathematics.
This definition of negative profiles of attitude within TMA suggests two new interrelated research issues. One the one hand, the development of observational tools aimed at identifying a student's profile of attitude towards mathematics, in particular at recognizing a possible negative component in this profile. On the other hand, the theoretical construction and experimentation of didactical actions, differentiated according to the different profiles of negative attitude identified in TMA, aimed at preventing or overcoming a negative attitude towards mathematics.
The TMA model, born as a model for students' attitude towards mathematics, appears suitable also to characterize attitude towards specific topics of mathematics (such as geometry, algebra,…) and to investigate about attitude towards mathematics held by different groups of people (such as teachers, adults, …).
For this reason, recently, the TMA model has been used also to study and analyse the attitude towards mathematics and its teaching of in-service and pre-service primary teachers (Coppola, Di Martino, Pacelli & Sabena, 2013).

Summing up and looking ahead

In mathematics education, the research on attitude has a very long tradition, based on the interest, shared by mathematicians, teachers and maths educators, in identifying a causal relationship between something called 'positive attitude' and achievement. In the first period of the research most of the studies aim at refining or developing measuring instruments, rather than at clarifying theoretical aspects.
With the evolution of Mathematics Education as a research field, and with the development of a specific research field on affect, also research on attitude towards mathematics evolves, identifying critical points in the previous phase, and posing the need for a theoretical framework as crucial issue in the agenda for future research. This

movement also results in a shift from a normative paradigm to an interpretive one. Attitude is no longer seen as an individual's trait, useful to predict his/her behaviour, but as an observer's construct, capable of suggesting an understanding of the individual's intentional actions in a complex context, as is the learning of mathematics: a multidimensional construct, that involves beliefs and emotions, acting as a bridge between them (Di Martino & Zan, 2011).

The development of research on attitude also suggests new issues to be explored, such as: constructing new observation tools, consistent with the interpretive approach and the multidimensional characterization of attitude; investigating attitude toward mathematics of different groups of individuals; identifying possible motives underlying a change of attitude; designing and testing didactical paths to prevent or modify attitude.

But a main issue remains the theoretical debate about the quality of research about attitude. As a matter of fact, the need of comparing results from different studies and different theoretical frameworks still is a crucial issue, even when studies with questionnaires and statistical analysis have re-placed by few case studies analysed with a qualitative approach. New paradigms and new methods require therefore the identification of new criteria for research quality: this is an important issue for future agenda of research in the affect field.

Despite the fact that many studies on attitude 'look ahead', taking into account the most important findings collected up to now, in our opinion some critical issues still remain.

The gap between the definition of the attitude construct and the methods used to assess it is far for being filled: many studies still use the term 'attitude' without defining it, or proposes questionnaires not consistent with the chosen characterization of attitude, and without clarifying the theoretical choices underlying them. Moreover, although the normative approach in the research on attitude has showed all its theoretical limits, many recent studies place themselves in a normative paradigm, even if, perhaps, not as a conscious choice made by the researcher.

This lack of a cumulative character in research on attitude is, in our opinion, one of its main weaknesses, a historical weakness that it has not been overcome. In 1976, in his updating of the research on attitude, Aiken wrote:

> Regardless of the efforts of this writer and others to bring to the educational research community periodic reviews of studies concerned with attitudes and anxiety toward mathematics, many investigators in this area continue to be unaware or unappreciative of previous research on the topic (…) This oversight is almost certainly due to a failure to search the relevant literature, the first step in any scientific inquiry (Aiken, 1976, p. 293)

More than thirty years later, we notice exactly the same phenomena, in a sort of theoretical and meta-theoretical déjà vu that, we are sure, has hardly limited the development of stronger results in the field. For that reason, we believe that tracing the 'story' of the construct attitude and discussing the results obtained is very significant.

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