On Dynamic Multiple Criteria Decision Making Models: A Goal Programming Approach

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Abstract

Dynamic multiple criteria decision making (DMCDM) represents an extension of classical multiple criteria decision making to a context in which all variables are depending on time. This complex decision making problem requires the development of methodologies able to incorporate different and conflicting goals in a satisfying design of policies. We formulate two different goal programming models, namely a weighted goal programming model and a goal programing model with satisfaction functions, for solving DMCDM models. We present an application of this methodology to analyze the trade-off between consumption and investment in a traditional Ramsey-type (1928) macroeconomic model with heterogeneous agents. For a specific realistic parameterization, such a model is solved by means of the proposed goal programming formulations.

1 Introduction

As society and the world as a whole become more and more complex, making good decisions becomes harder and harder. More frequently a decision maker (DM) is called to evaluate a set of alternatives in terms of a certain number of conflicting criteria. This is even more critical in an economic setup where the DM tries to allocate resources to his/her best use (going beyond the traditional resource availability constraints) and also needs to take into account several economic goals. Several authors in the literature have highlighted that human needs are incommensurable and thus economic benefits cannot be measured by a mere scalar number. Keeney and Howard (1976:19) state that *"in complex value problems consequences cannot be adequately described objectively by a single attribute"*. Just to provide a simple example, we can mention the recent concern for environmental issues; the DM has to plan the use of natural resources (such as water, land, and forestry) coping with several conflicting objectives (as, for instance, the decrease of the level of emissions of a power plant against the benefits of the power plant itself). Along this direction, André *et al.* (2009) have recently pointed out that policymakers do not seek to maximize a single function, but they are typically concerned about a bundle of economic, social and environmental variables or indicators, thus they try to design their policies to improve the performance of the economy as measured by multiple indicators.

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In reality, all problems and especially those related to economic issues cannot be summarized in a static framework, but carefully require to consider the evolution of the decision making process over time. For example, think about the simplest economic problem that every individual needs to face in his daily life, like the choice between consuming in order to achieve higher current utility or saving in order to invest his resources and possibly achieve higher utility levels in the future. This clearly requires taking into account the dynamic evolution of income over a certain horizon (possibly the whole lifespan) in order to determine the best allocation of a scarce resource (income) between its possible usages (consumption and investment). Even more complex it is the task for economic policymakers, since whenever trying to determine the best policy in order to pursue alternative economic goals, they need to account for the implications of such a policy on every single agent in the society. In addition, the presence of agents' heterogeneity makes the decision making process particularly difficult. Given such a dynamic nature of real world problems and the need to account for agents' heterogeneity, it is quite natural to rely upon dynamic multiple criteria decision making (DMCDM) in which for any feasible path the objective function provides a vector of values, representing the individual utility of every economic agent.

DMCDM models need to be understood in the Pareto sense, meaning that they search for optimal solutions with respect to the Pareto ordering cone. In the literature some results have been proved to characterize the optimal solutions of such models (see for instance, Khanh and Nuong (1988, 1989), and Ginchev *et al.* (2012)). In this chapter we do not provide any alternative optimality condition but we focus on practical approaches for solving these models instead. We propose two different goal programming (GP) models, namely a weighted GP (WGP) model and a GP model with satisfaction functions (GPSF), for approximating and solving a DMCDM program. We then illustrate this approach through a specific example in the context of macroeconomic policies in which we extend the classical Ramsey model analyzing the trade-off between consumption and investment choices by introducing a vector-valued utility to take into account agents' heterogeneity. Focusing on the Ramsey model allows us to exemplify the potential implications of the usage of DMCDM models to study economic problems. However, the proposed models can be straightforwardly adapted in order to analyze several other issues in which the problem is dynamic in nature and agents are heterogeneous with respect to certain characteristics. Apart from macroeconomic applications, others may include environmental policies, climate change agreements, and more broadly speaking differential and cooperative games (see Endwerda (2007) for a discussion of a dynamic multiple criteria approach applied to game theory).

The chapter is organized as follows. First we present the basic formulation of a DMCDM model, and then we recall some well-known GP models. After introducing two alternative specifications of the GP model, we present an illustrative example dealing with a multi agent macroeconomic model. Finally, we present some concluding remarks and propose directions for future research.

2 Multiple Criteria Decision Making

The general formulation of the multi-criteria model can be specified as follows: optimize $[J_1(x), J_2(x), ..., J_p(x)]$ under the condition that $x \in D$ where $J_i(x)$ represents the i-th objective function and D designates the set of feasible solutions (typically a compact subset of a normed vector space X). Let us define a vector function $J(x) := [J_1(x), J_2(x), ..., J_p(x)]$; according to this definition and by assuming that all objectives have to be minimized, a classical MCDM problem can be formulated as:

$$\begin{aligned} \min \quad & J(x) \\ s.t. \quad & x \in D \end{aligned} \tag{1}$$

As usual in multi-criteria optimization, a point $\hat{x} \in D$ is a global Pareto solution or efficient solution if for all $J(x) \subseteq J(\hat{x}) + (-R^p_+ \setminus \{0\})^c$ for all $x \in D$. In other words, a point $\hat{x} \in D$ is a Pareto solution if there is no $x \in D$ such that for all $J_i(x) \geq J_i(\hat{x})$ for all $i = 1, ..., p$ and $J_{i^*}(x) \geq J_{i^*}(\hat{x})$ for at least one $i^* \in \{1, ..., p\}$. The set of all Pareto solutions is called the Pareto frontier. Then a Pareto solution is never dominated by another feasible solution and for this reason it is called an undominated solution. The following results provide two conditions that characterize Pareto solutions (Sawaragi *et al.*, 1985).

Theorem 1. *Let $\alpha_i \in (0, 1)$, $\sum_{i=1}^{p} \alpha_i = 1$. Assume that $\hat{x} \in D$ is such that:*

$$\hat{x} \quad \in \quad \text{argmin}_{x \in D} \left\{ \sum_{i=1}^{p} \alpha_i J_i(x) \right\}$$

Then $\hat{x}$ is a Pareto optimal solution.

Theorem 2. *Suppose that Dis convex and $J_i(x)$ are convex for all $i = 1, ..., p$. Then for all Pareto optimal solutions $\hat{x}$ there exists $\alpha \in R^p$, $\alpha_i \in [0, 1]$, $\sum_{i=1}^{p} \alpha_i = 1$, such that:*

$$\hat{x} \quad \in \quad \text{argmin}_{x \in D} \left\{ \sum_{i=1}^{p} \alpha_i J_i(x) \right\}$$

The above Theorems 1 and 2 provide basic structure on the objective functions and the feasible solution set to characterize Pareto solutions in a multiple criteria problem. In particular Theorem 1 provides a sufficient optimality condition for Pareto optimality while Theorem 2 a sufficient one. It is worth noting that Theorem 2 is valid under the assumption of convexity.

3 Goal Programming

Within the multi-criteria decision aid paradigm, several usually conflicting criteria are considered simultaneously. The GP model is a well-known aggregating methodology for solving multi-objective programming problems allowing to take into account simultaneously several conflicting objectives. Thus the obtained solution through the GP model represents the best compromise that can be achieved by the DM. The GP model is a distance function where the unwanted positive and negative deviations, between the achievement and aspiration levels, are to be minimized. The GP model, first proposed by Charnes (1952) and Charnes *et al.* (1955), has been widely applied in several fields such as accounting, marketing, quality control, human resources, production, economics and operations management (Aouni *et al.,* 1997; Romero, 1991). The standard mathematical formulation of the GP model (Charnes, 1952) is as follows:

$$\begin{aligned} \min Z \quad &= \quad \sum_{i=1}^{p} \delta_i^+ + \delta_i^- \qquad (2) \\ s.t. \quad & \quad J_i(x) + \delta_i^+ - \delta_i^- = g_i, \quad i = 1, ..., p \\ & \quad x \in D \\ & \quad \delta_i^+, \delta_i^- \geq 0, \quad i = 1, ..., p \end{aligned}$$

where δ_i^+ and δ_i^- are, respectively, the positive and the negative deviations with respect to the aspiration levels (goals) g_i, $i = 1, ..., p$. The DM's appreciation of the positive and the negative deviations can be different based on the relative importance of the objective which can be expressed through the weights w_i^+ and w_i^- respectively. The mathematical formulation of the weighted GP (WGP) is as follows:

$$\begin{aligned} \min Z \quad &= \quad \sum_{i=1}^{p} w_i^+ \delta_i^+ + w_i^- \delta_i^- \qquad (3) \\ s.t. \quad & \quad J_i(x) + \delta_i^+ - \delta_i^- = g_i, \quad i = 1, ..., p \\ & \quad x \in D \\ & \quad \delta_i^+, \delta_i^- \geq 0, \quad i = 1, ..., p \end{aligned}$$

In decision making the role of the DM is crucial, and both how he thinks and decides, and what are his own values can significantly affect the decision making process. Usually a DM has a specific set of preferences which can be described through the notion of satisfaction functions. When such a system of preferences is introduced the GP model takes the following form:

$$\begin{aligned}
\max Z \quad &= \quad \sum_{i=1}^{p} w_i^+ F_i^+(\delta_i^+) + w_i^- F_i^-(\delta_i^-) \qquad (4)\\
s.t. \quad & J_i(x) + \delta_i^+ - \delta_i^- = g_i, \quad i = 1, ..., p\\
& x \in D\\
& 0 \leq \delta_i^+ \leq \alpha_{i\nu}^+ \quad i = 1, ..., p\\
& 0 \leq \delta_i^- \leq \alpha_{i\nu}^- \quad i = 1, ..., p
\end{aligned}$$

The satisfaction functions $F_i(\delta_i)$ allow the DM to express explicitly his preferences for any deviation between the achievement and aspiration levels of each objective: the general shape of the satisfaction functions is shown in Figure 1 (Martel and Aouni, 1990), where $F_i(\delta_i)$ is the satisfaction function associated with the deviation δ_i, α_{id} the indifference threshold, α_{io} the dissatisfaction threshold and $\alpha_{i\nu}$ the *veto* threshold.

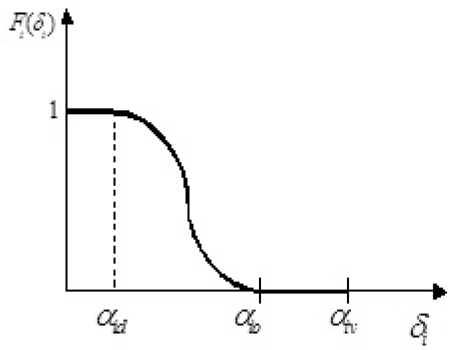

Figure 1: General form of the satisfaction function.

4 Dynamic Multiple Criteria Decision Making and Goal Programming

Denote with $X = C^1([a, b])$ the space of all differentiable paths defined on $[a, b]$, with U a set of controls, and with $f : R \times R^n \times R^n \to R^p$ and $h : R \times R^n \times R^n \to R^n$ two vector-valued smooth functions. Let us consider the following dynamic multi-criteria problem:

$$\begin{aligned}
\min \quad & \int_a^b f(t, x(t), u(t))dt \qquad (5)\\
s.t. \quad & \dot{x}(t) = h(t, x(t), u(t))\\
& x(a) = x_a\\
& x(b) = x_b\\
& u \in U
\end{aligned}$$

This represents a natural extension of classical optimal control problems to vector problems. We do not formulate any hypothesis on b, the above model can be assumed to be over either a finite or a infinite horizon.

Optimality conditions for (4) can be found, for instance, in Ginchev et al. (2012) and Engwerda (2007). They can be stated by using the optimality conditions presented in the previous case for an abstract optimization model and by defining

$$J_i(x,u) = \int_a^b f_i(t, x(t), u(t))dt.$$

We are now ready to formulate a GP model for solving (5). Let $g = (g_1, g_2, ..., g_p) \in R^p$ be a set of p goals corresponding to p functionals $\mathrm{J}_i(x,u)$. For this model, the standard mathematical formulation of the GP model is as follows:

$$\begin{aligned} \min Z &= \sum_{i=1}^{p} \delta_i^+ + \delta_i^- \qquad (6)\\ s.t. \quad & \int_a^b f_i(t, x(t), u(t))dt + \delta_i^- - \delta_i^+ = g_i, \quad i = 1, ..., p \\ & \dot{x}(t) = h(t, x(t), u(t)) \\ & x(a) = x_a \\ & x(b) = x_b \\ & u \in U \\ & \delta_i^-, \delta_i^+ \geq 0, \quad i = 1, ..., p \end{aligned}$$

where δ_i^+ and δ_i^- are, respectively, the positive and the negative deviations with respect to the aspiration levels (goals) g_i, $i = 1, ..., p$. An alternative model is the weighted goal programming that in this context can be formulated as follows:

$$\begin{aligned} \min Z &= \sum_{i=1}^{p} w_i^+ \delta_i^+ + w_i^- \delta_i^- \qquad (7)\\ s.t. \quad & \int_a^b f_i(t, x(t), u(t))dt + \delta_i^- - \delta_i^+ = g_i, \quad i = 1, ..., p \\ & \dot{x}(t) = h(t, x(t), u(t)) \\ & x(a) = x_a \\ & x(b) = x_b \\ & u \in U \\ & \delta_i^-, \delta_i^+ \geq 0, \quad i = 1, ..., p \end{aligned}$$

where w_i^+ and w_i^- are the weights corresponding to positive and negative deviations, respectively. The DM can express the relative importance of the objectives by providing a level of satisfaction of each positive and negative deviation. As a result, the GP model with satisfaction function is

$$\begin{aligned} \max Z &= \sum_{i=1}^{p} w_i^+ F_i^+ \left(\delta_i^+\right) + w_i^- F_i^- \left(\delta_i^-\right) \qquad (8)\\ s.t. \quad & \int_a^b f_i(t, x(t), u(t))dt + \delta_i^- - \delta_i^+ = g_i, \quad i = 1, ..., p \\ & \dot{x}(t) = h(t, x(t), u(t)) \\ & x(a) = x_a \\ & x(b) = x_b \\ & u \in U \\ & 0 \leq \delta_i^- \leq \alpha_{i\nu}^-, \quad i = 1, ..., p \\ & 0 \leq \delta_i^+ \leq \alpha_{i\nu}^+, \quad i = 1, ..., p \end{aligned}$$

The above two alternative formulations (7) - (8), after discretization of both integrals and differential equations, can be solved as static optimization problems.

5 An Example: the Ramsey Model with Vector-Valued Utility

For our purpose of exemplifying the usage of the two proposed GP models for solving DMCDM problems, the well-known Ramsey (1928) model may be useful. Indeed, it well fits formulation (2), since it summarizes the investment problem from a macroeconomic point of view as a traditional optimal control problem. The Ramsey (1928) model basically describes how a benevolent social planner (i.e., the DM) might decide what is the optimal level of consumption for the whole society by taking into account the fact that a larger consumption level tends to crowd out resources from investment opportunities: the more we consume today, the less we save and invest, thus the less resources we will have in the future to allow further consumption possibilities. The model is nowadays still the benchmark for assessing the impact of alternative macroeconomic policies on the long run development process of different economies. It has been extended along several directions in order to take into account also issues related to demography (Marsiglio, 2014), environment (Marsiglio, 2011), technological progress (La Torre and Marsiglio (2010)), human capital (Marsiglio and La Torre (2012a), (2012b)), and many other aspects relevant for macroeconomic goals.

The standard Ramsey model is a scalar problem in which the DM determines the best choice for the society as a whole which is totally summarized by the characteristics of the so-called "representative agent". In such a framework the society is totally homogeneous, in the sense that its members have all the same characteristics (preferences, endowments, and even relevant parameters), thus the optimization with respect to the representative agent coincides with the optimization for the whole society. Such a homogeneity in the characteristics of agents is clearly a strong simplification of reality, since in every society individuals differ in several ways. A more sensible description of the problem would thus require to allow for some heterogeneity in the characteristics of agents, and this can be straightforwardly done with the GP models we introduced in the previous section. Indeed, a simple way to account for agents' heterogeneity is assuming that the instantaneous utility function does not take a scalar form but a vector-valued one. This means that agents are identical for some aspect (capital endowments) but not for others (preferences). A vector-valued extension of the Ramsey model in Banach spaces has been recently discussed in Ginchev et al (2012) where the authors also provide necessary and sufficient optimality conditions.

In a Ramsey-type (1928) model, the social planner seeks to maximize social welfare by choosing the level of consumption and taking into account the dynamic evolution of capital. For the sake of simplicity we abstract from population growth and we normalize the population size to unity. The dynamic evolution of capital, coinciding with investments, depends on the difference between net (of replacement investments, with η being the depreciation rate of capital) output, $Y(t)$, and consumption, $C(t)$. Output is produced according to a Cobb-Douglas production function, $Y(t) = AK^{\alpha}(t)$, where A is a technological scale parameter and $0 < \alpha < 1$ represents the capital share of output. Whenever agents are homogeneous, the social welfare is defined as the discounted (ρ is the pure rate of time preference) sum of the instantaneous utilities of the representative agent; the instantaneous utility function is assumed to take a constant elasticity of substitution form, $U(C(t)) = \frac{C(t)^{1-\varphi}-1}{1-\varphi}$, where φ denotes the inverse of the intertemporal elasticity of substitution. However, when agents are heterogeneous and differ for their preferences, focusing on the representative agent is no longer possible. In such a framework we need to take into account the specific preferences of each single agent, and thus the social welfare function needs to reflect this, by attaching some weight to the utility of each agent. Denoting with $U_i(C(t))$ the instantaneous utility for agent i, in order to allow for some heterogeneity we assume that the parameter denoting the rate of time preference, ρ_i, and the intertemporal elasticity of substitution, φ_i, can differ from agent to agent. Thus, our vector-valued Ramsey model takes the form:

$$\begin{aligned} \max \quad & \left(\int_a^b U_1(C(t))e^{-\rho_1 t}dt, ..., \int_a^b U_p(C(t))e^{-\rho_p t}dt \right) \\ s.t. \quad & \dot{K}(t) = AK(t)^\alpha - \eta K(t) - C(t) \end{aligned} \tag{9}$$

The objective function in (9), describing the social welfare, takes values in R^p and depends on p instantaneous utility functions $U_i, i = 1, ..., p$ and different discount factors ρ_i. The dynamic constraint describes the evolution of physical capital over time, stating that for each t, output, $Y(t)$, is either consumed ($C(t)$) or invested ($\dot{K}(t) + \eta K(t)$). We assume that capital endowments are the same for each individual and since the capital market is the same for each agent, the dynamic evolution of capital is not agent-specific.

In order to solve the problem above, we rely on the two GP formulations earlier described. For the sake of simplicity we focus on the case in which we only have two agents, that is $p = 2$, meaning that the objective function takes values in R^2. The problem we are interested in can thus be formulated as follows:

$$\begin{aligned} \max_{C(t)} \quad & \left(\int_0^\infty \frac{C(t)^{1-\varphi_1} - 1}{1 - \varphi_1} e^{-\rho_1 t}dt, \int_0^\infty \frac{C(t)^{1-\varphi_2} - 1}{1 - \varphi_2} e^{-\rho_2 t}dt \right) \\ s.t. \quad & \dot{K}(t) = AK(t)^\alpha - \eta K(t) - C(t) \end{aligned} \tag{10}$$

In order to apply our GP models, we need first of all to determine the goal for each criterion. In order to do so, we proceed by solving the two single criterion problems separately, and using the value of the associated optimal objective functions to determine the goal to attach to the relevant criterion. Thus we consider two maximization problems separately, namely one for agent 1:

$$\begin{aligned} g_1 = \max_{C(t)} \quad & \int_0^\infty \frac{C(t)^{1-\varphi_1} - 1}{1 - \varphi_1} e^{-\rho_1 t}dt \\ s.t. \quad & \dot{K}(t) = AK(t)^\alpha - \eta K(t) - C(t) \end{aligned} \tag{11}$$

and one for agent 2:

$$\begin{aligned} g_2 = \max_{C(t)} \quad & \int_0^\infty \frac{C(t)^{1-\varphi_2} - 1}{1 - \varphi_2} e^{-\rho_2 t}dt \\ s.t. \quad & \dot{K}(t) = AK(t)^\alpha - \eta K(t) - C(t) \end{aligned} \tag{12}$$

The problems (11) and (12) can be analytically solved in order to determine the value of the goals g_1 and g_2 (see Smith, 2007). Once g_1 and g_2 have been determined, our two different specifications of the GP model can applied. Specifically, the WGP model can be constructed as follows:

$$\begin{aligned} \min Z \quad = \quad & w_1^+\delta_1^+ + w_1^-\delta_1^- + w_2^+\delta_2^+ + w_2^-\delta_2^- \\ s.t. \quad & \int_0^\infty \frac{C(t)^{1-\varphi_1} - 1}{1 - \varphi_1} e^{-\rho_1 t}dt + \delta_1^- - \delta_1^+ = g_1 \\ & \int_0^\infty \frac{C(t)^{1-\varphi_2} - 1}{1 - \varphi_2} e^{-\rho_2 t}dt + \delta_2^- - \delta_2^+ = g_2 \\ & \dot{K}(t) = AK(t)^\alpha - \eta K(t) - C(t) \\ & \delta_i^-, \delta_i^+ \geq 0 \quad \forall i \in \{1, 2\} \end{aligned} \tag{13}$$

Instead, the GPSF can be constructed as follows:

$$
\begin{aligned}
\max Z \quad &= \quad w_1^+ F_1^+\left(\delta_1^+\right) + w_1^- F_1^-\left(\delta_1^-\right) + w_2^+ F_2^+\left(\delta_2^+\right) + w_2^- F_2^-\left(\delta_2^-\right) \\
s.t. \quad & \quad \int_0^\infty \frac{C(t)^{1-\varphi_1}-1}{1-\varphi_1} e^{-\rho_1 t} dt + \delta_1^- - \delta_1^+ = g_1 \\
& \quad \int_0^\infty \frac{C(t)^{1-\varphi_2}-1}{1-\varphi_2} e^{-\rho_2 t} dt + \delta_2^- - \delta_2^+ = g_2 \\
& \quad \dot{K}(t) = AK(t)^\alpha - \eta K(t) - C(t) \\
& \quad 0 \leq \delta_1^+ \leq \alpha_{1v}^+ \\
& \quad 0 \leq \delta_2^+ \leq \alpha_{2v}^+ \\
& \quad 0 \leq \delta_1^- \leq \alpha_{1v}^- \\
& \quad 0 \leq \delta_2^- \leq \alpha_{2v}^-
\end{aligned} \tag{14}
$$

5.1 Numerical simulations

We now provide a numerical solution of our model by applying the GP specification in (13) and (14). For this purpose, we set the values of the parameters as follows: $A = 1$, $\rho_1 = 0.05$, $\rho_2 = 0.06$, $\varphi_1 = 2$, $\varphi_2 = 2.5$, $\eta = 0.05$, $\alpha = 0.33$, and $K_0 = 1$ (see Barro and Sala-i-Martin, 2004, for an economic justification of these parameters' values), and we use LINGO 14 for solving the relevant optimization problem. Under this parameterization, the model (10) reads as:

$$
\begin{aligned}
\max_{C(t)} \quad &= \quad \left(\int_0^\infty \frac{C(t)^{-1}-1}{-1} e^{-0.05t} dt, \int_0^\infty \frac{C(t)^{-1.5}-1}{-1.5} e^{-0.06t} dt\right) \\
s.t. \quad & \quad \dot{K}(t) = K(t)^{0.33} - 0.05K(t) - C(t)
\end{aligned} \tag{15}
$$

We focus first on the WGP specification, where the weights $w_1^+, w_1^-, w_2^+, w_2^-$ are assumed to take different values, in order to assess how attaching a different weight to each different criterion will affect the model's solution. For the sake of simplicity, we assume that positive and negative deviations receive the same weights. This means that if we attach a weight of 0.2 to the first agent ($w_1^+ = w_1^- = 0.2$) then we are attaching a weight of 0.8 to the second one ($w_2^+ = w_2^- = 0.8$). Specifically, we consider four different weights configurations: 0.2, 0.4, 0.6 and 0.8, meaning that $w_1^+ = w_1^- = 0.2, 0.4, 0.6, 0.8$ whenever $w_2^+ = w_2^- = 0.8, 0.6, 0.4, 0.2$. The WGP model can now formulated as follows:

$$
\begin{aligned}
\min Z \quad &= \quad w_1^+\delta_1^+ + w_1^-\delta_1^- + w_2^+\delta_2^+ + w_2^-\delta_2^- \\
s.t. \quad & \quad \int_0^\infty \frac{C(t)^{-1}-1}{-1} e^{-0.05t} dt + \delta_1^- - \delta_1^+ = g_1 \\
& \quad \int_0^\infty \frac{C(t)^{-1.5}-1}{-1.5} e^{-0.06t} dt + \delta_2^- - \delta_2^+ = g_2 \\
& \quad \dot{K}(t) = K(t)^{0.33} - 0.05K(t) - C(t) \\
& \quad \delta_i^-, \delta_i^+ \geq 0, \quad \forall i \in \{1, 2\}
\end{aligned} \tag{16}
$$

Since the problem is stated in discrete time, we need to proceed with its discretization in order to perform some numerical simulation. We approximate the infinite horizon integrals with a finite horizon T and the differential equations using classical numerical schemes as follows:

$$\begin{aligned}
\min Z &= w_1^+\delta_1^+ + w_1^-\delta_1^- + w_2^+\delta_2^+ + w_2^-\delta_2^- \\
s.t. \quad & \sum_{j=0}^{T} \frac{C(t)^{-1}-1}{-1} e^{-0.05t} dt + \delta_1^- - \delta_1^+ = g_1 \\
& \sum_{j=0}^{T} \frac{C(t)^{-1.5}-1}{-1.5} e^{-0.06t} dt + \delta_2^- - \delta_2^+ = g_2 \\
& K(j+1) - K(j) = K(j)^{0.33} - 0.05K(j) - C(j), \quad j = 0, ..., T-1 \\
& \delta_1^-, \delta_1^+, \delta_2^-, \delta_2^+ \geq 0
\end{aligned}$$

where the goals $g_1 = 2.07$ and $g_2 = 1.10$ are determined by the solution of the single criteria problems, as earlier discussed. The optimal dynamics of consumption, $C(t)$, and capital, $K(t)$, for each different values of the weights are shown in Figure 2.

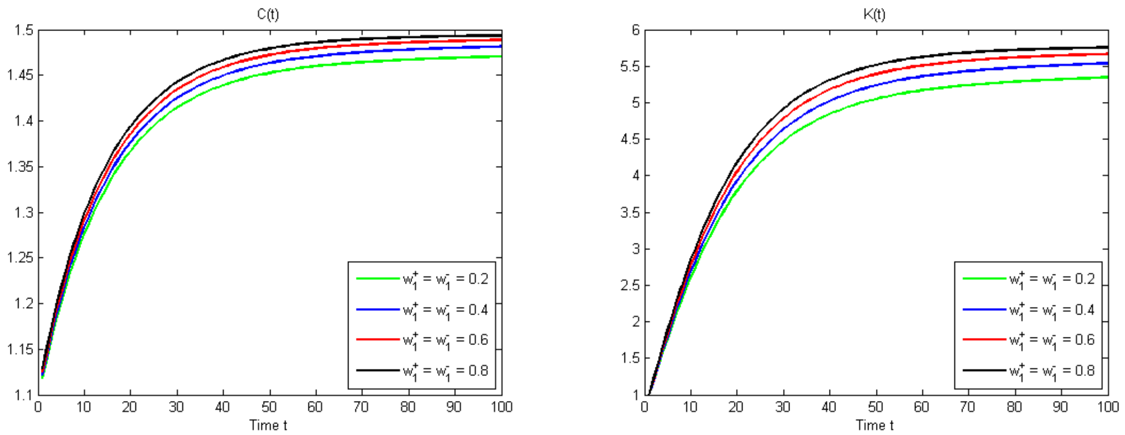

Figure 2: Optimal dynamics of consumption and capital (WGP)

We move now to the GPSF version of the model. Let us consider the following satisfaction function $F_\gamma(\delta) = \frac{1}{1+\gamma^2\delta^2}$. This function presents the desired properties (as in Figure 3) and it is trivial to verify that $F(0) = 1$, $F(\infty) = 0$, $F''(\delta) = 0 \Leftrightarrow \delta = \frac{1}{2\gamma}$ and that $0.9 \leq F(\delta) \leq 1$ if $0 \leq \delta \leq \frac{1}{3\gamma}$, $0 \leq F(\delta) \leq 0.1$ if $\delta \geq \frac{3}{\gamma}$ and $0 \leq F(\delta) \leq 0.01$ if $\delta \geq \frac{3}{\gamma}$. This means that this function shows a level of satisfaction between 90% and 100% when $0 \leq \delta \leq \frac{1}{3\gamma}$ and a level of satisfaction between 0% and 10% when $\delta \geq \frac{3}{\gamma}$. Natural candidates for the indifference threshold and the dissatisfaction threshold are, respectively, $\gamma_{id} = \frac{1}{3\gamma}$ and $\gamma_{io} = \frac{3}{\gamma}$. Let us assume the veto threshold $\gamma_{i\nu} = 2\gamma_{io} = \frac{6}{\gamma}$. As for the WGP model, we compare the impact of different relative importance in the two goals on the solution, considering exactly the same values of the weights. Using the above set of parameters, the GPSF model can be formulated as follows:

$$\begin{aligned}
\max Z &= \frac{w_1^+}{1+(\gamma\delta_1^+)^2} + \frac{w_1^-}{1+(\gamma\delta_1^-)^2} + \frac{w_2^+}{1+(\gamma\delta_2^+)^2} + \frac{w_2^-}{1+(\gamma\delta_2^-)^2} \qquad (17) \\
s.t. \quad & \int_0^\infty \frac{C(t)^{-1}-1}{-1} e^{-0.05t} dt + \delta_1^- - \delta_1^+ = g_1 \\
& \int_0^\infty \frac{C(t)^{-1.5}-1}{-1.5} e^{-0.06t} dt + \delta_2^- - \delta_2^+ = g_2 \\
& \dot{K}(t) = K(t)^{0.33} - 0.05K(t) - C(t) \\
& 0 \leq \delta_i^-, \delta_i^+ \leq \frac{6}{\gamma}, \quad \forall i \in \{1, 2\}
\end{aligned}$$

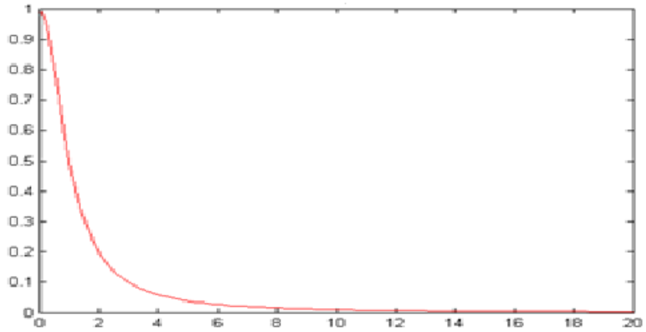

Figure 3: The satisfaction function F(δ).

where the goals are determined as earlier and therefore they are set as follows: $g_1 = 2.07$ and $g_2 = 1.10$. We also set $\gamma = 1$. In order to discretize the problem, we proceed as previously by approximating the infinite horizon integrals with a finite horizon T and the differential equations using classical numerical schemes as follows:

$$\begin{aligned}
\max Z \quad &= \quad \frac{w_1^+}{1+(\delta_1^+)^2} + \frac{w_1^-}{1+(\delta_1^-)^2} + \frac{w_2^+}{1+(\delta_2^+)^2} + \frac{w_2^-}{1+(\delta_2^-)^2} \qquad (18)\\
s.t. \qquad & \sum_{j=0}^{T} \frac{C(t)^{-1}-1}{-1} e^{-0.05t} dt + \delta_1^- - \delta_1^+ = g_1 \\
& \sum_{j=0}^{T} \frac{C(t)^{-1.5}-1}{-1.5} e^{-0.06t} dt + \delta_2^- - \delta_2^+ = g_2 \\
& K(j+1) - K(j) = K(j)^{0.33} - 0.05K(j) - C(j), \quad j = 0, ..., T-1 \\
& 0 \leq \delta_i^-, \delta_i^+ \leq 6, \quad \forall i \in \{1, 2\}
\end{aligned}$$

The optimal dynamics of consumption, $C(t)$, and capital, $K(t)$, for each different values of the weights are shown in Figure 4. By comparing Figure 2 with Figure reffig3, we can see that the results are qualitatively

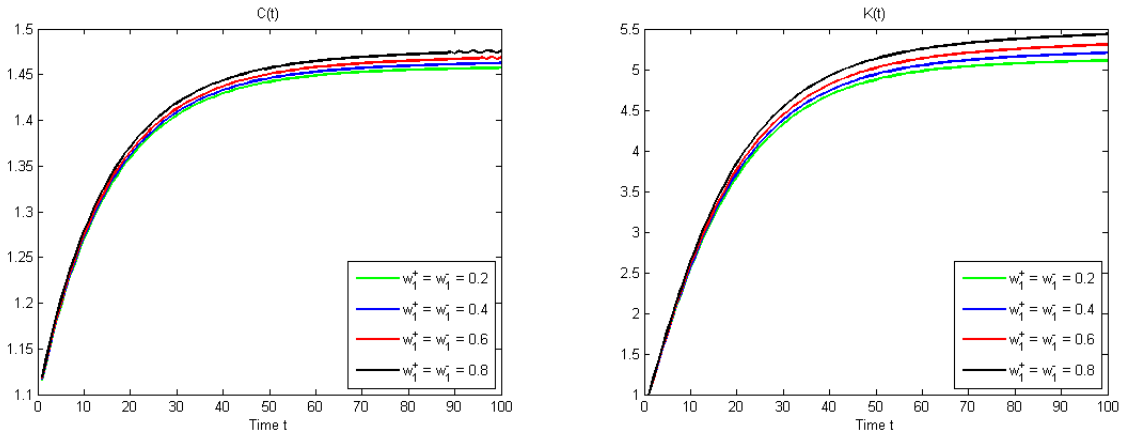

Figure 4: Optimal dynamics of consumption and capital (GPSF)

identical. Given the parameter values concerning the rate of time preference and the inverse of the intertemporal elasticity of substitution for the two agents, attaching a higher weight to the welfare of the agent 1 increases the overall consumption and capital stock in the economy.

6 Conclusions

In this chapter we have introduced two different formulations based on the GP philosophy for solving dynamic multi-criteria decision making problems. We have then presented an illustrative example in the area of macroeconomic policy, focusing on consumption and investment decisions, to show how this approach can be implemented when dealing with real world situations. The illustrated multi-criteria philosophy underlying the approach is consistent with the needs of policymakers to deal with dynamic problems with multiple goals to be simultaneously pursued even if they might have different importance. The numerical simulation developed for the vector-valued Ramsey model shows the goodness of this approach in the context of macroeconomic policy with vector-valued utility. This allows us to consider in a simple way how agents' heterogeneity may be encompassed in traditional macroeconomic models and how such a heterogeneity may affect the determination of optimal economic policies. A similar approach may be used to deal with issues which can be modeled as a dynamic problem and in which agents' are heterogeneous. Some specific examples include differential and cooperative games for analyzing environmental policy and climate change negotiations. For future research it might be interesting to combine our approach with a differential game setup in order to consider how agents' heterogeneity affects the potential trade-off between economic development and environmental preservation.

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