On the Relationship between Population Change and Sustainable Development

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Abstract

This paper investigates the relationship between population growth and economic growth, through the study of fertility choices and their effects on natural resources. It aims at analyzing the interactions between endogenous fertility choices and the environment and their link to the sustainable matter. We analyze a growth model driven by natural resources and without production, where agents have jointly to determine consumption and fertility, taking into account the effects of their decisions on the dynamics of natural resources. We adopt the most optimistic view on natural capital (it generates endogenous growth) and the weakest notion of sustainable paths (all variables are positive): in such a framework we expect that sustainable paths exist. We instead show that this is not always true. In fact, even if renewal capacity of natural resources is unbounded, not always a sustainable path can be found: this depends on the difference between the stationary fertility rate and the mortality rate. If the stationary fertility is lower than the mortality rate a sustainable path will not be found, and in such a case public intervention is necessary in order to address the economy along a sustainable path. This can be simply done through policies affecting public attention to environmental protection or the intensity of the dilution effect.

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1 Introduction

The issue of the relationship between population and economic growth has really ancient roots in the economic literature: Adam Smith and Malthus were among the first discussing the importance of controlling population growth in order to promote economic performance. After them, several theoretical and empirical studies investigated the relationship between population change and economic growth both from the economic and the demographic viewpoint, but a shared view has not arisen yet. In fact, as Bloom et al. (2003) summarize: *"...Though countries with rapidly growing populations tend to have more slowly growing economies..., this negative correlation typically disappears (or even reverses direction) once other factors... are taken into account"*. Three approaches have been proposed in order to study the issue: an optimistic, a pessimistic and a neutral view[1] (see Bloom et al., 2003). The most probably spread opinion is pessimistic

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[1]The optimistic view (Kuznets, 1960 and 1967, and Boserup, 1989; most recent analysis can be found in Jones, 2001, and Tamura, 2002) considers population as an important input to produce knowledge: the higher the population, the higher the probability new Isaac Newton were born. The neutral view (Bloom et al., 2003) instead has empirical foundation: there exists little cross-country evidence that population growth might either slow down or encourage economic growth.

(Solow, 1956; Becker and Barro, 1988; and Barro and Becker, 1989) and considers population as a threat for growth. This can be due to two different reasons: if the economy shows fixed resources and no sources of technical progress, in the long-run the (food) production activity will not be able to satisfy the pressure of population growth, leading per-capita resources to fall below a minimal subsistence level (Malthus, 1798); if the economy instead shows rapid population growth, then a large share of investment will be devoted to satisfy the needs of the increasing population (*investment-diversion effect* - Kelley, 1988), rather than to increase per-capita capital endowments. The proponents of this view base their argument on the idea that an increase in the population size leads to a dilution of available resources.

The topic of sustainable growth, instead, is a recent and growing issue in the economic growth literature. The possibility that deterioration of environmental quality, in particular caused by pollution, could inhibit economic growth was firstly suggested in the report to the Club of Rome entitled *Limits to Growth* (Meadows et al., 1972). The first recognition of the issue at international level was the creation by the UN General Assembly in 1985 of the Commission for Sustainable Development, chaired by the Prime Minister of Norway, Mrs Brundtland. The commission's report *Our Common Future* tried to emphasize that environmental protection is essential for economic development since the environment is an essential *'factor of production'* and source of important welfare services to people, even more in poor countries than in wealthy countries (World Commission on Environment and Development, 1987). There is wide agreement on the fact that sustainable development involves an integrated approach to economic, social and environmental processes; however, until now, the attention has been mainly focalized on the environmental and economic dimensions, addressing the social one (which can be mostly identified in the demographic dimension) only to a secondary role. In the current world, facing the uncertainties concerning the future of earth climate and environment, it is important to understand how finding a sustainable development path, where production, population and resources coexist without leading to an economic collapse. Such a situation in fact is not to be considered as unreal, as history can and should teach us. Typical examples are the collapse of the classical Maya civilization in the ninth century (Demarest, 2004), the dramatic decline of the Easter Island society (Flenley and Bahn, 2003) and the complete extinction of the Viking's colonies of Greenland (Diamond, 2005). However, what sustainable development really means is not clear: several definitions have been proposed and each of them underline different aspects of the matter; someone is too strong and someone else is too weak. The introduction of a clear notion of sustainability is still an open question and we propose an additional definition, which aim is to introduce a minimal requirement for sustainability.

Population growth, as firstly Malthus (1798) noticed, is an important factor of environment depletion: consumption activities deteriorate environment and more people exert higher pressure on environmental stock. Therefore, since the interaction between natural resources and population can be really important, the aim of this paper is to build a bridge between this two different kind of literatures. In fact, economic growth can be considered the main goal of current economies: however, its link with population dynamics and natural resources has often been underestimated[2]. Following Chichilnisky et al. (1995), we analyze the problem of sustainability studying an optimal growth model, where environment is represented by the stock of natural resources. We therefore assume there exist a one to one correspondence between environment and natural resources. With respect to Chichilnisky et al. (1995), we rely on discounted utilitarianism as a welfare criterion and we introduce population change and its linkage with environment (natural resources). Our paper is strictly related to Nerlove (1991), who studies the mutual relationship between population dynamics and the evolution of natural resources. With respect to him, we adopt an optimal growth framework (and not an OLG model) since sustainability issues have to be dealt with a long horizon approach; moreover, we explicitly model the population-environment relationship and we investigate under which conditions the economy is addressed along a sustainable path. We study the simplest model of endogenous growth, an AK

[2]Since there is still not a shared view on the relationship between population and economics (population growth is an important factor of environment depletion and environmental assets have a fundamental role for economic development), the interaction between these three factors deserves particular attention.

type model, driven by natural resources, where economic agents jointly determine their consumption level and their fertility rate. Agents' decisions concerning consumption and fertility deplete the natural resources, which represent a source of utility. Therefore, they have to take into account the pressure their choices exert to the environment, when trying to identify a possible sustainable development path, along which natural resources, population and growth coexist.

In section 2, we quickly review the issue of sustainability and its main implications for economic modeling. The attention is especially focused on the definition of sustainable development and on the choice of the welfare criterion to adopt in order to deal with such a matter. In section 3, we introduce the model economy in its general formulation, and derive the optimal paths for the control variables, consumption and fertility. Section 4 performs steady state analysis, studying a balanced growth path along which the fertility rate is constant and identifying the presence of possible sustainable path (defined as a path along which population, natural resources and consumption are positive, also asymptotically), and develops a comparative statics analysis, focalizing on policy implications. We show that even if renewal capacity of natural resources is unbounded, not always a sustainable path, where both population and natural resources coexist, can be found: this depends on the stationary fertility level. In particular, if it is lower than the mortality rate, the population will asymptotically disappear (implying that the economy will collapse) and the path followed by the economy is clearly not sustainable. We also show that with respect to other notions of sustainability (as Pezzey, 1997; and Arrow et al., 2004) our definition has the advantage of discriminating among different paths, labeling some of them as sustainable and some others as not. In section 5 we consider a special case of the model, that is an economy in which the stock of natural resources does not affect welfare, and we highlight the main differences. We show that also when utility depends only on consumption, the growth rate of population determines whether the path followed by the economy is sustainable or not, and the main results of the previous section still hold. Section 6, as usual, concludes.

2 Sustainable Development and Intertemporal Welfare

The developments of human activity in the last two centuries has dramatically changed the planet's climate, the biological mix and the natural resources. The main reasons of such impacts are related to the economy and population. Economic production uses energy, which is mainly obtained by fossil fuels, leading to carbon emissions. The emissions generated in the past century have consistently grown, irreversibly altering the climate of the planet. Population has also constantly grown, leading to higher and higher demand for production (of food and other consumable goods), waste and space, increasing the use of natural resources. These facts have remained without consideration for long time; only during the last decade of the XXI century the problem of ensuring a certain level of equity, among generations and among countries, emerged.

Probably the most important definition of sustainable development has been introduced by the Brundtland Commission, which labels sustainable development as development that *"satisfies the needs of the present without compromising the ability of future generations to meet their own needs"* (World Commission on Environment and Development, 1987). Even if particularly vague and cause of succeeding contentions (see for example Pearce et al., 1989; and Quiggin, 1997), this notion has been often used as a benchmark because it does not impose any constraints to growth and neither any particular duties to current generation, and moreover it is not formal at all. However, it clearly implies that sustainability mostly concerns two different but interrelated issues: respect of natural resources and intergenerational equity. The respect of natural resources is crucial to ensure equity among generations, because if each generation determines its consumption level without taking into account the effect implied on the future one, all resources will be exhausted soon and probably no future on the earth could be ensured. The intergenerational equity, instead, plays a central role on the evaluation of intertemporal welfare[3], and therefore on the identification

[3]See Heal (2005) for a survey of the issue.

of optimal allocations. Such an issue in standard macroeconomic (and in particular in growth) theory has generally been dealt with discounted (total) utilitarianism, which defines the social welfare as:

$$W = \int_0^\infty u(c_t) N_t e^{-\rho t} dt.$$

This means that social welfare equals the average utility, $u(c_t)$ where $c_t = \frac{C_t}{N_t}$ is per-capita consumption, multiplied by the population size, N_t, discounted by rate of time preference, $e^{-\rho t}$ where ρ is the pure rate of time preference. Notice that the introduction of discounting is necessary only for mathematical reasons, that is to ensure the bounded-ness of the objective function. But this mathematical necessity implies economic consequences, that is we are attaching less weight to future generations. As Ramsey (1928) commented, *"discounting of future utilities is ethically indefensible and arises purely from a weakness of the imagination"*. Of course, such a criterion cannot be used to deal with intergenerational equity, and therefore with sustainable issues. In order to avoid this, several attempts have been done in the literature. Some of them requires to adopt a different welfare criterion (as Ramsey, 1928; von Weizcker, 1967; Chichilnisky et al., 1995; and Chichilnisky, 1997) while some others to impose some additional constraints to the standard optimal control problem (as Pezzey, 1997; and Arrow et al., 2004).

Ramsey (1928), other than introducing special assumptions to ensure the sum converges, assumes that utility levels are bounded above; this allows him proposing to minimize the total difference over time between maximal utility and actual utility levels: $\int_0^\infty [b - u(c_t)] dt$, where b is a bliss point, the upper bound of the utility function. von Weizcker (1967) and others after him try to develop the overtaking approach: *"the overtaking criterion ranks as best the consumption sequence, if any, whose cumulative utility sum eventually exceeds that on any other path"* (Heal, 2005). These approaches ensure that we assign equal weight to all generations, but they have never been used, at least to our knowledge, in the literature. Another idea instead suggests that the sustainability issue has to be somehow linked with steady state and asymptotic behavior. For example, Chichilnisky et al. (1995) propose the Green Golden Rule as a welfare criterion to take into account sustainable matters. The Green Golden Rule consists of:

$$\max \lim_{t \to \infty} u(c_t) N_t,$$

that is, it represents the allocation maximizing the asymptotic (steady state) utility level, in order to determine the highest indefinitely maintainable utility level (rather than the highest consumption level, as implied by the standard Golden Rule). Again, Chichilnisky (1997) proposes the objective function to be a weighted average of discounted utility and asymptotic one:

$$W = \pi \int_0^\infty u(c_t) N_t e^{-\rho t} dt + (1 - \pi) \lim_{t \to \infty} u(c_t) N_t,$$

where $\pi \in (0, 1)$ is the weight assigned to discounted integral of utilities and $(1 - \pi)$ is that of long-run utility level, representing the sustainable utility level. The importance of the introduction of an asymptotic term is due to the fact that discounted utilitarianism attaches less weight to future generations in order to ensure the bounded-ness of the objective function and the role of the asymptotic term is therefore taking into account also long-run generations' welfare.

A completely different approach has instead been introduced in Arrow et al. (2004), which defines a path as sustainable if it implies non-decreasing welfare. The authors do not try to modify the standard welfare criterion, which remains discounted utilitarianism,

$$W = \int_0^\infty u(c_t) N_t e^{-\rho t} dt,$$

but just look for the imposition of some constraint in order to ensure sustainability, that is $\frac{\partial W}{\partial t} \geq 0$. However, a drawback of this definition is that paths satisfying both non-decreasing welfare and Pontryagin necessary conditions for consumption do not exist. In fact, optimal paths in neoclassical models do not satisfy this additional constraint. This is probably the reason why such an approach has never been used in the following literature. A similar approach can be found in Pezzey's (1997) survivable criterion, which proposes that the welfare derived from the standard dynamic maximization problem has to be higher than a minimal welfare associated to the survival of current population in order for the economy to be sustainable (survivable).

As this brief survey shows, which is the most convenient approach to deal with sustainability is still an open question. Following Pezzey (1997) and Arrow et al. (2004), we try to impose some constraints to the standard optimal control problem in order to ensure sustainability. With respect to them, we try looking for some minimal requirements for sustainability (therefore something much weaker than their notion), relying on the importance of asymptotic behavior, as suggested by Chichilnisky et al. (1995) and Chichilnisky (1997). In particular, we define a path as sustainable if it implies (strictly) positive values of all the economic variables, both in finite and infinite time (see Definition 2).

3 The Model

The economy is closed and composed of households that can only consume the unique good present in the economy (a natural good) and have to choose how much consumption and how many children to have. Therefore, population grows in accordance to household decisions[4]. There is no production and human choices of consumption and fertility decrease the stock of natural resources.

The representative household wants to maximize its lifetime utility function, which is the sum of its instantaneous utility function, depending both on per-capita consumption, c_t, and on the stock of natural resources, E_t, where $\frac{\partial u(\cdot)}{\partial c_t} > 0$, $\frac{\partial^2 u(\cdot)}{\partial c_t^2} < 0$ and also $\frac{\partial u(\cdot)}{\partial E_t} > 0$, $\frac{\partial^2 u(\cdot)}{\partial E_t^2} < 0$. In order to get a closed form solution, it is assumed to be iso-elastic:

$$u(c_t, E_t) = \frac{(c_t E_t^{\beta})^{1-\sigma}}{1-\sigma}, \tag{1}$$

where $\sigma \in (0,1)$ and $\beta \geq 0$. The utility function depends on the individual consumption level (households are not interested in aggregate consumption, but only in per-capita consumption) and on the stock of natural resources and the term β represents the weight of environment in agents utility (the green preferences), but does not depend on the fertility rate (having children or not does not affect the utility level). Notice that $\beta = 0$ represents the case in which environment is not a source of utility (the utility function depends only on consumption) and such a case will be analyzed in Section 5.

Population grows over time at a non constant rate, given by the difference between the endogenously determined birth rate, n_t, and the exogenous mortality rate, d:

$$\dot{N}_t = (n_t - d)N_t, \tag{2}$$

where both n_t and d are strictly positive.

The dynamics of natural resources depends on their renewal capacity, aggregate consumption and the dilution effect associated to population growth:

$$\dot{E}_t = R(E_t) - C_t - \phi(n_t)E_t \tag{3}$$

[4] The analysis of the causes of household choices is out of the goal of this work. In our model, fertility choices are endogenous as a result of the non-linear relationship between population growth and the environment, while why and how such fertility decisions are determined is not directly analyzed in the paper, since it is not among its main goals and such issue has been extensively studied in the endogenous fertility literature (see for example Barro and Becker, 1989; and Nerlove and Raut, 1997). Such a literature describes fertility choices as the result of the optimal balancing between the utility and disutility of having children, and several explanations of this trade-off are proposed. According to Nerlove and Raut (1997) distinction, our model should be labeled as a model of endogenous population change, rather than as model of endogenous fertility, since *"no decision-making mechanism is presupposed"*.

where $R(E_t)$ is the renewal capacity of the environment and $\phi(n_t)$ is the dilution function, related to population variations. We consider for simplicity the case in which renewal capacity is unbounded, that is $R'(E_t) > 0$, and we assume it is a linear function[5] of the stock of natural resources:

$$R(E_t) \;=\; rE_t. \tag{4}$$

The dilution effect in natural resources instead represents the pressure wielded by population growth on natural resources (and therefore economic growth). A diffused view on the relationship between population growth and economic growth sees population growth as detrimental for growth: in fact, the food production activity, which is mainly derived from natural resources, is overwhelmed by the pressures of population growth, and this can lead the available diet to fall below the subsistence level (Malthus, 1798). Therefore, it seems plausible to assume $\phi'(n_t) > 0$: as fertility increases, the pressure on natural resources increases too. Kelly and Schmidt (1995) shows that the impact of population on the economy depends on the level of economic development: the impact of population growth is negative for less developed countries, while it is positive for developed ones. Thus, this impact can change over time as the development proceeds. According to this result, therefore, the relationship between population and economic growth is non-monotonic. Non-monotonicity implies that such a relationship is non-linear[6]. We therefore consider a function non-linear in the fertility rate:

$$\phi(n_t) \;=\; an_t^b, \tag{5}$$

where $a > 0$ and $b > 0$ but $b \neq 1$. In particular, if $0 < b < 1$ ($b > 1$), an increase in the fertility rate (population size) decreases less (more) than proportionally the stock of natural resources. The presence of this non-linear function permits to have endogenous fertility even if fertility itself is not a source of utility. Notice that the environment is negatively affected by human choice through two different channels: consumption activity (human needs for life) requires the use of natural resources and fertility choices determine the size of population, which causes the weight of the pressure on environment.

The social planner maximizes the social welfare function of the economy under the economy resource constraint, the law of motion of demography and the initial conditions for natural resources and population:

$$\begin{aligned} \max_{c_t, n_t} \quad & W = \int_0^\infty u(c_t, E_t) N_t e^{-\rho t} dt \\ s.t. \quad & \dot{E}_t = rE_t - N_t c_t - an_t^b E_t \\ & \dot{N}_t = (n_t - d) N_t \\ & E_0,\ N_0 \text{ given} \end{aligned} \tag{6}$$

The planner objective function takes into account the size of current and future generations, showing inter-temporal altruism, represented by ρ, the rate of time preference (the lower the rate of time preference, the higher the planner's altruism towards later generations), and full intra-temporal altruism (it means that the weight assigned by the planner to each member of the same generation is the same: the weight of each individual is independent of the size of the generation).

[5]The choice of a linear function is reductive, but it permits us to characterize the steady state of our economy as a balanced growth path, simplifying computational problems. However, choosing another kind of unbounded function should not lead to different results. Moreover, the linear specification represents the most optimistic view on environmental regeneration and therefore it is an interesting benchmark for our analysis. Of course, if we consider a bounded renewal capacity, as in a logistic function, the outcome of the model can dramatically change.

[6]Since natural capital is the force driving endogenous growth, the $\phi(\cdot)$ function represents the pressure wielded by population change both on the environment and on the economic performance. Because no other linkage is present in our model between demography and economic growth, according to Kelly and Schmidt (1995) we assume such a function to be non-linear.

3.1 Optimal Paths

From the social planner maximization problem, we can derive the Hamiltonian function:

$$\mathcal{H}_t(c_t, E_t) = \frac{(c_t E_t^\beta)^{1-\sigma}}{1-\sigma} N_t e^{-\rho t} + \lambda_t \left[r E_t - N_t c_t - a n_t^b E_t \right] + \mu_t (n_t - d) N_t$$

and the first order necessary conditions:

$$\frac{\partial \mathcal{H}_t(\cdot)}{\partial c_t} = 0 \quad \rightarrow \quad (c_t E_t^\beta)^{-\sigma} E_t^\beta N_t e^{-\rho t} = \lambda_t N_t \tag{7}$$

$$\frac{\partial \mathcal{H}_t(\cdot)}{\partial n_t} = 0 \quad \rightarrow \quad \mu_t N_t = b \lambda_t a n_t^{b-1} A_t \tag{8}$$

$$\frac{\partial \mathcal{H}_t(\cdot)}{\partial E_t} = -\dot{\lambda}_t \quad \rightarrow \quad \beta (c_t E_t^\beta)^{-\sigma} c_t E_t^{\beta-1} N_t e^{-\rho t} + \lambda_t \left[r - a n_t^2 \right] = -\dot{\lambda}_t \tag{9}$$

$$\frac{\partial \mathcal{H}_t(\cdot)}{\partial N_t} = -\dot{\mu}_t \quad \rightarrow \quad \frac{(c_t E_t^\beta)^{1-\sigma}}{1-\sigma} e^{-\rho t} - \lambda_t c_t + \mu_t (n_t - d) = -\dot{\mu}_t \tag{10}$$

together with the initial conditions E_0 and N_0, the state equations:

$$\dot{E}_t = r E_t - N_t c_t - a n_t^b E_t \tag{11}$$

$$\dot{N}_t = (n_t - d) N_t \tag{12}$$

and the transversality conditions (TVCs):

$$\lim_{t \to \infty} \quad E_t \lambda_t = 0 \tag{13}$$

$$\lim_{t \to \infty} \quad N_t \mu_t = 0 \tag{14}$$

Solving the system of FOCs, we can obtain the optimal paths of consumption and fertility:

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma} \left[\sigma \beta \frac{c_t N_t}{E_t} + [1 + \beta(1-\sigma)](r - a n_t^b) - \rho \right] \tag{15}$$

$$\frac{\dot{n}_t}{n_t} = \frac{1}{b-1} \frac{c_t N_t}{E_t} \left[(1+\beta) - \frac{\sigma}{b(1-\sigma) a n_t^{b-1}} \right]. \tag{16}$$

Equation (15) depends positively on the ratio between aggregate consumption and stock of natural resources and negatively on the fertility rate. In particular, a rise in the fertility rate leads to a non-proportional reduction in the growth rate of per capita consumption and this is due to the dilution effect, which is non-linear. Notice that, if $b > 1$, the growth rate of consumption is a concave function of the fertility level and the fertility rate maximizing consumption growth results to be null.

Equation (16) instead depends on the ratio between aggregate consumption and environmental stock and on the rate of fertility. The signs of these relations cannot be determined a priori: in fact, they crucially depends on b (if it is higher or lower than one) and on the term in the squared brackets. It is interesting to notice that the choice of the consumption level, determining the size of the ratio term, affects the growth rate of fertility.

The TVC (14) implies that the growth rate of natural resources is bounded above:

$$\gamma_E < r - a n_t^b + \beta \frac{c_t N_t}{E_t} \tag{17}$$

while the TVC (15) implies that the ratio between aggregate consumption and natural resources is positive.

4 Steady State Analysis

The growth rates of the per-capita consumption, natural resources, population and fertility rate are:

$$\gamma_c = \frac{1}{\sigma}\left[\sigma\beta\frac{c_t N_t}{E_t} + [1+\beta(1-\sigma)](r - a n_t^b) - \rho\right] \tag{18}$$

$$\gamma_E = r - \frac{c_t N_t}{E_t} - a n_t^b \tag{19}$$

$$\gamma_N = n_t - d \tag{20}$$

$$\gamma_n = \frac{1}{b-1}\frac{c_t N_t}{E_t}\left[(1+\beta) - \frac{\sigma}{b(1-\sigma)a n_t^{b-1}}\right] \tag{21}$$

We now analyze possible equilibrium paths considering a balanced growth path, along which the growth rate of fertility is null.

Definition 1: (*Balanced Growth Path, BGP*) a balanced growth path, BGP, or steady state equilibrium, $(\overline{c}, \overline{n}, \overline{E}, \overline{N}, \gamma_c, \gamma_n, \gamma_E, \gamma_N)$, is a sequence of time paths, $\{c_t, n_t, E_t, N_t\}_{t\geq 0}$, along which all economic variables grow at constant rates. A BGP is said non-degenerate if c_t and E_t grow at non negative rates.

Along the BGP, the fertility rate, n_t, must be constant, $n_t = \overline{n}$: this means that the growth rate of fertility is null:

$$\overline{n} = \left[\frac{\sigma}{b(1+\beta)(1-\sigma)a}\right]^{\frac{1}{b-1}} \tag{22}$$

and the growth rate of population can be positive, negative or null in accordance to the difference[7] between $\overline{n}$ and d:

$$\gamma_N = \overline{n} - d. \tag{23}$$

Consequently, the growth rate of per capita consumption is:

$$\gamma_c = \frac{1}{\sigma}\left[\sigma\beta\frac{\overline{c}\overline{N}}{\overline{E}} + [1+\beta(1-\sigma)](r - a\overline{n}^b) - \rho\right] \tag{24}$$

and that of environment is:

$$\gamma_E = r - \frac{\overline{c}\overline{N}}{\overline{E}} - a\overline{n}^b \tag{25}$$

since the growth rate of environment must equalize the growth rate of aggregate consumption[8], otherwise $\gamma_E = -\infty$ or it would violate the TVC (14):

$$\gamma_E = \gamma_C = \gamma_c + \gamma_N. \tag{26}$$

This implies that per-capita variables, consumption and natural resources, grow at the same rate $\gamma = \gamma_c$.

In order to have endogenous growth, we have a lower bound for r:

$$r > a\overline{n}^b + \frac{\rho}{1+\beta(1-\sigma)} - \frac{\sigma\beta}{1+\beta(1-\sigma)}\frac{\overline{c}\overline{N}}{\overline{E}}, \tag{27}$$

[7] In fact, no condition a priori imposes restrictions on the gap between the stationary fertility and the exogenous mortality rate.

[8] In fact, if $\gamma_C > \gamma_E$ we would asymptotically have $\gamma_E = -\infty$, while if $\gamma_C < \gamma_E$, we would have $\gamma_E = r - a n_t^b$, violating equation (17).

while, in order to ensure bounded objective function instead,we have an upper bound for r:

$$r < a\overline{n}^b + \frac{\rho - \sigma(\overline{n} - d)}{(1-\sigma)(1+\beta)}. \tag{28}$$

Therefore, along the BGP the stationary fertility rate is positive, the economic growth rate is positive as well, while that of population can be positive, negative or null:

$$\overline{n} = \left[\frac{\sigma}{(1+\beta)(1-\sigma)ab}\right]^{\frac{1}{b-1}} \tag{29}$$

$$\gamma = \frac{1}{\sigma}\left[\sigma\beta\frac{\overline{c}\overline{N}}{\overline{E}} + [1+\beta(1-\sigma)](r - a\overline{n}^b) - \rho\right] \tag{30}$$

$$\gamma_N = \overline{n} - d. \tag{31}$$

If the fertility is lower than mortality rate, the population growth is negative and in the long-run all individuals will disappear: the population will continue to decrease until its complete disappearance but this would lead to have an high rate of growth during the life of the economy (the lower $\overline{n}$, the higher γ). If natality and mortality rate perfectly offset, the population will reach a positive stationary equilibrium level. If, instead, the birth rate is higher than the death rate, the population size will continue to rise, leading to a lower growth rate.

Proposition 1: *along the BGP, the following results hold:*
(i) if $b > 1$ ($b < 1$), the stationary fertility level is a positive (negative) function of the elasticity of substitution, σ, while it is a negative (positive) function of the green preferences, β, and of the dilution effect parameter, a;
(ii) the growth rate of the economy depends positively on the consumption-natural resources ratio, $\frac{C_t}{E_t}$, and negatively on the stationary fertility rate, $\overline{n}$;
(iii) population growth is a positive function of the stationary fertility level, $\overline{n}$ and a decreasing function of the mortality rate, d.

Proof: The result just derives from the partial derivatives of (29), (30) and (31), respect to the main parameters. ■

Among all possible paths, we are interested in a sustainable one, along which population, natural resources and growth coexist. However, as previously discussed, how sustainable path has to be interpreted is still not clear, even if several definitions have been proposed in the literature. Following Pezzey (1997) and Arrow et al. (2004), we introduce a definition of sustainable paths, rather than modifying the welfare criterion to adopt. With respect to them, we try to introduce the weakest notion, which could represent a minimal requirement for sustainability. Following Chichilnisky et al. (1995) and Chichilnisky (1997), we rely on the importance of asymptotic behavior. We define as sustainable every path along which all economic variables are strictly positive, requiring that this condition is satisfied also asymptotically.

Definition 2: (*Sustainable Development Path*) a sustainable development path is a sequence of time paths, $\{c_t, n_t, E_t, N_t\}_{t\geq 0}$, along which all economic variables are (strictly) positive, $c_t, n_t, E_t, N_t > 0$, and also asymptotically (strictly) positive, $\lim_{t\to\infty} c_t, \lim_{t\to\infty} n_t, \lim_{t\to\infty} E_t, \lim_{t\to\infty} N_t > 0$.

This definition is particularly weak: it does not concern growth rates but requires only that the variables are not addressed along a collapsing path. We require that population, consumption and environment (not their growth rate) are positive along the time horizon, but also at steady state. This looks like a minimal requirement in order to consider a path as sustainable: paths violating our definition cannot be labeled as sustainable in a stronger sense. In fact, a path along which consumption, population size and/or natural resources collapse cannot ensure the ability of future generations of satisfying their own needs, which is the most diffused (Brundtland Commission) definition of sustainability.

Notice that since the renewal capacity of natural resources is unbounded we can find a sustainable paths, where both population and natural resources coexist and moreover the economy grows. However, such a path does not always exist: it depends on the difference between the stationary fertility level and the mortality rate. If the former is lower than the latter, population will asymptotically disappear and no path can be found where both population and natural resource coexist: this means that agents endogenously decide the collapse of the society. Therefore, we have just proved:

Proposition 2: *the development path along which the economy is addressed is sustainable if the stationary fertility rate is at least as high as the mortality rate, that is if* $\overline{n} \geq d$, *otherwise it is not, that is the case in which* $\overline{n} < d$.

This is quite surprising: the model is really optimistic in terms of natural resources regeneration capacities and really weak in the notion of sustainability. This means that even in the most optimistic framework, if we consider also the interaction between population and environment, the existence of a sustainable path does not have to be taken for granted. However, in the case $\overline{n} < d$, the planner can intervene affecting $\overline{n}$ or d in order to switch the economy to a sustainable path (see next subsection). Notice that our definition of sustainability leads to a more realistic result than those we would obtain adopting another notion introducing sustainability as an additional constraint to the dynamic maximization problem (i.e. Pezzey, 1997; Arrow et al., 2004). Remember that Arrow et al. (2004) defines as sustainable a path along which welfare is non-decreasing over time, while Pezzey's (1997) define as survivable a path characterized by a welfare level higher than the minimal welfare allowing the survival of the current population. It is straightforward verifying that along the BGP, the welfare is decreasing over time, because of the necessity of ensuring boundedness of objective function[9]: this means that according to Arrow et al. (2004) formulation, no path is sustainable. It is also easy to see that if the minimal welfare associated to the survival of population is sufficiently low[10], then the steady state welfare level will always be higher than this: according to Pezzey's (1997) notion, all paths are sustainable (survivable). With respect to these notions, our definition of sustainability has the advantage of discriminating among different paths, labeling some of them as sustainable and some others as not.

We can notice that along the BGP, where the fertility rate is constant, the dynamic behavior of the economy is the same as in a standard AK model. Moreover, as in standard AK model, the model does not show any transitional dynamics: the economy lies along its BGP since time 0 (a similar result is obtained in Palivos and Yip, 1993, who show that an AK model with endogenous fertility does not show transitional dynamics; see Appendix A for more details). In fact, at time 0 if the fertility rate is chosen equal to $\overline{n}$, the growth rate of the economy, given is equation (34) is constant (in fact, the consumption-natural resources ratio has to be constant because of the TVC (14)); moreover, if the economy is not along its BGP from time 0, it will never converge to it.

[9] In fact, since the economy lies on its equilibrium path from time 0 (see Appendix A), the derivative of the (integrand in the) welfare function is: $[(1-\sigma)\gamma_c + \beta(1-\sigma)\gamma_E - (\rho - \overline{n})]Ae^{[(1-\sigma)\gamma_c + \beta(1-\sigma)\gamma_E - (\rho-\overline{n})]t}$, where $A = \frac{\overline{c}^{1-\sigma}\overline{E}^{\beta(1-\sigma)}\overline{N}}{1-\sigma}$. Condition (28) which ensures the fact that the welfare function is bounded, essential for the integral in W to be well defined, also ensures that the term in the square brackets is negative and therefore the whole time derivative is negative as well.

[10] The value to attribute to the minimal welfare permitting actual population to survive is almost arbitrary. If this is too high, the welfare level along the BGP will never be higher than this and we will obtain the same conclusion of Arrow et al. (2004): no path will be sustainable. If this is low instead, all paths will be sustainable.

4.1 Comparative Statics

We now perform an exercise of comparative statics in order to better understand the role of some parameters on the economic and population growth rates and identify the main policy[11] implications, underlying especially how policies affecting the stationary fertility level (or the mortality rate) should be introduced in order to address the economy along a sustainable path. In particular, we consider changes in the mortality rate, in the dilution effect parameter and in the green preferences.

4.1.1 Changes in the Mortality Rate

Suppose a new policy (like public expenditures in health or incentives to private health care, assuming such a policy can be achieved at zero cost) has just been introduced and its effect is to lower the mortality rate, d, affecting therefore the population growth. If the economy were initially along the BGP, this shock would have the effect[12] of shifting the economy from a BGP to another one. Along the new BGP the economic growth rate and the stationary fertility rate would remain unaffected while net population growth changes.

Suppose originally the stationary fertility were lower than the mortality rate[13]: this means, according to Proposition 2, that the path along which the economy is evolving is not sustainable. Then along the new BGP population growth would increase but it could be positive, negative or null, in accordance to the magnitude of the change. If such a change were strong enough, then population growth would be positive and as a result the economy is switched from a not sustainable path to a sustainable one. Therefore the introduction of a new policy aiming at increase public or private spending in health care, through the effect of lowering the mortality rate, can be used in order to reach a desired level of population, permitting the planner to switch the economy to sustainable paths.

4.1.2 Changes in the Intensity of the Dilution Effect

Suppose a new policy, as the introduction of public expenditure to improve environment protection (assuming that it can be achieved at zero cost), has the effect of decreasing a, that is it lowers the cost for the natural resources to maintain the same stock of population. If the economy were initially along the BGP, along the new one the economic growth, population growth and the steady state fertility level change. If $b > 1$, a drop in a leads first of all to a shift from a fertility rate to an higher new one, while the fertility growth rate continues to be null. This implies an higher population growth and a lower economic growth. If instead $b < 1$, the new stationary fertility rate will be lower and also the economic growth rate will be lower.

Suppose, again, that originally the stationary fertility were lower than the mortality rate, such that the economy do not lie on a sustainable path. The planner, through this kind of policies, can increase $\overline{n}$ enough to permit the shift of the economy to a sustainable path: this can be done by decreasing (increasing) a, if $b > 1$ ($b < 1$).

[11]Notice that in our model only a natural good is present, and economic activities, as production and public expenditures, cannot be easily encompassed in such a framework. Therefore, public intervention can be viewed as a mere exogenous shock affecting some parameters.

[12]Short-run and long-run effects coincide, as in the standard AK model: every shock in the economy translates in a jump of the (economic and population) growth rates, of the stationary fertility level or both. In fact, at any shock the fertility rate can be adjusted in order to lie directly on the new BGP.

[13]Such a situation is consistent with several industrialized economies (as for example Italy), in which the growth rate of (domestic) population is lower than its replacement rate. The result that the overall demographic growth is positive is just due to increasing migration flows (not present in our model since economies are closed). Other examples of how this situation can be relevant for the economic system are given by the collapse of the Maya civilization (Demarest, 2004), the decline of the Easter Island society (Brander and Taylor, 1998) and the complete extinction of the Viking's colonies of Greenland (Diamond, 2005).

4.1.3 Shifts in the Green Preferences

Suppose a policy oriented to awake households to environmental problem or to change the priority of agents is introduced. Its effect would be an increase in β. If the economy were initially along the BGP, this change would have the effect of shifting the economy from a BGP to another one. The stationary fertility level[14] and therefore population growth and economic growth rates will be different. If $b > 1$ an increase in β decreases the stationary fertility rate, leading to a lower population growth and an higher economic growth. If instead $b < 1$ fertility will increase and population growth will increase too while the effect of such a shock on economic growth is ambiguous.

Suppose that originally the stationary fertility were lower than the mortality rate, so that the dynamic path followed by the economy is not sustainable. Then, the green preferences can be influenced by pro-environment policies in such a way to lead the economy on a sustainable path: this can be done increasing (decreasing) β if $b > 1$ ($b < 1$).

5 Environment not Source of Utility

If $\beta = 0$, the stock of natural resources does not affect welfare and such a situation is an extreme case of the model. In this case the optimal paths of consumption and fertility simplify in:

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}\left[r - an_t^b - \rho\right] \tag{32}$$

$$\frac{\dot{n}_t}{n_t} = \frac{1}{b-1}\frac{c_t N_t}{E_t}\left[1 - \frac{\sigma}{b(1-\sigma)an_t^{b-1}}\right]. \tag{33}$$

We can notice that the growth rate of per-capita consumption is constant as in the standard AK model, unless the presence of the term representing the dilution effect. In this framework, along the BGP the stationary fertility rate is:

$$\overline{n} = \left[\frac{\sigma}{b(1-\sigma)a}\right]^{\frac{1}{b-1}} \tag{34}$$

while the economy growth rate is positive and that of population can be positive, negative or null:

$$\gamma = \frac{1}{\sigma}\left[r - a\overline{n}^b - \rho\right] \tag{35}$$

$$\gamma_N = \overline{n} - d. \tag{36}$$

With respect to the case in which $\beta \neq 0$, in such a framework the stationary fertility level is higher if $b > 1$ (and lower if $b < 1$). Since environmental stock does not affect their welfare, agents do not take into account the interaction between fertility choices and natural resources dynamics. In fact, an higher fertility rate decreases more the stock of natural resources, leading to a lower economic growth and an higher population growth. Notice that also in this case, Proposition 2 holds: the development path is sustainable if the stationary fertility rate is at least as high as the mortality rate. This means that the introduction of the environment (natural stock) in the instantaneous utility function is not essential for such a result to hold. However, since in the case $b > 1$, the stationary fertility rate is higher, the possibility of the economy to lie on a non-sustainable path will be less likely. In fact, fixed the other parameter values, the mortality rate has to be higher for having a negative growth rate of population.

[14] Notice that, considering the case where $b > 1$, a really high β could lead to reach the per-capita consumption growth maximizing null fertility level. In such a case, population will decrease along the BGP (implying that population asymptotically collapses) while economic growth will be maximal. Moreover, $\beta = 0$ represents the case where environment is not a source of utility. Therefore, β can be interpreted as a crucial policy parameter: promoting attention to environment, the planner can indirectly affect economic and population growth, through the direct effect of manipulating the stationary fertility rate.

6 Conclusion

The Brundtland Commission defined sustainable development as development that *"satisfies the needs of the present without compromising the ability of future generations to meet their own needs"*. The goal of our paper is to study the situations under which a sustainable path can be reached, focalizing our attention also on the interaction between population growth, environment and economic growth. With respect to previous works, we introduce a formal definition of sustainable path which permits us to take into account long-run generations' welfare without modifying the standard welfare criterion (the utilitarian approach). We define a path as sustainable if it implies strictly positive values of all economic variables, both in finite and infinite time (see Definition 2). We analyze a growth model driven by natural resources and without production, where agents have jointly to determine consumption and fertility, taking into account the effect of their decisions on the dynamics of natural resources. In our model, even if renewal capacity of natural resources is unbounded, not always a sustainable path, where both population and natural resources coexist, can be found: this depends on the difference between the stationary fertility level and the mortality rate. A sustainable path can be found only if the stationary fertility is higher or equal to the mortality rate.

Along the BGP, the growth rate of the economy and population ultimately depend on the stationary fertility level. Suppose the stationary fertility is lower than the mortality rate; then public intervention can be necessary in order to address the economy along a sustainable path. In fact, the planner can directly intervene in the economy determining both population and economic growth through policies aiming at affecting the stationary fertility rate. This can be done mainly with two different kind of policies: one affecting the dilution effect parameter and one affecting green preferences. The outcome of these policies is similar: both changes in the dilution effect and variations in the green preferences modify the population growth and affect economic growth therefore they can be adopted in order to reach the desired population level and in order to alter economic growth. In particular, such policies can be used in order to reach a sustainable path: suppose the stationary fertility is lower than the mortality rate; the planner can affect fertility or mortality with the right policy[15] in order to cancel such a gap and permit a sustainable path to exist.

For further research, we propose to focalize the attention on a more realistic framework, where also production and accumulation of physical capital play an active role in the economy. In particular, a two sector growth model, a-la Uzawa-Lucas, where natural resources have to be allocated between physical and natural production can do it. Another interesting line of research can be represented by the introduction of bounded renewal function, as a logistic one: in such a case is not obvious that a sustainable path exists at all.

A On the Transitional Dynamics

We can study the stability of the BGP, introducing the intensive variable $\chi_t = \frac{c_t N_t}{E_t}$ and studying the system in (χ_t, n_t). The law of motion of χ_t and n_t are:

$$\frac{\dot{\chi}_t}{\chi_t} = (1+\beta)\chi_t + \frac{(1-\sigma)(1+\beta)}{\sigma}(r - a n_t^b) - \frac{\rho}{\sigma} + n_t - d \qquad (37)$$

$$\frac{\dot{n}_t}{n_t} = \frac{1}{b-1}\chi_t \left[(1+\beta) - \frac{\sigma}{b(1-\sigma) a n_t^{b-1}} \right]. \qquad (38)$$

[15] For example, if $b > 1$, the fertility rate can be increased augmenting public expenditure to improve environmental protection or decreasing public care of environment; if $b < 1$, the policies have to be of the opposite sigh. The mortality rate, instead, can be reduced through incentives to private health care.

Therefore, the equilibrium of this system is given by $E = (\overline{\chi}, \overline{n})$, where:

$$\overline{\chi} = \frac{\rho}{\sigma(1+\beta)} - \frac{1-\sigma}{\sigma}(r - a\overline{n}^b) - \frac{\overline{n} - d}{1+\beta} \tag{39}$$

$$\overline{n} = \left[\frac{\sigma}{b(1+\beta)(1-\sigma)a}\right]^{\frac{1}{b-1}}. \tag{40}$$

Notice that $\overline{n} > 0$ if $0 < \sigma < 1$ while $\overline{\chi}$ is if $\overline{n} > \frac{b(1-\sigma)(\rho+\sigma d) - b(1-\sigma)^2(1+\beta)r}{\sigma^2 - b\sigma(1-\sigma)}$.

Linearizing the system, we obtain the Jacobian matrix, $J(\chi_t, n_t)$:

$$\begin{bmatrix} 2(1+\beta)\chi_t + \frac{(1-\sigma)(1+\beta)}{\sigma}(r - an_t^b) - \frac{\rho}{\sigma} + n_t - d & \chi_t[1 - \frac{b(1-\sigma)(1+\beta)an_t^{b-1}}{\sigma}] \\ \frac{1}{b-1}[(1+\beta)n_t - \frac{\sigma}{b(1-\sigma)an_t^{b-2}}] & \frac{1}{b-1}\chi_t[(1+\beta) + \frac{\sigma(b-2)}{b(1-\sigma)an_t^{b-1}}] \end{bmatrix}, \tag{41}$$

which evaluated at steady state is:

$$J(\overline{\chi}, \overline{n}) = \begin{bmatrix} (1+\beta)\overline{\chi} & 0 \\ 0 & (1+\beta)\overline{\chi} \end{bmatrix}. \tag{42}$$

Since $\overline{\chi}$ is positive, it is straightforward to notice that both eigenvalues are positive, implying that the system never reaches the steady state (namely its BGP), unless the initial choice for n_t is such that the fertility rate coincides with its stationary level from time 0.

Therefore, the economy lies immediately on the BGP, otherwise will never converge to it. If $n_0 = \overline{n}$, the model behaves as a standard AK model: there is no transitional dynamics and from $t = 0$ the model is in its steady state. If instead $n_0 \neq \overline{n}$, the economy does not converge to the BGP and the path followed by the economy does not show balanced growth. This means the economy, in order not to show diverging trajectories, lies on the BGP from time 0, meaning that the model does not show transitional dynamics.

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