Preventing learning difficulties in arithmetic: the approach of the PerContare project

Anna Baccaglini-Frank reports on activities which have been designed to sup[illegible] pupils who have been labelled as 'dyscalculic'.

Dyscalculia is not yet well-defined at an international level. In their critical review of the last 40 years of research on "mathematical learning disability (MLD)", Lewis and Fischer (2016) systematically analyse the methodological criteria used to identify MLD, and conclude that there is great variability in the criteria used to identify and classify MLD and that many studies do not control for non-cognitive factors when identifying students with MLD among those with persistent low achievement in mathematics.

We are sceptical about how dyscalculia is diagnosed in the Italian context, and even unsure about what dyscalculia really is and so hoped that the careful design of teaching materials, together with their appropriate implementation, could lead to higher achievement for most students on tests such as those used for diagnosing dyscalculia. We obtained funding for a collaboration between the 'University of Modena and Reggio Emilia' and a non-profit association, ASPHI, for a 3-year project, called "PerContare".

The PerContare project

The PerContare project aimed to develop effective inclusive teaching strategies and materials to support primary school teachers in addressing learning difficulties, especially of pupils potentially at risk of being diagnosed with dyscalculia. The project was a collaboration between a team of mathematics educators and a team of psychologists. The first objective of the project was to design and test teaching strategies and material to help all students in 1^{st} and 2^{nd} grade (ages 6-8) develop adequate numerical competence and it is this goal that I focus on in this article, introducing two of the activities proposed for the beginning of 1^{st} grade. I offer examples from two classrooms in which the material was trialled and briefly describe some theoretical grounding for the activities.

Classroom snapshots

The two snapshots show activities aimed to foster the internalisation of part-whole relations and strategies for composing and decomposing numbers. The first activity makes use of plastic rings and beans, while the second activity is an example of practices fostered in the project, that include particular uses of fingers.

Studies suggest that perceiving pattern and structure is a fundamental way of thinking that should be fostered in young children (See for example Mulligan and Mitchelmore, 2013). Part-whole relations help "organize children's knowledge about the ways in which material around them comes apart and goes together" (Resnick et al. 1991, p. 32). The internalisation of the part-whole relation between quantities entails understanding of addition and subtraction as actions (Schmittau, 2011), and recognizing that numbers are abstract units that can be partitioned and then recombined in different ways to facilitate numerical (also mental) calculation. Hands and fingers can be used to foster development of the part-whole relation, in particular with respect to 5 and 10, in a naturally embodied way. Literature from the fields of neuroscience, developmental psychology, and mathematics education suggests that using fingers for counting and representing numbers (Brissiaud, 1992) can have a positive effect on the development of numerical abilities and of number-sense.

The two snapshots come from two different classes. The videos were recorded during the teachers' regular classroom activity, which sometimes included interventions by a researcher (me) to work on specific topics, as in the case of the second excerpt. The excerpts discussed below are from videos taken between October and November of the students' first grade (school in Italy starts in mid September, so these are episodes that occurred at the beginning of the children's 1^{st} grade).

The "Beans in Rings" Activity

This activity is used to work on decomposing numbers in various ways, using beans and plastic rings.

1. Some beans are placed in a first ring, and a child is asked to count them, as shown in Figure 1. The same game can be played with other numbers of beans.

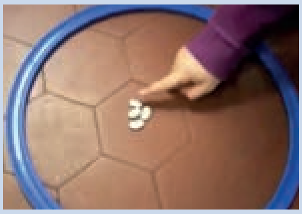
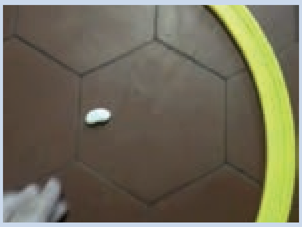
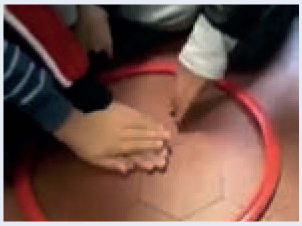
2. The teacher picks up the beans and separates them into two parts, one held in each hand, without showing the class. One part is placed in a second ring and uncovered (Figure 2), while the second part is placed in a third ring and covered up by a second child's hand (Figure 3). The second and third rings are placed below the first ring; the three

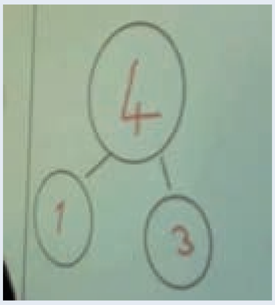
rings form an isosceles triangle.

3. The child who counted the beans initially is asked to “guess” how many beans are covered up.
4. The other children in the class are asked if they agree with the guess. The answer is then checked through counting (Figure 4).
5. Finally, the child who figured out the partition is asked to reproduce the situation analogically and symbolically in predefined diagrams on the Interactive White Board (IWB), as shown in Figure 5 and in Figure 6.

The teacher proposes the activity to work on decompositions of the number four.

Excerpt 1

What is said	*What is done*
1. *Teacher* (to G.): Count them. G: One, two, three, four.	Four beans are placed in a ring. G counts the beans pointing to each one with her finger (Figure 1).  Figure 1: The child is counting the beans in the first ring.
2. *Teacher*: Very good! So now I’m going to move them. I’m hiding them.	The teacher picks up the beans and separates them into two parts, one held in each hand, without showing the class.
3. *Teacher*: …and I’m putting here like this.	The teacher places one bean in a second ring (Figure 2).  Figure 2: The teacher puts the beans of one of the two parts into a new ring and leaves them uncovered.
4. *Teacher*: And over there I ask A. to hide them for me. Don’t peak! …Keep them well covered, A., eh!	The rest of the beans are placed in a second ring and covered up by A.’s hand (Figure 3).  Figure 3: The beans in the third ring are covered up.
5. *Teacher*: So, G., how many beans do you think A. has under his hand?	
6. *G.*: (immediately) Three.	G. looks right at the camera, not at the beans in the rings.
7. *Teacher*: Three? Should we check? 8. *Class*: Yes! 9. *Teacher*: Andrea can you show us?	A. uncovers the beans and the children count the beans, whispering, some pointing to the beans as they check.  Figure 4: After guessing at the number of covered up beans these are shown and counted.
10. *Teacher*: Very good G.! So, G., can you come draw, can you write on the whiteboard what happened?	G. draws three red dots (corresponding to the beans) in the top circle, and then one dot in the first circle connected to the top one, on the left. She does this without hesitation.
11. *G.*: (stops drawing dots) Teacher: And over there? How many did we have under A.’s hand? 12. *G.*: Three. 13. *Teacher*: OK. One, two and three (as G. draws them)	G. hesitates after drawing the dot in the circle on the left. Then the teacher prompts her with a question about the beans under A.’s hand, and she quickly answers and draws three dots in the circle on the right (Figure 5).  Figure 5: The analogical diagram completed by G.

What is said	*What is done*
14. *Teacher*: Now can you do it for me with the numbers over here on the side, G.?	G. starts by writing "1" in the circle on the lower left.
15. *Teacher*: There there is one. …There under A.'s hand we had…	G. then moves over to the circle on the lower right and writes "3".
16. *Teacher*: And up there can you write how many beans there were all together? …Over there what shall we write?	G. needs to climb up on a chair to reach the top circle on the interactive whiteboard. Then she hesitates and does not write anything. G. still does not write anything.
17. *Teacher*: The beans all together that were in the first ring, G.	
18. *G.*: We had four. … When we hadn't hidden any of them yet there were four.	However G. still does not write anything in the top circle.
19. *Teacher*: So then let's write four!	Figure 6: The symbolic diagram completed by G.
20. *G.*: (writes "4" in the top circle as in Figure 6)	
21. *Teacher*: Ok, thank you G.! Now, children let's try to do it also in our notebooks.	

Comments

G appears to be able to mentally predict how many beans are under her classmate's hand after one has been shown and left in the first circle. She responds very quickly (line 6), without counting, indicating that she seems to have internalised the decomposition of four into one and three. On the other hand, some classmates prefer to count to make sure (lines 7-9). This kind of difference in children's performances was quite typical in the classes where the material was experimented. Over time, the performances became more homogeneous, though mediation of the teachers, who were asked to do the following: invite more frequently to work children who seemed to be "slower" in mastering the shared strategies, and ask "faster" children to explain and motivate their own strategies as well as those of other children.

When she gets up to write on the white board, G. shows no difficulty representing the original bean situation analogically, and the 1 bean in the representation of the second ring (line 10). However she hesitates before drawing the three dots in the representation of the third ring, and does this only after the teacher prompts her to remember the situation (line 11). It is interesting to notice that to complete the analogical diagram G. proceeds in the same order as what physically happened with the beans. This is not what happens for the symbolic diagram, that she starts completing from the lower two circles representing the rings with one and three beans (lines 14 -15). This time she hesitates before filling in the top circle (lines 16-17). We note that the task requires representing at once (simultaneously) events that happened over time: at no point in time were there four beans in one ring and one and three beans in the other two rings! Indeed G. states (line 18): "We had four. …When we hadn't hidden any of them yet there were four." as if she needed to emphasize that the four beans were there only *before* any were hidden.

I chose this excerpt not only because it give a glimpse of what happened in the actual classrooms and allowed me to concretely show how all students were invited to participate during mathematics classes,

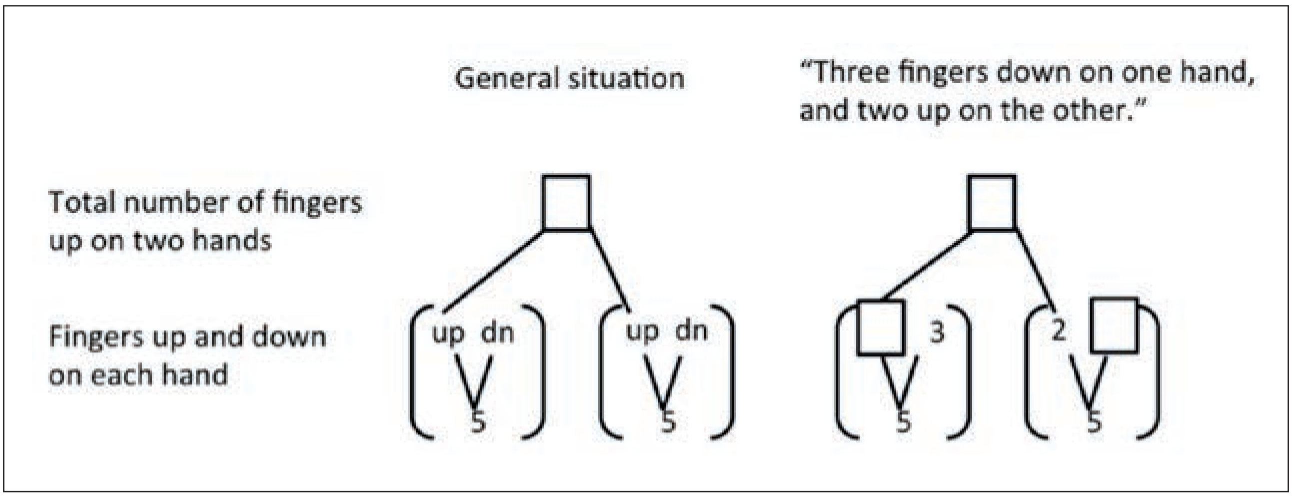


Figure 7: Representation of the part-whole relation in the "fingers game".

but also because the issue of managing time, and eliminating it in mathematical representations, is an inherent difficulty of learning mathematics that frequently gets overlooked (Sinclair, 2017). However it can cause difficulty even for the high achieving students like G.

The “Hidden Fingers Game”

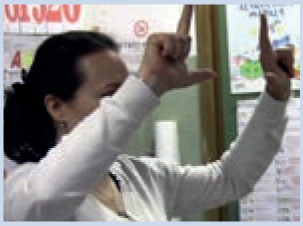
Below is a description of this game, proposed to start in the early months of children’s 1st grade.

1. The teacher describes a configuration of fingers and says: “On one hand I have N_1 fingers up/down and on the other hand I have N_2 fingers up/down”, while keeping her hands behind her back (See Figure 8). Then she asks: “What number am I making with the fingers that are up?”
2. The children try to produce the configuration described on their fingers (See Figure 9) and discover the number of fingers raised.
3. After seeing, listening to and comparing the children’s answers the teacher shows the configuration on her fingers (See Figure 10), highlighting that it is (usually) not the only possible one corresponding to the description.

The general situation presented in this game is described by the diagram in Figure 7, which shows precisely how part-whole relations can come into play in determining the total number of fingers raised.

In excerpt 2 the researcher (author of this paper) is visiting a first grade classroom, in November, and she proposes the “hidden fingers game”.

Excerpt 2

What is said	*What is done*
1. *Researcher*: So, on one hand I have three fingers lowered…three fingers lowered.	 Figure 8: The teacher describes a configuration of fingers while keeping them behind her back. She keeps both hands behind her back and starts a second round of the game. The children watch and listen, smiling.
2. *Researcher*: and on the other hand I have two raised. 3. *(a few) children*: Two!... Three!	
4. *Researcher*: No, how many fingers are raised? Do it with your hands.	The children start adjusting their fingers to match the configuration described verbally by the researcher.
5. *Researcher*: So one hand has three lowered, and the other has two raised. How many fingers are raised? 6. *Children*: Four, two… four! 7. *Researcher*: Let’s see how different people did it. Do it with your fingers.	The children try to produce the configuration described verbally on their fingers (Figure 9). Many of them repeat the configuration verbally as they try to adjust their fingers. Some children yell out answers, seemingly, without working on their fingers. The researcher waits and watches as the whole class figures out what the configuration might be. The children do not seem to be copying from one another. Most children show configurations of four fingers, two raised on each hand.  Figure 9: The children’s attempts.
8: *Researcher*: Very good! Three lowered and two raised.	The researcher has noticed all children’s fingers raised (even though not all children have raised their fingers in a correct configuration), so she shows a possible correct answer.  Figure 10: The teacher shows a possible correct configuration.

What is said	What is done
9. *Children*: Four!	
10. *Researcher*: Four! Very good! Let's see how you did it.	The researcher suggests comparing children's solutions, including those that gave results different from four.

Comment

This was the first time the children had played this game. They had only answered two easier finger configurations previously. During the first sessions of the game, children needed prompting (lines 4, 7) to produce possible configurations on their hands. Seeing the children work, (for example, the actions after words in line 7 and before line 8) was insightful for me because it allowed me, at a glance, to check all students' answers, sometimes even getting insight into their solution strategies. In later sessions, the children's solutions would be discussed before giving the correct (teacher-approved) answer. In this game, the part-whole relation becomes embodied: ten is decomposed into five and five, and five is decomposed in all possible ways on the children's hands. Indeed, this game seems to foster internalisation of part-whole relationships especially relative to the quantity five. Moreover, the game accomplishes the goal of using fingers in different ways to represent numbers within ten. The game was proposed at least three times a week in the experimental classrooms, and I believe it played a significant role in helping students avoid the development of persistent difficulties in arithmetic.

Concluding thoughts

Early results suggest that the teaching materials and strategies proposed in PerContare can contribute substantially to reducing the number of false positives in the diagnoses of dyscalculia. Such reduction seems, indeed, to be fruit of the implementation of activities like the ones presented in the examples. I expect that if we had the opportunity to zoom into the participation of initially "slower" students in sequences of episodes like the ones shown we would notice the key role played by our request in the design of the activities to foster participation and dialogue between all students. However, for now, this will remain a hypothesis to investigate in a future study. What has been confirmed by this study is the possibility that at least half of the Italian children who perform significantly below average on standardized diagnostic tests could perform normally or above average if only the mathematics education offered to them were different, and more in line with what was developed within PerContare.

My personal opinion is that, for the time being, in educational settings such as the Italian one, it is more fruitful to stop searching for *who* "dyscalculics" are, trying to label them while it is still unclear what the label stands for in each particular case, and instead concentrate on *why* some students fail in certain domains of mathematics (which?) and study what can be done to avoid such persistent and, in many cases, permanent failure.

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Anna Baccaglini-Frank is ~~a lecturer~~ at the University of Pisa, Italy.