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Interest rate targets and speculative attacks on public debt

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Abstract

The application of exchange rates target zones modeling to interest rates allows interpreting the puzzles that emerged with the public debt euro area crisis, namely the non-linear behavior of the interest rates and the fact that some stand-alone countries, not belonging to the euro area, have not been subject to speculative attacks in spite of equally large public debt-to-GDP ratios. As a matter of fact, this model shows that in the case of a non-credible upper threshold for the interest rate (that may be due to both the lack of room for increasing further the required government primary surplus and/or the absence of a monetary authority acting as a lender of last resort), the resulting public debt unsustainability determines an interest rate non linearity and makes the crisis possible for public debt levels that would be stable in the presence of a credible interest rate target.

Keywords: euro area crisis, interest rates, target zones, public debt, speculative attacks

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1. Introduction

After the outbreak of the euro area crisis, it was observed that some countries, like the UK, had not been affected by speculative selling in spite of the fact that their public debt-to-GDP ratio was even higher than the one of countries subject to attacks (as in the case of Spain). It was also identified a non-linear behavior of the interest rate with respect to public debt-to-GDP ratio, having been the interest rates rather low during the initial, 'honeymoon' years of EMU, and jumping up suddenly at some point during the crisis. As for the explanation of the first point, De Grauwe (2012) refers to the fact that a stand-alone country, like the UK, can rely on the potential for debt monetization guaranteed by its national central bank. This option, which is essential to reassure the markets and avoid the occurrence of self-fulfilling speculative attacks, was known to be unavailable to euro area countries, who cannot rely on the ECB to operate as a lender of last resort, given that she is explicitly forbidden by the Maastricht Treaty to act as such. As a result, euro area countries are exposed to destabilizing expectations that may lead to a default even if their public debt-to-GDP ratio is not substantially different from that of stand-alone ones. The non-linearity can be explained instead by concluding that the crisis was driven by negative self-fulfilling expectations, rather than by diverging economic fundamentals (De Grauwe and Ji, 2013a).

By adopting a model of heterogeneous markets' expectations about the uncertain level of the country's maximum feasible primary budget surplus, Tamborini (2015) provides a rigorous but also very intuitive explanation for both puzzles. He shows that the risk premium on public debt rises dramatically when the primary surplus which is necessary to grant debt stabilization approaches its maximum feasibility limit. It would be possible to conclude, then, that it is not the absolute size of public debt that matters for its sustainability, but rather the potential of a country to stabilize it.

In this paper I argue that a similar conclusion can be reached by applying the exchange rates target zones modeling technique to interest rates. I do this by showing that the presence of a non-credible upper threshold for the highest level of the interest rate that a government can stand while guaranteeing public debt stability (given the highest possible level of primary surplus that it can run and given the absence of a monetary authority playing the reassuring role of lender of last resort) provides the market with a clear indication as to the potential for a successful speculative attack, thereby encouraging it. This means that the speculative attack against public debt will take place at a level that would have guaranteed stability if the interest rate threshold could be credibly met. As a matter of fact, if either the fiscal authority could avoid the growth of public debt by running a primary surplus, or the central bank could reassure the market by standing ready to buy public debt, the interest rate would not be allowed to increase above the sustainability threshold level and no speculative attack would take place.

The paper is structured as follows. In par. 2 I review the relevant literature on target zones and on the euro area crisis and provide the motivation of the paper. In par. 3 I introduce the interest rate on public debt (par. 3.1) and define its upper feasibility threshold (par. 3.2). Par. 4 presents the model to be solved when considering respectively the 'honeymoon' case in which the stability interest rate threshold is expected not to be overtaken, so that public debt will be stabilized (4.1) and the 'divorce' case in which it is expected instead to be overtaken, so that public debt becomes unsustainable (4.2). Some policy observations are proposed in par. 5 and some final remarks close the paper in par. 6.

2. Literature review and motivation of the paper

The celebrated 'honeymoon' model on exchange rate target zones, presented by Paul Krugman at the end of the 1980s and published eventually in 1991 (Krugman, 1991), generated a vast amount of literature.[1] It proved that the imposition of credible floating bands could stabilize the exchange rate within the target zone thanks to the beneficial effect of the expected marginal intervention by the central bank. The more the exchange rate (e_t) would move towards the upper edge of the band ($\bar{e}$), then, the more it would be expected to be pushed back by the reflecting barrier within which it is allowed to fluctuate. Economic fundamentals, then, would enjoy what Krugman (1991) dubbed as a 'honeymoon'. The main thrust of the paper, namely the stabilizing property of a target zone, however, was soon disproved by Bertola and Caballero (1992). They showed that the adoption of a target zone (TZ) for the exchange rate might generate a 'divorce' (D) rather than a 'honeymoon' (H) if the defense of the margins of the band was not perfectly credible. A credible target zone (TZ, C) allows reaching the top of the band in correspondence of a state of economic fundamentals [2]($f_t^{TZ,C} = f_t^{FF} + H$) that would be much weaker than the one (f_t^{FF}) that would be reached with freely floating exchange rates (e_t^{FF}), therefore enjoying a 'honeymoon'. The expectation of an exchange rate realignment, namely of a non-credible target zone (TZ, NC), instead, would push it up towards the top of the band – therefore confirming the feared realignment – even with a relatively strong state of economic fundamentals ($f_t^{TZ,NC} = f^{FF} - D$): this is the case of the 'divorce'. Figure 1 represents what has been described above.

I claim in this paper that a similar approach is possible in analyzing the 2010-2012 waves of speculative attacks against the public debt of some countries belonging to the euro area.

In particular, Tamborini (2015) shows how the credibility of the fiscal consolidation required in order to stabilize public debt may be questioned by the feasibility of the primary surplus that a government can run. The expectation of an upper threshold that limits the social feasibility of the primary surplus – due to the existence of an upper limit on the revenues that a government can excise from its citizens and of a lower limit on the expenditures that it can cut – works exactly as the upper band of a target zones regime. As a matter of fact, the presence of such a limit on the primary surplus also implies a limit to the interest rate on public debt that the government and/or the central bank can pay while granting public debt sustainability.[3] A 'divorce' which is similar to the one identified by Bertola and Caballero (1992) emerges in this case and interest rates increase rather than decrease if a too high, therefore not feasible, primary surplus is required for public debt stability. It is worth stressing immediately, however, that the interest rate may be stabilized not only thanks to the primary surplus of the government, but also thanks to the central bank operating as lender of last resort, thereby buying the excess of supply of government bonds in the market. Tamborini (2015) considers a framework of heterogeneous agents' beliefs, in which the feasibility limit for the primary surplus is not known with certainty.[4] [5] Each agent has its own expectation about this limit. However, any time the primary surplus increases, a larger share of agents will think that it is approaching its feasibility limit. An always larger share of agents, then, will require to be compensated by a higher interest rate for the risk of public debt default that they get exposed to in holding domestic public debt. The presence of different primary surplus feasibility limits, but also the availability of different sources through which the stability of public debt could be assured (for example the availability of money financing – as argued by De Grauwe (2012) – or of federal financing) explains why some countries were subject to a speculative attack (like the euro area crisis countries) and some others were not (like the UK, or even the USA) in spite of similar public debt-to-GDP ratios.

Tamborini's model also explains why the interest rate behaved non-linearly, by reacting very little to low levels of public debt-to-GDP ratios, while responding dramatically to higher levels, that were approaching their maximum sustainability limit.

The same conclusions, however – namely both the destabilizing effects of a non-feasible primary surplus and the non-linear behavior of the interest rate on public debt – can be obtained by adapting the 'divorce' target zone model *à la* Bertola and Caballero (1992) to the case of speculation against public debt.[6]

The generic state of economic fundamentals, that was considered in the target zones literature in determining the value of the exchange rate, finds an obvious analogue in the public debt-to-GDP ratio, which affects the level of the interest rate to be paid on it. Both the approach and the conclusions are impressively similar, as I am going to show below.

[Figure 1 near here]

3. The interest rate on public debt and its upper threshold

3.1 The interest rate on public debt

A linearized version of the interest rate parity for public debt bonds is as follows:

$$i_t = \bar{r} + RP_t, \quad (1)$$

where:

$$RP_t = \alpha b_t + \beta \frac{E[di_t]}{dt}, \quad (2)$$

so that:

$$i_t = \bar{r} + \alpha b_t + \beta \frac{E[di_t]}{dt}. \qquad (3)$$

(1) tells that the interest rate, i_t, can be thought as determined by a riskless reference interest rate, $\bar{r}$ (that could also be considered as a policy instrument for public debt stabilization in the hands of the central bank) and by a risk premium, RP_t. In turn, the latter depends on two elements that have been identified in the literature, as reported in (2). The first one is the absolute size of the public debt-to-GDP ratio (Corsetti *et al.*, 2014), due to the fact that the higher b_t, the lower the potential to respond to negative shocks hitting the economy. The sensitivity of the interest rate with respect to b_t is measured by parameter α. The second part, instead, is the one with self-fulfilling features. The lower the expected sustainability of public debt, the higher the expected future variation of the interest rate, that in turn affects the current interest rate level with a weight given by parameter β.

3.2 The interest rate upper threshold on public debt

The government, together with the central bank, who operates through the instrument which is under her control, $\bar{r}$, and through the possible monetization of public debt,[7] have to respect an upper threshold for the nominal yield on public bonds in order to guarantee public debt sustainability.[8]

In the case of EMU, the concern of the ECB for the public debt stability of euro area countries is proved by the repeated declarations of its president, Mr. Mario Draghi, to justify the bank's intervention precisely with the need to reduce the high interest rates – unjustified by the state of economic fundamentals - that threatened public debt sustainability and in the end undermined the orderly running of monetary policy.

The interest rate has also a lower threshold (that I am going to ignore below in the solution of the model, as it also done in the exchange rates target zones literature), that may be thought as the zero lower bound (although central banks have shown that it is even possible to go below it). The value taken by i_t can be identified, then, as follows:

$$
\begin{aligned}
&i_t = \bar{i}^* \text{ if } i_t \geq \bar{i}^* \\
&i_t = \tilde{i}_t \text{ if } \underline{i}^* < i_t < \bar{i}^* \qquad (4) \\
&i_t = \underline{i}^* \text{ if } i_t < \underline{i}^*
\end{aligned}
$$

where $\bar{i}^*$, $\underline{i}^*$, $\tilde{i}_t$ and i_t represent respectively the upper and the lower thresholds for the interest rate, the interest rate that would obtain when it fluctuates within the announced bands, and the interest rate prevailing in case no commitment is taken.

The value of $\bar{i}^*$ may be thought as implicitly determined by the public debt solvency equation, as it will be discussed below.

The standard public debt dynamics, namely the continuous time variation of the public debt-to-GDP ratio, db_t, when federal solidarity is not possible - as it is the case in the euro area - is as follows:[9]

$$db_t = f_t dt - m_t dt + (i_t - g_t) b_t dt + \sigma dz. \qquad (5)$$

The term f_t is the primary public deficit-to-GDP ratio (namely e_t - t_t, where e_t is government's fiscal expenditure-to-GDP and t_t are government revenues-to-GDP) and the term m_t is the monetization rate of public debt (see Tamborini, 2015). The term $(i_t - g_t)b_t$ is the service on the debt as a ratio of GDP. The stochastic component of public debt-to-GDP growth is supposed to follow a Brownian motion (or Wiener process), σdz. Parameter σ represents the instantaneous standard deviation of the

Brownian motion and the term dz is the Brownian motion variation which is so characterized:

$$dz = \chi\sqrt{dt}, \tag{6}$$

where χ is a random variable which is independently, identically and normally distributed, with 0 mean and variance equal to 1, and dt is an infinitesimal time variation.

The fiscal authority operates in order to stabilize the steady state value of public debt-to-GDP, $\bar{b}$. For that to happen, by imposing $db_t = 0$ in (5), assuming that no help can come from the monetary side, namely that $m_t = 0$ (Tamborini, 2015), it turns out that the long run primary surplus/deficit (calculated in the steady state, in which there are no stochastic shocks) has to cover the long term service on the debt:

$$f^* = (g^* - i^*)\bar{b}, \tag{7}$$

where the symbol * refers to long term values.

If $i^* > g^*$, so that $(g^* - i^*) < 0$, it follows that $f^* < 0$, namely the government needs to run a budget surplus $s^* = -f^*$ in order to stabilize public debt.

By rearranging (7), then, we also have that:

$$i^* = g^* + \frac{s^*}{\bar{b}} \tag{8}$$

Where i^* indicates the long term interest rate that assures a steady state public debt-to-GDP ratio, $\bar{b}$, for a given long term primary surplus, s^* and a long term GDP rate of growth, g^*.

As argued by Tamborini (2015), however, the primary surplus that can be run by a government is not unbounded. It is determined by comparing the cost of solvency (positively related to s^*) with the cost of default (negatively related to s^*), so as to be determined at the point at which the two equate. The resulting government's maximum feasible long-run primary surplus is indicated with $\bar{s}^*$.

I can observe that such a maximum feasible $\bar{s}^*$, in turn, determines $\bar{i}^*$, which is the maximum long run interest rate that a government can afford to pay on its public debt in order to grant public debt sustainability and that it is targeting (since the government could afford to pay a higher interest rate on its debt, while granting public debt sustainability, only if $\bar{s}^*$ increased - or if g^*increased and/or if $\bar{b}$ decreased). From the definitions given above it follows that for public debt sustainability to be feasible it must be that $s^* \leq \bar{s}^*$, thereby implying that $i^* \leq \bar{i}^*$.

As a matter of fact, a too high fiscal contraction (implying a too low government expenditure and/or too high government revenues) may not be socially acceptable because it would imply a too large reduction of public expenditure or a too high rise of government revenues, and in that case it will not be feasible, so as to cause a higher rather than lower interest rate, as shown by Tamborini (2015).[10]

So, the $\bar{i}^*$ that the government (who has to face $\bar{s}^*$) can credibly defend is determined by (8') below:

$$\bar{i}^* = g^* + \frac{\bar{s}^*}{\bar{b}}. \qquad (8')$$

By considering (1), (8') implies:

$$\overline{RP}^* = g^* + \frac{\bar{s}^*}{\bar{b}} - \bar{r}^*, \qquad (8'')$$

where $\overline{RP}^*$ reflects the highest level of the risk premium that a government can stand for given $\bar{s}^*$, $\bar{r}^*$, g^* in order to keep $\bar{b}$ at its steady state value.[11] The variable $\bar{r}^*$ is the lowest possible long run interest rate level that the central bank can pick (that might coincide with the ZLB, but that might even be negative) while still assuring public debt stability.

Let us recall, however, that we have assumed so far $m_t = 0$, an hypothesis implying the absence of a lender of last resort. When the central bank is expected instead to stand ready to honor the public debt, in the case of stochastic shocks an additional (monetary) channel of debt stabilization would be available, together with the one provided by the primary surplus of the fiscal authority, as we are going to consider in the discussions below.

4. The interest rate target model

As we have seen above, the government stabilizes public debt in the steady state for a given maximum feasible level of $\bar{s}^*$, for g^* and for $\bar{i}^*$. Such a stabilization, however, does not rule out other variations of public debt. This means that the public debt-to-GDP ratio might still increase, due to the process of stochastic shocks that it may be subject to:

$$db_t = \sigma dz \tag{9}$$

What this implies is that public debt may still increase above its steady state value because of a process of stochastic shocks and in turn this may increase the risk premium above the level that the government can stand to guarantee public debt solvency.

This is due to the fact that there are no more ammunitions to defend the stability of public debt, namely neither government's extra primary surplus is feasible, nor the central bank is capable or willing to guarantee that the interest rate will not increase above $\bar{i}^*$: that is the case if $s^* > \bar{s}^*$, namely the primary surplus that would be necessary to satisfy the public debt solvency equation exceeds the one that would be feasible in social terms and/or if the interest rate cannot be stabilized because the central bank is not able or not willing to act as 'buyer of last resort' ($m_t = 0$), as we have assumed in the previous paragraph. As soon as

$\tilde{i}_t$ exceeds $\bar{i}^*$, then, public debt is not sustainable anymore and this generates an explosive spiral between interest rates and public debt.

If the government is expected instead to provide the (feasible) primary surplus which is necessary to guarantee the stability of public debt and/or if the central bank intervenes by buying public debt, so as to support with her demand the price of bonds, so that $m_t > 0$, the interest rate may well remain within the band ($\tilde{i}_t < \bar{i}^*$). The interest rate, then, would be stabilized and this would produce a 'honeymoon' which is similar to the one identified by Krugman (1991).

According to De Grauwe and Ji (2013a), this is the case of stand-alone countries, that are capable to guarantee the solvency of public debt by managing money supply through their domestic central bank, thereby reassuring the markets.

A similar conclusion, however, obtained in the euro area when the ECB decided to play the function of lender *(or buyer)* of last resort, or it would have obtained if a federal guarantee had stood ready to provide the additional resources that were necessary to stabilize public debt.

To summarize, then, our public debt model is composed by the two following equations:

$$\tilde{i}_t = \bar{r} + \alpha b_t + \beta \frac{E(d\tilde{i}_t)}{dt} \qquad (3')$$

$$db_t = \sigma dz \qquad (9)$$

This is a system that reminds quite closely the original Krugman (1991) target zones model, later on amended by Bertola and Caballero (1992). In what follows, I am going to show that it allows obtaining an interpretation of the euro area crisis that resembles closely the conclusions reached by Tamborini (2015) and De Grauwe and Ji (2012, 2013a).

4.1 Feasibility of government intervention: the 'honeymoon'.

In order to solve (3') and (9), let us follow the target zones literature by assuming a generic functional form for the $\tilde{i}_t$ as a function of b_t. We will then use it in order to have a closed form solution of the differential equation (3'). Observing the latter, we can take $\tilde{\imath}_t$ to be a function of the debt-to-GDP ratio.

$$\tilde{i}_t = q(b_t) \tag{10}$$

We can now use this equation to calculate the expected interest rate variation. In order to do so, let's expand our (stochastic) equation in a Taylor-type series, by calculating Ito's differential:

$$d\tilde{\imath}_t = q'(b_t)E(db_t) + \frac{1}{2}q''(b_t)E(db_t)^2 \tag{11}$$

From the definition of db_t in (9), it turns out that $(db_t)^2 = \sigma^2\chi^2 dt$. By considering the expected values and by dividing by the infinitesimal temporal variation, we obtain Ito's Lemma:

$$\frac{E(d\tilde{i}_t)}{d_t} = \frac{1}{2}q^{''}(b_t)\sigma^2, \tag{12}$$

given that $E[db_t]/dt = 0$ and $E[db_t]^2/dt = \sigma^2$. By replacing (12) into (3') we have, then:

$$\tilde{i}_t = q(b_t) = \bar{r} + \alpha b_t + \beta\frac{1}{2}q^{''}(b_t)\sigma^2 \tag{13}$$

This is a differential equation of the second order, whose generic solution is of the kind:

$$\tilde{i}_t = q(b_t) = \bar{r} + \alpha b_t + A_1 e^{\lambda_1 b_t} + A_2 e^{\lambda_2 b_t} \tag{14}$$

Let's take the second order derivative of (14) above in order to have a value for

$$q''(b_t) = \lambda_1^2 A_1 e^{\lambda_1 b_t} + \lambda_2^2 A_2 e^{\lambda_2 b_t} \quad (15)$$

So that by replacing it into (13), it gives:

$$\tilde{i}_t = q(b_t) = \bar{r} + \alpha b_t + \beta \frac{\sigma^2}{2} (\lambda_1^2 A_1 e^{\lambda_1 b_t} + \lambda_2^2 A_2 e^{\lambda_2 b_t}). \quad (16)$$

By comparing (16) with (14), we have:

$$A_1 e^{\lambda_1 b_t} + A_2 e^{\lambda_2 b_t} = \beta \frac{\sigma^2}{2} (\lambda_1^2 A_1 e^{\lambda_1 b_t} + \lambda_2^2 A_2 e^{\lambda_2 b_t})$$

$$A_1 e^{\lambda_1 b_t} \left(\lambda_1^2 \beta \frac{\sigma^2}{2} - 1 \right) + A_2 e^{\lambda_2 b_t} \left(\lambda_2^2 \beta \frac{\sigma^2}{2} - 1 \right) = 0 \,.$$

$$A_1 e^{\lambda_1 b_t} \left(\lambda_1^2 \beta \frac{\sigma^2}{2} - 1 \right) = 0 \text{ and } A_2 e^{\lambda_2 b_t} \left(\lambda_2^2 \beta \frac{\sigma^2}{2} - 1 \right) = 0 \,. \quad (17)$$

We can calculate now λ_1 and λ_2 by solving the following two equations: $\left(\lambda_1^2 \beta \frac{\sigma^2}{2} - 1 \right) = 0$ and $\left(\lambda_2^2 \beta \frac{\sigma^2}{2} - 1 \right) = 0$.

We have as a solution, then:

$$\lambda_{1,2} = \pm \sqrt{\frac{2}{\beta \sigma^2}} \quad (18)$$

This means that we have two complementary solutions which satisfy our 2nd order differential equation: $\tilde{i}_t^{\,c1} = A_1 e^{\lambda_1 b_t}$ and $\tilde{i}_t^{\,c2} = A_2 e^{\lambda_2 b_t}$.

We add them up to obtain the general solution, with λ_1 and λ_2 defined as above:

$$\tilde{i}_t = q(b_t) = \bar{r} + \alpha b_t + A_1 e^{\lambda_1 b_t} + A_2 e^{\lambda_2 b_t}$$

Let us ignore the (implicit) lower band, thereby ignoring the coefficient A_2 (the appropriateness of this assumption, which is not necessary and is only made in order to focus on the behavior of the upper part, might even be confirmed by the fact that the zero lower bound does not seem to be binding anymore, given that central banks have even experienced negative interest rates). Let us determine, then, the constant A_1.

In order to close the model in the case that we are considering, in which there is still room for the primary surplus to stabilize public debt and/or the central bank can operate as a lender of last resort, we can use the so called 'smooth pasting' condition, that was also used to close the first generation of target zones models, initiated by Krugman (1991) and showing the stabilizing effects of the imposition of fluctuation bands on exchange rates. This is realized by the tangency condition described below (see Figure 2). It should be considered, however, that while in Krugman's target zones model the exchange rate is assumed to be under the exclusive control and responsibility of the central bank, in the case that I am considering, both the fiscal and the monetary authority affect the determination of the interest rate, as I am going to further point out below.

Intuitively, the expectation that the current interest rate, $\tilde{\iota}_t$, will not be allowed to exceed the level $\bar{\iota}^*$, is based on the expectation that $\tilde{\iota}_t$ will be reduced through a reduction of b_t, which in turn is possible thanks to a higher primary surplus (as long as it is expected to be feasible) and/or through a reduction of the public debt in the hands of the private sector, because of the higher demand coming from the central bank. So, when $\tilde{\iota}_t$ reaches $\bar{\iota}^*$, the former is expected not to overtake it, therefore remaining within the band, no matter whether b_t will be increasing or decreasing: this can only be the case if a tangency condition occurs, as in Figure 2.

So, let us impose the 'smooth pasting' condition:

$$\frac{d\tilde{\iota}_t}{db_t} = \alpha + \lambda_1 A_1 e^{\lambda_1 b_t} = 0 \tag{19}$$

From which it follows that:

$$A_1 = \frac{-\alpha e^{-\lambda_1 b_t}}{\lambda_1} < 0. \tag{20}$$

We have, then:

$$\tilde{i}_T = \bar{r} + \alpha\bar{b} - \frac{\alpha}{\lambda_1} \quad (21)$$

And it also turns out that:

$$b_T = \bar{b} + \frac{1}{\lambda_1} \quad (22)$$

Where $\bar{b} = \frac{\bar{\imath}^* - \bar{r}^*}{\alpha}$ is the public debt-to-GDP ratio that obtains when the interest rate reaches $\bar{\imath}^*$ by following its linear path, without resenting of the imposition of the upper band.

Considering that at time T, $\tilde{\imath}_T \leq \bar{\imath}^*$, by replacing (18) into (22) (still with $A_2 = 0$) we have that:

$$b_T = \bar{b} + \frac{1}{\lambda_1} = \bar{b} + \sqrt{\frac{\beta\sigma^2}{2}} \quad (23)$$

The difference between b_T and $\bar{b}$, which is given by $\frac{1}{\lambda_1} = \sqrt{\frac{\beta\sigma^2}{2}}$, represents what Krugman defined as the 'honeymoon effect' and tells us by how much it is possible to have an increase of the public debt-to-GDP ratio while keeping $\tilde{\imath}_T \leq \bar{\imath}^*$.

What the 'smooth pasting' condition suggests is that the more $\tilde{\imath}_t$ gets close to $\bar{\imath}^*$, the more the primary surplus is expected to increase, so as to guarantee public debt sustainability (until it reaches its upper feasibility constraint), and/or the more the central bank is expected to intervene demanding public debt, thereby increasing its price and reducing the interest rate on it below or at the level of $\bar{\imath}^*$. It is expected, then, that any increase in public debt due to the stochastic shocks will be compensated either by a higher primary surplus or by a central bank intervention.

Figure 2 provides a graphical representation of these results.

However, all of this would only apply if there is expected room for the primary surplus to stabilize public debt and/or if the central bank has (or is expected to have) enough room for buying public debt, so that $\tilde{\iota}_T \leq \bar{\iota}^*$. This condition realized both during the initial, 'honeymoon' years of EMU, and after the celebrated 'whatever it takes' Draghi speech.

When this is not the case, however, we have to consider a different closing condition, that we are going to deal with in the next paragraph.

[Figure 2 near here]

4.2 Non-feasibility of government and central bank intervention: the 'divorce' effect

The literature on target zones also helps us understanding what the outcome would be in the situation in which the government cannot commit credibly to guarantee public debt solvency and/or the central bank is unavailable to act as a 'buyer of last resort'. As a matter of fact, the 'smooth pasting' solution would only apply in the case in which there is full confidence that the interest rate $\tilde{\iota}_t$ will not be allowed to increase above the upper target, $\bar{\iota}^*$, namely when public debt will be sustainable. This means that there is confidence that there will be enough room for the primary surplus to maintain public debt solvency and/or for the monetary authority to intervene and buy public debt so as to keep $\tilde{\iota}_t$ below $\bar{\iota}^*$.[12]

The opposite result, however, would occur in the case in which there is an expectation of a not sufficient room for fiscal consolidation, as suggested by Tamborini (2015), and/or of no availability of the other possible instruments. Tamborini (2015) points out that in some countries, during the euro area crisis, with the increasing of public debt (for a given interest rate and GDP rate of growth), the primary surplus that was required for its stabilization was approaching its upper feasibility limit, at which there was the expectation of no chances to further stabilize it. The more s^* was getting close to $\bar{s}^*$, then, the lower the confidence in public debt stabilization and the more likely the speculative attack was, given also the absence of a central bank playing the role of lender of last resort.

In this situation, when $\tilde{\imath}_t$ hits the interest rate threshold, $\bar{\imath}^*$ the former would be allowed to increase above the latter. Let us recall that such an interest rate threshold is determined by the maximum level of the primary surplus that the government can run (and by the constraint on the availability of other instruments of public debt financing) satisfying public debt stability. In that case, the more the interest rate moves up towards the top of the band, the higher the probability to get there.

In order to analyze the situation in the case in which there is no certainty that the limit imposed by the interest rate threshold will be met, let us follow Bertola and Caballero (1992) and the so called 'divorce effect' (as opposed to the Krugman's 'honeymoon' one) that they identified. In this situation, in which the fiscal surplus that would be necessary to assure public debt stability, s^*, will be higher than the maximum one that the country can stand, $\bar{s}$, and/or in which the central bank is not be able to prevent the increase of the interest rate, because she does not operate as lender of last resort, the debt stability condition will not be satisfied. This expectation produces destabilizing rather than stabilizing effects.

The public debt-to-GDP ratio can be assumed as fluctuating between 0 and the maximum level of public debt ($\bar{b}$) which is obtained in correspondence with the maximum feasible interest rate that assures public debt stability ($\bar{\imath}^*$), while the center of the band of the economic fundamental can be taken as equal to $\bar{b}/2$.

An arbitrage argument provides the closing equation. When the interest rate reaches its upper threshold, $\bar{\imath}^*$, determined by considering the corresponding maximum level that the public debt can reach in order to remain stable, $\bar{b}$, its value has to be equal to the expected one resulting from the weighted probabilities of the two different events that may take place. There is a probability p that neither the government, nor the central bank will have enough resources to stop b_t to grow above $\bar{b}$, nor to avoid the interest rate, then, to exceed its stability threshold. This implies that public debt may be allowed to exceed its target, and the interest rate will jump accordingly.

So, one possibility is that when the value of public debt-to-GDP ratio reaches $\bar{b}$, it will be allowed to move up by the size $\delta > 0$ and due to the higher interest rate to be charged on it, it will jump to the center of the new fluctuation band, namely to $\bar{b} + \delta/2$.

Of course, there is also the complementary probability (1-p) that public debt will not be allowed to increase, by keeping it at or below $\bar{b}$. The primary surplus adjustment at the margin might be such as to move its floating band down by ε. The public debt-to-GDP ratio will then go back to the center of a new floating band, $\bar{b} - \frac{\varepsilon}{2}$, which is included between $\bar{b} - \varepsilon$ and $\bar{b}$.

It turns out, then, that the arbitrage equation is as follows:

$$p\, \tilde{\imath}_t(\bar{b} + \tfrac{\delta}{2}, \bar{b} + \tfrac{\delta}{2}) + (1-p)\tilde{\imath}_t(\bar{b} - \tfrac{\varepsilon}{2}, \bar{b} - \tfrac{\varepsilon}{2}) = \tilde{\imath}_t(\bar{b}, \tfrac{\bar{b}}{2}). \qquad (24)$$

(where in the expression $\tilde{\imath}_t(x, y), x$ refers to the current value taken by the fundamental, and y refers to the value taken by the fundamental at the center of the band).

This generalizes the result obtained by Bertola and Caballero (1992), who consider instead the case in which an exchange rate devaluation takes place and a new floating band of equal size, $\bar{b}$, rather than of a generic size δ, is adopted, so that their case is nested within the more general case that is considered here, by assuming that $\delta = \bar{b}$. Following them in considering a symmetric fluctuation band centered on point c and implying that A_1 = - A_2 and λ_1 = - λ_2, the solution for the optimal control variable becomes:

$$\tilde{\imath}_t = q(b_t, c) = \ \bar{r} + \alpha b_t + Ae^{-\lambda(b_t - c)} - Ae^{\lambda(b_t - c)}. \qquad (25)$$

The same result is obtained if we adopt the same simplifying assumption we took above, namely considering only the upper band and ignoring the lower one, as I will do in what follows.

Let us consider, then, only one band, as it was done in order to solve for the 'smooth pasting' case:

$$\tilde{\imath}_t(b_t, c) = \bar{r} + \alpha b_t + Ae^{\lambda(b_t - c)}. \qquad (26)$$

We have that:

$$p[\bar{r} + \alpha\left(\bar{b} + \frac{\delta}{2}\right) + A] + (1-p)\ [\ \bar{r} + \alpha\left(\bar{b} - \frac{\varepsilon}{2}\right) + A] = \bar{r} + \alpha\bar{b} + Ae^{\lambda\frac{\bar{b}}{2}}, \qquad (27)$$

from which it follows that:

$$A = \frac{[p\alpha\left(\frac{\delta+\varepsilon}{2}\right) - \alpha\frac{\varepsilon}{2}]}{e^{\lambda\frac{\bar{b}}{2}} - 1}. \qquad (28)$$

This also means that $A \geq 0$ iff $[p\left(\frac{\delta+\varepsilon}{2}\right) - \frac{\varepsilon}{2}] \ \geq 0$, that is iff:

$$p \ \geq \frac{\varepsilon}{\delta+\varepsilon}, \qquad (29)$$

which corresponds to the case made by Bertola and Caballero (1992) for $\delta = \varepsilon = \bar{b}$.

Figure 3 represents in a simplified way what has been shown analytically above (with *D* referring to the 'divorce' effect).

The intuition for this 'divorce' result is quite straightforward. The more public debt is expected to be allowed to increase as a result of stochastic shocks (the larger δ), the lower the *p* required for the interest rate to increase as a result of a speculative attack. In turn, this depends on the anticipated lack of resources for stabilizing public debt, namely on the lack of possibility to run a sufficiently large primary surplus and/or of the lack of a central bank operating as a lender of last resort. Of course, the opposite holds for ε.

What all this suggests is that programs of fiscal consolidation have to be confronted with reality and their feasibility should be verified, as also argued by Tamborini (2015). However, it also means that the central bank plays a quite crucial role in reassuring the market and avoiding self-fulfilling crises.

The results obtained above confirm the non-linearity of the interest rate and public debt for some euro area countries, as driven by the expectation of the feasibility of public debt stabilization. The results of 'smooth pasting' and 'honeymoon' applies to the UK and the USA, while the case of 'divorce' is the one of euro area crisis countries. It should be recalled, however, that public debt stabilization also has to do with the behavior of the monetary authority, as I have stressed repeatedly, and with GDP growth (g^*) and federal solidarity, that have not been considered explicitly in this paper.

[Figure 3 near here]

5. Policy implications

Let us summarize the results that have been obtained in this paper and let us draw some possible policy implications from them. What has emerged in the analysis above is that the stabilization of public debt is only possible if the interest rate is not expected to exceed its upper stability threshold. This can be obtained both with the fiscal authority running a feasible (and therefore credible) fiscal consolidation, as shown clearly by Tamborini (2015), and/or with the monetary authority reassuring the market by standing ready to buy government debt. Either way will avoid public debt growth and will stabilize market expectations, so as to avoid the interest rate to exceed its upper stability threshold.

It is easy to observe that these two different options reflect the two different views that characterize the analysis and interpretation of the euro area crisis.[13] Boitani and Tamborini (2016), for example, referring to Bastasin (2015) compare the view suggesting that the crisis has been the sole responsibility of the fiscal authorities (who, as a result, would have to follow the prescriptions of 'market discipline'), with the view of a crisis driven by 'institutional fragility', suggesting in particular that the crisis has its origins mainly in the absence of a lender of last resort.

What emerges from the model presented in this paper is that both of them play a role, meaning that, especially when there is no more room for fiscal consolidation (which is the case explicitly considered by Tamborini, 2015), the existence of a lender of last resort becomes essential for avoiding self-fulfilling, liquidity crises.

The 'market discipline' view implies that the solution to the euro area crisis has to be found in the virtuous behavior of the fiscal authorities. This channel, however, may fail because of the unfeasibility of the fiscal consolidation effort which is required in order to stabilize public debt. In that case, the role played by institutions in preventing a liquidity crisis becomes particularly relevant: the presence of a lender of last resort would provide the expectation that the interest rate will be stabilized below its upper threshold, and this would result in a self-fulfilling stabilized public debt-to-GDP ratio.

At this stage it is also possible to compare the implications of the model proposed by Tamborini (2015) with those resulting from the present interest rate target model. The former focuses explicitly on fiscal policy, showing nicely how fiscal discipline may turn out to be counterproductive when it is not feasible (and therefore not credible). Precisely for that reason, however, Tamborini's model leaves the role of the interest rate and of monetary policy on the background, concluding that, since fiscal consolidation may fail, "the second solution (…) consists of setting a ceiling on the interest rate (…). This can be accomplished by absorbing the excess supply of bonds of the pessimist investors" for whom the public debt-to-GDP ratio would exceed $\overline{b}$ (Tamborini, 2015, p. 1172).

In the interest rate target model that I have presented above, instead, the role of the interest rate in granting public debt sustainability is addressed explicitly and both the role of fiscal policy ('market discipline' view) and of monetary policy ('institutional fragility' view) emerge as equal possible determinants of the actual interest rate, $\tilde{i}_t$, to be compared with the highest possible one, $\overline{i}^*$, which needs not to be exceeded in order to assure public debt sustainability.

6. Concluding remarks

In this paper I have shown that the exchange rate target zones modeling could be applied also to the euro area crisis in order to obtain some theoretical results that are in line with those obtained by Tamborini (2015) and that also explain the empirical evidence shown by De Grauwe and Ji (2013a). As a matter of fact, I have argued that in the case of a non-credible upper threshold for the interest rate (resulting either from the lack of room for increasing primary surplus further and/or from the absence of a lender of last resort), the resulting public debt unsustainability increases the interest rate on it, thereby anticipating the crisis at lower levels of public debt, compared to the case in which no binding constraint on the primary surplus were present and/or a lender of last resort existed.

Although the approach of heterogeneous markets expectations taken by Tamborini (2015) allows to interpret the puzzles that emerged with the euro area crisis, I have shown that this is also possible with a more traditional approach, based on the methodology resulting from target zones modeling. This seems to be a more general approach that allows to consider both the role of fiscal policy and that of monetary policy in affecting the interest rate on public debt and in keeping it below its maximum stability level.

In this paper I have followed the technique used by Krugman (1991) in proving the 'honeymoon' effect of a stabilizing interest rate target on public bonds, and the one used by Bertola and Caballero (1992) in showing the 'divorce' effect when that target cannot be credibly defended. In the solution of the model, I have departed from the latter, however, in two respects. First of all I have simplified the model by ignoring the lower band, rather than assuming symmetric bands, which would have been inappropriate in the case that I have been examining. Second, differently from Bertola and Caballero (1992), who assume that the new expected bands for the state of economic fundamentals are of the same s'ize as the old ones, I have considered a more general case for both the size of debt increase when the threshold lacks credibility, and the size of debt reduction when the primary surplus is expected to be sufficient to stabilize it. Bertola and Caballero's (1992) 'divorce' conclusion, then, turns out to be nested within my more general case.

Further research is awaiting in order to investigate, among other things, the role that GDP growth, the possibility of federal funding, and the strategic interaction between government and central bank may play in stabilizing public debt and avoiding the occurrence of a confidence crisis on it.

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[1] Krugman and Miller (1991) contains a first set of important contributions to this literature.

[2] One can think of the state of economic fundamentals as represented, for example, by money supply in a standard monetary model of exchange rate determination.

[3] Castro et al. (in press) consider the issue of fiscal sustainability under the lenses of population ageing.

[4] Gros (2012) also considers a model with an uncertain upper threshold for fiscal policy. Thanks to an anonymous referee for pointing this to my attention.

[5] Anufriev *et al.* (2013) also consider heterogeneous expectations, although in the setting of a frictionless DSGE model, and analyze their effects on macroeconomic stability with different interest rate rules with respect to inflation.

[6] Fiscal policy issues have already been analyzed by using the target zones technology. See for example Miller, Skidelsky and Weller (1990), Bertola (1990) and Bertola and Drazen (1993).

[7] The strategic relationship between the two, in order to reduce the interest rate, is not discussed here and will be the object of future research.

[8] Cantore *et al.* (in press) also consider the fiscal and monetary policy interaction for the management of a public debt crisis.

[9] The deterministic version of the stability condition reported in (1) above can be derived easily by considering the dynamic equation of public debt:

$$\frac{dB_t}{dt} = (E_t - T_t) - M_t + i_t B_t$$

where B_t is the steady state level of public debt, E_t is constant level of government expenditure, T_t is the constant level of taxation - so that $(E_t - T_t)$ is the constant primary deficit F_t - M_t is the amount of direct public debt monetization by the central bank, and i_t is the constant nominal interest rate to service the public debt. From the equation above, by dividing through by the nominal GDP, thereby considering the public debt/GDP ratio, it follows that:

$$\frac{db}{dt} = f_t - m_t + (i_t - g_t)b_t$$

Low case letters refer to the ratio of the respective capital letter with GDP, and g_t is the rate of growth of nominal GDP.

[10] Viegas and Ribeiro (2016) analyze the welfare consequences of some fiscal consolidation episodes in the European Union.

[11] Notice that with a little model modification, the argument would not change if we reasoned directly in terms of the risk premium, $\overline{RP}^*$, rather than in terms of the interest rate.

[12] We are ignoring here the role of economic growth, since a higher g^* would also help stabilizing $\tilde{\imath}_t$ below $\bar{\imath}^*$.

[13] However, I have only been able to see this thanks to the suggestion of an anonymous referee.

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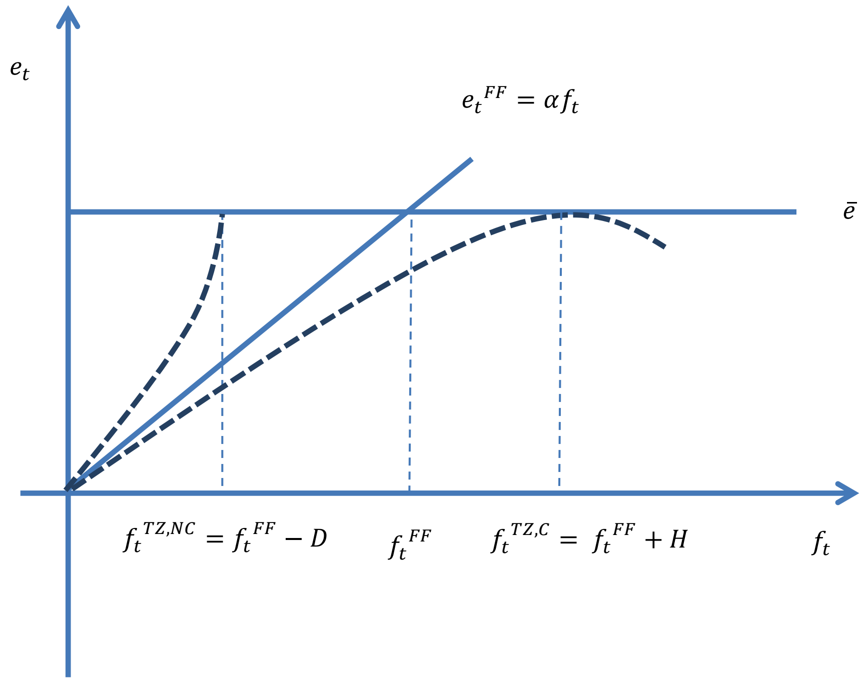

Figure 1: The 'honeymoon' (H) and 'divorce' (D) effect respectively with the adoption of a credible (TZ, C) and not credible (TZ, NC) exchange rate target zone. The upper margin of the TZ is identified with $\bar{e}$. f_t^{FF} indicates the level of economic fundamentals at which e_t would hit $\bar{e}$ in a free float regime.

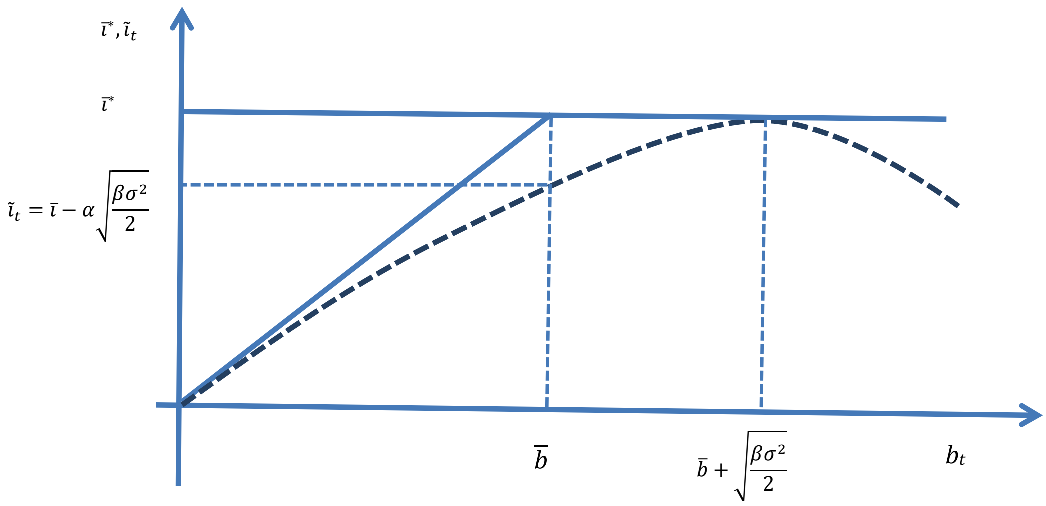

Figure 2: The 'smooth pasting' condition.

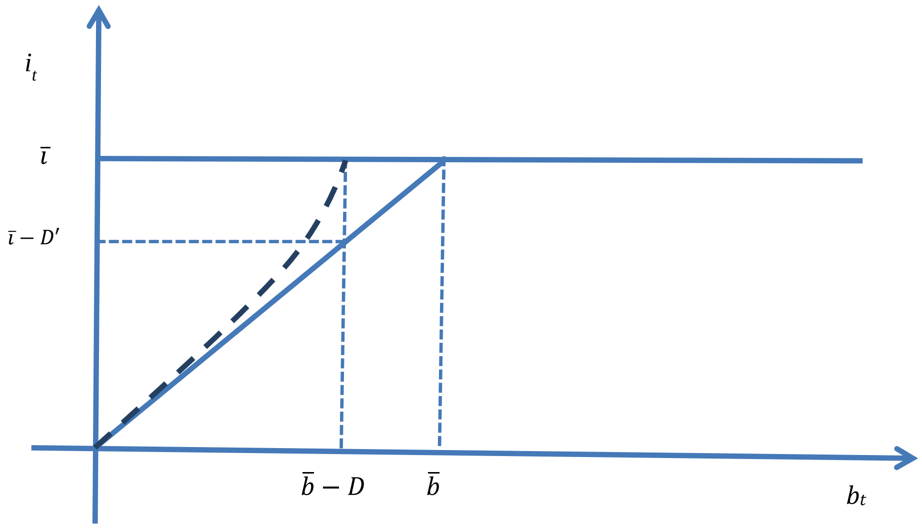

Figure 3: The 'divorce' effect (*D*) in the case of a non-feasible fiscal consolidation