Population and Geography Do Matter for Sustainable Development*

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Abstract

We analyze the spatio-temporal dynamics of a simple model of macroeconomic geography in which demography and pollution dynamics mutually affect each other. Pollution by reducing the carrying capacity of the natural environment, which determines the maximum amount of people a given location can effectively bear, crucially affects labor force dynamics which in turn alter the amount of resources available for abatement activities aiming to reduce pollution. Such mutual links determine the eventual sustainability of the development process in different locations and economies, and spatial interactions further complicate the picture. We show that neglecting the existence of mutual feedback between population and pollution leads to misleading conclusions about the eventual sustainability of a specific location. We also show that even neglecting the existence of spatial externalities can lead to misleading conclusions about the sustainability of different regions in the spatial economy. This suggests thus that both the nature of the population and pollution relationship and geographical factors may play a critical role in the process of sustainable development.

Keywords: Geography, Population, Pollution, Spatial Dynamics, Sustainable Development
JEL Classification: C60, J10, O40, Q50

1 Introduction

The topic of sustainability has become very popular over the last decades, and both academics and policymakers are nowadays more interested than ever in understanding how to address the economy along a sustainable development path (Solow, 1974; Stokey, 1998; UNEP, 2012). Sustainability has been traditionally defined as the ability to satisfy *"the needs of the present without compromising the ability of future generations to meet their own needs"* (WCED, 1987). Thus, such a notion requires to take into account three different critical dimensions[1] of the sustainable matter: an economic, an environmental and a demographic dimension. Clearly, economy and environment are strictly interconnected, since economic activities are the

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[1]The UN (2005) recognize that sustainable development needs to be based on three interdependent and mutually reinforcing "pillars": an economic, and environmental and a social pillar. The social pillar is the least defined and understood dimension of sustainability; in our discussion and following analysis this is interpreted in terms of sustainability of the human population. We thus focus on demographic growth and its relation with the other two (economic and environmental) pillars in order to shed some light on the mutual feedback among such three dimensions of sustainable development.

main source of environmental degradation and environmental quality may feed back on economic capabilities. But also demography plays a crucial role in the sustainable problem: population growth demands rising economic activities and exacerbates environmental problems, and at the same time it is affected by both economic and environmental outcomes, through food production and health conditions, respectively. Despite these clear links among such three dimensions of sustainability, most of the efforts in literature have been devoted to the analysis of the relation between the economic and environmental dimensions (see Xepapadeas, 2005, for a survey of the growth and environment relationship), often completely neglecting population growth and its mutual feedbacks with both economy and environment, even if a growing consensus has recently emerged on the fact that population clearly matters for sustainable development[2] (UNFPA, 2012). Thus, the goal of this paper is to shed some light on the role of population change in the sustainable discourse, by developing a simple model in which population and pollution dynamics mutually interact. Specifically, our contribution to extant literature is twofold: firstly, we explicitly allow for some mutual dynamic interactions between the population size (and thus the labor force) and the pollution stock; secondly, we also allow for the demographic and environment relation to be not only time but also space dependent.

The existence of some important relation between the human population and its hosting natural environment has been known for centuries. Malthus (1798) is the first to conjecture that fast population growth is not sustainable, since exercising excessive pressure on the availability of food supplies. The rise of technological progress in agricultural production has weakened the Malthusian argument, which over time, following Kahn et al. (1976), has been replaced by the cornucopian view that population growth by promoting technical change may even tend to increase food availability (see Panayotou, 2000, for a survey of alternative theories on the population and environment relation). As a result the concerns for the role of human population in the (sustainable) development process have vanished and very few attempts have been made to characterize the extent to which human population and the natural environment are effectively compatible from a sustainability point of view. By focusing only on the population and environment relation, some works show that the existence of sustainable paths where humans and the natural environment coexist does not have to be taken for granted at all (Nerlove, 1991; Marsiglio, 2011), while others show that sustainability can be achieved if humans adapt their behavior according to the state of the environment (Berck et al., 2012). By taking into account also economic factors, some recent works have shown the importance of demographic policies (de la Croix and Gosseries, 2012; Marsiglio, 2017) and technological progress (Boucekkine et al., 2014) in order to favor sustainable development. Our paper contributes to this literature by considering how spatial spillovers impact on all the three dimensions of the sustainable matter.

The importance of spatial interactions for economic activities and outcomes has been recognized only recently by the economic geography literature. Krugman's (1991) seminal work firstly identifies spatial externalities as the main source of regional differentiation, explaining why some regions might end up becoming an industrialized core and others an agricultural periphery; such a core-periphery pattern by being self-reinforcing may give rise to path dependent outcomes strongly affecting the development process of different regions for long periods of time. Only more recently an economic geography approach has been introduced in other setups, giving rise to spatial macroeconomics and spatial ecological economics models. A growing number of studies has analyzed the effects of spatial spillovers on economic growth (Brito, 2004; Camacho and Zou, 2004; Camacho et al., 2008; Boucekkine et al., 2009; 2013a, 2013b) and natural resource management (Brock and Xepapadeas, 2008, 2010; Brock et al., 2014b; Camacho and Pérez–Barahona, 2015). Only recently some attempts have been made to characterize the role of such spatial effects on the process of sustainable development, by analyzing the mutual economy and environment relation (Brock et al., 2014a;

[2]A recent report of the United Nations Population Fund stresses the importance of demographic challenges for sustainability issues and calls for more efforts in order to better understand its implications for development policies. *"In sum, while changes in population size have important implications for sustainable development, other population dynamics, which have received even less attention, have too. How many people will be added to the world matters; it also matters where they will live, how old they are and what they do, produce and consume"* (UNFPA, 2012).

La Torre et al., 2015). Our paper contributes to this literature by focusing on the role of demography and its spatial patterns in the process of sustainable development.

This work tries thus to combine together two different streams of literature: the sustainability and the macroeconomic geography literatures. From the latter we borrow the framework for our analysis: we consider, indeed, a spatial macroeconomic model with environmental and demographic interactions; the setup most similar to ours is La Torre et al.'s (2015), but differently from them we allow for population growth and labor migration to play a specific role in the dynamic evolution of the spatial economy. From the former, instead, we borrow the interest in analyzing whether and under which specific contexts sustainable development paths may exist; to the best of our knowledge, the presence of a spatial dimension in our analysis makes our paper not comparable to any other existing work, since spatio-temporal dynamics have never been discussed from a similar sustainability point of view thus far. Our main results allow to stress that: (i) by taking into account the population and pollution relationship, it is particularly difficult to identify sustainable development paths along which human population and the natural environment mutually coexist, and (ii) by neglecting the existence of spatial externalities, the conclusions about the sustainability of the development path followed by certain regions analyzed in complete isolation from neighboring regions may be completely misleading. Indeed, related to the first type of result, we can show that neglecting the mutual feedback between demographic and environmental outcomes precludes us from identifying the existence of a sustainable problem; this is due to the fact that such a view on the population and environment relation would lead us to conclude that sustainability would be naturally achieved and thus there is no need at all to worry about sustainable development. Related to the second result, instead, it may happen that some regions, which in absence of spatial interactions are meant to develop along a sustainable path, because of spatial externalities will be brought in an unsustainable status characterized by unlimited pollution growth and extinction of human population because of the interaction with neighboring regions developing along a unsustainable trajectory. These results suggest that despite the little attention received in literature thus far the relation between population and environment, along with its geographical characteristics, are essential elements to take into account in order to plan sustainable development and design appropriate policies.

The paper proceeds as follows. Section 2 introduces our baseline spatio-temporal dynamic model, which for the sake of simplicity abstracts from capital accumulation, and thus it is summarized by two partial differential equations describing the evolution of human population and pollution, respectively. We explicitly analyze how the demographic and environmental outcomes change both over time and across space in the case in which spatial externalities are either present or absent. Specifically, in section 3 we focus on the role of population growth (with no spatial externalities), showing that abstracting from the mutual population and pollution feedback leads to conclusions completely different from those obtained by taking into account such a mutual relation. In section 4 we focus on the role of spatial extenalities, showing that neglecting the existence of spatial interactions may distort completely our conclusions, which may end up being more optimistic in an a-spatial than in a spatial framework. Section 5 relaxes our assumption of no capital accumulation and thus extends the baseline model by introducing a third partial differential equation describing the evolution of capital. We show that the introduction of capital accumulation only complicates the analysis but does not substantially change our qualitative results. As usual section 6 concludes and presents directions for future research. Mathematical technicalities are discussed in the appendices A and B.

2 The Baseline Model

We consider a simple model of macroeconomic geography in which for the sake of simplicity agents consume all their disposable income (we shall remove this assumption in section 5) and inelastically supply labor. Since there is no unemployment, the population size and the labor force perfectly coincide, thus the terms population and labor force (or simply labor) will be used interchangeably in what follows. We assume that economic production activities generate pollution and abatement activities financed by income taxation reduce the amount of pollution in the economy, such that production has a net beneficial effect on pollution (via abatement). Pollution, by affecting the carrying capacity of the natural environment in which human population lives, determines the evolution of the labor force, which is an essential input in the production of final output. Differently from the extant spatial macroeconomic literature, population grows over time and its dynamics is therefore affected by pollution; at the same time pollution evolves over time and is affected by the population size; thus human population and the natural environment strongly affect each other through the pollution channel. We assume a continuous space structure to represent that the spatial economy develops along a linear city (see Hotelling, 1929), where the population (labor force) is mobile across different locations and pollution, even if generated in a specific location, diffuses across the whole economy as in La Torre et al. (2015). We denote with $L(x,t)$ and $P(x,t)$ respectively the population size and pollution stock in the position x at date t, in a compact interval $[x_a, x_b] \subset \mathbb{R}$, and $t \geq 0$. We also assume that the initial population and pollution distribution, $L(x,0)$ and $P(x,0)$, are known and there is no migration or pollution flow through the borders of $[x_a, x_b]$, namely the directional derivative is null, $\frac{\partial L(x,t)}{\partial x} = \frac{\partial P(x,t)}{\partial x} = 0$, at $x = x_a$ and $x = x_b$.

Output is produced according to a Cobb-Douglas production function employing a constant amount of capital, K, and labor as $Y(x,t) = AK^{\alpha}L(x,t)^{1-\alpha}$, where $A > 0$ is a scale parameter measuring the total factor productivity and $0 < \alpha < 1$ represents the capital share of income; without loss of generality the capital stock is normalized to unity, $K \equiv 1$. Pollution increases with the emissions generated by economic activity and decreases according to natural factors; specifically, economic output generates emissions which increase the stock of pollution at a rate $\beta > 0$, while the natural decay rate of pollution is $\delta_P > 0$. We assume that such a difference is positive, $\beta > \delta_P$, such that because of anthropogenic activities pollution tends to accumulate over time (La Torre et al., 2017). However, the local government collects taxes proportional to income, at the rate $0 < \tau < 1$, in order to finance abatement activities; the tax revenue, $R(x,t) = \tau Y(x,t)$, is entirely devoted to reduce emissions, and in particular the rate of pollutant emissions is lowered by a decreasing and convex abatement function $M[R(x,t)]$ with $M'(\cdot) < 0$ and $M''(\cdot) > 0$. The abatement function is assumed for the sake of simplicity to take the following form $M[R(x,t)] = \frac{1}{1+R(x,t)}$, implying that with no economic activities, $Y(x,t) = 0$, since there are no resources to finance abatement, pollution will tend to grow at its exogenous and constant rate, $\beta - \delta_P$. The local population, entirely consumes its disposable income, $C(x,t) = (1-\tau)Y(x,t)$, and grows over time. We assume that it evolves according to a logistic equation, where $\Omega > 0$ represents the carrying capacity of the natural environment, which is affected by pollution flows, through a decreasing and convex feedback function $F[P(x,t)]$ with $F'(\cdot) < 0$ and $F''(\cdot) > 0$. This captures the fact that pollution, by putting under stress the natural ecosystem, acts as an hindrance to the development of human population, which in turn is the primary source of pollution reduction. Such a feedback function is assumed to take a form similar to the abatement function, $F(x,t) = \frac{1}{1+\theta P(x,t)}$ with $\theta \geq 0$ being a scale parameter.

The spatio-temporal dynamic model can thus be summarized by the following system of two partial differential equations:

$$\frac{\partial L(x,t)}{\partial t} = d_L \frac{\partial^2 L(x,t)}{\partial x^2} + L(x,t)\left[\frac{\Omega}{1+\theta P(x,t)} - L(x,t)\right] \tag{1}$$

$$\frac{\partial P(x,t)}{\partial t} = d_P \frac{\partial^2 P(x,t)}{\partial x^2} + P(x,t)\left[\frac{\beta}{1+\tau A L(x,t)^{1-\alpha}} - \delta_P\right] \tag{2}$$

Equation (2) describes the evolution of pollution over time and across space. Pollution accumulation is driven by the characteristics of the environment, which suggest that the self-cleaning capacity of the natural environment, δ_P, is not enough to offset the human-induced pollution growth rate, β (La Torre et al., 2017). However abatement activities allow to lower the pollutant emissions, which net of abatement activities are equal to $\frac{\beta}{1+\tau A L^{1-\alpha}}$; if abatement is effective, the net (of abatement) growth of pollution is negative, and the pollution stock will decrease over time. The spatial externality is taken into account by the diffusion term: the intensity of the diffusion process is measured by the coefficient of diffusion $d_P \geq 0$, measuring the extent to which pollution no matter where it is originally generated spreads across the whole spatial economy (La Torre et al., 2015). Equation (1) describes the evolution of the human population over time and across space. In absence of pollution, the population size would grow according to a logistic law with constant carrying capacity, Ω (Verhulst, 1838). By taking into account the negative pollution externality, the demographic law of motion is still logistic, but the maximum value of the population size that the natural environment can bear is represented by the term $\frac{\Omega}{1+\theta P}$: pollution thus decreases the carrying capacity. As for the case of pollution, the spatial externality is represented by the diffusion term, where $d_L \geq 0$, represents the diffusion coefficient, measuring the extent to which population tends to migrate across different locations in the spatial economy.[3] Equations (1) and (2) allow us to analyze how human population and pollution are mutually related in a spatial context, formally taking into account two of the three dimensions (environment and population) of the sustainable problem. We shall introduce the third dimension (economy) in section 5, where we will consider the implications of capital accumulation on our setup.

Note that our framework is substantially different from extant works in the macroeconomic geography literature along two main directions. (i) Most of the papers focus on the spatial spillovers arising from capital accumulation (Brito, 2004; Camacho and Zou, 2004; Camacho et al., 2008; Boucekkine et al., 2009; 2013a, 2013b; La Torre et al., 2015), while we analyze the spatial spillovers associated with pollution and population growth. While no other work has accounted for the spatial implications of population growth, the effects of pollution diffusion are discussed only by La Torre et al. (2015), who analyze how pollution and capital accumulation are mutually related. In their setup static and dynamic spatial externalities are both essential determinants of the economic and environmental performance of specific regions; even if not explicitly mentioning sustainability to some extent their work allows to identify eventual sustainable paths and how the development pattern may vary from region to region within the spatial economy. Differently from them, we wish to understand the nature of the feedback effects between population growth and pollution, thus taking into account capital accumulation is not our main goal. However, as we shall show in section 5, the introduction of capital in our setup will not change our main conclusions, thus it seems convenient to present the model first in its simplest possible form.[4] (ii) Moreover, most of the papers in this literature analyze a framework with optimizing agents, which however raises issues related to the formulation of the associated optimal control problem (see Boucekkine et al., 2013a, 2013b, for a discussion of how a typical macroeconomic model extended to a spatial setup gives rise to an ill-posed problem). Even abstracting from agents' optimization, our model is able to capture in a neat and interesting way the main channels through which population and geography matter in the sustainability debate, thus it seems convenient to keep the analysis as simple as possible.

Our framework is also substantially different from extant works in the sustainability literature. The papers most closely related to ours are Berck et al.'s (2012) and Boucekkine et al.'s (2014) who characterize population dynamics through a logistic differential equation to account for feedback effects between human population and the environment. While in both these studies the feedback is unidirectional (from environment to population in Berck et al., 2012; and from population to environment in Boucekkine et al., 2014), in our model population and environment mutually affect each other. Moreover, none of these works takes into account the effects of spatial spillovers which is instead the main focus of our analysis.

[3]Note that in our setting the only determinant of the interlocation spatial dynamics of pollution and population is the interlocation concentration differential of pollution and population, respectively. Specifically, pollution (population) tends to spread out from locations with high concentrations to locations with low concentrations of pollution (population).

[4]It is also possible to show that from a qualitative point of view our results are robust to alternative formulations. Other than the introduction of capital accumulation, also allowing the consumption good to be tradable across location or abatement activities to be financed via lump sum (rather than proportional) taxation or else abatement to be independent of output would not substantially modify our conclusions.

3 No Diffusion: the Role of Population Dynamics

In order to look at the mutual interactions between population and pollution in the simplest possible way, we first analyze the behavior of the above system without diffusion, but preserving the spatial structure. This allows us to comment in a simplified setup on the role that population might play in the sustainable debate, but also to compare the outcome with what arises in the diffusion case which we will analyze later in section 4. As we shall see, not considering the interaction between population dynamics and pollution, as it has generally been done thus far in the sustainability literature, leads to substantially misleading conclusions about the eventual sustainability of the development process of specific locations or economies. In the case with no diffusion, that is $d_P = d_L = 0$, the system of partial differential equations (1) and (2) boils down to the following parametric system of ordinary differential equations:

$$\frac{dL(t)}{dt} = L(t)\left[\frac{\Omega}{1+\theta P(t)} - L(t)\right] \tag{3}$$

$$\frac{dP(t)}{dt} = P(t)\left[\frac{\beta}{1+\tau A L(t)^{1-\alpha}} - \delta_P\right] \tag{4}$$

The system (3) - (4) is characterized by several parameters, each of which could be space dependant, but for the sake of simplicity, and in order to emphasize the implications of spatial externalities, we assume that they are all spatially homogeneous. Any position x needs to be interpreted as a specific location, thus a set of adjacent locations can be interpreted as a region in the spatial economy. Note first that the system (3) - (4) is actually a system of ordinary differential equations, since each point in the spatial domain has its own time dynamics, but there is no interaction between adjacent locations. Next proposition offers a concise description of the properties of this system, stating that $\forall x \in [x_a, x_b]$ the system (3) - (4) has a trivial unstable equilibrium along with two non-trivial and somehow stable equilibria.

Proposition 1. *Suppose that $\beta - \delta_P < \tau A \delta_P \Omega^{1-\alpha}$. Then the system (3) - (4) has three equilibria, $E_1 = (0,0)$, $E_2 = (\Omega, 0)$ and $E_3 = (\overline{L}, \overline{P})$, where:*

$$\overline{L} = \left(\frac{\beta - \delta_P}{\tau A \delta_P}\right)^{\frac{1}{1-\alpha}} \tag{5}$$

$$\overline{P} = \frac{1}{\theta}\left[\frac{\Omega}{\left(\frac{\beta-\delta_P}{\tau A \delta_P}\right)^{\frac{1}{1-\alpha}}} - 1\right] \tag{6}$$

The origin $E_1 = (0,0)$ is unstable, $E_2 = (\Omega, 0)$ is asymptotically stable, while $E_3 = (\overline{L}, \overline{P})$ is saddle-point stable.

Proof. See Appendix A. ■

Proposition 1 fully characterizes the possible long run outcomes of our model economy. The technical condition in Proposition 1 requires that abatement activities are effective enough in their goal of reducing the growth rate of pollution; indeed, what such a condition means is that the otherwise positive net (of natural decay) growth rate of pollution, $\beta - \delta_P > 0$, thanks to environmental protection efforts is lowered enough to become negative, $\frac{\beta}{1+\tau A P \Omega^{1-\alpha}} - \delta_P < 0$. If this condition was not met, all environmental efforts would be pointless since pollution would always be meant to permanently grow, no matter the amount of resources devoted to abatement; clearly abatement activities in such a framework would make no sense. Provided that such a technical condition is met, apart from the trivial and not interesting equilibrium, there exist two different equilibria: E_2 is an "idyllic equilibrium" in which pollution is completely null and the population stabilizes at the level implied by the environmental carrying capacity, while E_3 is a more "realistic equilibrium" in which both population and pollution stabilize at strictly positive levels. Even if the idyllic equilibrium is asymptotically stable, the possible outcomes in our model economy are all but obvious and sustainable development cannot be taken for granted. Indeed, a crucial role in this context is played by the other non-trivial (realistic) equilibrium which to some extent determines whether the economy will tend to converge to the idyllic outcome or not.

In order to look at why this might be the case, let us analyze the demographic and pollution dynamics through the phase diagram in Figure 1. In the left panel, the blue stars identify the three equilibria, while the red curves show the joint evolution of population and pollution starting from arbitrary initial conditions (L_0, P_0). The black curve represents instead the unique trajectory allowing pollution and population to converge to the equilibrium $E_3 = (\overline{L}, \overline{P})$, that is it represents its unidimensional stable manifold. It is straightforward to notice that the stable manifold of the saddle behaves as a separatrix between the basin of attraction of the idyllic equilibrium E_2, and the region of the plane where the diverging trajectories are doomed to reach $(0, +\infty)$ eventually. This means that according to the value of the initial conditions (L_0, P_0) two different outcomes are possible[5]: as long as (L_0, P_0) lies below the stable manifold of the saddle then the idyllic equilibrium will be achieved over the long run; however, as (L_0, P_0) lies above this curve the idyllic equilibrium will no longer be reached, and in such a framework pollution will continue to grow permanently and, because of its effect on the carrying capacity, the population will asymptotically disappear. Clearly, while the former case represents an outcome which can be deemed as sustainable (since human population and the environment coexist), the latter needs to be deemed as unsustainable (the environment gets infinitely polluted and human population vanishes). Therefore, despite the existence of an asymptotically stable idyllic equilibrium the economy does not necessarily need to achieve a sustainable outcome, and whether this happens or not depends on the initial level of both human population and pollution. In Figure 1, the right panel represents both the stable and unstable manifolds of the saddle, and as it should be clear from the left panel, the unstable manifold gives rise to an heteroclinic orbit connecting the saddle itself to the idyllic equilibrium.

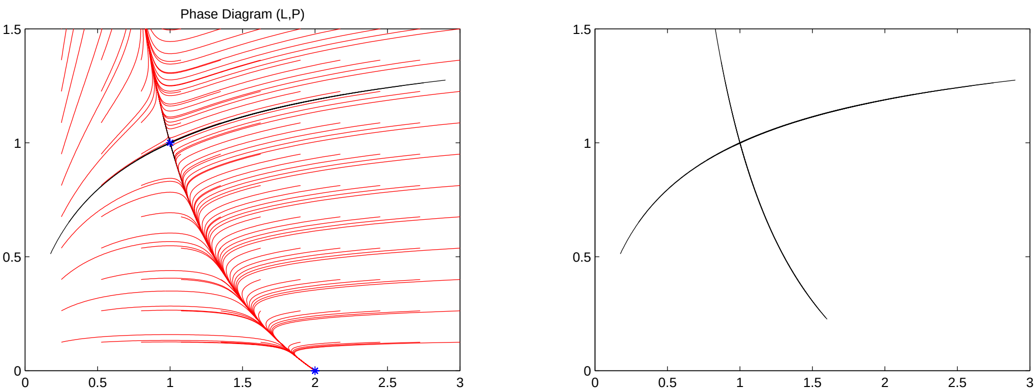

Figure 1: Phase diagram of population and pollution in the (L, P) plane (right panel), and stable and unstable manifolds of the saddle-point stable equilibrium E_3 (right panel).

From the above discussion it is clear that analyzing the size of the basin of attraction of the idyllic equilibrium is essential in order to understand whether the economy is likely to develop along a sustainable or unsustainable path. The phase diagram above is shown for a given set of parameter values satisfying the technical condition in Proposition 1. However, whenever $\beta - \delta_P$ changes, getting closer to zero or closer to its upper bound $\tau A \delta_P \Omega^{1-\alpha}$, the size of the basin of attraction of the idyllic equilibrium drastically changes.

[5]Note that a third outcome is possible: (L_0, P_0) lies exactly on the stable manifold and thus over the long run the more realistic equilibrium E_3 will be reached; also such a possibility gives ruse to a sustainable outcome. Since there exists a unique path allowing for convergence to such an equilibrium, we restrict our analysis to the discussion of whether the economy will tend to converge to an idyllic or a catastrophic status.

In the extreme case in which $\beta - \delta_P$ approaches zero, such a basin of attraction gets particularly large and eventually covers the entire first orthant. In the other extreme case in which $\beta - \delta_P$ approaches its upper bound, the basin of attraction shrinks and eventually becomes negligible. This result is illustrated in Figure 2, where the value of $\beta - \delta_P$ gradually increases from the top-left to the bottom-right. What this result means is simply that the possibilities to achieve sustainable development crucially depend both on natural features to a large extent out of human control (represented by the parameters β, δ_P, Ω), and on economic elements which to some extent can be affected by policymakers (given by τ).

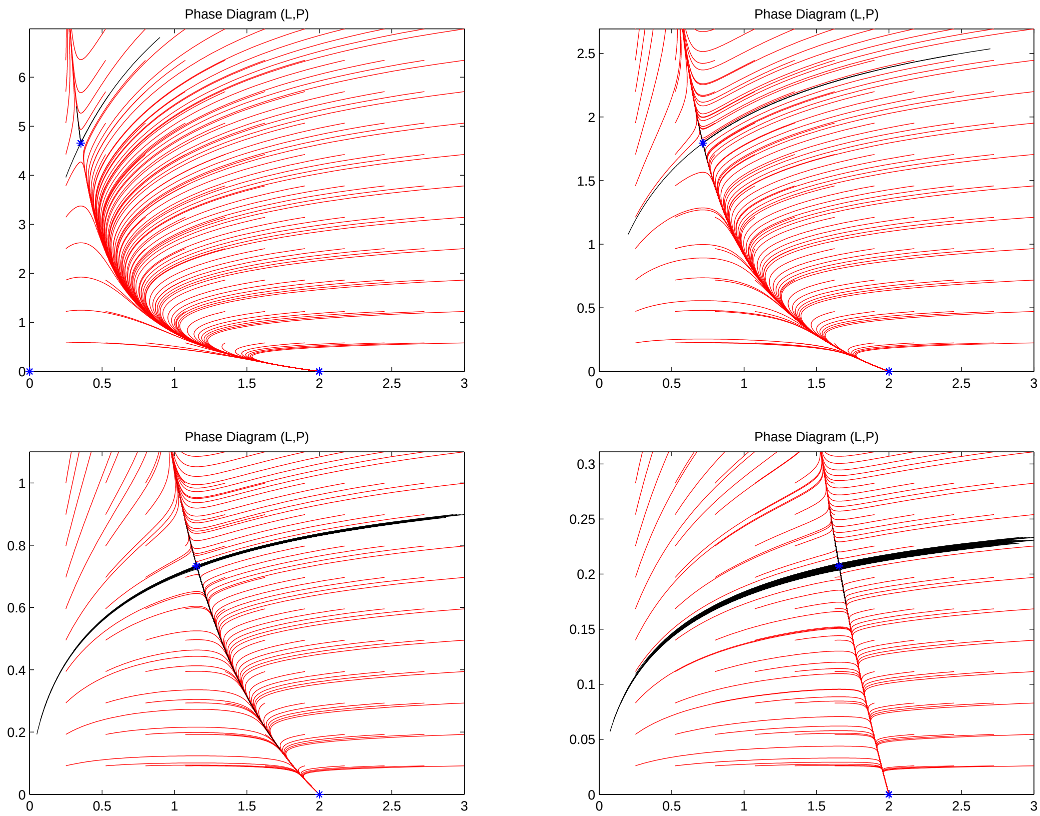

Figure 2: Phase diagram of population and pollution in the (L, P) plane with different value of the parameter β, gradually rising from top left to bottom right. Note the contraction of the basin of attraction of E_2, from the different scales of the vertical axis in each panel.

After showing that even if an idyllic asymptotically stable equilibrium exists it is not possible to take sustainable development for granted, we now compare this outcome with what we would obtain in absence of demographic and environmental interactions, as it has generally been done in literature. This can be seen by setting $\theta = 0$, which means that pollution does not feed back to population dynamics, which thus is completely exogenous and independent of pollution dynamics. In such a case the two differential equations in (3) - (4) are coupled only in one direction: from population to pollution, but not viceversa. In this framework, it is straightforward to note that the results substantially change, since the economy admits a unique non-trivial equilibrium.

Proposition 2. *Suppose that $\theta = 0$ and $\beta - \delta_P < \tau A \delta_P \Omega^{1-\alpha}$. Then the system (3) - (4) has two equilibria, $E_1 = (0, 0)$ and $E_2 = (\Omega, 0)$. The origin $E_1 = (0, 0)$ is unstable, and $E_2 = (\Omega, 0)$ is asymptotically stable.*

Proposition 2 shows that in the $\theta = 0$ case, there is an important difference with respect to what discussed before, and this is due to the fact that the saddle (the realistic equilibrium) completely vanishes. Thus, apart from the unstable trivial equilibrium, the economy admits a unique non-trivial equilibrium, represented by the idyllic equilibrium with no pollution and population size determined by the environmental carrying capacity. In such a framework, there is a unique possible outcome for our model economy, that is reaching the idyllic outcome over the long run. In such a case, there is no need to worry about sustainable outcomes, since the economy will naturally develop along a sustainable path. Such a result is substantially different from what discussed before, and clearly illustrates the limits of neglecting the existence of mutual feedbacks between population and pollution: we will be brought to conclude that to some extent sustainability can be taken for granted. Note that the result is consistent with our above discussion about the phase diagram in Figure 1. Indeed, there is no topological variation of the phase space when θ changes, but what simply happens is that there is a drastic expansion of the basin of attraction of E_2 as θ falls. As θ tends to zero, the stable manifold of the saddle E_3 shifts infinitely upward such that the basin of attraction of the stable equilibrium E_2 includes the entire first orthant; this can be intuitively seen from (6) which shows that $\overline{P}$ tends to infinity and thus the basin of attraction of E_2 becomes infinitely large. This means that, if initial conditions are strictly positive, no matter their values, then the population converges to Ω while pollution converges to zero (provided that the technical condition in Proposition 2 is met).

By comparing Proposition 1 and Proposition 2, it is clear the extent to which taking or not taking into account the mutual interactions between population change and pollution modifies the conclusions about the eventual process of sustainable development in specific locations or economies. When such interactions are considered, the economy may either converge to the idyllic equilibrium with no pollution and a positive population level, or alternatively diverge to a situation in which the pollution stock dramatically increases bringing population to disappear asymptotically. When the population and pollution interactions are not taken into account the economy will automatically converge to the idyllic equilibrium with no pollution and a positive population level. Clearly, while in the former case sustainability cannot be taken for granted, in the latter it can; this suggests that the current debates about sustainable development and the associated policy recommendations are often misleading since lacking a global view on the different dimensions of the sustainable problem.

4 Diffusion: the Role of Geographical Factors

We now turn to the analysis of the full model in which diffusion and thus spatial externalities are explicitly taken into account. In particular, we wish to understand whether the presence of such spatial interactions can alter our previous conclusions about the overall sustainability of the spatial economy. As we shall see the introduction of spatial externalities, conveyed by diffusion effects, alter not only the short run dynamics but also the long run steady states of pollution and population, meaning that not considering any spatial implication might lead to misleading conclusions about the eventual sustainability of the development process of different locations or regions, and even of the entire spatial economy.

Since we now allow for diffusion, the system (1) - (2) is a system of two partial differential equations (PDEs). It is well known that analyzing PDEs in order to obtain analytical solutions is very complicated but in this case, by borrowing from the mathematics literature, we can derive some interesting analytical results about the long run behavior of population and pollution. Let us first focus on the case in which pollution does not feed back on population, that is $\theta = 0$; in such a case, equation (1) reduces to the following expression:

$$\frac{\partial L(x,t)}{\partial t} = d_L \frac{\partial^2 L(x,t)}{\partial x^2} + L(x,t)\left[\Omega - L(x,t)\right], \tag{7}$$

which gives rise to a PDE widely discussed in the mathematics literature, known as the “Fisher equation”. The behavior of this equation is thus known and this allows us to characterize the long run behavior of our system, not only when $\theta = 0$ but also when $\theta > 0$. As discussed in appendix B, the above equation admits two equilibria $\overline{L}(x) = 0$ and $\overline{L}(x) = \Omega$, with the former being unstable while the latter being asymptotically stable, meaning that in absence of pollution feedback, in the long run human population will naturally achieve its carrying capacity. When $\theta > 0$, by applying a classical comparison theorem for parabolic PDEs (see Friedman, 2008, as reported in Proposition 7) it is possible to show that $L(x,t)$ is bounded from above by $\overline{L}(x)$ for all x and t, meaning that in presence of pollution feedback, at any moment in time and in any location across space human population is bounded from above from the population dynamics that would result in absence of pollution feedbacks. Therefore, in the long run in every location the population size will be at most equal to its carrying capacity, and the extent to which human population will fall below its carrying capacity crucially depends on the pollution level. For what concerns equation (2), by applying again a comparison theorem we can show that $P(x,t)$ is bounded from above by the solution of the following PDE:

$$\frac{\partial P(x,t)}{\partial t} = d_P \frac{\partial^2 P(x,t)}{\partial x^2} + P(x,t)\left[\frac{\beta}{1+\tau A L_{inf}^{1-\alpha}} - \delta_P\right], \tag{8}$$

where $L_{inf} = \inf_{x,t} L(x,t)$, and whenever $\frac{\beta}{1+\tau A L_{inf}^{1-\alpha}} < \delta_P$ its solution $\overline{P}(x)$ converges to zero. Similarly, $P(x,t)$ is bounded from below by the solution of the following PDE:

$$\frac{\partial P(x,t)}{\partial t} = d_P \frac{\partial^2 P(x,t)}{\partial x^2} + P(x,t)\left[\frac{\beta}{1+\tau A L_{\text{sup}}^{1-\alpha}} - \delta_P\right], \tag{9}$$

where $L_{sup} = \sup_{x,t} L(x,t)$, and whenever $\frac{\beta}{1+\tau A L_{\text{sup}}^{1-\alpha}} > \delta_P$ its solution $\overline{P}(x)$ diverges to infinity. We can thus summarize these results in the following proposition.

Proposition 3. *As $t \to +\infty$ and for any $x \in [x_a, x_b]$, two cases are possible: (i) if $\frac{\beta}{1+\tau A L_{inf}^{1-\alpha}} < \delta_p$ then $\overline{P}(x) = 0$ and $\overline{L}(x) = \Omega$, while (ii) if $\frac{\beta}{1+\tau A L_{\text{sup}}^{1-\alpha}} > \delta_P$ then $\overline{P}(x) \to \infty$ and $\overline{L}(x) = 0$.*

Proposition 3 identifies some (sufficient) conditions allowing to characterize the two possible outcomes for our spatial economy, in which either (i) pollution converges to zero and human population to its carrying capacity, or (ii) pollution diverges to infinity and human population completely disappears. Clearly the former case represents a situation in which the economy develops along a sustainable path, while the latter an unsustainable situation leading to a catastrophic outcome. By recalling that we are focusing on a situation in which the growth rate of pollution is larger than its decay rate, that is $\beta > \delta_P$, the above proposition states that: (i) sustainability can be achieved whenever the minimal abatement within the spatial economy is effective enough in order to make the net growth rate of pollution become negative; similarly, (ii) an unsustainable outcome is achieved whenever the maximal abatement within the spatial economy is not effective enough. Note that the effectiveness of abatement crucially depends on the (minimum or maximum) population level, which by determining income ultimately determines the amount of resources available for environmental protection activities.

These results are consistent with those discussed in the previous section, confirming that according to whether the growth rate of pollution, net of its natural decay and abatement activities, is positive or negative, the stock of pollution can achieve an idyllic (zero pollution) or a catastrophic (extremely high pollution) situation; this thus determines whether the population size is able to stabilize at a level consistent with the environmental carrying capacity or tends to collapse. However, an important difference with respect to what discussed earlier applies. While in absence of spatial externalities, the long run outcome in each location and each region depends on the their specific initial level of population and pollution (see Proposition 1), by accounting for such spatial spillovers allows us to conclude that this is not actually true. Indeed, thanks to the role of diffusion which tends to smooth spatial differences out (Boucekkine et al., 2009; La Torre et al., 2015), the long run outcome in the whole economy is spatially homogeneous (i.e., either a sustainable or a catastrophic outcome everywhere within the entire spatial domain) meaning that every location and region will achieve the same outcome independently of their specific initial condition. Specifically, the effectiveness of abatement activities in every location (i.e., the maximum and the minimum effectiveness) determines which outcome the entire spatial economy will experience.[6] As we shall clarify through some numerical simulations in a while, this intuitively suggests that geographical factors need to be crucially taken into account in the sustainability debate.

In order to illustrate the implications of the results just discussed, we now proceed with some numerical simulations. The parameters' values have been set in order to satisfy the parameter restriction in Proposition 1 and also in order to make the figures as clear as possible; however, it is possible to show that even under different parametrizations our qualitative results will not differ from the two scenarios (consistent with those pointed out in Proposition 3) that we shall discuss. The parameter values employed in our simulations are summarized in Table 1. Note that in our setting the initial conditions of population and pollution,

x_a	x_b	d_L	d_P	θ	β	Ω	τ	A	α	δ_P
-1	1	0.1	0.1	1	2	2	0.5	2	0.33	1

Table 1: Parameter values employed in our simulations.

namely $L(x, 0)$ and $P(x, 0)$, are the only source of spatial heterogeneity. Moreover, given that diffusion acts as a convergence mechanism (Boucekkine et al., 2009; La Torre et al., 2015) and there is no divergence mechanism to counteract on it (as for example in La Torre et al., 2015), the initial spatial heterogeneity will tend to be completely wiped out over time, meaning that intuitively the spatial economy will achieve either a sustainable or catastrophic outcome homogeneously in space (as discussed in Proposition 3). Note that in order to look at the implications of spatial externalities we need to compare the long run behavior of a two dimensional system, that is a point in $\mathbb{R}^2$, $\{L(+\infty), P(+\infty)\}$, with the long run behavior of an infinite dimensional system, that is a function in $\mathcal{C}^2$, $\{L(x, +\infty), P(x, +\infty)\}$. Indeed, what discussed in the previous section by abstracting from spatial interactions can be interpreted as simply the outcome in terms of population and pollution of one single location in the spatial economy (i.e., a point in $\mathbb{R}^2$). By taking into account spatial interactions, each single location is no longer independent from other locations, and thus its outcome in terms of population and pollution depends also on the outcome of others locations (i.e., a function in $\mathcal{C}^2$).

In order to illustrate all possible outcomes, we consider two alternative spatial configurations for the initial conditions of L and P. Without loss of generality we shall assume that the initial condition for pollution is spatially homogeneous while that for population is spatially heterogenous. Specifically, we shall assume that $P(x, 0) = 1$ with $L(x, 0) = 0.1 + \frac{1.8}{1+e^{-10(x)}}$ and $L(x, 0) = 0.1 + \frac{1.8}{1+e^{-10(x+0.5)}}$ in the first and second configuration, respectively; such specifications for the initial conditions allow us to obtain in the simplest possible way some smooth degree of heterogeneity within the spatial economy, but it is possible to show that considering other configurations will not modify our qualitative results. The rationale behind such two alternative configurations can be explained from Figure 3, which summarizes the model's outcome that we discussed before in the two dimensional framework. Remember that from what discussed in the previous section the pair of initial conditions for L and P determines whether the economy will converge to a sustainable equilibrium (initial conditions below the stable manifold of the saddle) or will diverge towards an unsustainable outcome (initial condition above the stable manifold). By keeping the initial condition of P exactly the same across the entire spatial domain, the initial condition for L is the only determinant of whether we lie below or above the stable manifold of the saddle at a given location x. In the figure our two spatial initial conditions configurations for L are represented by the blue curves while the red horizontal line represents the threshold value above/below which (given the initial value of P) we effectively are below/above the stable manifold of the saddle, such that in absence of spatial externalities we can expect

[6] It may be natural to think that the speed at which population diffuse across space depends on the pollution level in some specific location. However, it is possible to show that our results extend in a similar fashion to the case in which the diffusion coefficients d_L and d_P are functions of population and pollution, provided that such functions are bounded from above and below by positive constants.

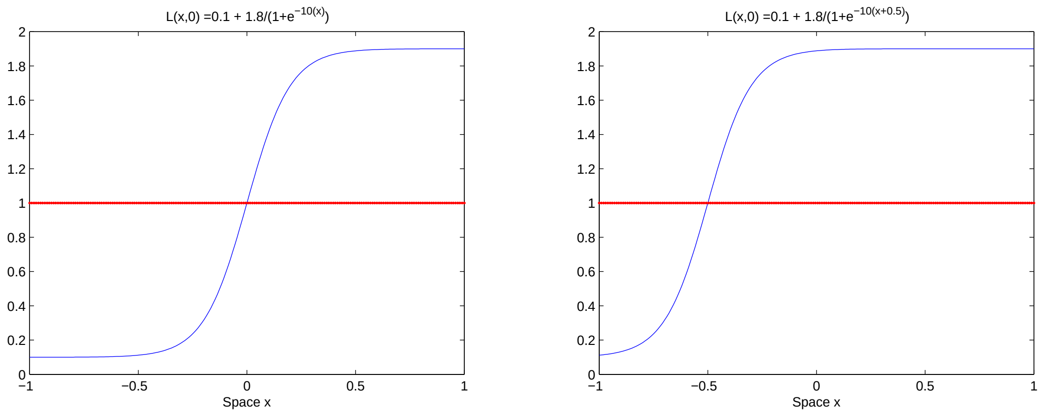

Figure 3: Initial population distributions for the two scenarios considered in the following numerical simulations: $L(x,0) = 0.1 + \frac{1.8}{1+e^{-10(x)}}$ (left), $L(x,0) = 0.1 + \frac{1.8}{1+e^{-10(x+0.5)}}$ (right)

convergence or divergence according to where the blue curve lies with respect to the red line. Note that given the parameter values in Table 1, without loss of generality, the threshold level for $L(x,0)$ is conveniently normalized to one. Both our initial conditions configurations allow the initial condition for L to span from points below to others above the red line, which separates the basin of attraction of the idyllic equilibrium E_2 from the basin of attraction of the catastrophic outcome ($L = 0, P = +\infty$); thus, such configurations of initial conditions imply that the spatial economy is divided in two main regions, one in which we can expect unsustainable outcomes (below the red line, in the left region) and one in which we can expect sustainable outcomes (above the red line, in the right region). Note that while in the converging region the case (i) of Proposition 3 applies, in the diverging region case (ii) applies, meaning that we cannot predict a priori which of the two possible scenarios (i.e., sustainable or catastrophic outcome) will arise within the entire spatial economy. Note also that the only difference in our two configurations is the relative size of the two regions which the spatial economy is divided into: while in the first configuration (left panel) the size of the diverging region is as large as that of the converging region, in the second (right panel) the size of the converging region is larger than that of the diverging region.

Figures 4 and 5 represent the results of our simulations for the first and the second initial conditions configuration, respectively. Their purpose is to visualize the consequences of the introduction of some spatial interaction between locations and regions on the dynamics and long run behavior of population and pollution with respect to the no-diffusion scenario. Both Figure 4 and Figure 5 are composed by four panels: on the left we present the evolution of $L(x,t)$ (top panel) and $P(x,t)$ (bottom panel) in the absence of spatial interactions, while on the right we introduce diffusion and show how the dynamics of $L(x,t)$ (top panel) and $P(x,t)$ (bottom panel) changes. Even if the parameters and initial conditions are the same in both left and right panels, we can observe that the spatio-temporal evolutions are dramatically different.

Recall that in the no-diffusion scenario the initial condition of L determines whether each location will be able to achieve a sustainable or unsustainable outcome, exactly as we have discussed before. In Figure 4 we can notice that the entire right region (the locations in which $x > 0$, such that $L(x,0) > 1$) converges to the idyllic equilibrium in which pollution is null and population is determined by the carrying capacity; the left region (the locations in which $x < 0$) instead entirely diverges towards an outcome characterized by infinite pollution and no population. Overall, assessing whether the spatial economy develops along a sustainable path is not possible, since while one region achieves a sustainable outcome the other clearly achieves an unsustainable outcome. With the introduction of spatial externalities, the results are substantially different. Indeed, because of the effects of diffusion, even the region which in absence of spatial interactions is able to follow a sustainable development pattern is eventually brought to diverge and achieve an unsustainable

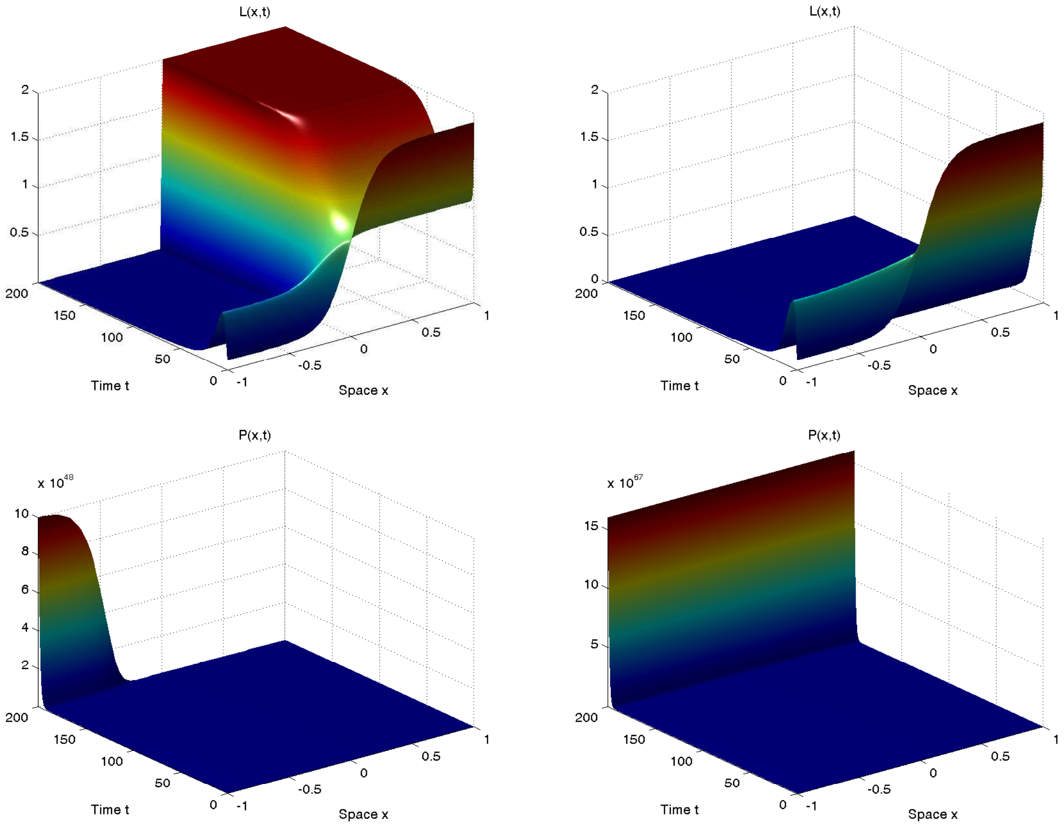

Figure 4: Spatio-temporal dynamics of population (left) and pollution (right) in the case without (left) and with (right) diffusion.

outcome characterized by an economic collapse (extinction of human population and infinitely high pollution level). In such a framework we can undoubtedly claim that the spatial economy follows overall an unsustainable trajectory which will lead the human population to completely disappear (because of the effect induced by pollution on the environmental carrying capacity) over the long run.

The results illustrated by Figure 5 are qualitatively the opposite of those just presented. Without diffusion, the entire right region (the locations in which $x > -0.5$, such that $L(x, 0) > 1$) converges to the idyllic equilibrium in which pollution is null and population is determined by the carrying capacity; the left region (the locations in which $x < -0.5$) instead entirely diverges towards an outcome characterized by infinite pollution and no population. Also in this case assessing the development process of the spatial economy from a sustainability point of view is not possible. The introduction of spatial externalities again completely changes the results, but differently from what discussed in the previous case, now both the regions achieve a sustainable outcome in which human population reaches its carrying capacity and pollution is completely null. In this case, when spatial interactions are taken into account, the spatial economy overall develops along a sustainable path with no pollution at all over the long run. Consistently with our theoretical conclusions, these two examples clearly show that the sustainable outcome of specific regions or economies may dramatically change when spatial heterogeneity and spatial interdependence are taken into account, and results which may seem obvious when we analyze locations or regions in complete isolation can be dramatically changed when we take into account their spatial interactions.

Our graphical analysis (which is however consistent with our theoretical predictions) allows us to derive some interesting conclusions. The size of the basin of attraction of the idyllic equilibrium dramatically changes with the introduction of diffusion, and this happens even in a context in which there is no other

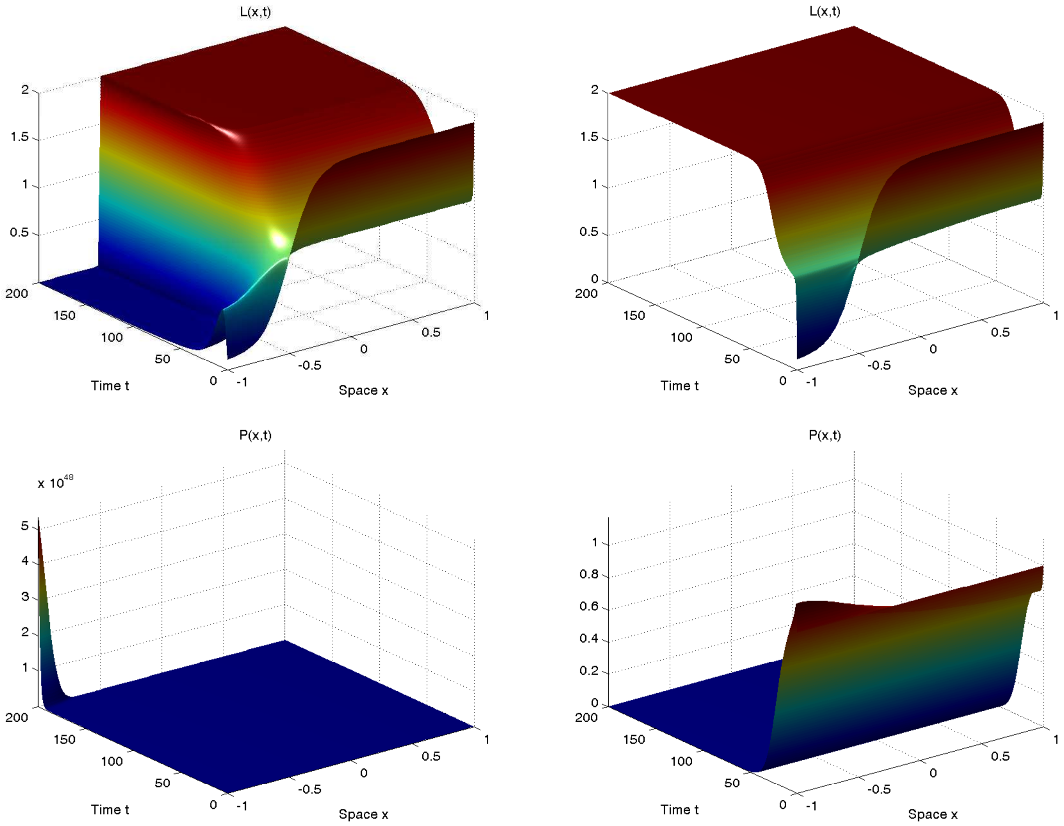

Figure 5: Spatio-temporal dynamics of population (left) and pollution (right) in the case without (left) and with (right) diffusion.

source of spatial heterogeneity apart from what induced by the spatial configuration of initial conditions. This means that even those regions which could be deemed as virtuous (non-virtuous) regions since developing along an apparent sustainable (unsustainable) path if analyzed in complete isolation, may be brought to an unsustainable (sustainable) outcome by non-virtuous (virtuous) regions simply because of spatial externalities. An important policy implication of this lies in the fact that for single regions or economies trying to plan sustainable development in isolation without coordinating efforts with others may be completely pointless, since pollution is a phenomenon with no geographical barriers and thus spatial externalities are likely to affect at least to some extent each single region and economy. This type of result confirms that the eventual spatial interdependence across regions and economies does matter in the sustainability debate. Since geographical characteristics have never been discussed in the sustainability literature thus far, our conclusions clearly call for the need of additional efforts to analyze the issue with more depth.

5 The Extended Model with Capital Accumulation

After presenting the model's results in the simplest possible form, by reducing the analysis to the mutual interactions between population and pollution, we now extend our baseline setup to introduce capital dynamics. This allows us to formally take into account all the three dimensions of the sustainable problem, such that economic, demographic and environmental phenomena are affecting one another. In order to introduce capital dynamics we follow the basic approach proposed firstly by Solow (1956), assuming thus that agents consume only an exogenously given fraction of their disposable income and the remaining portion is saved and used for capital investments. The spatio-temporal dynamics of the fully fledged model is thus described by the following system of three partial differential equations:

$$\frac{\partial K(x,t)}{\partial t} = d_K \frac{\partial^2 K(x,t)}{\partial x^2} + s(1-\tau)\frac{AK(x,t)^{\alpha}L(x,t)^{1-\alpha}}{1+\phi P(x,t)} - \delta_K K(x,t) \quad (10)$$

$$\frac{\partial L(x,t)}{\partial t} = d_L \frac{\partial^2 L(x,t)}{\partial x^2} + L(x,t)\left[\frac{\Omega}{1+\theta P(x,t)} - L(x,t)\right] \quad (11)$$

$$\frac{\partial P(x,t)}{\partial t} = d_P \frac{\partial^2 P(x,t)}{\partial x^2} + P(x,t)\left[\frac{\beta}{1+\tau AK(x,t)^{\alpha}L(x,t)^{1-\alpha}} - \delta_P\right] \quad (12)$$

where $0 < s < 1$ denotes the saving rate, $\delta_K > 0$ the depreciation rate of capital, and $d_K \geq 0$ the diffusion coefficient, measuring the extent to which capital tends to flow across different locations in the spatial economy. Note that the production function takes again a Cobb-Douglas form, but now capital is no longer assumed to be constant (and normalized to unity) but it evolves over time and across space. Moreover, the amount of resources available for capital investment is affected by pollution through a decreasing and convex damage function $D[P(x,t)]$ with $D'(\cdot) < 0$ and $D''(\cdot) > 0$, representing the impact of environmental outcomes on economic performance, as in La Torre et al. (2015); such a damage function, consistently with the abatement and feedback functions, is assumed to take the following form, $D(x,t) = \frac{1}{1+\phi P(x,t)}$ with $\phi \geq 0$ being a scale parameter.

It is possible to show that results very similar to those discussed in our benchmark model apply, as summarized in the following proposition (whose proof relies on the same argument based on a comparison approach discussed in the previous section).

Proposition 4. *As $t \to +\infty$ and for any $x \in [x_a, x_b]$, two cases are possible: (i) if $\frac{\beta}{1+\tau AK_{inf}^{\alpha}L_{inf}^{1-\alpha}} < \delta_P$ where $K_{inf} = \inf_{x,t} K(x,t)$, then $\overline{P}(x) = 0$, $\overline{L}(x) = \Omega$ and $\overline{K}(x) = (\frac{s(1-\tau)A\Omega^{1-\alpha}}{\delta_K})^{\frac{1}{1-\alpha}} > 0$; (ii) if $\frac{\beta}{1+\tau AK_{sup}^{\alpha}L_{sup^{1-\alpha}}} > \delta_P$ where $K_{sup} = \sup_{x,t} K(x,t)$, then $\overline{P}(x) \to \infty$, $\overline{L}(x) = 0$ and $\overline{K}(x) = 0$.*

Proposition 4 is qualitatively identical to Proposition 3, showing that according to the effectiveness of abatement activities (which now depends also on the maximal or minimal capital level) two alternative situations are possible: either (i) a sustainable outcome in which pollution is null, human population achieves its carrying capacity and capital a positive constant level, or (ii) an unsustainable outcome in which pollution diverges to infinity bringing both human population and capital to collapse to zero. Note that exactly as in the previous section, these two possible outcomes are homogeneous within the spatial domain, meaning that the entire spatial economy will achieve either a sustainable or unsustainable outcome.

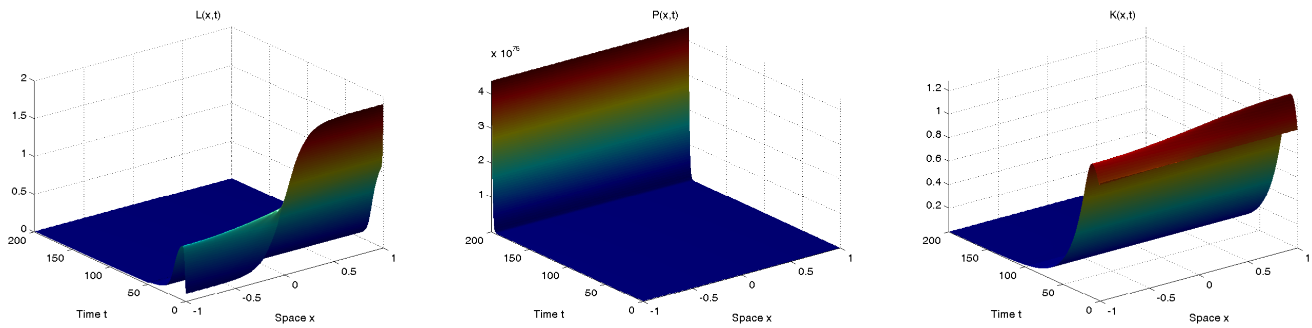

Figure 6: Spatio-temporal dynamics of population (left), pollution (center) and capital (right) in the case with diffusion.

In order to clarify our theoretical results above, we now briefly proceed with a numerical simulation of the system (10) - (11) - (12) in order to illustrate the two scenarios outlined in Proposition 4. We rely on the same parameter values employed in the previous section (see Table 1), apart from the new parameters which are set as follows: $\delta_K = 0.05$, $d_K = 0.1$, $\phi = 1$ and $s = 0.2$. In order to make the comparison between a framework with and without spatial interactions as clear as possible, we rely on the same initial conditions configurations as in Figure 4 and Figure 5, setting $K(x, 0) = P(x, 0) = 1$ for the sake of simplicity. Our simulation results with the two alternative initial conditions configurations are shown in Figure 6 and Figure 7, respectively, which clearly show that the introduction of capital does not modify the qualitative (transitional and long run) behavior of population and pollution.

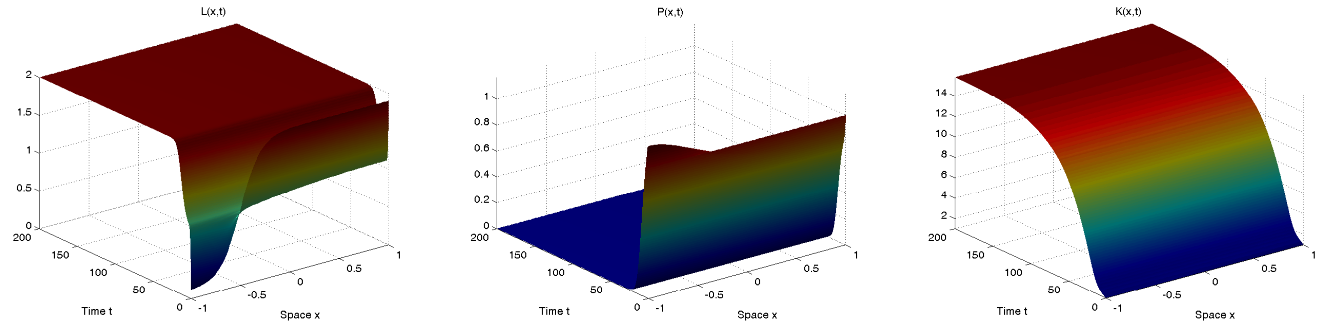

Figure 7: Spatio-temporal dynamics of population (left), pollution (center) and capital (right) in the case with diffusion.

In both the cases, the spatio-temporal evolution of capital closely mimics the dynamics of population, and in the long run it achieves either a zero level indicating the overall collapse of the spatial economy (Figure 6), or a positive level indicating overall sustainability for the spatial economy (Figure 7). Intuitively, this is due to the fact that pollution determines human population dynamics which, since labor is an essential input in the production of the unique consumable good, ultimately drives economic production (which is also affected by pollution as well) ruling capital dynamics. In the first case, since the pollution stock achieves an extremely high level, human population over the long run disappears, economic production will be null and thus capital will converge to zero; in the second case, since pollution vanishes over the long run, human population stabilizes at its carrying capacity level, economic production will be positive and so will the capital level. These results exactly confirm our previous conclusions derived in a simplified setup: mutual demographic-pollution feedbacks and spatial interactions do matter for sustainable development.

6 Conclusion

The topic of sustainable development has become quite popular lately, both among policymakers and academics. The discussions about sustainability have thus far mainly focused on two (economic and environmental) dimensions of the problem only, and the third (population) has barely been considered. Our paper proposes a more comprehensive approach to the sustainable problem by analyzing simultaneously the three dimensions of sustainable development, along with the implications of geographical heterogeneity and spatial externalities. Our results highlight two important types of conclusions: the mutual population and environment feedback matter for sustainable development, and also geographical considerations and spatial interactions do. Indeed, we show that neglecting the existence of mutual feedback between population and pollution leads to distorted conclusions about the process of sustainable development. We also show that neglecting the existence of spatial interactions between regions and economies leads to conclusions substantially different from those obtained by allowing for spatial externalities. Neglecting the role of both population and geography may lead to more optimistic results on the prospects of sustainable development, suggesting that the approach followed thus far in the sustainability literature has undermined the importance and the role of economic, environmental and demographic policies in addressing our society along a sustainable path.

Our paper represents only a preliminary attempt to analyze a very delicate and complicated problem, thus we cannot expect our analysis to be exhaustive. Several issues still need to be analyzed in order to build a unified approach allowing to understand the several channels through which economy, environment and population mutually interact. For example, understanding to what extent policymakers should revise their approach to deal with the sustainable problem in order to account for spatial interactions along with population-environment feedbacks is an important open question. Extending the analysis in order to consider the associated optimal control problem along the lines of La Torre et al. (2015) is a priority for future research; however, in order to meaningfully deal with this issue additional efforts from a computational point of view to develop reliable algorithms are needed (Boucekkine et al., 2009). Also the potential role of climate change in inducing poverty and worsening health conditions as suggested by a recent report from the World Bank (see Hallegatte et al., 2016) is likely to complicate the picture, further delocalizing the effects of environmental processes and increasing the degree of spatial interdependence. Extending the analysis to explicitly take into account climate change and its role in affecting economic, environmental and demographic outcomes is another priority for future research. These additional challenging tasks are left for future research.

A Proof of Proposition 1

The equilibria of the system of ordinary differential equations (3) - (4) correspond to the solutions of the following nonlinear system of algebraic equations:

$$0 = L(t)\left[\frac{\Omega}{1+\theta P} - L(t)\right] \tag{13}$$

$$0 = P(t)\left[\frac{\beta}{1+\tau A L(t)^{1-\alpha}} - \delta_P\right] \tag{14}$$

It is straightforward to verify that the origin is an equilibrium $E_1 = (0, 0)$. Straightforward algebra leads to the other two equilibria $E_2 = (\Omega_x, 0)$ and $E_3 = (\overline{L}, \overline{P})$ with:

$$\overline{L} = \left(\frac{\beta - \delta_P}{\tau A \delta_P}\right)^{\frac{1}{1-\alpha}} \tag{15}$$

$$\overline{P} = \frac{1}{\theta}\left[\frac{\Omega}{\left(\frac{\beta-\delta_P}{\tau A \delta}\right)^{\frac{1}{1-\alpha}}} - 1\right] \tag{16}$$

The parametric restriction in Proposition 1 guarantees that $\overline{L} > 0$ and $\overline{P} > 0$; indeed, $\beta - \delta_P > 0$ ensures that $\overline{L} > 0$, while $\Omega^{1-\alpha}\tau A \delta_P > \beta - \delta_P$ that $\overline{P} > 0$.

The stability property of the origin, E_1 cannot be analyzed via the traditional linearization method. It is however possible to show that the trajectories are eventually escaping from a circular sector surrounding the origin, provided that the radius of this sector is small enough. For this purpose, let us express the vector of the initial condition as:

$$L(0) = L_0 \tag{17}$$

$$P(0) = P_0 = vL_0 \tag{18}$$

where $v = \tan(\theta)^{-1}$ defines implicitly the direction of the vector of initial conditions (L_0, P_0), whose angle with respect to the L axis is θ. The idea is to show that, $\forall v \in (0, +\infty)$, the following vector field:

$$\frac{dL}{dt} = L_0\left[\frac{\Omega}{1+\theta v L_0} - L_0\right] \tag{19}$$

$$\frac{dP}{dt} = vL_0\left[\frac{\beta}{1+\tau A L_0^{1-\alpha}} - \delta_P\right] \tag{20}$$

has both positive components eventually, when $L_0 \to 0$. When L_0 tends to zero, equations (19) - (20) can be written as:

$$\frac{dL}{dt} \simeq L_0\Big[\Omega - L_0\Big] \tag{21}$$

$$\frac{dP}{dt} \simeq vL_0(\beta - \delta_P) \tag{22}$$

Given that $\Omega > 0$, the quantity on the RHS of equation (21) will eventually become positive, no matter the value of v. As for equation (22), the RHS is always positive $\forall v \in (0, +\infty)$, provided that the parametric restriction required by Proposition 1 is met. It remains to explore the extreme case where $v = 0$ or $v = +\infty$, that is the axes $P = 0$ and $L = 0$ respectively. When $P = 0$, the RHS of equation (3) is eventually positive, in the limit $L_0 \to 0$, as shown before, while in equation (4) the RHS is identically null. When $L = 0$, the RHS in equation (3) is null, while the RHS in equation (4) is positive, because $\beta - \delta_P > 0$ by assumption. The trajectories are eventually escaping from a circular sector in the positive orthant around the origin.

For what concerns the other two equilibria, linearization can be applied. The associated Jacobian matrix is given by:

$$J(L,P) = \begin{bmatrix} \frac{\Omega}{1+\theta P} - 2L & -\frac{\theta \Omega L}{(1+\theta P)^2} \\ -\frac{\beta P(1-\alpha)\tau A L^{1-\alpha}}{(1+\tau A L^{1-\alpha})^2 L} & \frac{\beta}{1+\tau A L^{1-\alpha}} - \delta_P \end{bmatrix} \tag{23}$$

Let us start with $E_2 = (\Omega, 0)$. The Jacobian matrix (23) evaluated at E_2 reads as follows:

$$J(\Omega, 0) = \begin{bmatrix} -\Omega & -\theta\Omega^2 \\ 0 & \frac{\beta}{1+\tau A \Omega^{1-\alpha}} - \delta_P \end{bmatrix}$$

Since the term $a_{2,2}$ is negative thanks to the parametric restriction required by Proposition 1, the determinant is positive and the trace is negative, thus the equilibrium E_2 is asymptotically stable. Finally we consider $E_3 = (\overline{L}, \overline{P})$. Simple inspection of the Jacobian matrix (23) shows that the terms $a_{1,2}$ and $a_{2,1}$ are both positive under Proposition 1. The Jacobian matrix (23) evaluated at E_3 in this case becomes:

$$J(\overline{L}, \overline{P}) = \begin{bmatrix} -\left(\frac{\beta-\delta}{\tau A \delta_P}\right)^{\frac{1}{1-\alpha}} & < 0 \\ < 0 & 0 \end{bmatrix}$$

Both the determinant and the trace are negative, thus the equilibrium E_3 is saddle point stable.

B Basic Facts on Partial Differential Equations

The aim of this appendix is to recall only few results that are useful to analyze our model. For more details and information, refer to Polyanin (2002). The following PDE:

$$\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2} + [1 - u(x,t)]u(x,t) \qquad (x,t) \in [a,b] \times (0, +\infty) \tag{24}$$

is known as *Fisher equation* or *logistic equation* in literature. Traveling-wave solutions are known for this equation and take the form:

$$u(x,t) = \frac{1}{\left[1 + C\exp\left(-\frac{5}{6}at \pm \frac{1}{6}\sqrt{6a}x\right)\right]^2} \tag{25}$$

and

$$u(x,t) = \frac{1 + 2C\exp\left(-\frac{5}{6}at \pm \frac{1}{6}\sqrt{-6a}x\right)}{\left[1 + C\exp\left(-\frac{5}{6}at \pm \frac{1}{6}\sqrt{-6a}x\right)\right]^2} \tag{26}$$

where C is an arbitrary constant. The following proposition provides a convergence result.

Proposition 5. *For the Fisher equation with Neumann boundary conditions* $\frac{\partial u(x_a,t)}{\partial x} = \frac{\partial u(x_b,t)}{\partial x} = 0$*, two solutions exist with* $0 \leq u(x) \leq 1$*:* $u(x) = 0$ *and* $u(x) = 1$*. For continuous initial conditions* $u(x,0) = u_0(x)$*,* $0 \leq u_0(x) \leq 1$*, the solution satisfies* $\lim_{t\to+\infty} u(x,t) = 1$*, uniformly in* $x \in [x_a, x_b]$*; except for* $u(x) = 0$*, if* $u_0(x) = 0$ *(everywhere).*

The following linear PDE:

$$\begin{aligned}
\frac{\partial u(x,t)}{\partial t} &= d\frac{\partial^2 u(x,t)}{\partial x^2} + cu(x,t) && (27)\\
\frac{\partial u(x_a,t)}{\partial x} &= \frac{\partial u(x_b,t)}{\partial x} = 0 && (28)\\
u(x,0) &= u_0(x) && (29)
\end{aligned}$$

is known as *heat equation with linear source term.* When $c = 0$ it collapses to the classical heat equation and through the substitution $u(x,t) = e^{ct}w(x,t)$ it can be reduced to the heat equation in the unknown w. It admits a closed form solution given by:

$$u(x,t) = e^{ct}\left[a_0 + \sum_{n\geq 1} a_n e^{-d\left(\frac{n\pi}{x_b-x_a}\right)^2 t}\cos\left(\frac{n\pi x}{x_b-x_a}\right)\right] \tag{30}$$

where:

$$\begin{aligned}
a_0 &= \frac{1}{x_b-x_a}\int_{x_a}^{x_b} u_0(x)dx && (31)\\
a_n &= \frac{2}{x_b-x_a}\int_{x_a}^{x_b} u_0(x)\cos\left(\frac{n\pi x}{x_b-x_a}\right)dx && (32)
\end{aligned}$$

The following proposition summarizes the possible outcomes associated with the above equation.

Proposition 6. *One of the following scenario can occur:*

- *If* $c > 0$ *and* $d \geq 0$ *then* $u(x,t) \to \infty$ *whenever* $t \to +\infty$ *and for any* $x \in [x_a, x_b]$*,*
- *If* $c = d = 0$ *then* $u(x,t) = a_0 + \sum_{n\geq 1} a_n \cos\left(\frac{n\pi x}{x_b-x_a}\right)$ *for any* $x \in [x_a, x_b]$*,*
- *If* $c = 0$ *and* $d > 0$ *the* $u(x,t) \to a_0$ *for any* $x \in [x_a, x_b]$*,*
- *If* $c < 0$ *and* $d \geq 0$ *then* $u(x,t) \to 0$ *whenever* $t \to +\infty$ *and for any* $x \in [x_a, x_b]$*.*

In order to study the behavior of PDEs with form different from the Fisher and the linear equation, we can rely on a comparison method. The following comparison theorem (Friedman, 2008) is useful in order to obtain some insights on the long run behavior of a general PDE.

Proposition 7. *Let* $u(x,t)$ *be smooth and suppose that:*

$$\begin{aligned}
\frac{\partial u(x,t)}{\partial t} - d\frac{\partial^2 u(x,t)}{\partial x^2} &\geq -cu(x,t) \quad \text{in } (a,b)\times(0,T) && (33)\\
\frac{\partial u(x,t)}{\partial n} &\geq 0, \quad \text{on } \{a,b\}\times(0,T) && (34)\\
\rho(0,x) &\geq 0 \quad \text{in } (a,b) && (35)
\end{aligned}$$

where d *is a positive real number and* $c \in R$*. Then* $u(x,t) \geq 0$ *in* $(a,b)\times(0,T)$*.*

Now consider the following PDE:

$$\frac{\partial u(x,t)}{\partial t} = d\frac{\partial^2 u(x,t)}{\partial x^2} + c[u(x,t)]u(x,t) \quad (36)$$

$$\frac{\partial u(x_a,t)}{\partial x} = \frac{\partial u(x_b,t)}{\partial x} = 0 \quad (37)$$

$$u(x,0) = u_0(x) \quad (38)$$

Note that this PDE includes both the Fisher and the linear equations as particular cases. Let us suppose that $c_{\min} \leq c[u(x,t)] \leq c_{\max}$ for any x and t. By applying the above comparison theorem to the above equation leads to the following result.

Proposition 8. *If $c_{\min} > 0$, then $u(x,t) \to \infty$ whenever $t \to +\infty$ and for any $x \in [x_a, x_b]$. If, instead, $c_{\max} < 0$, then $u(x,t) \to 0$ whenever $t \to +\infty$ and for any $x \in [x_a, x_b]$.*

Note that the results summarized in this appendix contain all the information that we need to analyze the behavior of the system of PDEs discussed in the body text.

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