Population and Pollution Interactions in a Spatial Economic Model

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Abstract

We analyze the spatio-temporal dynamics of a simple model of economic geography in which population and pollution dynamics are mutually interdependent. Pollution by reducing the carrying capacity of the natural environment, which determines the maximum amount of people a given location can effectively bear, affects labor force dynamics which in turn alter pollution emissions. Such mutual links determine the development path followed by different locations, and spatial interactions further complicate the picture. We show that neglecting the existence of spatial externalities can lead to misleading predictions about the development path followed by different locations in the spatial economy.

Keywords: Population Dynamics, Pollution, Spatial Model, Sustainability
JEL Classification: C60, J10, O40, Q50

1 Introduction

Sustainable development has become a very popular research topic lately, and the main research question in this context consists of understanding how to address the economy along a sustainable development path (Solow, 1974; Stokey, 1998). Sustainability ultimately requires to satisfy *"the needs of the present without compromising the ability of future generations to meet their own needs"* (WCED, 1987), demanding thus to take into account the population and environment relation. The channels through which the human population affects the natural environment in which it lives and how in turn the environment may alter the evolution of human population have been long discussed in literature since Malthus' (1798) seminal work (see among others, Nerlove, 1991; Marsiglio, 2011, 2017). However, none of the existing works is able to relate the issue to geographical and spatial characteristics, since they all assume that the economy is simply a unique point in space and thus eventual heterogeneities are completely ruled out. This is clearly a strong simplification of reality. while understanding the implications of geographical heterogeneity on the development path followed by a spatial-extended notion of economy is a very active and recent research topic, following Krugman's (1991) seminal work (see Camacho and Zou, 2004; Boucekkine et al., 2009; Xepapadeas, 2010). The goal of this paper consists of analyzing the population and environment relation from a spatial point of view, taking into account thus that the dynamics of population and the environment mutually affect each other not only over time but also across space.

Our work thus combine together two different streams of literature: the sustainability and the economic geography ones. From the latter we borrow the analytical framework by considering a spatial economic

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growth model with environmental and demographic interactions; the setup most similar to ours is La Torre et al's (2015), but differently from them we allow for population growth and labor migration. From the former, instead, we borrow the interest in understanding whether sustainable development can effectively occur; Nerlove (1991) and Marsiglio (2011) are closely related to our work, but differently from them our focus is not on natural resources but on pollution and we do not restrict our analysis to the temporal dynamics only since we allow also for spatial interactions. Our main results show that by neglecting the existence of spatial spillovers the possible predictions about the development path followed by different locations in the spatial economy may be misleading, suggesting thus that geographic externalities may be an important determinant of economic development.

The paper proceeds as follows. Section 2 introduces our spatio-temporal dynamic model, summarized by two partial differential equations. In section 3 we derive some analytical results in absence of spatial diffusion, while in section 4 we focus on the fully-fledged model in which spatial diffusion plays an active role. Section 5 concludes and presents directions for future research.

2 The Model

We consider a simple model of economic geography in which agents consume all their income and inelastically supply labor. Since there is no unemployment, the population size and the labor force perfectly coincide. Economic production generates pollution which by affecting the carrying capacity of the natural environment in which human population lives determines the evolution of the labor force, which is an essential input in the production of final output. We assume a continuous space structure to represent that the spatial economy develops along a linear city (see Hotelling, 1929), where the population is mobile across different locations and pollution, even if generated in a specific location, diffuses across the whole economy (La Torre et al., 2015). We denote with $L(x,t)$ and $P(x,t)$ respectively the population size and pollution stock in the position x at date t, in a compact interval $[x_a, x_b] \subset \mathbb{R}$, and $t \geq 0$. We also assume that the initial population and pollution distribution, $L(x,0)$ and $P(x,0)$, are known and there is no migration or pollution flow through the boundary of $[x_a, x_b]$ namely the directional derivative is null, $\frac{\partial L(x,t)}{\partial x} = \frac{\partial P(x,t)}{\partial x} = 0$, at $x = x_a$ and $x = x_b$ (Anita et al., 2013; Capasso et al., 2010).

The economic and environmental setup to a large extent resembles La Torre et al.'s (2015), but differently from theirs, our model focuses on the dynamic evolution of population and its interaction with pollution. Output is produced according to a Cobb-Douglas production function employing capital and labor as $Y(x,t) = AK(x,t)^{\alpha}L(x,t)^{1-\alpha}$, where $A > 0$ denotes the total factor productivity and $0 < \alpha < 1$ the capital share of income. We abstract from capital accumulation and without loss of generality the capital stock is normalized to unity, $K(x,t) = 1, \forall x, t$. Production activities generate emissions which increase linearly the stock of pollution and $\theta > 0$ measures the degree of such environmental inefficiency. These emissions are dampened by (spatially heterogeneous) public abatement activities, which reduce a share $u(x) \in [0,1]$ of total emissions, thus $1 - u(x)$ represents unabated emissions. Apart from abatement activities, the pollution stock tends to decrease at the constant rate $\delta_P > 0$ representing the natural decay rate of pollution. Agents are subject to (location-specific) proportional income taxation, $\tau(x) > 0$, which is used to finance the abatement activities needed to reduce the environmental effects associated with pollution; agents are assumed to consume completely their disposable income, implying that $C(x,t) = [1 - \tau(x)]Y(x,t)$. We assume that the (local) government wishes to maintain a balanced budget at any point in time, such that the tax revenue is totally devoted to reduce pollution. At location x the tax revenue is $T(x,t) = \tau(x)Y(x,t)$, while abatement activities, $M(x,t)$, decrease a certain share of pollution, $u(x) \in [0,1]$, by employing a certain amount of not consumed output with the following cost $M(x,t) = \mathcal{C}[u(x)]Y(x,t)$, where $\mathcal{C}(\cdot)$ is the cost function of abatement activities, taking the following form $\mathcal{C}[u(x)] = 1 - [1 - u(x)]^{\epsilon}$ with $\epsilon > 1$ (Bartz and Kelly, 2008). By equating the tax revenue and abatement we obtain a one-to-one relationship between the tax rate and the share of abated emissions, $\tau(x) = \mathcal{C}[u(x)]$, implying that consumption is given by the following expression: $C(x,t) = [1-u(x)]^{\epsilon} Y(x,t)$.. Population evolves according to a logistic equation, where $L^c(x) > 0$ represents the (spatially heterogeneous) carrying capacity of the natural environment, which is affected by pollution flows, through the following damage function $D(x,t) = \frac{1}{1+BP(x,t)^{\beta}}$ with $B > 0$ being a scale parameter and $\beta > 0$ measuring the magnitude of the pollution externality on population dynamics. Note that the share of abatement activities rules the economic-environmental trade off: a larger abatement improves the environmental outcome (by reducing pollution) at the cost of deteriorating the economic one (by reducing consumption).

The spatio-temporal dynamic model can thus be summarized by the following system of two partial differential equations:

$$\frac{\partial P(x,t)}{\partial t} = d_P \frac{\partial^2 P(x,t)}{\partial x^2} + \theta[1-u(x)]AL(x,t)^{1-\alpha} - \delta_P P(x,t) \tag{1}$$

$$\frac{\partial L(x,t)}{\partial t} = d_L \frac{\partial^2 L(x,t)}{\partial x^2} + \left[\frac{L^c(x)}{1+BP(x,t)^{\beta}} - L(x,t)\right] L(x,t) \tag{2}$$

Equation (1) describes the evolution of pollution over time and across space. The engine of pollution accumulation is represented by economic production activities; a fraction of the emissions is abated from the outset, through cleaning activities represented by term $1-u$, while a constant part of the pollution stock is eliminated by the self-cleaning capacity of the natural environment, represented by $\delta_P P$. The spatial externality, representing the extent to which the outcome in specific locations affects the outcomes in other locations as well, is captured by the diffusion term: the intensity of the diffusion process is measured by the diffusion coefficient $d_P \geq 0$, quantifying the extent to which pollution no matter where it is originally generated spreads across the whole spatial economy (La Torre et al., 2015).

Equation (2) describes the evolution of the human population over time and across space. In absence of pollution, the population size would grow according to a logistic law with constant carrying capacity L^c (Verhulst, 1838). By taking into account the negative pollution externality, the demographic law of motion is still logistic, but the maximum value of the population size that the natural environment can bear is represented by the term $\frac{L^c}{1+BP^{\beta}}$. As for the case of pollution, the spatial externality is represented by the diffusion term, where $d_L \geq 0$, represents the diffusion coefficient, measuring the extent to which population tends to migrate across different locations in the spatial economy.

3 The Model with No Diffusion

We first analyze the behavior of the above system without diffusion, but preserving the spatial structure. This allows us to compare the outcome with what arises in the diffusion case which we will analyze in the next section. In the case with no diffusion, that is $d_P = d_L = 0$, the partial differential equations (1) and (2) boil down to the following parametric system of ordinary differential equations:

$$\frac{dP(t)}{dt} = \theta[1-u]AL(t)^{1-\alpha} - \delta_P P(t) \tag{3}$$

$$\frac{dL(t)}{dt} = \left[\frac{L^c_x}{1+BP(t)^{\beta}} - L(t)\right] L(t) \tag{4}$$

The system (1) - (2) is characterized by several parameters, each of which could be space dependant, but we restrict our analysis to the effects of spatial heterogeneity on L^c. It is thus quite natural to suppose that the carrying capacity of the natural environment can vary across different locations, and thus it is reasonable to expect some spatial heterogeneity due to such inherent characteristic of specific locations. Since we are especially interested in discussing the implications of the population and environment relation, understanding the specific spatial characteristics of such a parameter, $L^c_x = L^c(x)$, is essential to comment on the interplay between human population and the natural environment. Specifically, this parameter captures the pollution feedback on population, and we wish to analyze how the location-specific carrying capacity L_x^c along with the diffusion terms $d_i \frac{\partial^2}{\partial x^2}$ where $i = L, P$, shape the time evolution of population and pollution. Note first that the system (3) - (4) is actually a continuous set of systems of ordinary differential equations, because of the presence of the space dependant parameter L_x^c: each point in the spatial domain has its own time dynamics, but there is no interaction between adjacent locations. Next proposition offers a concise description of the properties of this continuous set of systems, stating that $\forall x \in [x_a, x_b]$ the system (3) - (4) has a unique and stable non-trivial equilibrium.

Proposition 1. *The system (3) - (4) admits a unique nontrivial equilibrium,* $(\overline{P}, \overline{L}) \in \mathbb{R}^2_{++}$, $\forall x \in [x_a, x_b]$*:*

$$\begin{aligned} \overline{P} &= \left[\frac{\theta A(1-u)}{\delta_P}\right] \overline{L}^{1-\alpha} \\ \overline{L} &= RootOf\left\{ B\left[\frac{\theta A(1-u)}{\delta_P}\right]^{\beta} L^{(1-\alpha)\beta+1} + L - L_x^c = 0 \right\}. \end{aligned}$$

Moreover $(\overline{P}, \overline{L})$ *is asymptotically stable.*

Proposition 1 can be proved by using a classical linearization approach. The Jacobian matrix associated with the non-trivial equilibrium, $J(\overline{P}, \overline{L})$, is given by:

$$J(\overline{P}, \overline{L}) = \begin{bmatrix} -\delta_P & A(1-\alpha)(1-u)\overline{L}^{-\alpha} \\ -\beta B L_x^c \overline{L}\overline{P}^{\beta-1}(1+B\overline{P}^{\beta})^{-2} & -\overline{L} \end{bmatrix} \tag{5}$$

It is not difficult to determine the signs of each element. a_{11} is obviously negative. Since $\overline{L}$ and $\overline{P}$ are both positive, a_{12} is positive while a_{21} and a_{22} are both negative. It follows that both the eigenvalues of the Jacobian matrix are negative.

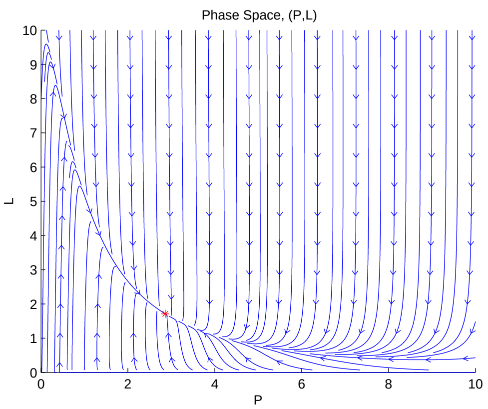

Figure 1: Phase portrait in the no diffusion case ($d_P = d_L = 0$).

Figure 1 represents the phase portrait for the following parametrization: $u = 0.5$, $\theta = 0.2$, $\delta_P = 0.05$, $A = 1$, $B = 1$, $\alpha = 0.33$, $\beta = 1.5$ (see La Torre et al., 2015), showing that whatever is the pair of initial conditions, (P_0, L_0), the system converges to its unique nontrivial equilibrium. The existence of a steady state in which both human population and pollution attain a strictly positive value suggests that despite the pollution feedback on population dynamics each location in the spatial economy develops along a trajectory which could be deemed as sustainable in some minimal sense. In absence of spatial interactions, the spatial economy is overall able to proceed its process of economic development along a smooth path, independently on the spatial parameter L^c_x. Even if an analytical expression for the steady state values cannot be obtained, it is possible to infer from the steady state expressions above how they do depend on such a spatial parameter and thus how the heterogeneity in the carrying capacity is likely to affect the long run equilibrium of both population and pollution.

4 The Model with Diffusion

We now turn to the analysis of the full model in which diffusion and thus spatial externalities are explicitly taken into account. In particular, we wish to understand whether the presence of such spatial interactions can alter our previous predictions about the development path followed by different locations in the spatial economy.. Given the spatial structure of the economy, the analysis of transitional dynamics can be performed only numerically, thus we now focus on numerical simulations in order to illustrate the spatial implications of pollution accumulation and population growth. Even if the numerical simulations that follow are based upon a specific set of parameters and initial conditions, reported in (**??**), it is possible to show that, since the nontrivial equilibrium is unique (See La Torre et al., 2015, for a discussion of how the presence of spatial externalities differently affect the system dynamics in the case of unique or multiple equilibria), even under different parametrizations the following qualitative results will hold true.

$$\begin{cases} u = 0.5,\ \theta = 0.2,\ x_a = -1,\ x_b = 1,\ \delta_P = 0.05, \\ A = 1,\ B = 1,\ \alpha = \frac{1}{3},\ \beta = 1.5,\ d_P = 0.1,\ d_L = 0.1, \\ P(x,0) = 1 + x,\ L(x,0) = 1 + x, \\ \sigma^2_{L^c} = 0.1,\ L^c = 10,\ L^c(x) = L^c e^{-\frac{x^2}{\sigma^2_{L^c}}}. \end{cases} \tag{6}$$

Most parameters take the same values as in La Torre et al. (2015) consistently with empirical evidence (see references therein), apart from those which are set to unity without loss of generality, and those which are specifically set in order to make our graphical illustrations as clear as possible. The share of abated emissions u is a candidate to be a control variable, that is a policy variable optimally chosen by the social planner in order to keep under control the level of pollution stock and thus to limit its impacts on population. We do not analyze the associated optimal control problem, thus for the sake of simplicity we assume that it takes the central value in the control space, namely $u = 0.5$. We assume the initial distribution of pollution, $P(x,0) = P_x = 1 + x$, to mimic to the initial distribution of population, $L(x,0) = L_x = 1 + x$. We set the carrying capacity as follows $L^c(x) = L^c e^{-\frac{x^2}{\sigma^2_{L^c}}}$, meaning that in the central locations it is larger than in the lateral ones. The results of our simulations are shown in the Figures 2 and 3.

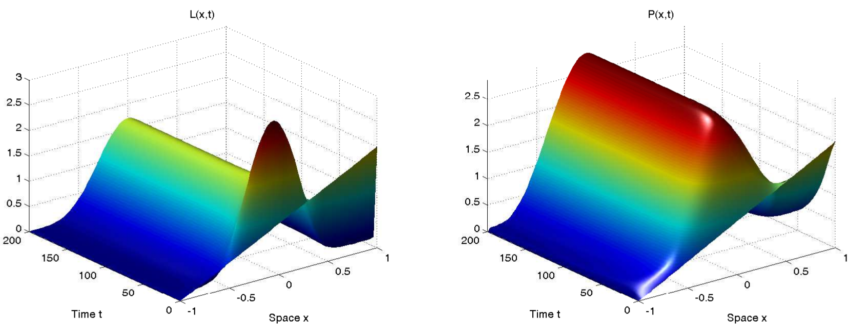

Figure 2: Evolution of pollution and population: no diffusion case ($d_P = d_L = 0$).

Figure 2 describes the evolution over time and across space of population (left panel) and pollution (right panel) in the case in which diffusion is absent, that is $d_P = d_L = 0$, consistently with what discussed in section 3. Given the shape of $L^c(x)$, it is clear that the central locations, where a higher carrying capacity is assumed, establish their primacy over time. There is no interaction among locations (no spatial externality), and for each location x the system (3) - (4) reaches its non-trivial and stable steady state. Figure 3 presents the same simulations in the case in which there is diffusion, that is $d_P = d_L > 0$. The overall dynamics of the system (1) - (2) is analogous to what seen before but there are notable differences that underline the role of diffusion as a spatial externality, justifying thus the introduction of a spatial model to the study of the dynamic relation between pollution and population. Indeed, even if the shape of the initial condition for both population and pollution increases linearly from the leftmost to the rightmost locations, the spatial profiles of both population and pollution over time change to end up mimicking the spatial pattern of the carrying capacity, which by being the only spatially-dependent parameter completely determines the spatial pattern at the equilibrium.

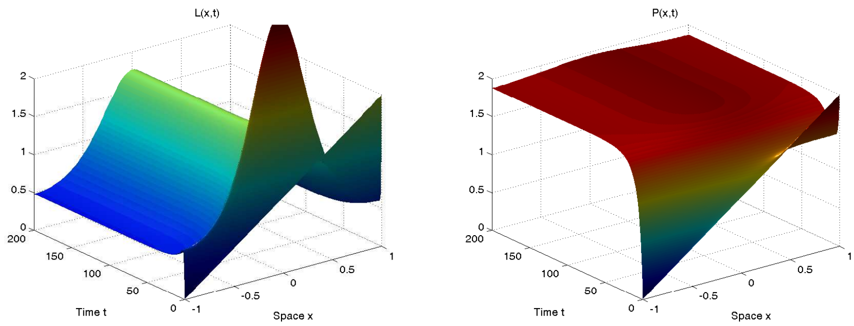

Figure 3: Evolution of pollution and population: diffusion case ($d_P = d_L = 0.1$).

By comparing Figure 2 and Figure 3, it is possible to notice that diffusion has a twofold effect on the dynamics and steady states of pollution: the central and the lateral locitons witness less and more pollution accumulation, respectively, with respect to the case without diffusion. This is because of the inherent tendency of diffusion to smooth differences out (Boucekkine et al., 2009; La Torre et al, 2015). Pollution diffusion does have a beneficial effect for the initially most polluted locations and a detrimental effect for the less polluted ones: ignoring spatial externalities can thus result in macroscopic modeling errors, since not only the dynamics, but even the steady states are affected by this type of spatial interaction. It is also clear that when diffusion is present the overall population becomes larger: on the one hand, the central locations reach a higher demographic concentration, on the other hand, the lateral locations are an order of magnitude bigger, with respect to the no-diffusion scenario. This is apparently in contrast with what happens to pollution: pollution has a negative impact on the growth of population via its carrying capacity dampening factor. At the steady state, in the central locations there is less pollution such that the population concentration tends to increase: the reaction term, $(\frac{1}{1+BP^{\beta}})$, prevails on the smoothing tendency of diffusion, $d_L \frac{\partial^2}{\partial x^2}$. In the lateral locations we would expect a symmetrical behavior, that is pollution to increase while population to decrease; what instead happens is that diffusion prevails on reaction and the population in the lateral locations can benefit from migration from the central ones. Clearly, the introduction of diffusion enriches the dynamics and affects the steady states: the overall effects are the results of the dynamical tension between the reaction and the diffusion components of the system (1) - (2).

In figure 4 we show the long run per capita pollution in the case with no (left panel) and with (right panel) diffusion. At the beginning of the time horizon per capita pollution is identically equal to 1 across the spatial domain in both the cases by assumption. In both the cases, over time per capita pollution increases everywhere, but the locations who suffer the most are lateral ones, due to the extremely low value of the environmental carrying capacity that tends to keep population down. A few words on the long run spatial distribution of per capita pollution in the two different frameworks are needed. Per capita pollution is bounded, such that in the long run the spatial economy can be considered sustainable. Comparing the left

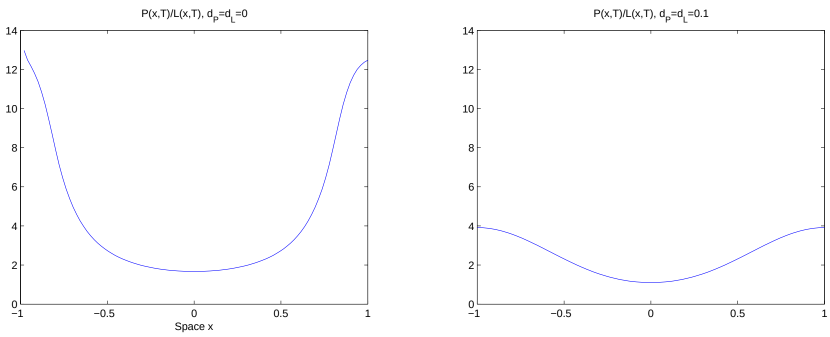

Figure 4: Steady state per capita pollution: no diffusion and positive diffusion cases.

and the right panels, it is clear that the central locations performs better in terms of per capita pollution, in both scenarios: the combined effect of reaction and diffusion previously mentioned turns out to be favorable to the central locations, as long as per capita pollution is deemed to be a proxy of the health status of the environment. The major difference between the two cases results in the higher level of pollution per capita taken in the no-diffusion scenario. As seen before, both pollution and population increase in the lateral locations when spatial externality are taken into account; however, now we can compare such relative increases: population increases more than pollution, resulting in lower per capita pollution than in the no-diffusion case.

5 Conclusion

This paper analyzes the mutual interactions between population and pollution in a spatio-temporal dynamic economic geography model. We develop a dynamic macroeconomic model to analyze the extent to which population and pollution may affect each other not only over time but also across space. We show that the population and pollution feedback may be important in order to assess the development path that different locations in the spatial economy will follow. This means that neglecting any spatial implication may give rise to misleading predictions about the environment and population relation. Thus, from a policy point of view, spatial externalities represent an important aspect which deserves further attention. Indeed, the analysis performed in this paper cannot be considered exhaustive, since important issues have not be taken into account. Specifically, the pure dynamic setup of the model does not allow us to assess how optimally defined policies may alter our conclusions about the development path followed by single locations in the spatial economy. Extending the analysis in order to consider the associated optimal control problem along the lines of La Torre et al. (2015) is a priority for future research.

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