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Abstract

We develop an analytically tractable agent-based model of heterogeneous agents to characterize how social interactions within a neighborhood determine the dynamic evolution of its ethnic composition. We characterize the conditions under which integration or segregation will occur, which depends on how the social externality parameter compares with the net benefit from leaving. Minority segregation may result from the process of tipping, and in such a case we identify three possible tipping channels: two are related to exogenous shocks (migration flows and changes in tipping points) and one is related to the endogenous probabilistic features of our framework (endogenous coordination across agents). This characterization of integration and segregation conditions yields interesting policy implications for both social and urban planning policies that favor desegregation.

Keywords: Racial Segregation, Social Interactions, Tipping
JEL Classification: C60, D70, J15

1 Introduction

Despite significant improvements in the last few decades, racial discrimination is still a pervasive aspect of our society today (Glaeser and Vigdor, 2001; Logan et al., 2004). It manifests in several ways ranging from labor market discrimination to residential segregation. From a historical perspective, residential segregation has received growing attention since Schelling's (1969) seminal paper. Several works document the extent to which urban segregation occurs and negatively affects the wellbeing of the segregated groups by limiting their access to education, employment opportunities and health outcomes, along with favoring poverty and criminal behavior (Galster, 1987; Orfield and Eaton, 1996; Shihadeh and Flynn, 1996; Cutler and Glaeser, 1997; Williams and Collins, 2001; Card and Rothstein, 2007). From a theoretical perspective, segregation outcomes have been explained from two different points of view, stressing the role of economic and social factors. Classical economic arguments going back to Tiebout (1956) and Rosen (1974) state that neighborhood sorting is a driver of residential segregation in the presence of heterogeneity in households' incomes and preferences, which ultimately determine their willingness to pay for location characteristics. According to this view, the provision of public goods in specific neighborhoods can generate segregation through its effects on prices and housing demand (McGuire, 1974; Card et al., 2008; Kollmann et al., 2018). Social arguments originating in Schelling's (1969, 1971, 1978) works state that social interactions at microeconomic level is a cause of segregation at macroeconomic level. According to this view, even moderate

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individuals' preferences for ethnic isolation can yield segregation in the entire neighborhood (Zhang, 2004a, 2004b, 2011; Pancs and Vriend, 2007; Grauwin et al., 2012). Clearly, these alternative theories emphasize how different policies can be used to promote desegregation: on the one hand, economic theory suggests that urban planning policy, taking the form of public investments in specific neighborhoods, is a natural answer; on the other hand, social interactions theory identifies in social policy, aiming to promote integration between different ethnic groups, as another solution. Few attempts have also been made to bridge these two theories by developing a mixed approach in which some social interaction mechanism (i.e., taste for ethnic isolation) is introduced in a classical economic setup of neighborhood sorting (Becker and Murphy, 2000; Sethi and Somanathan, 2004; Banzhaf and Walsh, 2013). By following this last branch of literature, the goal of this paper consists of developing a framework in which economic and social factors jointly contribute in determining single households' decisions which ultimately drive eventual segregation outcomes. Different to extant works, rather than extending an economic setting to account for some social elements, we introduce some economic factors in a simple social interactions framework. Indeed, as pointed out by Arrow (1998), among the possible explanations of segregation, social interactions appear to provide the most convincing argument.

We build on the social interactions theory as outlined in Schelling's works (1969, 1971, 1978) in order to introduce some economic mechanisms to determine the extent to which economic and social factors may interact to determine segregation. Schelling's main conclusion is that even if the majority and minority groups have some degree of preference for integration, individual decisions could lead to aggregate outcomes of predominantly segregated neighborhoods. In particular, following an in-migration of a minority group, provided that the minority share exceeds a critical threshold the neighborhood will "tip" to being composed entirely of the minority population.[1] The existence of such non-linearities and tipping behaviors has been firstly demonstrated empirically by Card et al. (2008) who, by applying a regression discontinuity approach to analyze tract-level data within US cities between 1970–2000, show that the majority flees as soon as a neighborhood becomes 5–20% minority populated. A number of later studies applying the same methodology confirm the existence of tipping over different time periods and in other countries as well[2] (Alden et al., 2015; Shertzer and Walsh, 2016; Kollmann et al., 2018). However, very little is known about the determinants of such dynamic effects and only few attempts have been made to characterize tipping. Most of the theories proposed thus far focus on simple static approaches based upon traditional economic arguments (Card et al., 2008; Heal and Kunreuther, 2010; Banzhaf and Walsh, 2013), while attempts to discuss the role of social interactions in dynamic settings are limited (see Zhang, 2011). However, as suggested by Schelling (1971), *"the analysis of 'tipping' phenomena [...] requires explicit attention to the dynamic relationship between individual behavior and collective results"*. Therefore, we aim to contribute to this latter literature in order to provide a dynamic explanation of tipping. Unlike previous works which present some modifications of Schelling's original model and are particularly complicated to the extent to preclude us from understanding the main mechanisms in place (see Zhang, 2011, who analyzes Schelling's setup form an evolutionary game point of view), we develop an alternative framework which maintain the basic features of Schelling's model but turns out to yield simpler, intuitive and analytically-tractable solutions.

To this end, we need to understand the two main features of Schelling's model that need to be preserved. Schelling is often considered as one the fathers of agent-based modeling, as his analysis represents one of the earliest agent-based model examples (Epstein and Axtell, 1996; Zhang, 2004a). In Schelling's setup there exists some degree of preference for racial isolation, which is essential to yield segregation. These

[1]The phenomenon of tipping has been discussed firstly by Grodzins (1957), who claim that: *"for the vast majority of white Americans a tipping point exists"*. He is also the first to suggest that the process of tipping is irreversible, a conclusion supported by Duncan and Duncan (1957) in their study of the experience of Chicago between 1940 and 1950, and by following theoretical works (Zhang, 2011).

[2]Tipping points have been shown to exist also in other discrimination and segregation contexts, including schooling (Caetano and Maheshri, 2017) and employment (Pan, 2015).

two features will be maintained in our framework which relies upon social interactions models based upon a random utility setting (Brock and Durlauf, 2001; Blume and Durlauf, 2003). The presence of agents' heterogeneity gives rise to an agent-based model and the presence of a social externality yields preference for ethnic isolation. In contrast to Schelling's model and its following refinements (Zhang, 2011), this framework can be fully analytically solved giving rise to closed-form results, which determine not only the long run equilibrium outcomes but also the tipping point, which to the best of our knowledge represents an important novelty in this literature. Specifically, our approach describes the dynamic evolution of the majority share in a neighborhood in which individuals are heterogeneous in their degree of preference towards own-group neighbors. This permits us to characterize the conditions under which either segregation or integration will occur, and in the former case, whenever segregation results from tipping, to identify three possible channels through which tipping may occur. This characterization of integration and segregation conditions yields interesting policy implications for social and urban planning policies that favor desegregation.

This paper proceeds as follows. Section 2 introduces our social interactions framework to describe how single households decide whether to continue residing in a certain neighborhood by taking into account both economic factors (i.e., individual benefits and costs), along with social factors (i.e., the behavior of other own-group households through a social externality). As a matter of expositional simplicity and in line with previous literature, we shall refer to the major ethnic group as "whites" and to the minority as "blacks". The aggregation of single households' decisions allows us to characterize the neighborhood outcome in terms of the share of white flight (i.e., the share of white households leaving the neighborhood). Section 3 analyzes the asymptotic-population dynamics, which is entirely summarized by a differential equation describing the evolution of white flight. In this setting we can explicitly identify the long run equilibrium outcomes, which may represent either segregation or integration. In the case of minority segregation we determine under which conditions this may result from tipping and we identify two possible mechanisms related to exogenous shocks (migration flows and changes in tipping points) for tipping to occur. Section 4 analyzes the finite population dynamics, in which white flight is stochastic and characterized by some transition probabilities. In such a probabilistic setting, we show that our asymptotic long run predictions may not always be accurate and that a further potential tipping channel, related to endogenous coordination across agents, may emerge. Section 5 concludes and suggests directions for future research.

2 The Model

Consistent with previous literature, we consider a setting in which the housing supply is fixed. Specifically, we focus on a neighborhood D (for district) endowed with a large number N of dwellings, each of which can be occupied by either one white or one black household. Let us assume that initially all dwellings are occupied by whites and each white household $i = 1, ..., N$ attempts to maximize the utility associated with his residential choice. White households are heterogeneous in their degree of preference towards own-group neighbors. The utility function of any white household i is associated with the choice $\omega_i = \{0, 1\}$ such that $\omega_i = 1$ ($\omega_i = 0$) denotes that the household leaves (stays in) the neighborhood. The decision to leave or stay is determined by the utility associated with residing in that neighborhood. This utility depends on three elements: a private and a social component, which are common to all households, and an idiosyncratic component, which is household-specific. The private component is given by the net benefit of leaving the neighborhood $b - c \in \mathbb{R}$, determined by the difference between benefits $b > 0$ and costs $c > 0$. The social component is associated with the choice of other (white) households and is equal to $J\tilde{x}_i^e$, where $\tilde{x}_i^e$ is the expectations of household i about others' mean choices such that $\tilde{x}_i^e = \frac{1}{N-1}\mathbb{E}[\sum_{j \neq i} \omega_j]$, and $J > 0$ is a scale factor measuring the magnitude of such an expectation in each household's utility. The idiosyncratic component is given by ε_i, which is a household-specific random term independently and identically distributed (i.i.d.) across households, determining the specific type of each single household. The utility function of each white household is, therefore, given by the following expression:

$$u_i(\omega_i) = \omega_i \left(b - c + \varepsilon_i + J\tilde{x}_i^e\right). \tag{1}$$

The utility associated with the decision to leave is $u_i(1) = b - c + \varepsilon_i + J\tilde{x}_i^e$ while the utility to stay is $u_i(0) = 0$. Clearly, as long as the utility associated with staying in the neighborhood is larger than the utility associated with leaving, the household will continue to stay in the neighborhood. In other words, whenever $u_i(1)$ is larger (smaller) than zero, the white household will leave (stay).

Note that the utility level depends on a number of factors. (i) The private component $b - c$ measures the net private utility associated with leaving the neighborhood D. This may be thought of as the net utility obtained from a wide range of sources, including the differential benefits between amenities in other neighborhoods and those in neighborhood D, and the costs related to relocation decisions. The differential benefits may take into account housing prices, population density, degree of safety, location, availability of public transport, school, parks or leisure facilities. The costs may include accommodation search, relocation fees (related to selling their property for homeowners, buying a new property, depositing bonds, or interrupting a current lease for tenants). We do not restrict a priori how benefits and costs compare such that net benefits can be positive, negative or null. Note that the private component term captures economic factors related to the availability of private and public goods, consistent with what is discussed by extant economics literature. (ii) The social component $J\tilde{x}_i^e$ captures the (positive) externality generated by the decision of other (white) households where J determines the magnitude of such an externality. Specifically, J quantifies the importance of a social norm measuring the extent to which conforming to the behavior of other whites is desirable from the point of view of the single household. We can think of the utility arising from this social component as the support and benefits of informal networks in the local community shared by people of the same ethnic group: a larger share of whites leaving will reduce the strength of such informal networks providing each white household with a stronger incentive to leave the neighborhood as well. Therefore, the social component captures social factors similar to those discussed in the social interactions literature; indeed, such a social externality by incentivizing white households to mimic the behavior of other whites determines to some extent their preference for ethnic isolation. (iii) The idiosyncratic component ε_i captures the individual degree of preference towards own-group neighbors, related to ethnic prejudice. Similar to what is discussed in the mixed economic-social interactions literature, we assume that such a degree of preference towards own-group neighbors is different from household to household, determining heterogeneity in the characteristics of the white population. This suggests that, everything else equal, according to their specific type (the specific value of ε_i) different households may prefer leaving the neighborhood, while others staying. The type of each household, ε_i, is determined by the realization of i.i.d. random shocks drawn from a common distribution $\eta(z) = \mathbb{P}(\epsilon_i \leq z)$.

As in Brock and Durlauf (2001), we can show that the utility function (1) can be cast into a probabilistic choice model such that:

$$\mathbb{P}(\omega_i = 1 |\, \tilde{x}_i^e) = \eta\left(b - c + \varepsilon_i + J\tilde{x}_i^e\right). \tag{2}$$

By following the same approach as in Blume and Durlauf (2003), we can recast the above model into a dynamic probabilistic choice model. Let us define $x_t^N = \frac{1}{N}\sum_{i=1}^{N}\omega_{i,t}$ as the proportion of white households at time t who leave the neighborhood, which we will refer to as the *"white flight share"* (see Banzhaf and Walsh, 2016). Then, as in the static model, white households decide whether to leave or stay in the neighborhood at any given time t with the following probability:

$$\mathbb{P}(\omega_{i,t+\Delta t} = 1 |\, \omega_{i,t}, x_t^N) = \eta\left(b - c + \varepsilon_i + Jx_t^N\right). \tag{3}$$

The Markovian dynamics of the stochastic process $x^N \equiv (x_t^N)_{t\geq 0}$ in (3), representing the evolution of the white flight share, cannot be explicitly analyzed in a finite dimensional population framework. Therefore, in order to derive some analytical results in the next section we will focus on an infinite dimensional population version of the model, and we will get back to its finite population version in the following section. This will help understand the nature of neighborhood dynamics by clearly identifying when either integration or segregation due to tipping will occur, and in the latter case to distinguish three alternative tipping channels.

Note that our model is substantially different from Schelling's (1969, 1971, 1978) original setup but at the same time it maintains its peculiarities, even if in a simplified fashion. In particular Schelling has developed two alternative frameworks to discuss residential segregation, the spatial proximity model and the bounded–neighborhood model (see Zhang, 2011, for a concise summary of their characteristics), both of which define the neighborhood from a spatial perspective. Such a spatial structure makes the dynamic analysis of segregation outcomes particularly cumbersome and precludes the possibility of obtaining intuitive closed-form results. In order to increase the degree of tractability, in our model the neighborhood is completely a-spatial, namely it is a point in space where the entire (white and black) population resides. By analyzing the evolution of the white flight share we can understand the population dynamics within the neighborhood.

3 Asymptotic Dynamics

As mentioned above, even though analyzing the (stochastic) finite population dynamics is not possible, it is possible to analyze the (deterministic) dynamics associated with its asymptotic version in which the number of households N is infinitely large. By following the same argument as in Blume and Durlauf (2003) and Marsiglio and Tolotti (2018), it is possible to derive the following convergence result.

Proposition 1. *Suppose that $\eta(\cdot)$ is absolutely continuous. Then, the sequence of stochastic processes x^N converges almost surely to $x \equiv (x_t)_{t\geq 0}$, where x_t is the solution of the following differential equation:*

$$\dot{x}_t = \eta\left(b - c + Jx_t\right) - x_t, \tag{4}$$

with fixed and given initial condition $x_0 \in (0,1)$.

Proposition 1 states that in such an asymptotic framework we can describe the evolution of the white flight share, which is entirely determined by the net benefits of leaving and the social externality parameter. Moreover note that (4) describes the aggregate behavior of whites, and through aggregation we lose track of the behavior of each individual household (and their type). Since the aggregate outcome in the neighborhood is entirely described by a differential equation, this can be explicitly analyzed in order to characterize the long run equilibrium outcomes.

For ease of illustration, in the following we will focus on household types uniformly distributed over the unit interval. This allows us to explicitly derive a number of conclusions, differently from previous literature which fails to provide a neat characterization of neighborhood outcomes. However, apart from the eventual loss of closed-form expressions for the equilibrium outcomes, the same qualitative results will hold true also whenever the distribution of types will be single-picked[3]. Under the uniform distribution assumption, $\eta(z)$ is given by the following expression:

$$\eta(z) = \begin{cases} 0 & \text{if} \quad z < 0 \\ z & \text{if} \quad 0 \leq z < 1 \\ 1 & \text{if} \quad z \geq 1 \end{cases} \tag{5}$$

[3] In similar settings of social interactions, it is common to assume that the distribution of types is logistic since it describes social phenomena well (Anderson et al., 1992). More generally, by relying on any unimodal continuous distribution function, different from the uniform, the results will be qualitatively similar to those we will derive under a uniform assumption, apart from the lack of closed-form expressions for the equilibrium outcomes (Blume and Durlauf, 2003; Marsiglio and Tolotti, 2018). It seems convenient to discuss our model's implications in the simplest possible setup.

In such a setting it is straightforward to show that the equilibrium outcome depends on the size of the social externality parameter (J) and the size of the net benefits ($b - c$). If $J < 1$ then there always exists a unique stable equilibrium, $\overline{x}^*$, while if $J > 1$ there might exist either a unique stable equilibrium $\overline{x}^*$ or a multiplicity of equilibria $\overline{x}_L < \overline{x}_M < \overline{x}_H$ with the middle being unstable and the extremes stable. The next two propositions summarize all the possible outcomes.

Proposition 2. *Assume that the social externality is small ($J < 1$). If the net benefit is negative ($b - c < 0$) complete segregation of the majority will occur ($\overline{x}^* = 0$); if the net benefit is positive but small ($0 < b - c < 1 - J$) integration will occur ($\overline{x}^* = \frac{b-c}{1-J}$), while if the net benefit is positive and large ($0 < b - c < 1 - J$) complete segregation of the minority will occur ($\overline{x}^* = 1$).*

Proposition 3. *Assume that the social externality is large ($J > 1$). If the net benefit is negative and large ($b - c < 1 - J < 0$) complete segregation of the majority will occur ($\overline{x}^* = 0$); if the net benefit is negative and small ($1 - J < b - c < 0$) there exists a tipping point ($\overline{x}_M = \frac{c-b}{J-1}$) determining whether complete segregation of the majority ($\overline{x}_L = 0$) or minority ($\overline{x}_H = 1$) will occur, while if the net benefit is positive ($b - c > 0$) complete segregation of the minority will occur ($\overline{x}^* = 1$).*

Proposition 2 and Proposition 3 show that according to the specific parameter configurations the neighborhood might end up in a situation of either complete segregation or integration, and which equilibrium outcome effectively occurs is determined by both economic (i.e., net benefit) and social (i.e., social externality) factors. Intuitively, independent of the size of the social externality, if the net benefit is small enough no white will leave and the neighborhood will remain populated entirely by whites. Alternatively, if the net benefit is large, all whites will leave and the neighborhood will become completely populated by blacks. Such trivial outcomes are not interesting for our analysis, thus in what follows we will not formally discuss them. More interesting is understanding nontrivial equilibrium outcomes, which depend on the relative size of economic and social factors, namely on how the social externality parameter compares with the net benefit. There are two possible non-trivial outcomes: (i) integration of whites and blacks; (ii) blacks segregation eventually resulting from tipping. Note that in both non-trivial cases the net benefit, given by $b - c$, takes an intermediate value (not too large and not too small). Integration, which is characterized by the neighborhood with a mixed composition of whites and blacks, can occur only if the equilibrium is unique and the social externality is small ($J < 1$). Black segregation can occur only if there is a multiplicity of equilibria and the social externality is large ($J > 1$), provided that the initial white flight share exceeds the tipping point ($\overline{x}_M$), represented by the middle unstable equilibrium. Only in this latter framework tipping can eventually occur.

Specifically, tipping refers to a situation in which the neighborhood starts from a white status-quo ($x_0 < \overline{x}_M$) and then it eventually tips to become predominantly black ($\overline{x}_H = 1$). This requires that the white flight share changes basin of attraction by crossing the tipping point (see Zhang, 2011, for a similar discussion of the tipping process). The next proposition summarizes the situations under which this can occur.

Proposition 4. *Tipping may occur as a result of either (i) in-migration of blacks, or (ii) a fall in the tipping point.*

Proposition 4 outlines two channels through which tipping may occur, and in both cases this can only be due to exogenous shocks. (i) Clearly, an in-migration of blacks is accompanied by an out-migration of whites and thus, by reducing the overall share of whites in the neighborhood, it also increases the white flight share. This is consistent with the interwar US experience where migration flows have been a major determinant of racial segregation (Wright, 1986; Shertzer and Walsh, 2016; Kollmann et al., 2018). (ii) Further, a reduction in the tipping point, caused by a rise in either the net benefit from leaving or in the social externality, can directly induce tipping. This is consistent with the post-war US experience where housing price considerations have played a major role in black segregation (Boustan, 2010; Brooks, 2011; Card et al., 2008). Note that these two channels for tipping are similar to those already presented in the economics literature and in the original Schelling's (1971) work, even if they have been derived in more informal ways or in less formalized frameworks. The existence of these two channels confirms that our theory is supported by empirical evidence which explains real world experiences. However, to the best of our knowledge, no other work has thus far explicitly characterized the tipping point, and its closed-form expression clearly showing that it depends both on economic and social factors allow to explain the large variation in tipping points estimates across space and over time (Card et al., 2008; Kollmann et al., 2018) as it may simply be due to some parameter difference. Moreover, in the empirical literature on tipping (Card et al., 2008; Alden et al., 2015; Shertzer and Walsh, 2016; Kollmann et al., 2018), a common assumption underlying the regression discontinuity approach is that tipping points are exactly the same in all neighborhoods within a city (Banzhaf and Walsh, 2013). However, our model suggests this is unlikely to be the case due to differential amenities across neighborhoods, and therefore in certain neighborhood white flight might not take place till the black share exceeds particularly large values (a similar conclusion is presented in different contexts by Banzhaf and Walsh, 2013; and Caetano and Maheshri, 2017).

There is no other work which has identified in such a neat and intuitive way as ours the conditions under which either integration or tipping may occur. What our analysis suggests is that even a small difference in either economic (net benefit) or social (social externality) factors can yield diametrically opposite segregation outcomes, which has important policy implications. For example, suppose that we observe a neighborhood in which growing black segregation occurs as a result of tipping, which is the case whenever $1-J < b-c < 0$. Clearly, in this context understanding how to intervene with specific policies in order to favor desegregation is a concern for local policymakers, and integration may be achieved whenever $0 < b - c < 1 - J$. In order to effectively achieve integration two different types of policy intervention are needed, affecting the net benefit and the social externality, respectively. First, the net benefit from leaving for the majority needs to increase from negative to positive; this can be implemented through specific urban planning policies aimed to increase the provision of public goods (like safety, public transport, school, parks or leisure facilities) in the neighborhood, which by increasing the potential returns from selling a dwelling located in the neighborhood D incentivizes homeowners to leave. Second, the social externality needs to decrease from above to below unity; this can be implemented through social policies aimed to favor multiculturalism and tolerance with respect to diversity in the local community. Provided that these policies affect the size of the net benefit and the social externality sufficiently to ensure that the inequality $0 < b - c < 1 - J$ holds, then it will be possible for the neighborhood to experience integration. Note that urban planning and social policies alone, which are typically identified as alternative natural solutions of racial segregation issues (Pancs and Vriend, 2007; Banzhaf and Walsh, 2013), are not enough to ultimately promote integration and could even promote further segregation. Other studies have raised concerns about the effectiveness of urban planning policies (NEJAC, 2006; Banzhaf and McCormick, 2012; Sieg et al., 2004; Banzhaf and Walsh, 2013) and social policies (Pancs and Vriend, 2007; Zhang, 2011); different from these studies which point to either the role of prices or the role of social interactions in explaining such a counterintuitive result, our model suggests that this may be due to how economic and social factors interact in determining households' residential choices.

4 Finite Dynamics

Thus far we have explicitly characterized the possible equilibrium outcomes within a neighborhood, but when studying social dynamic processes it is essential not to rely exclusively on equilibrium analysis (Pancs and Vriend, 2007). Recall that we have focused only on the deterministic asymptotic version of the stochastic finite population framework characterized by transition probabilities as in (3), however such an approach may lead to loss of important pieces of information in the presence of multiple equilibria as discussed in Marsiglio and Tolotti (2018).[4] Indeed, in this case, due to the probabilistic nature of the model and the presence of two locally stable equilibria, namely $\overline{x}_L$ and $\overline{x}_H$, it may well be possible that, despite our deterministic theory suggests that as long as $x_0 < \overline{x}_M$ the long run equilibrium will be $\overline{x}_L = 0$, the equilibrium effectively achieved will be $\overline{x}_H = 1$ meaning that the neighborhood will endogenously tip giving rise to black segregation (without any migration flow or change in the tipping point). This feature is related to the metastability properties of the two stable equilibria, whenever metastability is defined as in Benaim and Weibull (2003).[5]

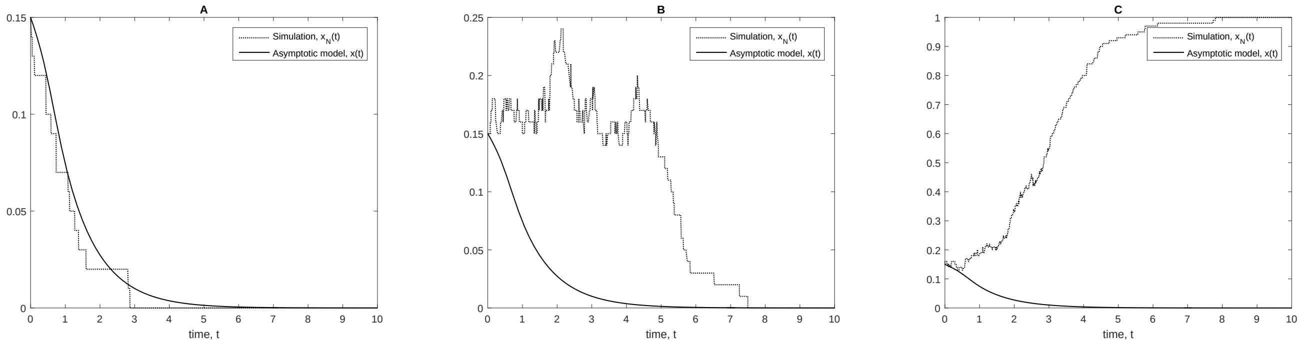

Figure 1: White flight share dynamics in the asymptotic and finite population model. The simulated white flight share x_t^N shows different types of behavior: converging straight towards the expected equilibrium (A), experiencing a transient noisy phase before converging to the expected equilibrium (B), converging towards the unexpected equilibrium (C).

We illustrate this outcome in Figure 1 which represents the evolution of the white flight share as predicted by the asymptotic deterministic model (solid curve) and resulting from a simulation of the stochastic finite population model (dotted curve). We arbitrarily set the parameter values as follows: $J = 2$, $b - c = -0.2$, $x_0 = 0.15$, $N = 100$, implying that the tipping point is $\overline{x}_M = 0.2$, consistently with the estimates of Card et al. (2008). However, it is possible to show that the results will be qualitatively identical in any other parametrization giving rise to multiplicity of equilibria, that is $1-J < b-c < 0$. Note that since $x_0 < \overline{x}_M$ we would expect, consistent with our asymptotic theory, that white flight will not occur and the neighborhood will remain entirely populated by whites, that is $\overline{x}_L = 0$. In panel A, we show the typical situation in which the white flight share converges straight towards the expected equilibrium. In panel B we show a situation in which the simulated trajectory oscillates for a transient period around the tipping point and, finally, converges towards the expected equilibrium. In panel C, in contrast, we show that the white flight share converges towards the unexpected equilibrium $\overline{x}_H = 1$: such a shift from the basin of attraction of one equilibrium to the other occurs endogenously through coordination across agents. This allows us to state the following result.

Proposition 5. *Tipping may also occur as a result of endogenous coordination across the majority.*

Proposition 5 outlines a third channel through which tipping may occur, but differently from the two identified in Proposition 4, this arises endogenously. Note that endogenous coordination across whites is a

[4]In presence of a unique equilibrium this type of problem does not occur, since in such a setting the stochastic finite population dynamics closely mimics the deterministic asymptotic dynamics (Marsiglio and Tolotti, 2018).

[5]Metastability refers to a situation in which, even if a process starts in the basin of attraction of a particular equilibrium, there exists a "tunneling time" that is a time in which the process leaves this basin. However, a precise and unique definition in economics does not exist: according to Benaim and Weibull (2003) a process is metastable if there exists a unique tunneling time (i.e., when the process leaves its original basin of attraction, it then remains outside it forever), while according to Marsiglio and Tolotti (2018) a process is metastable if there exist infinite tunneling times (i.e., when the process leaves its original basin of attraction, it then keeps returning into and leaving it). Note that in our framework neighborhood tipping is due to metastability meant as in Benaim and Weibull (2003), but not as in Marsiglio and Tolotti (2018).

peculiarity of our framework and deals with its probabilistic properties. As long as the trajectory of the stochastic process is still in its transient phase (i.e., out of equilibrium), it can leave the basin of attraction of the theoretically predicted equilibrium and reach an unexpected equilibrium. To the best of our knowledge, there in no other work in which tipping occurs endogenously except for Zhang's (2011), in which tipping is driven by a probabilistic framework similar to ours. However, in his model black segregation is the only possible outcome, while in our setting white segregation or integration of blacks and whites may also occur. Moreover, following Schelling's (1969, 1971, 1978) model his analysis is performed in terms of the pair of different-ethnicity neighbors, while ours is in terms of the white share which is the variable commonly employed in the empirical analysis of segregation (Card et al., 2008). However, note that unlike the other two channels, tipping due to endogenous coordination across agents cannot be easily captured in empirical works, and this actually raises concerns about the robustness of the tipping points estimates obtained through the regression discontinuity approach.

The possibility that a neighborhood will endogenously tip even in the absence of exogenous shocks further stresses the relevance of our previous policy discussion. The need to implement specific policies in order to favor desegregation does not arise only when growing minority segregation occurs but also whenever there is multiplicity of equilibria such that a tipping point exists. In fact, even if the white flight share does not exceed the tipping point, black segregation may potentially occur via endogenous coordination. In this case combining urban planning and social policies might be the best strategy in order to effectively achieve desegregation.

5 Conclusion

There is an established literature arguing that social interactions may be an essential determinant of racial segregation. We build on Schelling's works (1969, 1971, 1978) by introducing some economic mechanisms in a social interactions framework to determine the extent to which economic and social factors jointly determine segregation. We develop a simple analytically tractable agent-based model to explicitly characterize and identify the conditions under which integration or minority segregation via tipping might occur as a result of optimizing households' behavior when choosing whether to leave or stay in a neighborhood. Their decision crucially depends on the relative size of economic (net benefits from leaving) and social (social externality) factors. We explicitly analyze the asymptotic population model to characterize the long run equilibrium and obtain a closed-form expression for the tipping point, which depends on both economic and social factors. We also analyze the finite population model to identify the possibility of crossing the basin of attraction of the (locally) stable equilibria. This allows us to determine three possible channels, two related to exogenous shocks (migration flows and changes in tipping points) and one to the model's endogenous probabilistic features (endogenous coordination across agents), through which tipping may occur. The characterization of integration and segregation conditions point to the need for intervention through urban planning and social policies to promote desegregation.

Despite its simplicity we believe that our model is striking and intuitive in capturing the possible determinants of racial segregation, but further research is needed to clarify the mechanisms underlying tipping. Our closed-form expression for the tipping point has the potential to inform future empirical research, since it provides some testable predictions which permit testing of our model's implications without relying on the regression discontinuity approach. Moreover, as discussed in Schelling (1971) and in the later social interactions literature, residential decisions often give rise to segregation clusters, meaning that there are some spatial patterns which need to be further examined. Bringing theory to the data in order to validate our conclusions and better explaining the spatial dynamics of racial segregation are left for future research.

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