Population and neoclassical economic growth: a new child policy perspective°

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**Abstract**

Using the basic OLG model of neoclassical growth with endogenous fertility, we show that a child tax can be used as a single instrument to actually raise population growth in the long run, while also raising per capita income.

*Keywords* Child tax; Fertility; OLG model

*JEL Classification* H24; J13; J18

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## 1. Introduction

An issue of major concern so far recognized in the economic literature deals with the merit of population control policies as an inducement to per capita income growth.[1] This line of reasoning dates back to Malthus (1798) and his followers – such as Stuart Mill (1965) –, and shows some recent extreme applications: for instance, the one-child per family policy used by the Chinese government (see Coale, 1981). It is worth noting that a crucial feature of the exogenous growth theoretical literature is the inverse relationship between per capita income and population growth. Indeed, armed with this theoretical trade-off, the policymakers in many developing countries (even under the advise of sovranational institutions) aimed at increasing per capita income through anti-natalist policies. Moreover, apart from the authority of either the classical economists or the neoclassical growth theory, it remains an important open question whether and how an anti-natalist policy, used essentially to raise the income of the currently living people, can be legitimate or not, especially for those countries ascribing a significant importance to the people's freedom to procreate.

In this paper we analyse and discuss the effects of child taxes on both macroeconomic and demographic variables in the basic overlapping generations (OLG) model of neoclassical growth (Diamond, 1965) with endogenous fertility. The main result of this paper is that a child tax can be adopted as a single instrument to actually raise population growth in the long run, while also raising per capita income. Therefore, (*a*) the trade-off between per capita income growth and population growth may be relaxed, and (*b*) in contrast with the anti-natalist policy prescriptions of the standard neoclassical growth theory, a child tax policy may be growth-enhancing by preserving a pro-natalist population view.

The remainder of the paper is organised as follows. In Section 2 we develop the model. In Section 3 we analyse and discuss the main results. Section 4 concludes.

## 2. The model

Identical agents live in a three-period OLG economy. Life is divided into childhood, young adulthood and old-age. During childhood individuals do not make economic decisions. Adult individuals belonging to generation $t$ ($N_t$) have a homothetic and separable utility function ($U_t$) defined over young-aged consumption ($c_t^{\ y}$), old-aged consumption ($c_{t+1}^{\ o}$) and the number of children ($n_t$),[2] as in Galor and Weil (1996). Only young-adult individuals join the workforce, and the labour supply is assumed to be constant and normalised to unity. As an adult, each young receives the competitive wage $w_t$ for each unit of labour. This income is used to consume, to bear children and to save. Raising children require a fixed amount of resources $m > 0$ per child (measured in units of consumption goods). Moreover, the government levies a constant per child tax ($\beta > 0$) and the revenues so collected are lump-sum rebated within the same working age (child bearing) generation ($\tau_t > 0$). Old-age individuals are retired and live on the proceeds of their savings ($s_t$) plus the accrued interest at the rate $r_{t+1}$.

Therefore, the representative individual entering the working period at time $t$ faces the problem:

$$\max_{\{c_t^{\ y}, c_{t+1}^{\ o}, n_t\}} U_t\left(c_t^{\ y}, c_{t+1}^{\ o}, n_t\right) = (1-\phi)\ln\left(c_t^{\ y}\right) + \gamma \ln\left(c_{t+1}^{\ o}\right) + \phi \ln\left(n_t\right), \tag{P}$$

subject to

$$\begin{aligned} c_t^{\ y} + s_t &= w_t + \tau_t - (m+\beta)n_t \\ c_{t+1}^{\ o} &= (1+r_{t+1})s_t \end{aligned},$$

where $0 < \gamma < 1$ is the subjective discount factor and $0 < \phi < 1$ captures the taste for children relative to material consumption when young.

The first order conditions are:

$$\frac{c_{t+1}^{\ o}}{c_t^{\ y}} \cdot \frac{1-\phi}{\gamma} = 1 + r_{t+1}, \tag{1}$$

$$\frac{c_t^{\ y}}{n_t} \cdot \frac{\phi}{1-\phi} = m + \beta\,. \tag{2}$$

[1] As emphasised by Ehrlich and Lui (1997, p. 232),“ Government interference has been justified on the normative argument that the exercise of free choice by parents does not necessarily lead to socially optimal outcomes, as well as on the positive argument that since per capita income is nothing but the ratio of output to population, $q = Q/N$, an effective way of permanently uplifting the latter above a miserable level of subsistence would be to lower the denominator.”.

[2] The variable $n$ represents the number of children with $n-1$ being the population growth rate.

Eq. (1) equates the marginal rate of substitution between working period and retirement period consumptions to their relative prices, whereas Eq. (2) equates the marginal rate of substitution between consuming when young and having children to the marginal cost of raising an extra child.

Using Eqs. (1) and (2) together with the lifetime budget constraint, we get the demand for children and the saving function, respectively:

$$n_t = \frac{\phi}{1+\gamma} \cdot \frac{w_t + \tau_t}{m+\beta}, \tag{3}$$

$$s_t = \frac{\gamma}{1+\gamma}(w_t + \tau_t). \tag{4}$$

As regards the public sector, the government levies a fixed per child tax and the revenues so collected are lump-sum rebated within the same child bearing generation according to the following per capita balanced budget formula:

$$\tau_t = \beta n_t, \tag{5}$$

where the left-hand side represents the government expenditure and the right-hand side the (child) tax receipts. Notice that agents act in an atomistic way and do not take Eq. (5) into account when deciding on the number of children and on the saving rate.[3]

Firms are identical and act competitively. Aggregate production takes place according to the constant returns to scale Cobb-Douglas technology $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$,[4] where $K_t$ and $L_t = N_t$ are, respectively, the capital input and the labour input combined at time $t$ to produce the output $Y_t$, $A > 0$ is a scale parameter and $0 < \alpha < 1$ is the distributive capital share. Therefore, the intensive form production function is $y_t = Ak_t^{\alpha}$, where $k_t := K_t / N_t$ and $y_t := Y_t / N_t$ are per capita capital and output, respectively. The price of final output is normalised to unity and capital depreciates fully at the end of each period. Therefore, profit maximisation leads to the following marginal conditions for capital and labour, respectively:

$$r_t = \alpha A k_t^{\alpha-1} - 1, \tag{6}$$

$$w_t = (1-\alpha) A k_t^{\alpha}. \tag{7}$$

Now, inserting Eq. (5) into Eq. (3) to eliminate $\tau_t$ and rearranging terms yields:

$$n_t = \frac{\phi\, w_t}{(1+\gamma)m + (1+\gamma-\phi)\beta}, \tag{8}$$

Combining Eqs. (4), (5) and (8), the saving function is:

$$s_t = \frac{\gamma(m+\beta)w_t}{(1+\gamma)m + (1+\gamma-\phi)\beta}. \tag{9}$$

Using Eq. (5) and knowing that $N_{t+1} = n_t N_t$, the equilibrium in goods as well as in capital markets is given by $n_t k_{t+1} = s_t$. Using Eqs. (8) and (9) to substitute out for $n_t$ and $s_t$ respectively, the equilibrium condition boils down to the following (constant) per capita stock of capital in the long run

$$k_{t+1} = k^*(\beta) = \frac{\gamma}{\phi}(m+\beta). \tag{10}$$

## 3. Income and fertility in the long run

### *3.1. Income*

From Eq. (10) it can easily be seen that a rise in the child tax monotonically increases the long-run per capita stock of capital, that is, $\frac{\partial k^*(\beta)}{\partial \beta} = \frac{\gamma}{\phi} > 0$. Therefore, the following proposition holds:

**Proposition 1.** *The long-run per capita income increases monotonically with the child tax.*

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[3] Notice that this policy does not weight upon the current elderly. The government in fact levies a fixed per child tax and uses the revenues so collected to finance a lump-sum subsidy within the same working-age (child-bearing) generation.

[4] The introduction of an exogenous rate of labour productivity growth would not have changed any of the results of the present paper; hence, it is not included here.

**Proof.** Since $y^*(\beta) = Ak^*(\beta)$ and $\partial k^*(\beta)/\partial\beta > 0$ for any $\beta > 0$, then Proposition 1 follows. **Q.E.D.**

The economic intuition is the following. *In primis*, since the (child) tax revenue is rebated as a lump-sum subsidy in the same working age (child-bearing) generation, it increases the income of current workers (savers) thus enhancing both savings and capital accumulation. *In secundis*, the offspring tax is expected to reduce the heads in the whole economy, and thus it may increase per capita income as well. However, in the next section we will see that the straightforward negative effect of child taxes on fertility is not always assured. In fact, in the long run, depending on the parameters of the problem, both the rate of population growth and per capita income can be increased at the same time with the child tax. In any case, Proposition 1 shows that the induced increase in savings is always higher than the rise in the rate of fertility.

*3.2. Fertility*

To analyse how the child tax affects the fertility rate, let us rewrite the steady-state demand for children as a generic function of $\beta$ as

$$n^* = n^*\{\beta, w^*[k^*(\beta)]\}. \tag{11}$$

The total derivative of Eq. (11) with respect to $\beta$ gives:[5]

$$\frac{dn^*}{d\beta} = \overbrace{\frac{\partial n^*}{\partial \beta}}^{-} + \underbrace{\overbrace{\frac{\partial n^*}{\partial w^*}}^{+} \cdot \overbrace{\frac{\partial w^*}{\partial k^*}}^{+} \cdot \overbrace{\frac{\partial k^*}{\partial \beta}}^{+}}_{+}. \tag{12}$$

Eq. (12) shows that the final effect of a rise in the child tax on the long-run individual fertility is ambiguous and depends on two counterbalancing forces: (*a*) a negative (direct) effect which reduces population growth by increasing the cost of raising an extra child, and (*b*) a positive (indirect) general equilibrium feedback effect which acts on fertility through the increased wage rate. In particular, a rise in the child tax increases the long-run stock of capital per person. A higher stock of capital in turn enhances wage income. Given the positive relationship between fertility and wages (namely a Malthusian Fertility Effect) the higher the wage rate, the higher the demand for children.

To analyse ultimately which of the two forces dominates, we now combine Eqs. (7), (8) and (10) to obtain:

$$n^*(\beta) = \frac{\phi(1-\alpha)A\left[\frac{\gamma}{\phi}(m+\beta)\right]^{\alpha}}{(1+\gamma)m + (1+\gamma-\phi)\beta}. \tag{13}$$

Therefore, the following propositions hold:

**Proposition 2.** *Let $\phi < \bar{\phi}$ hold. Then introducing a child tax increases the fertility rate in the long run.*

**Proof.** Differentiating Eq. (13) with respect to $\beta$ and evaluating it at $\beta = 0$ yields:

$$\frac{\partial n^*(\beta)}{\partial \beta}\Big|_{\beta=0} = \frac{n^*(0)\cdot(\phi - \bar{\phi})}{m(1+\gamma)},$$

so that

$$\begin{cases} \frac{\partial n^*(\beta)}{\partial \beta}\Big|_{\beta=0} > 0 \quad iff \quad \phi > \bar{\phi} \\ \frac{\partial n^*(\beta)}{\partial \beta}\Big|_{\beta=0} < 0 \quad iff \quad \phi < \bar{\phi} \end{cases},$$

where $\bar{\phi} := (1-\alpha)(1+\gamma)$ and $n^*(0)$ is the long-run fertility rate before applying the child tax. **Q.E.D.**

**Proposition 3.** *(1) Let $\phi < \bar{\phi}$ hold. Then $n^*(\beta) < n^*(0)$ for any $\beta > 0$.*

[5] Details are given in the Appendix.

*(2) Let* $\phi > \bar{\phi}$ *hold. Then* $n^*(\beta)$ *is an inverted U-shaped function of the child tax with* $\beta = \beta_n$ *being an interior global maximum, and* $n^*(\beta) > n^*(0)$ *for any* $0 < \beta < \hat{\beta}$ *and* $n^*(\beta) < n^*(0)$ *for any* $\beta > \hat{\beta}$ *where* $\hat{\beta} > \beta_n$.

**Proof.** The proof uses the following derivative:

$$\frac{\partial n^*(\beta)}{\partial \beta} = \frac{n^*(\beta) \cdot \left[m(\phi - \bar{\phi}) - \beta(1-\alpha)(1+\gamma-\phi)\right]}{(m+\beta) \cdot \left[(1+\gamma)m + (1+\gamma-\phi)\beta\right]}.$$

If $\phi < \bar{\phi}$ then $\frac{\partial n^*(\beta)}{\partial \beta} < 0$, and hence $n^*(\beta) < n^*(0)$ for any $\beta > 0$. This proves point (1).

If $\phi > \bar{\phi}$ then

$$\frac{\partial n^*(\beta)}{\partial \beta} \gtrless 0 \Leftrightarrow \beta \lesseqgtr \beta_n,$$

where

$$\beta_n := \frac{m(\phi - \bar{\phi})}{(1-\alpha)(1+\gamma-\phi)}, \tag{14}$$

represents the fertility-maximising child tax. Since $n^*(0) > 0$, $n^*(\beta)$ is a positive (negative) monotonic function of $\beta$ for any $0 < \beta < \beta_n$ ($\beta > \beta_n$) and $\lim_{\beta \to +\infty} n^*(\beta) = 0$, then there always exists a finite threshold value $\hat{\beta} > \beta_n$ such that $n^*(\hat{\beta}) = n^*(0)$, and thus $n^*(\beta) > n^*(0)$ for any $0 < \beta < \hat{\beta}$ and $n^*(\beta) < n^*(0)$ for any $\beta > \hat{\beta}$. This proves point (2). **Q.E.D.**

Proposition 2 reveals the existence of a threshold value of the parents' taste for children ($\phi = \bar{\phi}$) which discriminates against the effectiveness of the child tax as an instrument to disincentive individual fertility. In particular, if parents are relatively children interested ($\phi > \bar{\phi}$)[6] then, rather unexpectedly, the introduction of a per child tax increases the demand for children in the long run, i.e., the population growth rate is higher than before applying offspring taxes.

As a consequence, Proposition 3 shows that if the preference for having children is high enough ($\phi > \bar{\phi}$) then the fertility rate in the long run increases along with the per child tax if the latter is not fixed at too high a level, that is $0 < \beta < \beta_n$. In fact, when $\beta$ is low enough, the positive indirect general equilibrium effect due to the increased wage rate prevails on the negative direct effect due to the increased cost of children: hence, the population growth rate raises along with the child tax. Moreover, a fertility-maximising child tax does exist as well (see Eq. 14). By contrast, if the taste for children is low enough ($\phi < \bar{\phi}$), the fertility rate in the long run is always lower than in the absence of child taxation, since the negative direct effect owing to the higher cost of children always prevails.

The economic intuition behind Propositions 2 and 3 is the following. A rise in the child tax implies a substitution effect which reduces the demand for children by increasing the total cost of having an extra child. However, since the child tax revenue is rebated as a lump-sum subsidy within the same working age generation, a twofold effect emerges: (*a*) in the short run, a direct income effect that may partially counterbalance the substitution effect; and (*b*) in the long run, an enlargement of the budget constraint due to a higher per capita stock of capital (and thus a higher wage rate) which produces an overall income effect that may more then fully counterbalance the substitution effect.

Finally, given the result stated in Propositions 1 and 3, we have:

**Proposition 4.** *Let* $\phi > \bar{\phi}$ *hold. Then, in the long run the government can increase both the per capita income and the population growth rate with a per child tax policy such that* $0 < \beta \le \beta_n$.

**Proof.** The proof is obvious given the results stated in Propositions 1 and 3. **Q.E.D.**

*3.3. Partial and general equilibrium analyses of the child tax policy*

[6] Alternatively, if the technology of production is relatively capital oriented and/or individuals are relatively impatient.

For a better understanding of the long-run effects of the per child tax policy enunciated by Propositions 1-4, we now highlight the twofold effect at work: the (short-run) partial equilibrium effect and the (long-run) general equilibrium effect. In particular, at the moment of the introduction of the child tax, the wage earned by the young workers is kept constant because the stock of capital is unchanged. Since the child tax is rebated in a lump-sum way as a subsidy to the working age generation, the child tax policy does not shift the aggregate budget constraint faced by such a generation. However, it changes the marginal incentive of the young. The result is a shift along the lifetime budget constraint to a lower fertility rate and a higher saving rate, thus leading to a higher per capita stock of capital installed in the whole economy in the subsequent period. This represents the partial equilibrium effect at work, since the wage and the interest rate are momentarily unaffected, which is even coherent with the policy prescriptions of the traditional neoclassical growth theory, that is, the per capita income is increased and the population growth rate is reduced. However, the (short-run) partial equilibrium effect is not the end of the story. In an OLG context, in fact, the stock of capital affects both the wage and the interest rate in subsequent periods. Let us now detail step by step the dynamical effects of the child policy at work. Assume a per child tax is introduced at the beginning of period $t$: this leads – as expected – to a higher saving rate as well as a lower demand for children in the same period, while the per capita stock of capital and the wage rate both are kept unchanged. As a consequence of the higher saving rate and the lower fertility rate, a higher capital stock will be installed at the beginning of period $t+1$, $k_{t+1}$, and then also the wage rate, $w_{t+1}$, will be higher than that we had before adopting the child tax. Therefore, the saving rate and the fertility rate at time $t+1$ both will be higher than those we had at time $t$. In particular, the effect of higher wages on fertility may be high enough to uplift the population growth rate over the level we had at time $t-1$, in spite of the disincentive effect induced by the child tax. Remarkably, we note that the percentage increase of both savings and fertility is the same from period to period: hence, the capital per person will stay forever constant at the level established at time $t+1$ and, ultimately, the long-run per capita income and the fertility rate both will be higher than the corresponding values before applying the child policy (loosely speaking, in the long run the income effect prevails on the substitution effect).

To grasp the meaning of the mechanism described above and to further understand how the child tax reform affects savings, fertility, the per capita stock of capita and the wage rate over time, in the following table we present a numerical example based on parameter values chosen only for illustrative purposes, that is, $A=10$, $\alpha=0.60$, $\gamma=0.10$, $m=0.20$, $\phi=0.50$.[7] In particular, we assume that at time $t-1$ the policy is absent ($\beta=0$), while at time $t$ as well as in all subsequent periods $\beta=0.05$.

**Table 1**. Child tax: Macroeconomic and demographic effects.

| | $i=t-1$ | $i=t$ | $i=t+1$ | $i=t+j$, $j=2\ldots+\infty$ |
|---|---|---|---|---|
| $k_i$ | 0.04 | 0.04 | 0.05 | 0.05 |
| $w_i=w_i(k_i)$ | 0.5798 | 0.5798 | 0.6628 | 0.6628 |
| $s_i=s_i(w_i(k_i))$ | 0.0527 | 0.0579 | 0.0662 | 0.0662 |
| $n_i=n_i(w_i(k_i))$ | 1.3177 | 1.1596 | 1.3257 | 1.3257 |

Table 1 shows that when the child tax is introduced (time $t$), the existence of a partial equilibrium effect (*a*) increases the saving rate and (*b*) reduces the fertility rate, with both capital and wages being unchanged due to the OLG structure of the economy (see column 3). However, the existence of a general equilibrium effect implies that the generation entering the working period at time $t+1$ (as well as all subsequent generations)[8] will experience an increase in the stock of capital – and thus in wages – which more than counterbalances the negative partial equilibrium effect by uplifting definitively the population growth rate over the pre child tax level (time $t-1$), and by further expanding private savings (see columns 4 and 5).

Our paper therefore shows that the negative trade-off between (neoclassical) economic growth and population growth must be dramatically reconsidered in the light of the effects of child taxes. In fact, when the parents' taste for children is relatively high, a per child tax should be adopted as a single policy instrument to promote both per capita income and population growth.

[7] Notice that: (*a*) the value of the distributive capital share used here holds for several developing and developed countries (see Rodriguez and Ortega, 2006, Table A1); (*b*) the subjective discount factor generates a propensity to save around 10 per cent, which represents a realistic value for many countries (see OECD Economic Outlook 2008); (*c*) the cost of children has been calibrated to be almost $1/3$ of the workers' income; (*d*) $A$ is simply a scale parameter in the Cobb-Douglas production function. Notice also that this parameter set generates $\bar{\phi}=0.44$ and $\beta_n=0.05$.

[8] It is worth noting that from time $t+1$ onwards all variables are in steady state.

## 4. Conclusions

We discussed both macroeconomic and demographic effects of child taxation in the basic overlapping generations model of neoclassical growth with endogenous fertility. We showed that the negative trade-off between per capita income growth and population growth, which is a tenet of the standard neoclassical growth theory, is dramatically reconsidered. Indeed the main result of this paper is that under some plausible conditions, such as high preference for having children, high distributive capital share and low degree of thriftiness, a child tax should be adopted as a single instrument to actually raise population growth in the long run, while also raising the long-run per capita income. The policy implications are straightforward: in contrast with the current anti-natalist policies (aiming at reducing population growth to permanently uplift per capita income) suggested by the standard neoclassical growth theory and applied in many developing countries, the introduction of a child tax would provide a single instrument able of achieving a higher income per person along with a higher fertility rate. The economic interpretation of our findings is at least twofold: (*a*) a child tax can promote the (neoclassical) economic growth by preserving a pro-natalist population view, and (*b*) if for some exogenous reasons an anti-natalist population view existed, then every child could be subsidised rather than taxed.

## Appendix

Effects of the child tax on the rate of fertility:

$$\frac{\partial n^*}{\partial \beta} = \frac{-(1+\gamma-\phi)\phi w^*}{\left[(1+\gamma)m + (1+\gamma-\phi)\beta\right]^2} < 0\,, \tag{A1}$$

$$\frac{\partial n^*}{\partial w^*} = \frac{\phi}{(1+\gamma)m + (1+\gamma-\phi)\beta} > 0\,, \tag{A2}$$

$$\frac{\partial w^*}{\partial k^*} = \alpha(1-\alpha)A\left(k^*\right)^{\alpha-1} > 0\,. \tag{A3}$$

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