An OLG model of growth with longevity: when grandparents take care of grandchildren

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Abstract By assuming that grandparents take care of grandchildren, this paper aims at studying the effects of longevity on long-term economic growth in a model with overlapping generations and endogenous fertility. We show that an increase in longevity may: (*i*) reduce the long-term economic growth; (*ii*) increase the supply of labour, and (*iii*) cause fertility either to increase of decrease depending on the size of time spent by grandparents to rise grandchildren. These findings also hold in an endogenous growth setting à la Romer (1986).

Keywords Longevity; OLG model

JEL Classification J13; J22; O41

1. Introduction

A recent stylised fact is the decline in adult mortality that several countries around the world have experienced. The rise in life expectancy in developed countries resulted in a doubling of the ratio of life cycle years lived after 65 to years lived 20 to 64 (Livi-Bacci, 2006), and it is expected to increase further on in the near future (see the time evolution of the old-age dependency ratio in countries such as Italy, Japan and Spain as reported in Sinn, 2007, Figure 6, p. 10). However, the sharp increase in longevity has also augmented the share of leisure time that the elderly can devote to take care of grandchildren, which is sometime an almost entirely substitutes for parental care.

So far the economic growth literature, at least the models with overlapping generations (OLG), has retained that a positive relationship between longevity and economic growth exists (Ehrlich and Lui, 1991), essentially because of the increase in savings and capital accumulation that a longer life span causes. Indeed, Fanti (2009) shows in a Diamond's growth model augmented with endogenous fertility motivated by the so-called "weak form of altruism" (see Zhang and Zhang, 1998), that a rise in the life span increases savings and reduces the demand for children, so that capital accumulation increases more than when fertility is not an economic decision variable. Such a positive relationship seems to hold also when the textbook Diamond's OLG model is extended to account for endogenous fertility motivated by altruistic reasons with voluntary inter-generational bequests, social security and education (Zhang et al., 2001),[1] or when adult mortality is affected by public investments

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[1] For instance, Zhang et al. (2001, p. 485) claim: "A rise in longevity has direct effects on fertility, human capital investment, and growth, as well as indirect effects through increasing unfunded social security contributions…

in health (Chakraborty, 2004). In fact, "Yet health plays a role quite unlike any other human capital: by increasing lifespans it makes individuals effectively more patient and willing to invest, and by reducing mortality risks, it raises the return on investment… when life expectancy is low, individuals discount the future more heavily and are less inclined to save and invest.", Chakraborty, 2004, p. 120).[2]

However, some empirical works have shown that the relationship between longevity and economic growth may be hump-shaped, namely when life expectancy is fairly low (high), a increase in it causes economic growth to raise (fall), see, e.g., Maddison (1992), Kelley and Schmidt (1995) and Barro and Sala-i-Martin (2004). To the best of our knowledge, from the theoretical grounds the sole model that accords with these empirical facts is Zhang et al. (2003), which departs from the basic Diamond's model by extending Zhang et al. (2001) with accidental bequests. Indeed, it is just through the interaction between education and accidental bequests that a rise in longevity can actually reduce the rate of economic growth.

It should be noted that the above mentioned theoretical literature abstracts from another stylised fact recently evidenced: the role that grandparents have in raising grandchildren. As an example, ISFOL (2011) reveals that in Italy 36 per cent of women that belongs to the active population decide to have a child only whether some forms of child care assistance inside the home, provided by non-cohabitant family members, e.g., grandparents, exist. This phenomenon can actually produce macroeconomic effects through the reduction in the opportunity cost of children by parents and the rise in both the labour supply by women and the fertility rate.

The aim of this paper is to investigate how the existence of an exogenous provision of grandparental child care activity inside the home affects the relationship between longevity and long-term economic growth. The importance of grandmothering is proved to exist in human behaviour and it has implications with regard to their social organisation (Hawkes et al., 1998). For the purpose of analysing this question in an economic model, we consider a general equilibrium OLG model of neoclassical growth with exogenous longevity and endogenous fertility. By assuming a perfect annuities market and taking into account the evidence of the importance of the child care assistance by grandparents (Minkler and Fuller-Thomson, 1999; Hayslip and Kaminski, 2005; Hank and Buber, 2009), as a substitute for parental child care inside the home, we show that an increasing longevity (*i*) increases the supply of labour by the young parents, (*ii*) causes fertility either to increase of decrease depending on the size of the grandparental child rearing time, and (*iii*) can actually work as a growth-reducing device in a simple OLG model à la Diamond (1965). Moreover, we also find that a rise in the time devotes by grandparents to take care of grandchildren does not affect the labour supply of the young parents.

The value added of the present paper essentially lies in the provision of another explanation of the hump-shaped relationship between longevity and long-term growth, as evidenced by the empirical literature above mentioned.

The rest of the paper is organised as follows. Section 2 builds on the model and discusses the main results. Section 3 presents the conclusions. Appendix 1 analyses the case of endogenous growth à la Romer (1986) with grandparents that take care of grandchildren, and compares it with the model of neoclassical growth presented in the main text. Appendix B introduces a CIES (Constant Inter-temporal Elasticity of Substitution) utility function.

2. The model

The net effects of rising longevity on fertility tend to be negative, but positive on human capital investment and growth."

[2] See also Blackburn and Cipriani (2002).

Consider a general equilibrium OLG closed economy populated by perfectly rational and identical individuals. Life of each person within every generation is divided between childhood and adulthood. Economic decisions are exclusively made in the latter period of life, which is in turn divided between youth (working period) and old age (retirement period). As an adult, each individual has preferences towards material consumption and the number of children, as in Eckstein and Wolpin (1985), Eckstein et al. (1988) and Galor and Weil (1996), which are assumed to be a normal good. This is the so-called weak form of altruism towards children (see Zhang and Zhang, 1998), because parents directly derive utility from the number of children they have but do not enjoy from the utility drawn by their offspring. Young agents of generation t (N_t) have an endowment of one unit of time, a fraction of which ($0<h_t<1$) is supplied to firms while earning the wage w_t per unit of labour, and the remaining part ($0<q<1$) is spent to care for n_t descendants, with q being the exogenous fraction of time endowment that each parent devotes to raise a child (Galor and Weil, 1996; Morand, 1999).[3] We assume that each young individual dies at the onset of old age with an exogenous probability $1-\pi$, that is $0<\pi\leq 1$ is the probability of surviving at the end of youth. Moreover, we also assume that each grandparent (i.e., those belonging to generation $t-1$) devotes a constant fraction $0<z<1$ of her time endowment to take care of each grandchild, that is the provision of child care assistance inside the home by the old reduces the opportunity cost for parents to rear children. Since only πN_{t-1} old individuals are alive at time t, the labour supply of the young of generation t can definitely be written as follows:

$$h_t = 1 - n_t(q - z\pi), \tag{1}$$

where $z < q/\pi := z_1$ must hold as a technical condition to ensure the existence of a positive number of children. In this model, therefore, h_t is endogenous because individuals choose the number of children (i.e., n is an endogenous variable), once the values of q, z and π are fixed exogenously. Things would be different if both fertility and labour supply were endogenous variables (this is left for future research DA ELIMINARE APPENA ARRIVANO LE PROOF). Note that $h_t > 0$ implies $n_t < 1/(q - z\pi) := \overline{n}$, which is the maximum number of children that a young individual can give birth to, and the higher the time spent raising a child by the young (resp. old), the lower (resp. higher) $\overline{n}$. Then, the budget constraint of young individuals reads as follows:

$$c_{1,t} + s_t = w_t h_t = w_t[1 - n_t(q - z\pi)], \tag{2}$$

that is, the labour income is divided between consumption ($c_{1,t}$) and saving (s_t).

Old individuals retiree and live with the amount of resources saved when young plus the expected interest from period t to period $t+1$ at the interest factor R^e_{t+1}. The existence of a perfect annuities market (where savings are intermediated through mutual funds) implies that old survivors will benefit not only from their own past saving plus interest, but also from

[3] Note that in order to take into account the negative substitution effect – on fertility – of the female labour earnings due to the potential increase of women's labour force participation (see Mincer, 1963), it is possible to introduce DA MODIFICARE COSì a gender gap to differentiate the child bearing technology on the basis of the ability of male and female to raise children. In fact, it seems widely accepted that the low female labour participation exerts a depressing role on economic growth, while also being a reason of the inverse relationship between fertility and growth, because of the high opportunity cost (wage) of raising children in developed countries, which, in turn, increases with the growth rate.

the saving plus interest of those who have deceased.[4] Therefore, the budget constraint at time $t+1$ of the typical old individual that started working at time t can be expressed as follows:

$$c_{2,t+1} = \frac{R^e_{t+1}}{\pi} s_t \,, \tag{3}$$

where $c_{2,t+1}$ is consumption when old.

The typical individual of generation t chooses fertility and saving to maximise the expected lifetime utility function

$$U_t = \ln(c_{1,t}) + \pi \ln(c_{2,t+1}) + \gamma \ln(n_t)\,, \tag{4}$$

subject to (2) and (3) where $\gamma > 0$ is the parents' relative taste for children. Therefore, fertility and saving are respectively determined as follows:

$$n_t = n = \frac{\gamma}{(q - z\pi)(1+\pi+\gamma)}\,, \tag{5}$$

$$s_t = \frac{\pi w_t}{1+\pi+\gamma}\,. \tag{6}$$

From (6) we see that saving does not depend on the interest factor because of the hypothesis of logarithmic utility, so that the substitution effect and the income effect cancel each other out in that case (de la Croix and Michel, 2002). With the more general CIES (Constant Inter-temporal Elasticity of Substitution) utility, saving would depend on the interest factor and it would be increasing or decreasing in it depending on the relative value of the inter-temporal elasticity of substitution (i.e., the substitution effect and income effect are different).

In addition, it is clear that the number of children is constant (Eq. 5). Through (1) and (5), therefore, the labour supply is given by

$$h_t = h = \frac{1+\pi}{1+\pi+\gamma}\,, \tag{7}$$

and it is constant as well.

From (7) we have the following proposition.

Proposition 1. *[Labour supply]. (1) A rise in life expectancy (π) monotonically increases the supply of labour. (2) A rise in child-rearing time by the old people (z) does not affect the supply of labour.*

Proof. Since $\partial h / \partial \pi = \gamma / (1+\pi+\gamma)^2 > 0$ and $\partial h / \partial z = 0$, then Proposition 2 follows. **Q.E.D.**

Now, let $z_2 := q/(1+\gamma)$ and $z_3 := q/(3+\gamma)$ be two threshold values of z, where $z_3 < z_2 < z_1$. Then, from (5) the following proposition is established.

Proposition 2. *[Fertility]. (1) Let $0 < z < z_3$ hold. Then, the rate of fertility monotonically decreases with the rate of longevity. (2) Let $z_3 < z < z_2$ hold. Then, the rate of fertility is a U-shaped function of the rate of longevity. (3) Let $z_2 < z < z_1$ hold. Then, the rate of fertility monotonically increases with the rate of longevity.*

Proof. The proof uses the following derivative:

[4] This is a reasonable hypothesis to characterise developed economies where markets for annuities are largely adopted. It should be noted, however, that the results of the present paper also hold by assuming accidental bequests (Abel, 1985).

$$\frac{\partial n}{\partial \pi} = \frac{\gamma[z(1+\gamma) - q + 2z\pi]}{(q - z\pi)^2(1+\pi+\gamma)^2} \underset{>}{\overset{<}{}} 0 \Leftrightarrow \pi \underset{>}{\overset{<}{}} \pi_n := \frac{q - z(1+\gamma)}{2z}. \tag{8}$$

If $0 < z < z_3$, then $\pi_n > 1$ and $\frac{\partial n}{\partial \pi} < 0$ for every $0 < \pi < 1$. If $z_3 < z < z_2$, then $0 < \pi_n < 1$ and $\frac{\partial n}{\partial \pi} \underset{>}{\overset{<}{}} 0$ if and only if $\pi \underset{>}{\overset{<}{}} \pi_n$, where π_n is the fertility-minimising rate of longevity. If $z_2 < z < z_1$, then $\pi_n < 0$ and $\frac{\partial n}{\partial \pi} > 0$ for every $0 < \pi < 1$. **Q.E.D.**

Propositions 1 and 2 reveal that it is possible to jointly increase both the labour supply and fertility when longevity increases, provided that the time devoted to the child care assistance by grandparents is sufficiently high. In fact, a trade-off between labour supply and fertility exists when grandparents do not spend enough time for caring their grandchildren. However, the rise in the child care inside the home (grandparental effect) together with observed increase in longevity in developed countries, may represent a possible reason of the existence of a positive relationship between labour supply and fertility (Apps and Rees, 2004).

The value-added of this theoretical framework basically lies in Eq. (1), which describes the mechanics of the model and determines the labour supply of young persons, h_t, that is equal to the time endowment, 1, minus the constant fraction of time q spent by parents to care for n_t children net of the fraction of time z (multiplied by the proportion of old individuals alive in that generation, π) spent by grandparents to care for their n_t grandchildren. If $z = 0$, the model collapses to the standard OLG framework where parents take care of their children. Solving (1) for the fertility yields: $n_t = (1 - h_t)/(q - z\pi)$, where the denominator is positive by assumption. Now, assume that π increases, meaning a higher longevity. Labour supply increases as a result of (7). Given q (the time spent by parents to care for children), the number of children can increase only if z (the time spent by grandparents to care for grandchildren) is sufficiently high. We note Point 2 of Proposition 1 holds because of the assumption of is logarithmic utility. In Appendix 2 we discuss the more general case of CIES preferences, where we show that the behaviour of the labour supply in the short term with respect to z depends on the elasticity of substitution.

Firms are identical and markets are competitive. The (aggregate) constant returns to scale Cobb-Douglas technology is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where Y_t, K_t and $L_t = h_t N_t$ are output, capital and the time-t labour input, respectively, $A > 0$ and $0 < \alpha < 1$. Therefore, output per young ($y_t := Y_t / N_t$) is obtained as $y_t = Ak_t^{\alpha} h_t^{1-\alpha}$, where $k_t := K_t / N_t$ is the stock of capital per young person. Assuming that capital fully depreciates at the end of every period and output is sold at unit price, profit maximisation implies:

$$R_t = \alpha A (k_t / h_t)^{\alpha - 1}, \tag{9}$$

$$w_t = (1 - \alpha) A (k_t / h_t)^{\alpha}, \tag{10}$$

Knowing that $N_{t+1} = n_t N_t$, the equilibrium on the capital market is

$$n_t k_{t+1} = s_t. \tag{11}$$

Combining (5), (6), (7), (10) and (11) we get:

$$k_{t+1} = \frac{\pi}{\gamma}(1 - \alpha) A k_t^{\alpha} (q - z\pi)[h(\pi)]^{-\alpha}. \tag{12}$$

Fixed points of (12) are defined as $k_{t+1} = k_t = k$. Then the following proposition holds.

Proposition 3. *A rise in life expectancy ambiguously affects the long-term stock of capital.*

Proof. From (12) it can easily be seen that

$$k = k[\pi, h(\pi)]. \tag{13}$$

The total derivative of (13) with respect to k gives:

$$\frac{dk}{d\pi} = \overbrace{\frac{\partial k}{\partial \pi}}^{+/-} + \overbrace{\frac{\partial k}{\partial h}}^{-} \cdot \overbrace{\frac{\partial h}{\partial \pi}}^{+}, \tag{14}$$

and Proposition 3 follows. **Q.E.D.**

A reduction in adult mortality affects capital accumulation through a twofold channel which goes through (*i*) saving and fertility, and (*ii*) the labour supply. First, a rise in life expectancy directly increases saving and capital accumulation but, depending on the relative size of the time spent by grandparents to care for grandchildren (see Proposition 2), it ambiguously affects fertility. Moreover, it increases the labour supply (see Proposition 1), and this in turn tends to reduce the wage per unit of labour earned by the young parents and then capital accumulation reduces through this channel. Second, there exists a direct negative effect on capital accumulation due to the rise in the labour supply.

With regard to the output per young person in the long term, we find that

$$y = y\{k[\pi, h(\pi)], h(\pi)\}, \tag{15}$$

and the following proposition holds.

Proposition 4. *A rise in life expectancy ambiguously affects long-term neoclassical economic growth.*

Proof. Totally differentiation (13) with respect to k gives:

$$\frac{dy}{d\pi} = \overbrace{\frac{\partial y}{\partial k}}^{+} \cdot \left(\overbrace{\frac{\partial k}{\partial \pi}}^{+/-} + \overbrace{\frac{\partial k}{\partial h}}^{-} \cdot \overbrace{\frac{\partial h}{\partial \pi}}^{+} \right) + \overbrace{\frac{\partial y}{\partial h}}^{+} \cdot \overbrace{\frac{\partial h}{\partial \pi}}^{+}, \tag{16}$$

and Proposition 4 follows. **Q.E.D.**

Proposition 4 shows that a rise in life expectancy still remains ambiguous on the long-term economic growth even if the negative effect of it is weakened, as compared to the effect capital accumulation, by the positive effect on output per capital induced by the increase in the labour supply.

We now turn to numerical simulations (Table 1) to illustrate Proposition 3 and 4.[5]

Example 1. Parameter set: $A = 10$, $\alpha = 0.33$ (Gollin, 2002) and $q = z = \gamma = 0.3$.

Table 1. Long-term capital stock, output, labour supply, fertility and saving when longevity increases ($z = 0.3$).

π	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
k	0.529	1.238	1.843	2.235	2.362	2.211	1.803	1.196	0.505	0.194
y	6.896	9.24	10.648	11.45	11.753	11.58	10.894	9.569	7.235	5.291

[5] Table 1 also shows the corresponding values of both the labour supply and fertility rate. Note also that with this parameter set $z_2 = 0.23$ and $z_3 = 0.09$, while $\pi_k = 0.495$ and $\pi_y = 0.514$ which represent the capital- and output-maximising longevity rates, respectively.

h	0.785	0.8	0.812	0.823	0.833	0.842	0.85	0.857	0.863	0.866
n	0.793	0.833	0.892	0.98	1.111	1.315	1.666	2.38	4.545	8.888
s	0.42	1.031	1.646	2.191	2.624	2.909	3.005	2.849	2.296	1.727

The following result holds.

Result 1. *When the time spent by grandparents to care for grandchildren if sufficiently high, both capital accumulation and output per young person are inverted U-shaped functions of the rate of longevity.*

The inverse relationship between longevity and long-term growth described here accords with the empirical evidence presented by Maddison (1992), Kelley and Schmidt (1995) and Barro and Sala-i-Martin (2004). The net effect gets through four channels in this simple general equilibrium economy: (*i*) the labour supply, (*ii*) fertility (*iii*) saving and (*iv*) capital accumulation. When grandparents devote a relatively large amount of their time endowment to the child care assistance inside the home, a rise in longevity causes an increase in both the supply of labour and fertility (see Proposition 1 and 2, respectively). Saving instead first increases, because individuals expect to live longer, but then decreases because the rise in the labour supply reduces the wage. Due to the reduction in saving and the rise in fertility, capital accumulation becomes lower when longevity is large enough. Though the increase in the supply of labour causes a positive direct effect on output per young, the negative effect of the reduced capital accumulation dominates when longevity is high, and this definitely causes a reduction in economic growth in the long term.

Example 2. Parameter set: $A = 10$, $\alpha = 0.33$, $q = \gamma = 0.3$ and $z = 0$.

Table 2. Long-term capital stock, output, labour supply, fertility and saving when longevity increases ($z = 0$).

π	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
k	0.62	1.73	3.14	4.8	6.65	8.68	10.88	13.22	15.7	17
y	7.26	10.31	12.69	14.73	16.53	18.18	19.71	21.14	22.49	23.14
h	0.78	0.8	0.81	0.82	0.83	0.84	0.85	0.86	0.86	0.86
n	0.71	0.67	0.62	0.59	0.55	0.53	0.5	0.47	0.45	0.44
s	0.44	1.15	1.96	2.82	3.7	4.57	5.44	6.3	7.13	7.55

Example 3. Parameter set: $A = 10$, $\alpha = 0.33$, $q = \gamma = 0.3$ and $z = 0.1$.

Table 3. Long-term capital stock, output, labour supply, fertility and saving when longevity increases ($z = 0.1$).

π	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
k	0.59	1.56	2.68	3.87	5.06	6.22	7.31	8.32	9.22	9.62
y	7.14	9.97	12.05	13.72	15.11	16.29	17.29	18.14	18.86	19.18
h	0.78	0.8	0.81	0.82	0.83	0.84	0.85	0.86	0.86	0.86
n	0.74	0.71	0.69	0.67	0.66	0.65	0.65	0.65	0.65	0.65
s	0.43	1.11	1.86	2.62	3.37	4.09	4.77	5.4	5.99	6.26

Example 4. Parameter set: $A = 10$, $\alpha = 0.33$, $q = \gamma = 0.3$ and $z = 0.2$.

Table 4. Long-term capital stock, output, labour supply, fertility and saving when longevity increases ($z = 0.2$).

π	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
k	0.56	1.39	2.25	3.01	3.63	4.05	4.25	4.23	4	3.8
y	7.02	9.61	11.37	12.63	13.54	14.14	14.46	14.52	14.32	14.11
h	0.78	0.8	0.81	0.82	0.83	0.84	0.85	0.85	0.86	0.86
n	0.76	0.77	0.78	0.8	0.83	0.87	0.93	1.01	1.13	1.21
s	0.42	1.07	1.75	2.42	3.02	3.55	4	4.32	4.54	4.6

Result 2. *When the time spent by grandparents to cake for grandchildren is sufficiently low, both capital accumulation and output per young person monotonically increase with the rate of longevity. When the time spent by grandparents to care for grandchildren becomes higher, Result 1 holds.*

3. Conclusions

We studied the effects of longevity on long-term economic growth in an OLG model à la Diamond (1965) with endogenous fertility. As recent empirical works argued, it is natural to presume that grandparents help parents in the caring of their children. Until now the theoretical literature avoided to include this phenomenon in macroeconomic models. This paper represents a first attempt to fill this gap by allowing for the grandchildren care assistance by grandparents. In particular, by assuming perfect annuities market and grandparents that devotes an exogenous fraction of their time endowment to take care of grandchildren, by reducing the opportunity cost of children for the young parents, we showed the existence of an inverse relationship between longevity and economic growth. This holds in both the neoclassical growth context à la Solow (1956) and endogenous growth setting à la Romer (1986).

We avoid to include endogenous labour-leisure choices in the second period of life, the former being favoured by the reduced adult mortality (indeed, several governments are aiming at implementing policies to lengthen the age of retirement and/or promote the employment of the elderly). A natural extension of the present paper can be to introduce an endogenous time allocation for old people (for instance, through the choice of how much time to devote either to working activities or child care assistance), or, alternatively, include male and female labour participation. To this regard, several governments around the world are trying to implement policies aiming at increasing the female labour participation rate as a stimulus to economic growth. The existence of the VIA child care activity by grandparents, by alleviating the time-cost of children by women that belong to the active population VIA, seems to be to go in that direction. However, we conjecture that the final effect on long-term economic growth of a rise in female participation rates may be ambiguous because of the increase in fertility that it may cause.

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Appendix 1. Endogenous growth

This appendix builds on a model of endogenous growth à la Romer (1986) and compares the results of it with those of the model of neoclassical growth studied in the main text. With regard to the consumers' side, nothing changes with respect to the model of Section 2 so that

Propositions 1 and 2 continues to hold. With regard to the production side, we now assume that the technology of production faced by each firm $i = 1, \dots, I$ is the following:

$$Y_{i,t} = K_{i,t}^{\alpha}(A_{i,t}L_{i,t})^{1-\alpha} = B\,k_t^{1-\alpha}K_{i,t}^{\alpha}L_{i,t}^{1-\alpha}, \tag{17}$$

where $Y_{i,t}$, $K_{i,t}$ and $L_{i,t}$ represent the output produced, and capital and labour hired by firm i, respectively, $A_{i,t} := a(K_t / N_t)$ is an index of labour productivity of each single firm, which is assumed to depend on the average stock of capital per young person in the whole economy, $k_t = K_t / N_t$, and it is taken as given by firm i, $B := a^{1-\alpha} > 0$ is a scale parameter and $0 < \alpha < 1$. Since all firms are identical, by setting $L_{i,t} = L_t$, $K_{i,t} = K_t$ and $Y_{i,t} = Y_t$, aggregate production at t is described by the technology $Y_t = Bk_t^{1-\alpha}K_t^{\alpha}L_t^{1-\alpha}$, where Y_t, K_t and $L_t = h_t N_t$ are the aggregate values of output, capital and the time-t labour input, respectively, $B > 0$ and $0 < \alpha < 1$. Therefore, output per young person ($y_t := Y_t / N_t$) is obtained as $y_t = Bk_t h_t^{1-\alpha}$. By assuming that capital fully depreciates at the end of every period and output is sold at unit price, profit maximisation implies:

$$R_t = \alpha B h_t^{1-\alpha}, \tag{18}$$

$$w_t = (1-\alpha)Bk_t h_t^{-\alpha}, \tag{19}$$

The equilibrium in the capital market is still given by (11). Then, capital accumulation is the following:

$$k_{t+1} = \frac{\pi}{\gamma}(q - \pi z)(1-\alpha)Bk_t\left(\frac{1+\pi+\gamma}{1+\pi}\right)^{-\alpha}. \tag{20}$$

From (20) we obtain the constant growth rate of the economy, that can be written as follows:

$$g = \frac{\pi}{\gamma}(q - \pi z)(1-\alpha)B\left(\frac{1+\pi+\gamma}{1+\pi}\right)^{-\alpha} - 1. \tag{21}$$

We note that the growth rate of the economy is determined by (21) while fertility and the labour supply are constant and given by (5) and (7), respectively.

Starting from (21), numerical simulations (not reported here to save space) show that longevity affects both the labour supply and fertility according to the rules of Propositions 1 and 2 (this holds for different values of the time spent by grandparents to raise grandchildren). Of course, this is in accordance with the fact the introducing endogenous growth à la Romer (1986) does not modify the consumers' behaviour described in Section 2. In addition, we find that the behaviour of the rate of economic growth when longevity varies follows Results 1 and 2. This is summarised in Result 3.

Result 3. *When the time spent by grandparents to cake for grandchildren is sufficiently low, the growth rate of the economy monotonically increases with the rate of longevity. When the time spent by grandparents to care for grandchildren if sufficiently high, the growth rate of the economy is an inverted U-shaped functions of the rate of longevity.*

Appendix 2. CIES preferences

This appendix describes the model with grandmothering under CIES (Constant Inter-temporal Elasticity of Substitution) utility and fertility. For references with regard to the assumption of CIES preferences, see Michel and de la Croix (2002) and de la Croix and Michel (2002) with exogenous fertility, and Spataro and Fanti (2011) and Fanti and Gori (2013) with endogenous fertility.

The lifetime utility of the individual representative of generation t is given by the following CIES formulation:

$$U_t = \left(1-\frac{1}{\sigma}\right)^{-1} c_{1,t}^{1-\frac{1}{\sigma}} + \pi\left(1-\frac{1}{\sigma}\right)^{-1} c_{2,t+1}^{1-\frac{1}{\sigma}} + \gamma\left(1-\frac{1}{\sigma}\right)^{-1} n_t^{1-\frac{1}{\sigma}}, \tag{22}$$

where $\sigma > 0$ $(\sigma \neq 1)$ is the (constant) inter-temporal elasticity of substitution. The maximisation of (30) with respect to n_t and s_t subject to the budget constraints

$$c_{1,t} + \frac{\pi c_{2,t+1}}{R_{t+1}^e} = w_t[1 - n_t(q - z\pi)], \tag{23}$$

gives the following fertility and saving formulations:

$$n_t = \frac{\gamma^\sigma w_t}{w_t^\sigma (q - z\pi)^\sigma [1 + \pi(R_{t+1}^e)^{\sigma-1} + \gamma^\sigma w_t^{1-\sigma}(q - z\pi)^{1-\sigma}]}, \tag{24}$$

$$s_t = \frac{w_t \pi (R_{t+1}^e)^{\sigma-1}}{1 + \pi(R_{t+1}^e)^{\sigma-1} + \gamma^\sigma w_t^{1-\sigma}(q - z\pi)^{1-\sigma}}. \tag{25}$$

Of course, the threshold $z < q/\pi := z_1$ should hold to preserve the economic meaning of time constraint (1). Then, from (1) and (24) the labour supply is given by:

$$h_t = \frac{1 + \pi(R_{t+1}^e)^{\sigma-1}}{1 + \pi(R_{t+1}^e)^{\sigma-1} + \gamma^\sigma w_t^{1-\sigma}(q - z\pi)^{1-\sigma}}. \tag{26}$$

Different from the case of log-utility ($\sigma = 1$), fertility and the labour supply now depend on factor prices. From (26) the following proposition holds.

Proposition 5. *[Labour supply under CIES preferences]. (1) A rise in life expectancy (π) increases the supply of labour in the short term if and only if* $z > \frac{q(R_{t+1}^e)^{\sigma-1}}{\sigma - 1 + \sigma\pi(R_{t+1}^e)^{\sigma-1}}$. *(2) A rise in child-rearing time by the old people (z) increases (resp. decreases) the labour supply of the young parents in the short term if $\sigma > 1$ (resp. $\sigma < 1$).*

Proof. Since $\operatorname{sgn}\frac{\partial h}{\partial \pi} = \operatorname{sgn}\{z[\sigma - 1 + \sigma\pi(R_{t+1}^e)^{\sigma-1}] - q(R_{t+1}^e)^{\sigma-1}\}$ and $\operatorname{sgn}\frac{\partial h}{\partial z} = \operatorname{sgn}(\sigma - 1)$, the result follows. **Q.E.D.**

References

Abel A (1985) Precautionary savings and accidental bequests. Am Econ Rev 75:777–791

Apps P, Rees R (2004) Fertility, taxation and family policy. Scand J Econ 106:745–763

Barro RJ, Sala-i-Martin X (2004) Economic Growth. Second Edition. Massachusetts MA, MIT Press

Blackburn K, Cipriani GP (2002) A model of longevity, fertility and growth. J Econ Dyn Control 26:187–204

de la Croix D, Michel P (2002) A Theory of Economic Growth. Dynamics and Policy in Overlapping Generations. Cambridge University Press, Cambridge

Diamond PA (1965) National debt in a neoclassical growth model. Am Econ Rev 55: 1126–1150

Eckstein Z, Wolpin KI (1985) Endogenous fertility and optimal population size. J Public Econ 27:93–106

Eckstein Z, Stern S, Wolpin KI (1988) Fertility choice, land, and the Malthusian hypothesis. Int Econ Rev 29:353–361

Ehrlich I, Lui FT (1991) Intergenerational trade, longevity, and economic growth. J Polit Econ 99:1029–1059

Fanti L (2009) Longevity, fertility and Demographic Transition in an OLG model. Econ Bull 29:348–357

Fanti L, Gori L (2013) Fertility-related pensions and cyclical instability. J Popul Econ (2013) 26:1209–1232

Galor O, Weil DN (1996) The gender gap, fertility, and growth. Am Econ Rev 86:374–387

Gollin D (2002) Getting income shares right. J Polit Econ 110:458–474

Hank K, Buber I (2009) Grandparents caring for their grandchildren: findings from the 2004 survey of health, ageing, and retirement in Europe. J Fam Issues 30:53–73

Hawkes K, O'Connell JF, Blurton Jones NG, Alvarez H, Charnov EL (1998) Grandmothering, menopause, and the evolution of human life histories Proc Natl Acad Sci 95:1336–1339

Hayslip B Jr, Kaminski PL (2005) Grandparents raising their grandchildren: a review of the literature and suggestions for practice. Gerontologist 45:262–269

ISFOL (2011) Occupazione e maternità: modelli territoriali e forme di compatibilità. Roma: Istituto per lo Sviluppo della Formazione Professionale dei Lavoratori. I Libri del Fondo Sociale Europeo 148

Kelley A, Schmidt R (1995) Aggregate population and economic growth correlations: the role of the components of demographic change. Demography 32:543– 555

Livi-Bacci M (2006) A Concise History of World Population. Forth Edition. Malden MA, Wiley-Blackwell

Maddison A (1992) A long-run perspective on savings. Scand J Econ 94:181–196

Michel P, de la Croix D (2000) Myopic and perfect foresight in the OLG model. Econ Lett 67(1):53–60

Mincer J (1963) Market prices, opportunity costs, and income effects. In Christ CF (Ed.), Measurement in Economics: Studies in Mathematical Economics and Econometrics in Memory of Yehuda Grunfeld. Stanford CA, Stanford University Press

Minkler M, Fuller-Thomson E (1999) The health of grandparents raising grandchildren: results of a national study. Am J Public Health 89:1384–1389

Morand OF (1999) Endogenous fertility, income distribution, and growth. J Econ Growth 4:331–349

Romer PM (1986) Increasing returns and long-run growth. J Polit Econ 94:1002–1037

Sinn HW (2007) Introduction to: Europe and the demographic challenge. CESifo Forum 8:7–11

Solow RM (1956) A contribution to the theory of economic growth. Q J Econ 70:65–94

Spataro L, Fanti L (2011) The optimal level of debt in an OLG model with endogenous fertility. Ger Econ Rev 12(3):351–369

Zhang J, Zhang J (1998) Social security, intergenerational transfers, and endogenous growth. Can J Econ 31:1225–1241

Zhang J, Zhang J, Lee R (2001) Mortality decline and long-run economic growth. J Public Econ 80:485–507

Zhang J, Zhang J, Lee R (2003) Rising longevity, education, savings, and growth. J Dev Econ 70:83–101