Endogenous lifetime, accidental bequests and economic growth

Luciano Fanti • Luca Gori • Fabio Tramontana

Abstract This paper introduces the concept of unintentional bequests in a closed economy à la Chakraborty (2004) with overlapping generations. We show that scarce public investments in health can lead to poverty traps depending on the relative size of the output elasticity of capital. More importantly, the existence of unintentional bequests, rather than a market for annuities, means that health tax rates play a prominent role in determining the stability of the long-run equilibrium in rich economies. In fact, Neimark-Sacker bifurcations and endogenous fluctuations occur depending on the size of the public health system.

Keywords Accidental bequests; Endogenous lifetime; Health; OLG model

JEL Classification C62; I18; J18; O4

1. Introduction

Demography (fertility and longevity) has been recognised as playing a prominent role in economic growth and development (see, amongst many others, Becker and Barro, 1988; Mason, 1988; Barro and Becker, 1989; Fogel, 1994, 2004, de la Croix and Licandro, 1999; Galor and Weil, 1999, 2000; Galor and Moav, 2002; de la Croix and Doepke, 2003, 2004; Barro and Sala-i-Martin, 2004; Moav, 2005; Kraay and Raddatz, 2007). The economic causes and consequences of the reduction in both birth and mortality rates, observed in several developed countries in the recent decades (Livi-Bacci, 2006), have led economists to carry out thorough investigations regarding the interrelationships between demographic and macroeconomic outcomes (Cervellati and Sunde, 2005, 2011; Galor, 2005, 2010; Lorentzen et al., 2008). One reason for this is the tremendous impact on policies that the steadily declining number of young (active) people in total population as well as the steadily increasing number of elderly may have in the near future.[1] The burgeoning macroeconomic theoretical literature

L. Fanti
Department of Economics, University of Pisa, Via Cosimo Ridolfi, 10, I–56124 Pisa (PI), Italy
e-mail: lfanti@ec.unipi.it or fanti.luciano@gmail.com
tel.: +39 050 22 16 369; fax: +39 050 22 16 384

L. Gori (corresponding author)
Department of Law, University of Genoa, Via Balbi, 30/19, I–16126 Genoa (GE), Italy
e-mail: luca.gori@unige.it or dr.luca.gori@gmail.com
tel.: +39 010 209 95 03; fax: +39 010 209 55 36

F. Tramontana
Department of Economics and Quantitative Methods, University of Pavia, Via San Felice, 5, I–27100 Pavia (PV), Italy
e-mail: fabio.tramontana@unipv.it

[1] As an example, think of the provision of public pensions, which are mainly organised on a pay-as-you-go basis in several European countries: i.e., the income of current workers is taxed by the government to finance the benefits received by the current pensioners (see Fanti and Gori, 2012a). There are extensive debates between

that has focused on growth models with overlapping generations (OLG) and endogenous lifetime, has tried to shed light on the nature and causes of economic development by assuming either exogenous fertility or endogenous fertility.

With regard to the former class of models, Chakraborty (2004) introduces endogenous lifetime into the standard two-period OLG model by Diamond (1965) and considers a market for annuities. He assumes that the probability of surviving from the first period of life (youth) to the next (old age) depends on an individual's health status, which is improved through financing public investments in health. The main result provided by Chakraborty (2004) is that poverty traps can result from scarce health investments, because a shorter life span acts as a disincentive to save and accumulate capital further on. Chakraborty and Das (2005) build on a model with human capital, private health expenditure and intentional bequests (i.e., inter vivos transfers) to study the reasons why inequality persists between rich and poor countries. Bhattacharya and Qiao (2007) assume that an individual's lifetime is dependent on his/her health status which is, in turn, improved by private health investments accompanied by a tax-financed public health program. They show that the economy is exposed to endogenous fluctuations when the private and public inputs in the longevity function are complementary. Leung and Wang (2010) studied a model with a private system of health care services and found that saving and health complement each other. From a normative point of view, de la Croix and Ponthière (2010) show that the steady-state Golden Rule of capital accumulation in an economy with endogenous lifetime is lower than the standard Diamond's (1965). Finally, Fanti and Gori (2012b) introduce endogenous lifetime in an OLG small open economy with a perfect market for annuities and show that an increase in public health investment can actually reduce savings because of the counterbalancing forces at work. In fact, a rise in the labour income tax rate: (*i*) increases life expectancy, so that savings increase, and (*ii*) reduces the disposable income of the young workers, so that savings are reduced. They also show that the public health policy can represent an A–Pareto improvement (see Golosov et al., 2007 for the concept of A–efficiency and P–efficiency).

With regard to the latter class of models, i.e. those with endogenous lifetime and endogenous fertility, Blackburn and Cipriani (2002) show that there are regimes of development: the former characterised by low income, high fertility and a short life span, the latter by high income, low fertility and a long life span. Their model is in agreement with the empirical evidence of the Demographic Transition. Fanti and Gori (2010) extend Chakraborty's model (2004) to endogenous fertility under the hypothesis of a weak form of altruism towards children (see Zhang and Zhang, 1998), and show that (*i*) low and high regimes of development can co-exist, and (*ii*) an adequate child tax policy can effectively help people to permanently escape from poverty and maximise long-term welfare. Indeed, Varvarigos and Zakaria (2012) find that tax-financed public expenditure that complements private health investments provides an additional explanation for the decline in fertility throughout economic growth.

The distinctive feature of the introduction of longevity in the basic OLG model by Diamond (1965) is the treatment of the savings of the deceased people. There are two polar cases, which obviously also include intermediate cases: (*i*) perfect annuities markets, i.e., savings are fully annuitized. Old survivors benefit not only from their own past savings plus interest, but also from savings plus the interest of those who have deceased, and the savings are intermediated through mutual funds; (*ii*) no annuities markets, as in Abel (1985), i.e., the savings of the deceased become accidental or unintentional bequests[2] to their own offspring.

economists to find appropriate ways to reform the social security system (e.g., Boeri et al., 2001, 2002; Cigno, 2007; Cigno and Werding, 2007) because of concerns regarding population ageing.

[2] Other major bequest motives are altruism and exchange. While there is no consensus on which motive dominates (see, e.g., Altonji et al., 1997), Hurd (1997) argues that bequests are largely accidental.

While Chakraborty (2004) assumes the former hypothesis, in this paper we focus on the latter. This introduces an important new factor: the delayed levels of longevity rate also matter, so that the dynamics of the economy are characterised by a two-dimensional non-linear system instead of the one-dimensional system that describes the dynamics in Chakraborty's model (2004).

The main purpose of this paper is to study how public investments in health affect economic growth and stability of the steady state equilibria in an OLG economy à la Chakraborty (2004) by assuming unintentional bequests rather than a market for annuities. While in Chakraborty (2004), both the lowest and highest steady-state equilibria are always locally asymptotically stable with monotonic trajectories, we show that the existence of unintentional bequests left by the deceased to their offspring makes the financing of health care services responsible for the existence of non-monotonic (local) dynamics. Neimark-Sacker bifurcations and endogenous fluctuations occur when there are threshold effects of health investments on longevity. Moreover, global analysis reveals that increasing the health tax rate too much can have the undesirable effect of permanently entrapping an economy in poverty. This is because when two locally stable attractors coexist, trajectories can converge towards the origin even when starting from initial conditions that are in fact closer to strictly positive steady state.

The rest of the paper is organised as follows. Section 2 presents the model. Section 3 analyses the dynamics, and the local and global stability properties of an OLG model with endogenous lifetime, public health investments and unintentional bequests. Section 4 outlines the conclusions.

2. The model

2.1. Individuals

Consider a general equilibrium OLG closed economy populated by perfectly rational and identical two-period-lived individuals. Life is divided between youth and old age (as in Diamond, 1965). The former is a work period fixed with certainty, the latter is a retirement period, whose length is uncertain. Population is fixed and constant at N. We assume that the typical agent within every generation is either dead or alive at the beginning of the retirement period with probability $1-\pi$ and π, respectively. When he or she is young, the individual representative of generation t is endowed with one unit of labour inelastically supplied to firms, while receiving wage income w_t per unit of labour (used for consumption and saving purposes). In addition, the government collects wage income taxes at a constant rate $0<\tau<1$ to finance public health expenditure with a balanced budget. Since agents do not know when they will die, additional unintentional bequests can occur.[3] If the typical agent of generation t dies at the onset of old age (with probability $1-\pi_t$), his/her accumulated savings are in full bequeathed to his/her heirs. To keep the representative agent formulation tractable, the bequests

$$b_{t+1} = (1-\pi_t) R^e_{t+1} s_t \,, \tag{1}$$

where s_t is saving and $R^e_{t+1} := 1 + r^e_{t+1}$ represents the expected gross interest factor accrued from time t to time $t+1$ (with r^e_{t+1} being the expected interest rate), are assumed to be equally divided amongst all the young people in every generation. This means that the bequest dependent wealth distribution is uniform, as in Hubbard and Judd (1987). This

[3] Note that, unlike Chakraborty (2004), our model is developed by assuming unintentional bequests without a market for annuities.

assumption enables us to conduct a representative agent analysis and to specifically focus on the effects of changes in longevity.[4]

The work period budget constraint and the retirement period budget constraint of an individual that belongs to generation t read, respectively, as:

$$c_{1,t} + s_t = w_t(1-\tau) + b_t, \quad (2.1)$$

$$c_{2,t+1} = R^e_{t+1} s_t, \quad (2.2)$$

where $c_{1,t}$ and $c_{2,t+1}$ are young-age consumption and old-age consumption. Eq. (2.1) implies that bequests are equally allocated across all members within a certain generation. By taking factor prices and bequests as given, the individual representative of generation t chooses how much to save out of his/her disposable income to maximise the expected lifetime utility function

$$U_t = \ln(c_{1,t}) + \pi_t \ln(c_{2,t+1}), \quad (3)$$

subject to Eqs. (2.1) and (2.2), where the death contingent utility index is normalised to zero.[5] The first order conditions for an interior solution of the problem are given by:

$$\frac{c_{2,t+1}}{c_{1,t}} \cdot \frac{1}{\pi_t} = R^e_{t+1}. \quad (4.1)$$

Eq. (4.1) equates the marginal rate of substitution between consumption when young and when old with the interest factor determined on the capital market. From Eq. (4.1) it is clear that in an economy with accidental bequests, individuals take into account the effects of longevity on the inter-temporal substitution between consumption when young and consumption when old, that is they internalise the social benefits of an increase in individual longevity due to a rise in health spending. An increase in longevity makes it convenient to postpone consumption to the future. This represents the first difference between an economy with accidental bequests and an economy with a perfect market for annuities. Indeed, in the latter, each individual does not take into account the (social) benefits of an increase in public healthcare investments on (individual) health and longevity, because when a person dies at the onset of old-age, his/her savings are divided amongst all the members of a generation (the size of which being contingent on *average* longevity), so that the benefits of an increased life span on savings are too small to be taken into account by each individual in the market (see Fanti and Gori, 2012b), and the first order condition Eq. (4.1) would be modified to become $\frac{c_{2,t+1}}{c_{1,t}} = R^e_{t+1}$. In contrast, in an economy with unintentional bequests, the savings of a deceased person are equally bequeathed in full to his/her own descendants. Indeed, unlike an economy with unintentional bequests, when a (perfect) market for annuities exists, old survivors benefit not only from their own past savings plus interest, but also from the savings plus interest of those who have died. Savings are then allocated to mutual funds and invested in order to guarantee a gross return that depends on mortality rates of the surviving old agents (which are all annuitized).[6]

Combining Eqs. (2.1), (2.2) and (4.1), we obtain the following saving function:

[4] This can indeed be assumed in a context of exogenous fertility. Things would be different if fertility was endogenous.

[5] See, e.g., Abel (1985), Chakraborty (2004), Chakraborty and Das (2005), Pestieau et al. (2008), Chakraborty et al. (2010), Fanti and Gori (2012b) for similar formulations of expected utility functions.

[6] Every annuitant deposits his/her savings with a mutual fund. Savings are then invested by the fund to get a return factor (independent of longevity). Then: (*i*) if an annuitant is alive, he/she gets savings plus the return factor divided by the longevity rate; (*ii*) if an annuitant dies at the onset of old age, the contract with the fund ends and his/her savings are distributed between all the survived annuitants. The situation is different with accidental bequests, as savings of deceased are directly bequeathed to his/her descendants.

$$s_t = \frac{\pi_t}{1+\pi_t}[w_t(1-\tau)+b_t], \tag{4.2}$$

where b_t is determined by the one-period backward Eq. (1). As expected, the existence of accidental bequests positively affects savings.

2.2. Firms

Identical firms act competitively on the market. At time t, the homogeneous output Y_t is produced by combining capital (K_t) and labour ($L_t = N$ in equilibrium) through the constant returns to scale Cobb-Douglas technology $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where $A > 0$ is a scale parameter in the production function and $0 < \alpha < 1$ the output elasticity of capital. Since capital totally depreciates at the end of every period and output is sold at the unit price, profit maximisation implies that the interest factor and wage equal the marginal products of capital and labour, respectively, that is:

$$R_t = \alpha A k_t^{\alpha-1}, \tag{5}$$

$$w_t = (1-\alpha) A k_t^{\alpha}, \tag{6}$$

where $k_t := K_t / N$ is the stock of capital per young person.

2.3. The public health system and endogenous lifetime

We follow Chakraborty (2004) and assume that at time t, health investments $h_t N$, where h_t is the health investment per young person, are financed at a balanced budget with a labour income tax levied by the government at the (constant) rate $0 < \tau < 1$, so that the tax receipts are $\tau w_t N$. The health budget per young person can then be written as:

$$h_t = \tau w_t. \tag{7}$$

In addition, the survival probability at the end of youth of an individual born at time t, π_t, is assumed to positively depend on the individual's health status, which is in turn increased by health investments per young person h_t, so that $\pi_t = \pi(h_t)$. Following Blackburn and Cipriani (2002) and de la Croix and Ponthière (2010), we specialise this relationship with the following function:

$$\pi_t = \pi(h_t) = \frac{\gamma \Delta h_t^{\delta}}{1+\Delta h_t^{\delta}}, \tag{8}$$

where $\delta, \Delta > 0$, $0 < \gamma \leq 1$, $\pi'(h) > 0$, $\lim_{h\to\infty} \pi(h) = \gamma \leq 1$, $\pi''(h) < 0$ if $\delta \leq 1$ and $\pi''(h) \underset{<}{\overset{>}{-}} 0$ for any $h \underset{>}{\overset{<}{-}} h_T := \left[\frac{\delta - 1}{(1+\delta)\Delta}\right]^{\frac{1}{\delta}}$ if $\delta > 1$. The demographic parameter γ captures the intensity of the efficiency of health investments. A rise in γ may be interpreted as exogenous medical advances. The parameters δ and Δ determine both the turning point of $\pi'(h)$ and speed of convergence of the rate of longevity towards the saturating value γ. In particular, δ measures how an additional unit of health investment is transformed into higher longevity through health technology. If $\delta \leq 1$, $\pi(h)$ is concave for any h and, hence, no threshold effects exist, thus longevity increases less than proportionally from zero up to γ as h rises. If $\delta > 1$ the longevity function is S-shaped and threshold effects exist: i.e., longevity increases more (resp. less) than proportionally before (resp. after) the threshold h_T. This means that the more intense threshold effects are (high values of δ), the slower an additional unit of health investment is transformed into a higher life span when h is relatively low, while approaching the saturating value γ more efficiently and rapidly as h becomes larger (e.g. Martikainen et al., 2009 and Fioroni, 2010 for empirical evidence). In this case δ measures the intensity of threshold effects of the accumulated health capital as an inducement to higher longevity.

2.4. Equilibrium

Given the government budget Eq. (7), equilibrium the capital market can be written as:

$$k_{t+1} = s_t . \tag{9}$$

Combining Eqs. (4.2), the one-period backward Eqs. (1) and (9), equilibrium implies:

$$k_{t+1} = \frac{\pi_t}{1+\pi_t}[w_t(1-\tau) + (1-\pi_{t-1})R_t k_t], \tag{10}$$

which is independent of expectations about future factor prices. This means that the dynamics of capital are not contingent on expectations, as will be seen later.[7]

Now, using the equilibrium conditions in the factor markets Eqs (5) and (6), and knowing that the longevity function Eq. (6) can be expressed as $\pi(h_t) = \pi(\tau(1-\alpha)Ak^{\alpha}) := \Pi(k_t)$ by making use of Eqs. (7) and (8), capital accumulation is driven by the following second-order nonlinear difference equation:

$$k_{t+1} = \frac{\Pi(k_t)}{1+\Pi(k_t)} A k_t^{\alpha} \{(1-\alpha)(1-\tau) + \alpha[1-\Pi(k_{t-1})]\}, \tag{11}$$

which can also be written as follows:

$$k_{t+1} = \frac{\gamma z_1(\tau) A k_t^{\alpha(1+\delta)}}{1+(1+\gamma)z_1(\tau)k_t^{\alpha\delta}} \left[z_2(\tau) + \alpha \frac{1+(1-\gamma)z_1(\tau)k_{t-1}^{\alpha\delta}}{1+z_1(\tau)k_{t-1}^{\alpha\delta}} \right], \tag{12}$$

where $z_1(\tau) := \Delta[\tau(1-\alpha)A]^{\delta} > 0$ and $z_2(\tau) := (1-\alpha)(1-\tau) > 0$. Fixed points of the map defined in Eq. (12) are determined as $k_{t+1} = k_t = k_{t-1} = k$. They are represented by the roots of the following function:

$$\mathrm{B}(k) = \frac{\gamma z_1(\tau) A k^{\alpha(1+\delta)}}{1+(1+\gamma)z_1(\tau)k^{\alpha\delta}} \left[z_2(\tau) + \alpha \frac{1+(1-\gamma)z_1(\tau)k^{\alpha\delta}}{1+z_1(\tau)k^{\alpha\delta}} \right] - k = 0 . \tag{12.1}$$

3. Existence and local stability of fixed points

We now discuss the existence and stability of both the zero and positive steady states of Eq. (12), starting with the analysis of $k = 0$. The qualitative results of the model are different depending on the mutual relationship between the parameters α and δ. In addition, a crucial role for the health tax rate τ on (local) stability is established (see Section 3.1).

From Eq. (12.1) it is clear that $k = 0$ is a fixed point of the system described by Eq. (12). Non-zero equilibria are determined by interior solutions of the following equation:

$$G_1(k) = G_2(k) , \tag{12.2}$$

[7] See Michel and de la Croix (2000) and de la Croix and Michel (2002) for a discussion about differences in dynamic outcomes under myopic foresight and perfect foresight in OLG growth models with capital accumulation and two-period lived individuals.

where $G_1(k)=z_2(\tau)+\alpha\frac{1+(1-\gamma)z_1(\tau)k^{\alpha\delta}}{1+z_1(\tau)k^{\alpha\delta}}$ and $G_2(k)=\frac{k^{1-\alpha(1+\delta)}[1+(1+\gamma)z_1(\tau)k^{\alpha\delta}]}{\gamma z_1(\tau)A}$. The following proposition thus holds.

Proposition 1. *Let $0<\alpha<1/(1+\delta)$ hold. Then, one and only one positive steady state, $\bar{k}$, of the dynamic system Eq. (12) exists.*

Proof. Consider the functions $G_1(k)$ and $G_2(k)$, which are continuous for any $k>0$. Then,

$$G_1'(k)=\frac{-\alpha^2\delta\gamma z_1(\tau)k^{\alpha\delta}}{[1+z_1(\tau)k^{\alpha\delta}]^2 k}, \tag{12.3}$$

and

$$G_2'(k)=\frac{1-\alpha(1+\delta)+(1-\alpha)(1+\gamma)z_1(\tau)k^{\alpha\delta}}{\gamma z_1(\tau)Ak^{\alpha(1+\delta)}}. \tag{12.4}$$

It is easy to verify that $G_1(0)=z_2(\tau)+\alpha$, $G_1(+\infty)=\lim_{k\to+\infty}G_1(k)=z_2(\tau)+\alpha(1-\gamma)$, where $G_1(0)>G(+\infty)$, and $G_1'(k)<0$ for any $k>0$. If $0<\alpha<1/(1+\delta)$, then $G_2(0)=0$, $G_2(+\infty)=\lim_{k\to+\infty}G_2(k)=+\infty$ and $G_2'(k)>0$ for any $k>0$. Therefore, $G_1(k)=G_2(k)$ only once at $\bar{k}>0$ for any $k>0$. **Q.E.D.**

Proposition 2. *Let $1/(1+\delta)<\alpha<1$ hold. Then, if $\alpha\delta>1$ and $\gamma>\frac{[\alpha(1+\delta)-1](\alpha\delta+1)}{(1-\alpha)(\alpha\delta-1)}-1$, at most two positive steady states, $\bar{k}_2>\bar{k}_1$, exist.*

Proof. If $1/(1+\delta)<\alpha<1$, then $G_2(0)=\lim_{k\to 0}G_2(k)=+\infty$, $G_2(+\infty)=\lim_{k\to+\infty}G_2(k)=+\infty$, while studying the sign of G_2' in (12.4), it is easily seen that $G_2'(k)<0$ (resp. $G_2'(k)>0$) for any $0<k<k_{\min}$ (resp. $k>k_{\min}$), where $k_{\min}:=\left[\frac{\alpha(1+\delta)-1}{(1-\alpha)(1+\gamma)z_1(\tau)}\right]^{\frac{1}{\alpha\delta}}>0$ is the unique critical point (global minimum) of $G_2(k)$. The second derivatives are:

$$G_1''(k)=\frac{\alpha^2\delta\gamma z_1(\tau)k^{\alpha\delta}[z_1(\tau)(1+\alpha\delta)k^{\alpha\delta}-(\alpha\delta-1)]}{k^2[1+z_1(\tau)k^{\alpha\delta}]^3}, \tag{12.5}$$

and

$$G_2''(k)=\frac{\alpha k^{1-\alpha(1+\delta)}\left\{(1+\delta)[\alpha(1+\delta)-1]-z_1(\tau)(1-\alpha)(1+\gamma)k^{\alpha\delta}\right\}}{\gamma z_1(\tau)Ak^2}. \tag{12.6}$$

If $\alpha\delta>1$, from the sign of G_1'' in (12.5), we see that $G_1(k)$ has a unique inflection point $k_{flex}:=\left[\frac{\alpha\delta-1}{z_1(\tau)(\alpha\delta+1)}\right]^{\frac{1}{\alpha\delta}}>0$ and it is concave (resp. convex) for any $0<k<k_{flex}$ (resp. $k>k_{flex}$). Nothing can be said about the number of steady states if $k_{flex}<k_{\min}$. From the sign G_2'' in (12.6), we see that $G_2(k)$ has a unique inflection point as well, which is always larger than $k_{\min}$, so that $G_2(k)$ turns out to be convex for $0<k<k_{\min}$. Thus, if $k_{flex}>k_{\min}\Rightarrow\gamma>\frac{[\alpha(1+\delta)-1](\alpha\delta+1)}{(1-\alpha)(\alpha\delta-1)}-1$), then three cases are possible for values of k such that $0<k<k_{\min}$, where $G_2(k)$ is convex while $G_1(k)$ is concave:

Case 1: $G_1(k)$ and $G_2(k)$ have no intersections at all. Then the number of steady states must necessarily be zero because $G_1(k_{\min}) < G_2(k_{\min})$ and for $k > k_{\min}$, $G_2(k)$ starts increasing, so no intersections are possible;

Case 2: $G_1(k)$ and $G_2(k)$ has one intersection. Therefore, we must have that $G_1(k_{\min}) > G_2(k_{\min})$ and by considering that for $k > k_{\min}$, $G_2(k)$ is increasing and tends to infinity, the two curves must necessarily intersect one more time. So, we have two steady states;

Case 3: $G_1(k)$ and $G_2(k)$ have two intersections. This is the maximum number of intersection for two decreasing functions, one convex and the other concave. Similarly to Case 1, we now have that $G_1(k_{\min}) < G_2(k_{\min})$, and for the same argument, we cannot have further intersections. **Q.E.D**.

Remark 1. *For the remaining cases, it is not possible to analytically prove how many positive steady states can be founded. Nevertheless, a huge number of simulations permits us to be reasonably confident that scenarios with more than two positive steady states are unlikely to be observed.*

Propositions 1 and 2 and Remark 1 imply that for low values of the output elasticity of capital, the graphs of the maps G_1 and G_2 intersect only once at $\bar{k}$, while when the output elasticity of capital becomes larger: (*i*) the graph of G_2 lies above the graph of G_1 for any $k > 0$, and no positive steady state exists (this holds when the technological scale parameter A in the Cobb-Douglas production function is small), or (*ii*) the graph of G_2 intersects the graph of G_1 twice for any $k > 0$, and two steady states $\bar{k}_2 > \bar{k}_1 > 0$ exist (this holds when A is sufficiently large).[8] The propositions and the remark emphasise the importance of the mutual relationship between the parameters in both the production function and longevity function in determining different scenarios. When α is small, a unique regime of development does in fact exist (Diamond, 1965). When α becomes larger, two regimes of development (low and high) may appear and then initial conditions are important to determine whether an economy can converge towards the low regime of development or the high regime of development. This result is in line with Chakraborty (2004) and Bunzel and Qiao (2005). Of course, larger values of A work with the largest steady state because, everything else being equal, capital accumulation increases when the technological scale in the production function rises. Further, the higher δ (which, we recall, captures how an additional unit of health investment is transformed into higher longevity through the health technology), the lower the threshold of α beyond which two development regimes exist depending on the size of A.

Now, in order to study the local stability properties of the fixed points we transform the system of a single second order difference equation (12) into a system of two first order difference equations (e.g. Azariadis, 1993 and Grandmont et al., 1998). Let $x_t := k_{t-1}$ be a new supporting variable. Then Eq. (12) can be written as:

$$\begin{cases} k_{t+1} = \dfrac{\gamma z_1(\tau) A k_t^{\alpha(1+\delta)}}{1+(1+\gamma) z_1(\tau) k_t^{\alpha\delta}} \left[z_2(\tau) + \alpha \dfrac{1+(1-\gamma) z_1(\tau) x_t^{\alpha\delta}}{1+z_1(\tau) x_t^{\alpha\delta}} \right]. \\ x_{t+1} = k_t \end{cases} \tag{13}$$

[8] This is shown by several numerical experiments not reported in the paper.

The local stability of fixed points is studied by means of the linear approximation given by the Jacobian matrix of partial derivatives (J) evaluated at the generic steady state $\bar{k}$, which for the system (13) is:

$$J = \begin{pmatrix} J_k & J_x \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \left.\frac{\partial k_{t+1}}{\partial k_t}\right|_{k_t=\bar{k}} & \left.\frac{\partial k_{t+1}}{\partial x_t}\right|_{k_t=\bar{k}} \\ \left.\frac{\partial x_{t+1}}{\partial k_t}\right|_{k_t=\bar{k}} & \left.\frac{\partial x_{t+1}}{\partial x_t}\right|_{k_t=\bar{k}} \end{pmatrix}, \tag{14}$$

where

$$J_k = \frac{\gamma z_1(\tau)\alpha A \bar{k}^{\alpha(1+\delta)-1}}{1+(1+\gamma)z_1(\tau)\bar{k}^{\alpha\delta}} \cdot \left[1 + \frac{\delta}{1+(1+\gamma)z_1(\tau)\bar{k}^{\alpha\delta}}\right] \cdot \left[z_2(\tau) + \alpha \frac{1+(1-\gamma)z_1(\tau)\bar{k}^{\alpha\delta}}{1+z_1(\tau)\bar{k}^{\alpha\delta}}\right] > 0, \tag{15}$$

and

$$J_x = \frac{-\delta[\gamma\alpha z_1(\tau)]^2 A\bar{k}^{\alpha(1+2\delta)-1}}{[1+z_1(\tau)\bar{k}^{\alpha\delta}]^2 \cdot [1+(1+\gamma)z_1(\tau)\bar{k}^{\alpha\delta}]} < 0. \tag{16}$$

The trace and determinant of (14) are $T := Tr(J) = J_k > 0$ and $D := Det(J) = -J_x > 0$, respectively, so that the characteristic polynomial is:

$$\mathrm{P}(\lambda) = \lambda^2 - T \cdot \lambda + D, \tag{17}$$

whose discriminant $Q := T^2 - 4D$ can either be positive or negative. Therefore, complex eigenvalues can exist.

Bifurcation theory describes the way the topological features of the system (such as the number of stationary points or their stability) vary as parameter values change. For a system in two dimensions, the stability conditions ensuring that both eigenvalues remain within the unit circle[9] are:

$$\begin{cases} (i) \quad F := 1 + T + D > 0 \\ (ii) \quad TC := 1 - T + D > 0 \\ (iii) \quad H := 1 - D > 0 \end{cases}. \tag{18}$$

The violation of any single inequality in (18) leads to: (*i*) a flip bifurcation (a real eigenvalue that passes through -1) when $F = 0$; (*ii*) a fold or transcritical bifurcation (a real eigenvalue that passes through $+1$) when $TC = 0$; (*iii*) a Neimark-Sacker bifurcation (i.e., the modulus of a complex eigenvalue pair that passes through 1) when $H = 0$, namely $D = 1$, and $|T| < 2$.

As regards the stability properties of the zero equilibrium, we have the following proposition.

Proposition 3. *(1) Let* $0 < \alpha < 1/(1+\delta)$ *hold. Then,* $k = 0$ *is locally unstable. (2) Let* $1/(1+\delta) < \alpha < 1$ *hold. Then,* $k = 0$ *is locally asymptotically stable.*

Proof. If $0 < \alpha < 1/(1+\delta)$, then as k tends to 0 an eigenvalue of the Jacobian matrix tends to infinity, and the steady state results in an unstable fixed point. If $1/(1+\delta) < \alpha < 1$, then both the eigenvalues are zero. **Q.E.D.**

As Proposition 3 shows, the zero equilibrium of the dynamic system described by Eq. (12) can either be stable or unstable depending on the relative size of the output elasticity of capital. The existence (and the stability properties) either of a single positive fixed point or multiple

[9] If no eigenvalues of the linearised system around the fixed points of a first order discrete system lie on the unit circle, then such points are defined as being *hyperbolic*. Roughly speaking, at non-hyperbolic points topological features are not structurally stable.

positive fixed points strictly depends on whether $k=0$ is stable or unstable. Given Propositions 1 and 2 and Remark 1 regarding the existence of a unique fixed point or multiple fixed points, and Proposition 3 regarding the stability conditions of the zero equilibrium, in Subsection 3.1 we show through numerical simulations that the following results hold.[10]

Result 1. *If $0<\alpha<1/(1+\delta)$, then the dynamic system described by Eq. (12) admits two steady states $\{0,\bar{k}\}$, where $0<\bar{k}$, the former being locally unstable and the latter locally asymptotically stable.*

Result 2. *If $1/(1+\delta)<\alpha<1$, then the dynamic system described by Eq. (12) reasonably admits either the stable zero steady state alone (if the technological scale parameter A in the Cobb-Douglas production function is sufficiently small), or three steady states $\{0,\bar{k}_1,\bar{k}_2\}$ (if A is sufficiently large), where $0<\bar{k}_1<\bar{k}_2$, the first is locally asymptotically stable, the second is a saddle point and the third may be locally asymptotically stable or unstable.*

As is clear from Results 1 and 2, conditions regarding the stability of the largest steady state depend on the mutual relationship between technology parameters in both the production function (α) and health technology (δ). The more health investments are "smoothly transformed" into better health and higher longevity (low values of δ), the higher the importance of the output elasticity of capital in determining how the increase in wages affects health investments which, in turn, increase life expectancy (low mortality rates). When δ becomes larger, threshold effects become important and the fraction of wages needed to make investments in health efficient, becomes larger so that the "impetus to capital accumulation" due to "large life expectancy gains" (Chakraborty, 2004, p. 124) requires higher values for the capital stock. In this case, if an economy starts out with low capital stock values, both the wages earned by the young and the expenditure on health are sufficiently small, and thus do not provide an adequate stimulus to life expectancy and economic growth. However, even if an economy starts out with sufficiently high initial conditions, higher values of δ may mean that capital accumulation in the next period becomes lower than the current period, so that over-expenditure in health may actually reduce growth.

Result 3. *If $1/(1+\delta)<\alpha<1$ and $\delta>1$, then the local dynamics (in the neighbourhood of the largest steady state $\bar{k}_2$) can be oscillatory. Additionally, depending on the size of the health tax rate τ, Neimark-Sacker bifurcations and endogenous fluctuations occur.*

In addition, the following proposition holds:

Proposition 4. *A flip bifurcation can never occur.*

Proof. The proof is obvious as the sign of both the trace and determinant of J, evaluated at the generic steady state $\bar{k}$, are $T>0$ and $D>0$. Therefore, $F>0$ always holds. **Q.E.D.**

3.1. Stability of positive steady states and bifurcations

[10] Since the map is difficult to handle in a neat analytical form, the local stability analysis of the positive (largest) steady state is performed through computations (no closed-form expression of the fixed point exists). With regard to local and global analyses, given that the dynamic patterns are characterised by a possible rich set of complex scenarios, our aim is to describe some interesting outcomes regarding dynamics with no claim of generalisation.

3.1.1. Case $0 < \alpha < 1/(1+\delta)$

In this case the position of the eigenvalues of the Jacobian matrix J relative to the unit circle, evaluated at the unique positive steady state $\bar{k}$, are smaller than unity and the three conditions stated in (18) are fulfilled. This case is uninteresting from a dynamic point of view and thus we have not presented any numerical experiments.[11] We refer the reader to the dynamic analyses developed by Chakraborty (2004) with exogenous fertility, and Fanti and Gori (2010) with endogenous fertility.

3.1.2. Case $1/(1+\delta) < \alpha < 1$

In this case the position of the eigenvalues of the matrix J relative to the unit circle is unclear, and the positive steady state $\bar{k}_2$ can either be stable or unstable. Since the three conditions in (18) cannot easily be treated in a neat analytical form when they are evaluated at the positive steady states $\bar{k}_1$ and $\bar{k}_2$. Consequently, in order to illustrate Results 2 and 3 above, we resort to numerical simulations to show that Neimark-Sacker bifurcations and endogenous fluctuations can actually occur when there are threshold effects of health investments on longevity. We thus chose the following parameter configuration: $\gamma = 0.99$, $\Delta = 1$, $\delta = 20$, $\alpha = 0.4$ (which may be considered as an average value between the values of the output elasticity of capital in developed and developing countries, which, according to, for example, Kraay and Raddatz (2007), are $\alpha = 0.33$ and $\alpha = 0.5$, respectively)[12] and $A = 4.5$. Figure 1 represents the bifurcation diagram for τ with respect to the steady state values of the variable k for a simulation of system (13) starting from $k_0 = x_0 = 1$.

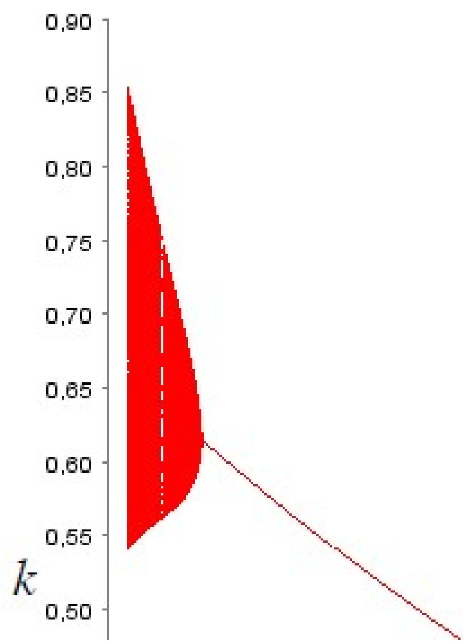

Figure 1. Bifurcation diagram for τ ($k_0 = x_0 = 1$): an enlarged view for $0.473 < \tau < 0.77$ and $0.1 < \bar{k} < 0.9$.

[11] Numerical experiments are of course available upon request.
[12] See Gollin (2002) for estimates on the output elasticity of capital in developed countries.

Figure 1 reveals that there is a double Neimark-Sacker bifurcation when the health tax rate varies. In fact, when $\tau = 0.4945$ we get $F = 2.9613 > 0$, $TC = 1.0403 > 0$, $H = 0$ and $T = 0.9604$. A second Neimark-Sacker bifurcation occurs when $\tau = 0.6859$, corresponding to which $F = 2.7186 > 0$, $TC = 1.2827 > 0$, $H = 0$ and $T = 0.7179$.

Simulations (not reported for reasons of space) reveal that when $\delta = 1$ (i.e. no threshold effects of health investments on longevity exist), Results 1 and 2 resembles Point (i) of Proposition 1 by Chakraborty (2004, p. 126) in a model with a perfect market for annuities. This means that: (*i*) there is one locally asymptotically stable steady state (as in Diamond, 1965) when $0 < \alpha < 1/2$, and (*ii*) there are two locally asymptotically stable steady states $\{0, \bar{k}_2\}$ when $1/2 < \alpha < 1$.[13]

If we slightly change the initial conditions from $k_0 = x_0 = 1$ to $k_0 = x_0 = 0.2$, Figures 2 (bifurcation diagram) and 3 (the largest Lyapunov exponent, $Le1$, plotted against the parameter τ) reveal that deterministic chaos occurs because there are ranges of values of the health tax rate for which the Lyapunov exponent is steadily positive when $0.75 < \tau < 0.7625$.

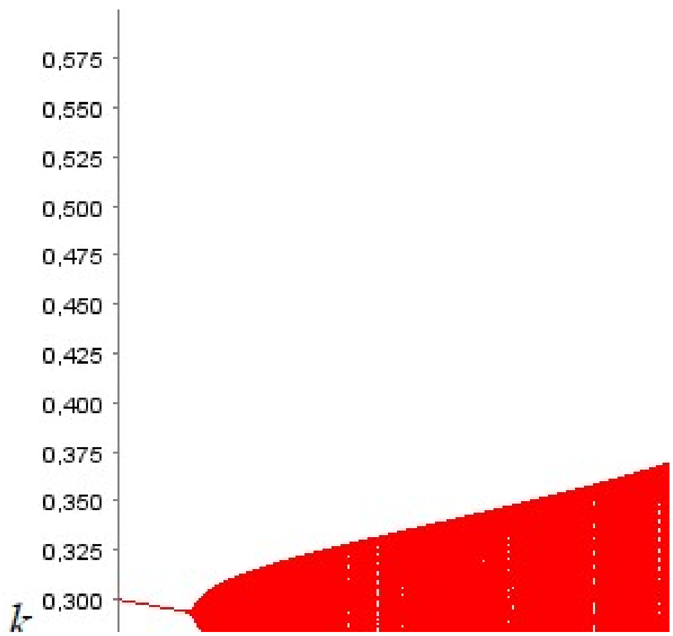

Figure 2. Bifurcation diagram for τ ($k_0 = x_0 = 0.2$): an enlarged view for $0.68 < \tau < 0.77$ and $0 < \bar{k} < 0.6$.

[13] When $1/(1+\delta) < \alpha < 1$ and $\delta > 1$ both the low and high steady states in the model by Chakraborty (2004) are locally asymptotically stable with monotonic trajectories.

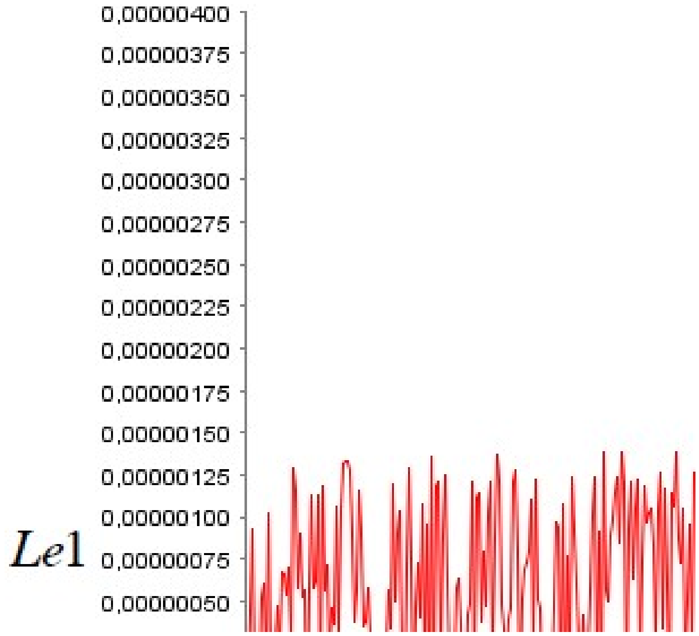

Figure 3. Largest Lyapunov exponent plotted against τ ($0.75 < \tau < 0.76$).

3.2. Sketch of global analysis

When two locally stable attractors coexist, trajectories may converge to one or the other, depending on the initial conditions. In our model, when the condition given in Result 2 holds, a low and a high valued attractors coexist and it is important to identify their basins of attraction (that is, the set of initial conditions leading to one attractor or to the other). The study of the basin configurations is called *global analysis* and differs from the local analysis because it does not consider the linearization of the map around a steady state, but considers the map in its original nonlinear formulation. Global analysis is a mix of analytical and numerical tools. This is quite important because, especially in discrete-time systems, it is not always true that initial conditions closer to one attractor characterize trajectories leading towards it. In our case this means that we are not sure whether or not when two locally stable attractors coexist, high (resp. low) initial values of the variables lead to convergence towards the high valued (resp. low valued) attractor. In fact, basins of attractions have either a simple or complicated structure. They can be *connected* (that is, only made up of a compact subset of the phase space containing the attractor itself) or *disconnected* (that is made up by the union of the subset of initial conditions around the attractor, called *immediate basin*, and by its disconnected preimages). We will show that even connected basins can have a complicated structure.

Given the complexity of our map, it is not possible to analytically detect global bifurcation values. It is still possible however to perform some computer-assisted simulations. Let us now consider a parameter configuration leading to coexistence of two locally stable attractors (Figure 4.a). This means that the condition given in Result 2 is fulfilled. In this case, besides the locally stable steady state $\{0\}$ and the saddle point $\{\bar{k}_1\}$, there is a locally attractive closed invariant curve[14] originating from the Neimark-Sacker bifurcation of the largest steady state

[14] For a map G defined on $U \subset \Re^n$, a subset $S \subset U$ is said to be invariant if $G^n(S) \subset S$ for any $n \in Z$. This invariant set can be a closed (i.e. containing all its accumulation points) curve and like any other invariant set can be locally stable. We refer the reader to Lorentz (1993) or Medio (1995), among others, for more formal and rigorous definitions.

$\{\bar{k}_2\}$. This means that of the possible initial conditions, we can identify those that give rise to trajectories converging to a quasi-periodic attractor. They form the basin of attraction of the closed invariant curve. In Figure 4.a the two basins of attraction are denoted by different colours. They are separated by the stable manifold of the saddle point and are connected. If we increase the value of τ, for instance, we can see that the basin configuration drastically changes and becomes more complicated. A basins structure, such as the one represented in Figure 4.b, is such that even starting from initial conditions closer to the closed invariant curve, (like initial conditions A or B) trajectories converge towards the origin. We will leave a more thorough analysis of global bifurcation for a more technical paper. For now we only highlight once again that whenever two locally stable attractors coexist, it is not obvious to which one a generic trajectory converges, even when starting very close to one of them.

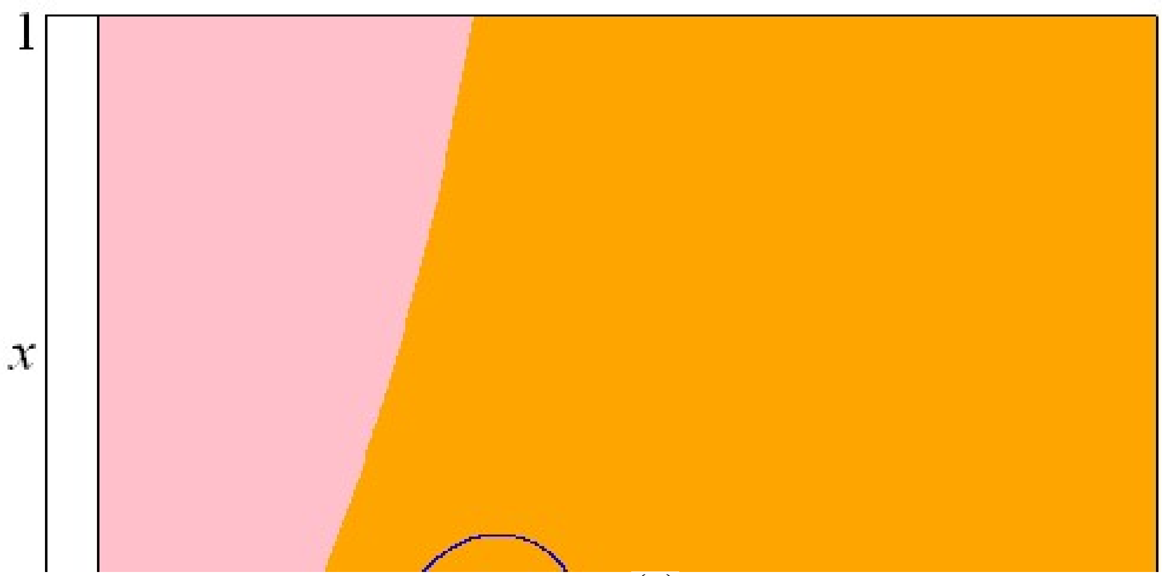

Figure 4. Basin of attraction of the steady state $\{0\}$ (pink) and of the closed invariant curve around $\{\bar{k}_2\}$ (orange). Parameter set: $A = 5.35$, $\alpha = 0.5$, $\gamma = 0.94$, $\delta = 6$, and $\Delta = 0.9$. The health tax rate τ is 0.7815 in (a) and 0.8 in (b).

4. Conclusions

We have studied the dynamic properties of a general equilibrium overlapping generations model with public health investments that affect the lifetime of people. We have shown that the existence of unintentional bequests, rather than a market for annuities, means that the dynamics of the economy is characterised by a two-dimensional discrete non-linear system, rather than the one-dimensional system in Chakraborty's model (2004). This introduces the possibility of dynamics characterised by oscillations, Neimark-Sacker bifurcations and deterministic chaos in rich economies (the largest steady state) when the health tax rate changes and the threshold effect of health investments on longevity exist. Of course, the prevailing regime of development (a unique positive long-run equilibrium or two long-run equilibria, the former known as poverty trap) depends on whether the zero equilibrium is stable or unstable. Our global analysis revealed that two locally stable attractors coexist, so that it is not obvious to which one a generic trajectory converges, even when starting very close to one of them. We believe that our results thus also represent a policy warning regarding the destabilising effect of the financing of (public) health investments, because over-expenditure can reduce growth even for those economies with a large initial capital stock. Differences in the initial conditions between countries, as well as the size of their public health programmes may help to explain the possible existence of poverty traps (Chakraborty, 2004). However, even when the initial conditions of different economies are similar, they will end up "looking very different" (Azariadis, 1993, p. 106) as regards both the macroeconomic and demographic variables, because endogenous fluctuations are possible (Bhattacharya and Qiao, 2007).

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