Economic growth and stability with public PAYG pensions and private intra-family old-age insurance

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Abstract This paper compares the steady state and dynamic outcomes of two historical alternatives as a means of old-age insurance – i.e., voluntary intra-family transfers from young to old members versus pay-as-you-go public pensions –, in a general equilibrium overlapping generations model with children as a desirable good. We show that the shift from a private system of old-age support to public pensions increases GDP per worker. Moreover, although in both cases the steady-state stock of capital, under myopic expectations, may be (globally) unstable depending on the size of the inter-generational transfer, we show that the existence of public pensions rather than private transfers considerably reduces the possibility of cyclical instability.

Keywords Endogenous fertility; Myopic foresight; OLG model; Private old-age support; Public PAYG pensions

JEL Classification C62; H55; J14; J18; J26

1. Introduction

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Several developed countries have introduced public pension programmes since the first part of the last century (in the US social security was introduced at the end of thirties).[1] In practice, the social security system is mostly unfunded or based on pay-as-you-go (PAYG) rules. While all societies have been historically contributed to the livelihood of the elderly through voluntary intra-family transfers from children to parents (see, for instance, Ehrlich and Lui, 1991), at higher stages of economic development, governments of Western countries have propagated the provision of public pensions in order to secure old-age consumption, because the organisation of societies and the structure of economies and institutions were suddenly changing.[2]

Since economists agree with the reducing-fertility effect of the introduction of public pensions as a substitute of intra-family gifts (Cigno, 1993), then for some developing countries public pensions might apparently be advised for the purpose of lowering fertility rates (see, e.g., the case of China). In contrast, for developed countries plagued by below-replacement fertility it has been pointed out that the existence of PAYG pensions is responsible of the observed fertility drop (see, amongst others, Boldrin et al., 2005; Cigno and Werding, 2007; Galasso et al., 2009).

In addition, the economic literature has also investigated the relationship between the introduction of public pensions (together with the private system of old-age backing) and economic development. Nishimura and Zhang (1992, 1995) and Cigno (1995) – by assuming backward altruism (i.e. from children to parents) – show that introducing a tax-financed social security system may increase utility in a partial equilibrium model. Zhang and Zhang (1995), by assuming backward altruism, and Wigger (1999), which instead takes both backward and forward altruism (i.e. from parents to children) into account, analyse a general equilibrium context of endogenous growth and find that the rate of growth of GDP per worker may increase along with the contribution rate to the public pension system.

Unlike previous studies, in this paper the decision to have children is based not only for supporting the old-aged but also on altruism towards children, in order to capture the rather realistic idea that parents directly derive utility from having children also when public pension are absent. Moreover, the assumption of children as a investment good, on which all this literature is based upon, is quite different from that at the basis of the choices of fertility in the modern approach (i.e. the new home economics, see Becker, 1960), where both the number and quality of children affect parents' utility. In order to be in line with the new home economics approach, in the present study we assume that the *number* of children enters parents' utility, according to the so called weak altruism towards children (see Zhang and Zhang, 1998). Therefore, having children is desirable in both cases of presence and absence of public pensions. In the former case, the government collects taxes on workers' income to provide pension benefits for the old. In the latter case, the young voluntarily transfer (due to, e.g., cogent social norms) a fraction of their income to support their parents. Following an established literature (see, e.g., Bental, 1989; Morand, 1999), such a fraction is assumed to be exogenously given.

Although individuals derive utility from having children under both alternatives of old age support, the incentive to have children is larger when private intra-family gifts are in existence, because the size of the benefit received when old depends on individual (rather than aggregate) fertility and this, in turn, contributes to reduce the cost of children. This hypothesis is different from: (*i*) Nishimura and Zhang (1992, 1995), Cigno (1995) and Zhang and Zhang (1995), which assume backward altruism (i.e., an individual derives utility from

[1] See, e.g., Cutler and Johnson (2004) and Caucutt et al. (2007).

[2] "It has been observed that at certain stage of economic growth and development a nation starts to consider the introduction of social security programs. In developing nations, there has been increasing discussion in popular press of introducing social security systems." (Zhang and Zhang, 1995, p. 441).

old age consumption of their parents), and (*ii*) Wigger (1999) where it is simultaneously assumed the existence of both types of altruism as well as the presence of both private and public support for the old-aged. Moreover, we differ from all these papers because in the present study we *separately*[3] compare the two alternatives as a means of old-age insurance rather than considering them together, in order to better capture the effects of the historical evolution from private to public systems of old-age support on both neoclassical economic growth and economic stability. Since the above mentioned literature has shown that the unambiguous effect of the introduction of public pensions and the rise in the contribution rates imply that young people reduce private transfers because the elderly get more support from pensions, then the importance of private transfers gets smaller when public pensions raises. Therefore, for simplicity, in this paper we consider a switch between the two transfer systems, although such a change may have been a gradual process during the stages of development.

This paper shows, in line with Zhang and Zhang (1995) and Wigger (1999), that the shift from a private system of old-age insurance to a public system of social security in an economy with overlapping generations (OLG) and endogenous fertility, always increases GDP per worker. Moreover, it is also shown that, under myopic foresight, both the subjective discount factor and taste for children are crucial to determine stability outcomes under of private intra-family transfers, while such parameters play no role when public pensions are in existence. The reason is that in the former case, the size of the benefit received when old depends on individual fertility, while in the latter case it depends on average fertility. This implies that under private transfers, the number of children is higher and saving is lower than under PAYG pensions.

The remainder of the paper is organised as follows. Section 2 describes the features common to a general equilibrium economy with both private old-age backing and public PAYG pensions. Section 3 (4) analyses and discusses the steady-state and dynamic outcomes under private old-age support (public PAYG pensions). Section 5 compares the two alternative systems of social security. Section 6 concludes.

2. The economy

Consider a general equilibrium OLG closed economy populated by a continuum of identical individuals and identical firms. Time is discrete and indexed by $t=0,1,2\dots$. Let N_t be the number of young members within every generation. We assume that population grows at rate $\overline{n}_{t-1}-1$, where $\overline{n}_{t-1}$ is the *average* or *aggregate* number of children in the overall economy at time $t-1$, so that $N_t=\overline{n}_{t-1}N_{t-1}$ is the equation that drives population growth period by period.

2.1. Firms

We assume that identical firms act competitively on the market. The representative firm produces Y_t units of final goods and services at time t through the Cobb-Douglas production function $Y_t=AK_t^{\alpha}L_t^{1-\alpha}$, where K_t and L_t are the aggregate capital stock and labour input

[3] Another reason to separately introduce the private and public systems of inter-generational transfer may also concern the different role of each system in different societies: for instance, while in European countries the shift between the two systems began to occur since the nineteenth century (see, e.g., Blackburn, 2002), in Asian societies the relevance of intra-family transfers is historically more pervasive, as noted by Yoon and Talmain (2001, p. 588): "Voluntary transfer from children to parents commonly happens in Asian countries (Korea, Japan, China etc.) because those countries, steeped in benign Buddhist traditions, depend heavily on the family system to provide the bulk of support for the elderly."

hired in that firm, respectively, $0<\alpha<1$ is the output elasticity of capital and $A>0$ is a scale parameter. Knowing that $L_t = N_t$ holds in equilibrium, production per worker can easily be expressed as $y_t = Ak_t^{\alpha}$, where $k_t := K_t / N_t$ and $y_t := Y_t / N_t$ are capital and output per worker, respectively.

Since capital totally depreciates at the end of every period and output is sold at unit price, profit maximisation implies that capital and labour are paid their respective marginal productivities, that is:

$$R_t = \alpha A k_t^{\alpha-1}, \tag{1}$$

$$w_t = (1-\alpha) A k_t^{\alpha}, \tag{2}$$

where $R_t = 1 + r_t$ and w_t represent the rental price of capital (with r_t being the interest rate) and the cost per unit of labour (the wage rate) at time t, respectively.

2.2. Individuals

Individuals are assumed to have identical preferences. Each generation overlaps for one period with the previous generation and then overlaps for one period with the next generation. Life of the typical agent is divided between childhood and adulthood. In the former period, an agent does not make economic decisions and then consumes a fixed fraction of resources from parents. In the latter period, she works and takes care of children when young, while being compulsory retired when old. Labour is inelastically supplied to firms by the young members of every generation and it is paid at the competitive wage w_t per unit of labour.

The individual representative of generation t has preferences towards material consumption over the life cycle and the number of children. The lifetime utility function takes the following logarithmic form:[4]

$$U_t = \ln(c_{1,t}) + \beta \ln(c_{2,t+1}) + \gamma \ln(n_t), \tag{3}$$

where $c_{1,t}$ and $c_{2,t+1}$ represent young-age consumption and old-age consumption, respectively, n_t is the *individual* number of children, $0<\beta<1$ is the subjective discount factor and $\gamma>0$ captures the parents' taste for children. Children are therefore assumed as a consumption good when public PAYG are in existence (because old age income depends on aggregate fertility), and both as an investment good[5] and consumption good under the assumption of voluntary intra-family gifts (because old age income is conditional on individual fertility in such a case).

As regards child-rearing activities, we assume that parents devote a fixed amount of resources $e>0$ to take care of a child, so that the cost of raising n_t descendant is simply given by en_t. Although this hypothesis would apparently be rather simplistic, we stress that the choice of modelling children costs as being fixed and exogenous is first due to the fact that the focus of the present study is the comparison of long-run and dynamic outcomes of two historical and polar alternatives as a means of old-age insurance: private voluntary intra-

[4] Logarithmic functions similar to Eq. (3) including the *number* of children as an argument, are employed, amongst others, by Eckstein and Wolpin (1985), Eckstein et al. (1988), Galor and Weil (1996), van Groezen et al. (2003), van Groezen and Meijdam (2008), Fanti and Gori (2009, 2010, 2012).

[5] This is the well known "old-age security hypothesis", which is relevant in economies where inter-generational transfers take place within the family (see Bental, 1989; Cigno 1992).

family transfers from the young to the old[6] (which have been observed especially at lower stages of economic development, i.e. when children represented a partial substitute for other saving opportunities[7]), and public PAYG pensions (which, in contrast, are used at higher stages of economic development). For less developed or developing economies, therefore, it is quite realistic to assume children costs as being a fixed portion of family income, while for developed economies, the cost of children can be represented by both a fixed share of (or proportional to) working income, to take the direct cost incurred to provide a certain number of commodities and services to nurturing children into account, and time costs, to capture the opportunity cost of bearing them (see, e.g., Boldrin and Jones, 2002). In order to make both cases comparable and analytically tractable, we assumed children costs and being fixed and exogenous throughout. It is important to note, however, that since in the present study parents are not truly altruistic in the Becker sense,[8] then under the assumption of private old-age support the problem that selfish parents would like to reduce to the minimum the nurturing cost per child may exist when the welfare of descendants does not enter the utility of parents and old-age income is contingent on the number of children raised. In such a case (i.e., endogenous cost of children) would be optimal for parents to starve their children. This is clearly pointed out by Cigno (1995) for the case of pensions conditional on individual fertility (see Cigno and Werding, 2007 for an interesting discussion on the topic). The assumption of exogenous children costs avoids to account of this concern. Indeed, as recognised by Cigno (1995, p. 171): "selfish parents will want to spend the absolute minimum for each child they have [when old-age income is contingent on the number of children], which is unlikely to be intertemporally efficient. [However,] The problem is by-passed by N–Z's [Nishimura and Zhang (1992)] assumption, common to Bental (1989), that the cost of children is exogenous. Another way out would be to make pensions conditional on the amount spent on children, but there is then the problem of monitoring such expenditures." Moreover, the assumption of exogenous children costs – either fixed or as a percentage of working income – belongs to a wide class of models, which includes, amongst others, Zhang and Zhang (1995), Wigger (1999), van Groezen et al., 2003; van Groezen and Meijdam, 2008; Fanti and Gori (forthcoming).

It could now be useful to briefly discuss the differences between weak and pure altruism towards children. Under the former assumption, parents derive utility from the number of children they have (see Galor and Weil, 1996; Zhang and Zhang, 1998). Under the latter assumption, the utility of descendants enters the utility function of parents (see Barro, 1974; Becker and Barro, 1988; Barro and Becker, 1989; Becker et al., 1990).[9] Indeed, if, for instance, the utility of parents directly depends on child quality expenditures (i.e., nurturing expenditures, education expenditures and so on), then the cost of children becomes an endogenous variable (see, e.g., Strulik, 2004). In such a case, a corner solution implying no quality expenditure is possible. In contrast, if preferences of parents are characterised by the so called weak altruism towards children, then the assumption of exogenous and positive expenditure ($e > 0$) to raise a child is necessary for an interior solution of the individual utility maximisation problem to exist.

We now turn to the study of long-run and dynamic outcomes of an OLG economy with: (*i*) private intra-family transfers from young to old members (Section 3), and (*ii*) public PAYG

[6] In particular, we follow an established literature (see Bental, 1989; Raut and Srinivasan, 1994; Chakrabarti, 1999; Morand, 1999), and assume that young individuals voluntary transfer a fixed proportion of their income to parents in order to support them when old.
[7] See, e.g., Neher (1971), Cain (1981, 1983).
[8] Indeed, each parent derives utility from the number of children raised, but she does not care about their utility (see Eq. 3).
[9] See Zhang and Zhang (1998) for a discussion of the trade off between quantity and quality of children.

pensions (Section 4). Then, we take explicitly the comparison of both alternatives into account.

3. Private old-age support

In this section we take the case of private intra-family transfers from young to old members as a means of old-age insurance into account. In particular, we assume that each young member of generation t voluntarily devotes an exogenous[10] fraction $0<\eta<1$ of wage income to support material consumption of parents (the size of which is determined, for instance, by social norms religious beliefs or cultural reasons), so that ηw_t is the cost incurred by the young and $\eta w^e{}_{t+1} n_t$ is the expected benefit received by the old, which positively depends on the *individual* number of children raised when young. Therefore, the budget constraints of both the working period and retirement period of an individual of generation t can respectively be written as follows:

$$c_{1,t}+s_t+en_t=w_t(1-\eta), \tag{4}$$

$$c_{2,t+1}=R^e_{t+1}s_t+\eta w^e{}_{t+1} n_t . \tag{5}$$

Eq. (4) implies that when young, wage income – net of transfers voluntarily devoted to support old-age consumption – is divided amongst material consumption, saving (s_t) and the (fixed) cost of raising n_t descendants. Eq. (5) reveals that consumption possibilities when old are constrained by both the amount of resources saved when young (times the interest factor R^e_{t+1}) and the expected benefit (gifts received from children).

The individual representative of generation t chooses fertility and saving to maximise the utility function Eq. (3) subject to both the working period budget constraint, Eq. (4), and the retirement period one, Eq. (5), and by taking factor prices and the contribution rate η as given. This implies that each child represents both an investment good and consumption good for a parent, with adult individuals being assumed to draw utility directly from the number of children raised (forward weak altruism). Therefore, the first order conditions for an interior solution are the following:

$$\frac{c_{2,t+1}}{c_{1,t}}\cdot\frac{1}{\beta}=R^e_{t+1}, \tag{6}$$

$$\frac{c_{1,t}}{n_t}\cdot\gamma=e-\eta\frac{w^e{}_{t+1}}{R^e_{t+1}} . \tag{7}$$

Eq. (6) equates the marginal rate of substitution between consumption when young and consumption when old to the expected interest factor determined on the capital market. Eq. (7) equates the marginal rate of substitution between consumption when young and the number of children to the expected (net) marginal cost of bearing an extra child. It is clear that the net marginal cost of children is obtained as the difference between the gross marginal cost of raising a child (e) and the present value of the expected benefit received when old. The higher the fraction of wage income devoted to support old-age consumption, the higher such the benefit will be entitled to the old-aged, and then the lower the net marginal cost of children. Indeed, since individuals know that the size of the (private) inter-generational

[10] This assumption accords with, e.g., Bental (1989) and Morand (1999). Other papers, such as Ehrlich and Lui (1991) and Azariadis and Drazen (1991) takes the size of the transfer as an endogenous variable. Moreover, it should be noted that in these papers it is implicitly assumed that the implicit contract that regulates the transfer mechanism between children and parents will be honoured in every period. The emerging of time consistency and self-enforcement problems of these contracts are discussed by Ehrlich and Lui (1991).

transfer is determined on the basis of their own fertility, the marginal cost of raising a child reduces as long as the contribution rate to the private system of old-age insurance increases.

Using the first order condition and the budget constraints, fertility and saving are respectively given by:

$$n_t = \frac{\gamma w_t (1-\eta)}{(1+\beta+\gamma)\left(e - \eta \frac{w^e_{t+1}}{R^e_{t+1}}\right)}, \tag{8}$$

$$s_t = \frac{w_t (1-\eta)}{(1+\beta+\gamma)\left(e - \eta \frac{w^e_{t+1}}{R^e_{t+1}}\right)} \left[\beta e - (\beta+\gamma)\,\eta \frac{w^e_{t+1}}{R^e_{t+1}}\right]. \tag{9}$$

3.1. Equilibrium and dynamics with myopic foresight and private old-age support

Knowing that population evolves according to $N_{t+1} = \overline{n}_t N_t$, market-clearing in the capital market is determined by the equality between investment and saving, that can be written in per worker terms as follows:

$$\overline{n}_t k_{t+1} = s_t . \tag{10}$$

Since $\overline{n}_t = n_t$ holds in a symmetric equilibrium, then by using Eqs. (8), (9) and (10) we get:

$$k_{t+1} = \frac{\beta e}{\gamma} - \frac{\beta+\gamma}{\gamma} \eta \frac{w^e_{t+1}}{R^e_{t+1}} . \tag{11}$$

In order to close the model, it is now necessary to specify the type of expectations formation mechanisms of individuals about factor prices. The two polar alternatives generally assumed when studying dynamic general equilibrium OLG economies are: (*i*) myopic foresight, and (*ii*) perfect foresight (see, e.g., Michel and de la Croix, 2000; de la Croix and Michel, 2002. In the present study we exclusively focus on the former mechanism for the interesting dynamic events that the model generates in such a case. Under myopic foresight, individuals expect that both the interest and wage rates at time $t+1$ depend on the stock of capital per worker at time t, that is:

$$\begin{cases} R^e_{t+1} = \alpha A k_t^{\alpha-1} \\ w^e_{t+1} = (1-\alpha) A k_t^{\alpha} \end{cases} . \tag{12}$$

Therefore, by exploiting Eqs. (11) and (12), the dynamic path of capital accumulation in an economy with private inter-generational transfers is described by the following first-order linear difference equation:

$$k_{t+1} = \frac{\beta e}{\gamma} - \eta \mathrm{B}_1 \mathrm{B}_2 k_t , \tag{13}$$

where $\mathrm{B}_1 := \frac{1-\alpha}{\alpha}$ and $\mathrm{B}_2 := \frac{\beta+\gamma}{\gamma} > 1$. Eq. (13) shows that the dynamic evolution of capital depends on two components: (*i*) a constant component (because of the assumption of fixed cost of children), and (*ii*) an inter-generational transfer component, due to the existence of intra-family gifts, the size of which being determined as the contribution rate to the private old-age insurance system (η) weighted by two coefficients: the former captures the relative importance between the share of labour and the share of capital in technology; the latter depends on preference parameters. As long as η raises, the disposable income of the young reduces together with the need to sustain old-age consumption, because the benefit received when old increases (crowding out effect), so that saving becomes lower through this channel.

The reduction in saving due to the inter-generational transfer effect is also weighted by a coefficient that measures the relative importance between the share of labour and the share of capital in technology.

Steady-state implies $k_{t+1} = k_t = k^*$. Then, the per worker long-run stock of capital is simply given by:[11]

$$k^*_{OAS} = \frac{\beta e \alpha}{\alpha\gamma + \eta(1-\alpha)(\beta+\gamma)}. \tag{14}$$

Below, we examine the stability properties of such an equilibrium point. First, let

$$\hat{\eta} = \hat{\eta}(\alpha, \beta, \gamma) := \frac{1}{\mathrm{B}_1 \mathrm{B}_2}, \tag{15}$$

$$\hat{\alpha} = \hat{\alpha}(\beta, \gamma) := \frac{\beta+\gamma}{\beta+2\gamma}, \quad 1/2 < \hat{\alpha} < 1. \tag{16}$$

be two threshold values of the contribution rate to the private system of old-age insurance and the capital share in technology, respectively. Then, the following proposition holds.

Proposition 1. *[OAS economy].*

(1) Let $0 < \alpha < \hat{\alpha}$ *hold. Then* $\hat{\eta} < 1$*, and:*

(1.1) if $0 < \eta < \hat{\eta}$*, trajectories are oscillatory and convergent to* k^*_{OAS}*, which is globally stable;*

(1.2) if $\eta = \hat{\eta}$*, an oscillation of constant amplitude emerges;*

(1.3) if $\hat{\eta} < \eta < 1$*, trajectories are oscillatory and divergent from* k^*_{OAS}*, which is globally unstable;*

(2) Let $\hat{\alpha} < \alpha < 1$ *hold. Then* $\hat{\eta} > 1$ *and, for any* $0 < \eta < 1$*, trajectories are oscillatory and convergent to* k^*_{OAS}*, which is globally stable.*

Proof. By differentiating Eq. (13) with respect to k_t gives:

$$\frac{\partial k_{t+1}}{\partial k_t} = -\eta \mathrm{B}_1 \mathrm{B}_2 < 0. \tag{17}$$

Then, $\frac{\partial k_{t+1}}{\partial k_t} \gtrless -1$ implies:

$$\frac{\partial k_{t+1}}{\partial k_t} = -\eta \mathrm{B}_1 \mathrm{B}_2 \gtrless -1 \Rightarrow \eta \lessgtr \hat{\eta}, \tag{18}$$

where $\eta = \hat{\eta}$ (defined by Eq. 15) is the threshold value of η below (beyond) which k^*_{OAS} is globally stable (unstable). In particular, $\hat{\eta} < 1$ ($\hat{\eta} > 1$) for any $0 < \alpha < \hat{\alpha}$ ($\hat{\alpha} < \alpha < 1$), where $1/2 < \hat{\alpha} < 1$ is defined by Eq. (16). Therefore, (*i*) if $0 < \alpha < \hat{\alpha}$ then $\hat{\eta} < 1$ and (*1.1*) $-1 < \frac{\partial k_{t+1}}{\partial k_t} < 0$ for any $0 < \eta < \hat{\eta}$, (*1.2*) $\frac{\partial k_{t+1}}{\partial k_t} = -1$ if and only if $\eta = \hat{\eta}$, and (*1.3*) $\frac{\partial k_{t+1}}{\partial k_t} < -1$ for any $\hat{\eta} < \eta < 1$. This proves point (1); (*ii*) if $\hat{\alpha} < \alpha < 1$ then $\hat{\eta} > 1$ and $-1 < \frac{\partial k_{t+1}}{\partial k_t} < 0$ for any $0 < \eta < 1$. This proves point (2). **Q.E.D.**

[11] The subscript OAS refers to "old-age support".

Moreover, from Eq. (15) we get the following proposition:

Proposition 2. *[OAS economy]. An exogenous positive shock on the output elasticity of capital (α), the taste for children (γ) [the individual subjective discount factor (β)] acts as an economic stabiliser [de-stabiliser].*

Proof. The proof is obvious from $\frac{\partial \hat{\eta}}{\partial \alpha} = \frac{\gamma}{(1-\alpha)^2(\beta+\gamma)} > 0$, $\frac{\partial \hat{\eta}}{\partial \gamma} = \frac{\alpha\beta}{(1-\alpha)(\beta+\gamma)^2} > 0$ and $\frac{\partial \hat{\eta}}{\partial \beta} = \frac{-\alpha\gamma}{(1-\alpha)(\beta+\gamma)^2} < 0$. **Q.E.D.**

4. Public PAYG pensions

In this section we take the case of public PAYG pension into account as an alternative to a private system of old-age support to transfer resources across generations. The rules of unfunded PAYG schemes imply that current workers finance pensions to current pensioners. Therefore, the aggregate pension expenditure at time t, $P_t = p_t N_{t-1}$, where p_t represents the per old pension expenditure in the same period and N_{t-1} is the number of young people born at the beginning of period $t-1$, which become older at time t, is constrained by the amount of tax receipt $\tau w_t N_t$, where $0 < \tau < 1$ is the contribution rate to the PAYG system levied on current workers' income. Since $N_t = \overline{n}_{t-1} N_{t-1}$, the per pensioner government accounting rule reads as follows:

$$p_t = \tau w_t \overline{n}_{t-1}, \tag{19}$$

which depends on average fertility.

The budget constraints of working period and retirement period of an individual of generation t in a PAYG-taxed economy are respectively given by:

$$c_{1,t} + s_t + e n_t = w_t (1-\tau), \tag{20}$$

$$c_{2,t+1} = R^e_{t+1} s_t + p^e_{t+1}. \tag{21}$$

Analogously to Eq. (4), Eq. (20) implies that wage income when young – net of taxes to finance pensions – is divided amongst material consumption, saving and the cost of bearing n_t children. Eq. (21) reveals that old-age consumption depends on both saving plus interests plus expected pension benefit p^e_{t+1}. From the point of view of individuals, the accounting rule Eq. (19) is not a budget constraint to be taken into account when maximising lifetime utility. This is the reason why p^e_{t+1} in Eq. (21) does not depends on the fertility rate (see, e.g., Cigno, 1995; van Groezen et al., 2003; van Groezen and Meijdam, 2008; Fanti and Gori, 2012). It is therefore important to stress that under PAYG pensions, fertility is endogenous in the sense that utility is directly affected by the presence of children, but – different from an economy with a private system to transfer resources within the family – children are now considered as a consumption good but not as an investment good. Indeed, the presence of a public system of social security implies that the benefit of having a child is too small to be internalised by parents (see Cigno, 1993) so that may free ride on pensions by having lower children (i.e., the pension benefit depends on aggregate fertility and it is entitled to an individual irrespective of the number of children she decides to raise). From a modelling point of view, but for a redefinition of variables, the difference between an economy with private transfers the young generation to the older one and public pensions, consists in including Eq. (19) into the retirement period budget constraint. This is similar to what Nishimura and Zhang (1992) do in an OLG model with generic utility and without production (partial equilibrium), and where young individuals are also allowed to choose the size of the transfer (backward altruism: i.e., an agent cares also about consumption of her parents when old), while in this paper it is fixed and constant (see Footnote 10).

The representative individual at time t chooses now fertility and saving in order to maximise Eq. (3) subject to Eqs. (20) and (21), and by taking factor prices and the tax rate τ and the pension accounting rule Eq. (19) as given. The first order conditions for an interior solution are, therefore, the following:

$$\frac{c_{2,t+1}}{c_{1,t}} \cdot \frac{1}{\beta} = R^e_{t+1}, \tag{22}$$

$$\frac{c_{1,t}}{n_t} \cdot \gamma = e. \tag{23}$$

Eq. (22) is identical to Eq. (6). Eq. (23) makes clear the difference between the hypotheses of private and public systems of old-age insurance. Indeed, Eq. (23) reveals that the marginal rate of substitution between young-age consumption and the number of children is now equal to gross marginal cost of bearing an extra child. This because with PAYG pensions do not take the positive externality of children into account and then now a rise τ does not contribute to reduce the marginal cost of children.

By setting aggregate fertility be equal to individual fertility in equilibrium, $\bar{n}_t = n_t$, and exploiting the first order conditions, the individual budget constraints and the one-period forward pension accounting rule Eq. (19), fertility and saving in a PAYG-taxed economy are respectively given by:

$$n_t = \frac{\gamma w_t (1-\tau)}{(1+\beta+\gamma)e - \gamma\tau \dfrac{w^e{}_{t+1}}{R^e_{t+1}}}, \tag{24}$$

$$s_t = \frac{w_t (1-\tau)}{(1+\beta+\gamma)e - \gamma\tau \dfrac{w^e{}_{t+1}}{R^e_{t+1}}} \left[\beta e - \gamma\tau \frac{w^e{}_{t+1}}{R^e_{t+1}}\right]. \tag{25}$$

Eqs. (24) and (25) deserve some comments and should be compared with Eqs. (8) and (9). In economies with private intra-family transfers and public pensions, a rise in the contribution rate affects fertility and saving differently. As regards the former, the effect is twofold under both insurance systems. Indeed, there exist: (*i*) a negative intra-generational effect because the disposable income of the young reduces, and (*ii*) a positive inter-generational effect because the present value of the benefit received when old raises. However, ceteris paribus as regards the contribution rate ($\eta = \tau$), the difference between the two alternatives consists in the size of the positive effect. Since individuals do not internalise the positive external effect of children when PAYG pensions are in existence, the rise in the benefit entitled to the old-aged when the contribution rate paid the young raises is higher when private transfers are implemented. This implies, ceteris paribus, that fertility tends to be higher under OAS than under PAYG transfers (see the denominators of Eqs. 8 and 24). As regards latter, in addition to the opposite effects (*i*) and (*ii*) above mentioned, there also exists a negative inter-generational effect when the contribution rate is increased under both alternatives. However, the size of it is higher when individuals privately transfer resources across generations than with public PAYG pensions. This implies, ceteris paribus, that saving tends to be higher under PAYG pensions than under OAS (see the term in brackets in Eqs. 9 and 25).

4.1. Equilibrium and dynamics with myopic foresight and public pensions

Market-clearing in the capital market is still determined by Eq. (10). Then, by using Eqs. (24) and (25) the equilibrium condition can now be written as follows:

$$k_{t+1} = \frac{\beta e}{\gamma} - \tau \frac{w^e_{t+1}}{R^e_{t+1}}. \tag{26}$$

By assuming myopic foresight (see Eq. 12), the dynamic path of capital accumulation in a PAYG-taxed economy is described by the following first-order linear difference equation:

$$k_{t+1} = \frac{\beta e}{\gamma} - \tau \mathrm{B}_1 k_t , \tag{27}$$

which is similar to Eq. (13): the only difference consists in a lower negative inter-generational transfer effect on capital accumulation than under OAS. This appears as a reduction (given by the term B_2) in the destabilising component.

Steady-state implies $k_{t+1} = k_t = k^*$. Therefore,

$$k^*_{PAYG} = \frac{\beta e \alpha}{\gamma[\alpha + \tau(1-\alpha)]}. \tag{28}$$

is the long-run per worker stock of capital in an economy with PAYG pensions.

Let

$$\hat{\tau} = \hat{\tau}(\alpha) := \frac{1}{\mathrm{B}_1}, \tag{29}$$

be a threshold value of the tax rate. Then, we have the following proposition.

Proposition 3. *[PAYG-taxed economy].*

(1) Let $0 < \alpha < 1/2$ hold. Then $\hat{\tau} < 1$, and:

(1.1) if $0 < \tau < \hat{\tau}$, trajectories are oscillatory and convergent to k^*_{PAYG}, which is globally stable;

(1.2) if $\tau = \hat{\tau}$, an oscillation of constant amplitude emerges;

(1.3) if $\hat{\tau} < \tau < 1$, trajectories are oscillatory and divergent from k^*_{PAYG}, which is globally unstable.

(2) Let $1/2 < \alpha < 1$ hold. Then $\hat{\tau} > 1$ and, for any $0 < \tau < 1$, trajectories are oscillatory and convergent to k^*_{PAYG}, which is globally stable.

Proof. Differentiating Eq. (27) with respect to k_t gives:

$$\frac{\partial k_{t+1}}{\partial k_t} = -\tau \mathrm{B}_1 < 0. \tag{30}$$

Then, $\frac{\partial k_{t+1}}{\partial k_t} \gtrless -1$ implies:

$$\frac{\partial k_{t+1}}{\partial k_t} = -\tau \mathrm{B}_1 \gtrless -1 \Rightarrow \tau \lessgtr \hat{\tau}, \tag{31}$$

where $\tau = \hat{\tau}$ (defined by Eq. 29) is the threshold value of the tax rate below (beyond) which k^*_{PAYG} is globally stable (unstable). In particular, $\hat{\tau} < 1$ ($\hat{\tau} > 1$) for any $0 < \alpha < 1/2$ ($1/2 < \alpha < 1$). Therefore, (*i*) if $0 < \alpha < 1/2$ then $\hat{\tau} < 1$ and (*1.1*) $-1 < \frac{\partial k_{t+1}}{\partial k_t} < 0$ for any $0 < \tau < \hat{\tau}$, (*1.2*)

$\frac{\partial k_{t+1}}{\partial k_t} = -1$ if and only if $\tau = \hat{\tau}$, and (*1.3*) $\frac{\partial k_{t+1}}{\partial k_t} < -1$ for any $\hat{\tau} < \tau < 1$. This proves point (1); (*ii*) if $1/2 < \alpha < 1$ then $\hat{\tau} > 1$ and $-1 < \frac{\partial k_{t+1}}{\partial k_t} < 0$ for any $0 < \tau < 1$. This proves point (2). **Q.E.D.**

Proposition 1 and 3 make clear the difference between private transfers and public pensions on stability of the long-run equilibrium. Since with PAYG pensions individuals do not internalise the positive external effects of children, the marginal cost of raising a child is higher than with OAS and this, in turn, implies that a rise in the contribution rate increases the size of the inter-generational transfer more than with PAYG pensions. Under OAS, individuals know that the benefit they will receive when old is computed on the basis of their own number of children (rather than average fertility). Since the weight of the present value of inter-generational transfer component on capital accumulation is higher under OAS than under PAYG, stability effects are different. This topic will be discussed in Section 5.

Moreover, from Eq. (29) we have the following proposition:

Proposition 4. *[PAYG-taxed economy]. An exogenous positive shock on the output elasticity of capital (α) [the taste for children (γ), the individual subjective discount factor (β)] acts as an economic stabiliser [is neutral on stability].*

Proof. The proof is obvious from $\frac{\partial \hat{\tau}}{\partial \alpha} = \frac{1}{(1-\alpha)^2} > 0$ and $\frac{\partial \hat{\tau}}{\partial \gamma} = \frac{\partial \hat{\tau}}{\partial \beta} = 0$. **Q.E.D.**

5. Private intra-family transfers versus public PAYG pensions

The aim of this section is to compare the steady-state and the dynamic outcomes under the two polar alternatives of old-age insurance analysed in the previous sections. For doing this, first we let $\eta = \tau$ hold.[12] Then, we compare Eq. (14) versus Eq. (28) as regards steady states, and Eq. (13) versus Eq. (27) as regards the dynamics of capital. The results are summarised in the following two propositions.

Proposition 5. *[Steady states]. Let $\eta = \tau$ hold. Then $k^*_{PAYG} > k^*_{OAS}$.*

Proof. The proof is straightforward by comparing Eqs. (14) and (28) when $\eta = \tau$. **Q.E.D.**

The reason why capital accumulation is higher with PAYG pensions rather than with private intra-family transfers is the following. As previously discussed, with a private system of old-age support, individuals know that they will receive the benefit at older ages on the basis of their own number of children, and this therefore tends to reduce the marginal cost of child rearing as compared with an economy with public PAYG pensions (where the benefit at older ages depends on aggregate fertility), so that the number of children under PAYG pensions is lower than under OAS (see, e.g., Cigno, 1993; Zhang and Nishimura, 1993). Moreover, ceteris paribus, the shift from private transfers to public pensions increases savings, because children represent a partial substitute for saving opportunities when old. As a consequence, since capital accumulation is the result of the ratio between saving and fertility, it results to be higher when a public (rather than private) system of social security is in place.

[12] This assumption follows Nishimura and Zhang (1992, 1995), Cigno (1995) and Zhang and Zhang (1995).

From a microeconomic point of view, a public pension system is welfare enhancing since it reduces adverse selection (i.e., it reduces the incentive not to have children or to reduce expenses in children nurture), pooling individuals facing different longevity risks. From a macroeconomic point of view, there are several papers that deal with this topic in the overlapping generations literature: Zhang and Zhang (1995) and Wigger (1999) study models of endogenous growth, while Nishimura and Zhang (1992, 1995) and Cigno (1995) are in partial equilibrium. In particular, all this papers take the case of backward altruism towards children (i.e., individual derive utility from material consumption of the old parents) into account, and compare the effects of the introduction of public PAYG pensions in an economy where private intra-family gifts are already in existence. Differently, we assume weak altruism towards children and compare the long-run effects of private gifts and public pensions as *alternative* ways to transfer resources across generations in a general equilibrium model of neoclassical growth. Of course, Proposition 5 accords with Zhang and Zhang (1995) and Wigger (1999) in the sense that the former authors find that the growth rate in an economy with public pensions and private intra-family gifts is higher than that of an economy where public pensions are absent. The latter author finds a similar result, while also showing that when the contribution rate to the PAYG system increases the relationship between the growth rate of the economy and the contribution rate may be hump shaped.

Proposition 6. *[Dynamics]. Let* $\eta = \tau$ *hold. Then, cyclical instability under OAS more likely occurs then under PAYG pensions.*

Proof. Since $\mathrm{B}_2 > 1$, then by comparing Eqs. (13) and (27) we get $\hat{\eta} < \hat{\tau}$. Moreover, since $1/2 < \hat{\alpha} < 1$, then for any $\eta = \tau$ the region of cyclical instability under PAYG pensions is always lower than under private old-age insurance. **Q.E.D.**

Proposition 6 follows from Propositions 1 and 3. Indeed, ceteris paribus, the relative weight of the inter-generational transfer component in capital accumulation is higher in an economy with a private system of old-age insurance than in an economy with public pensions. Since individuals know that the benefit entitled at older age is contingent on individual fertility, then they tend to substitute saving with children in an OAS economy. This is the economic reason why the slope of the capital accumulation locus is higher with respect to the case of public pensions.

5.1. A numerical exercise

When the output elasticity of capital is larger than one-half, Proposition 3 ensures that with public PAYG pensions the long-run equilibrium of an economy is stable irrespective of the size of the pension system. A numerical illustration would help to show that the stabilising role of PAYG pensions as a substitute of the intra-family gifts is relevant in actual economies.

For illustrative purposes we report the share of physical capital (α) for each of G7 economies from Bernanke and Gürkaynak (2001) and Pecchenino and Pollard, 2005) (see Table 1). Moreover, we observe, by following rather common estimates, that an average contribution rate to the PAYG system of about 16 per cent as well as an expected higher future contribution rate of about 28 per cent by the year 2040 may be realistic for some actual developed economies (see, e.g., Feldstein, 2005; Liikanen, 2007).

Table 1. Capital shares, α, for the G7 countries.

Canada	France	Germany	Italy	Japan	UK	US
0.32	0.26	0.31	0.29	0.32	0.25	0.26

Sources: Bernanke and Gürkaynak (2001) and Pecchenino and Pollard, 2005).

In the case of an average capital share of 0.3 and an average contribution rate around 0.16, it is easy to see, under PAYG pensions, that the stability condition (Eq. 29) is fulfilled since the destabilising factor is $\tau B_1 = 0.3733 < 1$. Moreover, by taking the less favourable case as regards the capital share into account (i.e., $\alpha = 0.25$ which refers to UK) and assuming a future expected high contribution rate of 28 per cent, such a condition still holds since $\tau B_1 = 0.84 < 1$.

Things are different if the support for the old-aged is privately provided within the family. In order to better understand this point, we now "calibrate" the model with private old-age support by choosing, in addition to $\alpha = 0.3$ and $\tau = \eta = 0.16$, other plausible parameters. As regards preferences, we assume $\beta = 0.6$ (the subjective discount factor), see Žamac (2007, p. 628), and $\gamma = 0.2$ (the taste for children). Furthermore, we set $A = 2$ and $e = 0.1$ to obtain a long-run fertility rate around the replacement level (e.g., 2.1 children per couple).[13] Then, it is easy to verify that the stability condition in an OAS economy, see Eq. (15), is violated (i.e., $\eta B_1 B_2 = 1.4933 > 1$).

To sum up we may conclude that under plausible parameter values, the shift from a private system of old-age support to public pensions tends to stabilise the long-run equilibrium. This can also be ascertained by looking at Figure 1, which is plotted for the above parameter values. The figure shows that the long-run stock of capital under PAYG ($k^*_{PAYG} = 0.218$) is stable ($\frac{\partial k_{t+1}}{\partial k_t} = -0.373$), while the long-run stock of capital under OAS ($k^*_{OAS} = 0.12$) is unstable ($\frac{\partial k_{t+1}}{\partial k_t} = -1.493$).

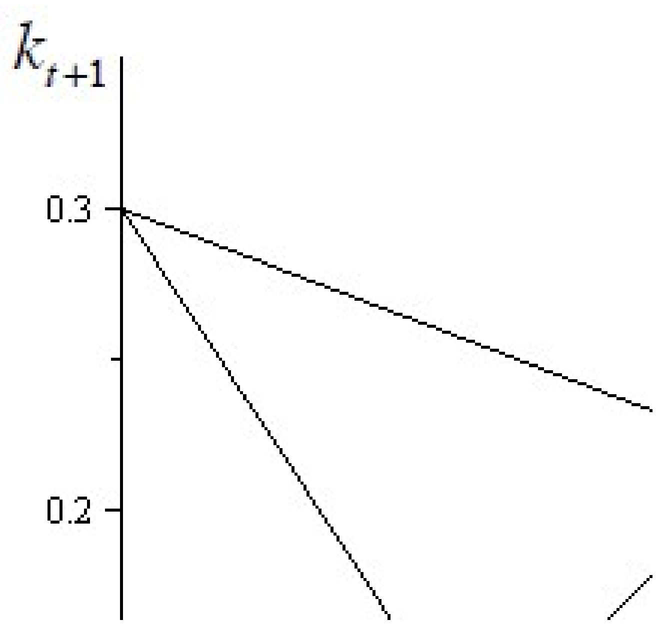

Figure 1. Capital accumulation loci under public pensions (PAYG) and private old-age support (OAS).

6. Conclusions

[13] The percentage of children costs is around 30 per cent of the equilibrium competitive wage.

This paper investigated the effects of two historical alternatives as a means of old-age insurance, i.e., voluntary intra-family transfers from young to old members versus pay-as-you-go public pensions, on steady-states and dynamic outcomes of a neoclassical overlapping generations growth model, by also assuming (in line with the new home economics) that children are a desirable good. As is known, the shift from the former to the latter insurance system has been commonly observed through the stages of economic development, especially between developing and developed countries.

This paper showed, in line with Zhang and Zhang (1995) and Wigger (1999), that public pensions rather than private transfers increases the steady-state stock of capital and output per worker. Moreover, we found, under myopic expectations, that with public pensions the parametric region of cyclical instability is lower than with private old-age backings.

Our findings suggest that the introduction of a public system of social security as a substitute to private intra-family insurance to secure old-age consumption, may also be justified by reasons that have not been so far taken into account: indeed, in addition to the rise of GDP per worker, public pensions reduce the possibility of cyclical instability with respect to private old-age support.

Finally, two limitations of our model should be acknowledged: the first comes from the use of specific utility and production functions; the second is that the paper abstracts from other important components of demography, that is adult mortality. Extending the model to incorporate both the former and latter points would be fruitful for future research.

Acknowledgements

The authors gratefully acknowledge an anonymous reviewer for valuable comments on an earlier draft. The usual disclaimer applies.

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