Adult mortality drops and the effects of the evolution from private intra-family gifts to public pensions

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1. Introduction

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In the recent decades dramatic increases in life expectancy have been observed in several countries in the world. The mortality reduction has been a very impressive phenomenon with a twofold feature (Lee, 1994; Zhang at al., 2001): (*i*) in the early stages of development, the mortality decline has mainly concerned infants. This has lead to a rapid population growth, (*ii*) in the later stages of development, mortality has continued to decline from already low levels, and most part of the gain in the years of life have concerned the older post-retirement ages (Lee-Tuljapurkar, 1997).[1]

It has been noted that the economic consequences of a further mortality decline may be of importance because a longer life span may favour a rapid increase in life-cycle saving and stimulate growth,[2] even if expensive programmes for the provision of pensions and health care services will be required for an increasing number of old individuals. In particular, there are growing concerns about the consequences of adult mortality reductions on the sustainability of PAYG pension systems. For this reason it is of importance to better understand whether the rise in longevity, which has accompanied in the course of historical evolution the diffusion of the public instead of the private support for older people, has been harmful for macroeconomic outcomes.

The question posed in this study is therefore the following: may the change from a private system of old-age insurance to a public system of social security, which has occurred in concomitance with a lengthening of life in the past century in developed countries, be considered harmful or beneficial for the long-term macroeconomic outcomes? The answer depends on the relative size of adult mortality and, in particular, in countries with high rate of adult mortality (underdeveloped and developing countries), the intra-family channel to support old members should be preferred to a public pension system.

The findings of this paper not only matter for the comparative analysis of different historical epochs and different economies characterised by different longevity rates, but they are of interest also for contemporary highly developed economies such as Italy. In fact, recent empirical contributions have provided evidence in favour of the old-age security motive in the contemporary Italian society. In particular, Billari-Galasso (2010), by using as a natural experiment the Italian pension reforms of the 90s, emphasise the relevance of the "old-age security" motive for contemporary fertility. The rather unexpected fact that in highly developed economies with public pension systems people have kids for their old-age support is confirmed also by Attanasio-Brugiavini (2003) and Bottazzi et al. (2006), which "are consistent with the existence of a strong old-age security motive in the contemporaneous Italian society." (Billari-Galasso, 2010, p. 4).

The results by Billari-Galasso (2010) also represent a stimulus to investigate the role of longevity – which has substantially increased in Italy in the recent decades – on the macroeconomics by comparing the old-age support motive to have children with the consumption motive to have children[3] (at least for developed countries).

In addition, the importance of the old-age security hypothesis in contemporary societies is not limited to Italy. For instance, Kağitçibaşi (1982) reports the results of interviews according to which this motive had been cited as "somewhat important" or "very important" by 32 percent of married German women and 27 percent of married U.S. women, while

[1] Moreover, fertility declined as well, thus reducing population growth rates and making populations older, especially in developed countries, where longer life has gone strictly hand in hand with very low fertility, with the result of a dramatic population aging. As noted by Zhang et al. (2001, p. 486) "during the past century, the ratio of life cycle years lived after 65 to years lived 20 to 64 has doubled in many industrial nations, due to mortality decline."

[2] However, Zhang at al. (2001, p. 486) note that there is little evidence with regard to this point.

[3] In fact, Billari-Galasso (2010, p. 5) note that "our empirical findings fail to support the traditional "consumption" motive, which has become the workhorse model of fertility choice in a recent literature on demographic transition and economic growth, or female labor force participation."

Rendall-Bahchieva (1998) provide an extensive documentation of the relevance of children for providing support to parents in contemporary U.S. Overall, it is argued that having children in order to be supported when old should be prominent in developed countries where family ties are relatively stronger such as Italy, Mexico, Poland, US and Spain.[4]

By using the words of Samuelson (1975a, p. 537) "for several generations people may benefit on a lifetime basis by having numerous children to support them well in their old ages, out of filial piety or by means of social security." Voluntary intra-family transfers from young to old people, either in cash or in kind, have been historically observed in virtually all societies and they have substantially contributed to the livelihood of the elderly, mainly in epochs where the public pension provision was absent (see Ehrlich-Lui, 1991). On the one hand, from a theoretical point of view, the intra-family transfers provide a reason to have children in order to secure old-age consumption. On the other hand, the preference for having children as a consumption good is also a well documented argument. In fact, according to the so-called new home economics, pioneered by Leibenstein (1957) and Becker (1960), children directly enter their utility functions like any other consumption good, or, in other words, people have a weak form of altruism towards children (Zhang-Zhang,1998).[5]

After the World War II the provision of pay-as-you-go (PAYG) public pensions have been largely implemented, especially in developed countries, so diminishing the importance of children as a means of old-age insurance. In these countries a sharp decline in fertility as well as a dramatic increase in longevity have also been observed (Dyson-Murphy, 1985; Coale-Watkins, 1986; Barro-Sala-i-Martin, 2003, with regard to fertility; Fogel, 1994, 1997; Easterlin, 1996; Livi-Bacci, 2006, with regard to life expectancy).

Recent studies on the impact of public PAYG pensions on income growth and fertility when the demand for children is due to an insurance motive with only backward altruism (from children to parents), are Zhang (1995a) and Zhang-Zhang (1995), which show that introducing unfunded public pensions leads to a decrease in fertility but an increase in per capita income growth, and Zhang (1995b), which focuses on the influence of public pensions on the choice of parents between the number of children and human capital investments in children. Wigger (1999) extends the previous studies by assuming that children serve also as a consumption good, the parents' consumption enters the lifetime utility function, and both intra-family gifts and public pensions are in existence. While all the above mentioned papers avoid to include longevity in the analyses, Zhang et al. (2001) and Zhang et al. (2003) examine the impacts of mortality decline on long-term endogenous (human capital driven) growth in a dynastic family model with and without an annuity market, respectively, while Bertocchi et al. (2010) tackle with the relation between longevity, pensions and retirement ages.

None of these papers, however, concentrates on a comparison – with an exogenously given probability of dying[6] – between the two historical alternatives as a means of old-age insurance in the benchmark general equilibrium neoclassical growth model with overlapping generations (OLG) à la Diamond (1965) extended with endogenous fertility under weak altruism towards children. Furthermore, the existing theoretical literature also avoid to

[4] These countries are the first five, respectively, among the OECD countries for their strong family ties (Alesina-Giuliano, 2010).

[5] In the economic literature, models contrasting the old-age insurance motive to have children with the "altruistic" motive are reviewed in Ehrlich-Lui (1997); see also Zhang-Zhang (1998) and van Groezen et al. (2003). The empirical evidence, instead, is controversial (Cigno-Rosati, 1996; Ehrlich-Lui, 1998).

[6] It should be noted that, while we only consider an exogenous mortality rate, other works take longevity as an endogenous variable but for purposes different from the present one. For instance, Balestra-Dottori (2009) assume that lifespan depends on health care expenditure and environmental quality. Then, they compare the long-term outcomes in an economy where agents vote over health spending and environmental maintenance with those obtained in an economy where health spending and environmental maintenance are chosen by a social planner.

analyse the effects on stability of changes in adult mortality under both types of transfer systems. In order to keep the analysis as simple as possible, in this paper we consider a standard OLG growth model[7] and assume that people regard their children as desirable in themselves. In addition, parents expect that their descendants will support them when old according to an exogenous gift proportional to wage income.[8] This paper differs from the preceding literature with regard to the assumption of forward altruism towards the number of children instead of the backward altruism assumed in the above mentioned papers. Then, we investigate the effect of changing private intra-family transfers with public PAYG pensions on economic growth and stability when longevity varies.

We find that while the fertility-reducing effect of PAYG systems is confirmed, changing the private old-age insurance system with a public system of social security in countries with relatively small rates of adult mortality (developed countries) increases capital accumulation and GDP per worker, while also acting as an economic stabiliser. In contrast, the use of PAYG pensions in societies where adult mortality is large enough reduces economic growth and may destabilise the long-term equilibrium. This suggests that the historical evolution from private to public old-age backing observed in Western countries and the reduction in adult mortality have promoted economic growth and stability. Moreover, the policy suggestion is that to the extent that the mortality reduction is a widespread phenomenon also in developing countries, due, for instance, to the pervasive diffusion of medical enhancements, an introduction of PAYG pensions to substitute intra-family transfers towards older people reduces over-population and increases economic growth, while also promoting stability of the long-term equilibrium.

In addition to the OLG growth literature with demographic behaviours, this paper contributes to the deterministic business cycle literature (Grandmont, 1985), and shows the different dynamic effects of changes in longevity with respect to two alternatives to transfer resources inter-generationally.[9]

The rest of the paper is organised as follows. Section 2 develops the models with private intra-family gifts and public pensions. Sections 3 and 4 study the steady-state and dynamic features of the economy, respectively. Section 5 compares the effects of a change in longevity under the two alternatives to transfer resources from young to old people with respect to both the long-term income and stability outcomes. Section 6 outlines the conclusion.

2. The economy

2.1. Firms

Firms are identical and act competitively on the market. Aggregate production at t (Y_t) takes place by combining capital (K_t) and labour ($L_t = N_t$ in equilibrium) according to the constant returns to scale Cobb-Douglas technology $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where $A > 0$ is a scale parameter and $0 < \alpha < 1$ is the output elasticity of capital. Defining $k_t := K_t / N_t$ and $y_t := Y_t / N_t$ as capital

[7] This is in contrast with the above mentioned literature which assume an endogenous growth setting.

[8] This follows, for instance Bental (1989), Morand (1999) and Strulik (2004), who assume that each child gives an exogenous transfer to her parent. There is, of course, a problem regarding the enforceability of such implicit contracts (Ehrlich-Lui, 1997), but that issue is beyond of the scope of the present paper. In any case, we note that an implicit contract is credible and time consistent if cheating young adults receive nothing from their children when old (Tamura, 2000).

[9] Note that, for simplicity, we developed a linear dynamic model whose outcome may be, of course, only a globally stable or unstable equilibrium. However, even a very slight modification, such as the assumption of a monetary instead of fixed child cost, by introducing a non-linearity in the dynamic model, allows to generate persistent business cycles.

and output per worker, respectively, the intensive form production function may be written as $y_t = Ak_t^{\alpha}$.

By assuming total depreciation of capital at the end of every period and the output is sold at the unit price, profit maximisation implies that the marginal product of capital and labour equal the gross interest factor and wage rate, respectively, that is:

(1) $$R_t = \alpha A k_t^{\alpha-1},$$

(2) $$w_t = (1-\alpha) A k_t^{\alpha}.$$

2.2. Individuals

Consider a general equilibrium OLG closed economy populated by identical individuals. Life is divided into three stages: childhood, young age and old-age. In the first period, individuals do not make economic decisions. In the second period, individuals work and raise children. In the third period, they are retired. Young individuals of generation t (N_t) inelastically supply labour to firms and receive the competitive wage w_t per unit of labour. This income is used to consume ($c_{1,t}$), to save (s_t) and to take care of children. With regard to childrearing activities, we assume that parents devote a positive fixed amount of resources e to raise a child,[10] so that the cost of raising n_t children is en_t. We also assume that young individuals survive to the working period with an exogenous probability $0 < \pi < 1$ (i.e., $1-\pi$ is the probability of dying at the end of youth). The existence of a perfect annuity market implies that consumption of old survivors ($c_{2,t+1}$) is given not only from their own past saving plus expected interest (R_{t+1}^e), but also from saving plus interest of those who have deceased.

Preferences of individuals of generation t consumption when young, consumption when old and the number of children[11] are described by the following lifetime expected utility function:

(3) $$U_t = \ln(c_{1,t}) + \pi\beta \ln(c_{2,t+1}) + \gamma \ln(n_t),$$

where $0 < \beta < 1$ is the subjective discount factor and $\gamma > 0$ the parents' relative taste for children.

The individual representative of generation t chooses how much to save out of his/her disposable income and the number of children in order to maximise the expected lifetime utility index (3) subject to different budget constraints according to whether the way to transfer resources as a means of old-age insurance is:

(*i*) a voluntary intra-family gift from young people to old people (Eqs. 4 and 5), or

(*ii*) a pay-as-you-go public pension system (Eqs. 7 and 8).[12]

[10] As recognized by De Tray (1983) and Morand (1999, 334), "there is no precise consensus on the particular functional form for the cost of children to the family, but because child rearing takes both time and material resources, economists such as Becker-Barro (1988) and Barro-Becker (1989) have postulated" a form for the cost of raising one child which includes a part linked with wages (a proxy for the time-cost of producing a child) and a fixed part measuring the good-cost associated with feeding the child. For simplicity, only the fixed cost is often considered irrespective of which alternative way to transfer resources as a means of old-age insurance is assumed (following for instance Bental (1989) and van Groezen et al. (2003), in which voluntary intra-family transfers from young to old people, and pay-as-you-go financed public pensions are considered, respectively).

[11] See Eckstein-Wolpin (1985), Galor-Weil (1996), van Groezen et al. (2003) and van Groezen and Meijdam (2008).

[12] Since mathematics is standard and results are routine, we develop the model in a sketched way and present only the final short-run and long-run results.

We note that in the former case children are also desired for old age-support reasons, while in the latter case only for the consumption motive.

Intra-family gifts. We assume that the members of the young generation at time t voluntarily devote a fixed fraction $0 < d < 1$ of wage income to support material consumption of when old, so that $\pi d w_t$ is the expected cost to each young and $d w_{t+1}^e n_t$ is the expected benefit received to the surviving old (Ehrlich-Lui, 1991; Fanti-Gori, 2012). Therefore, the budget constraints of both the working period and retirement period are respectively given by

(4) $$c_{1,t} + s_t + e n_t = w_t(1 - \pi d),$$

(5) $$c_{2,t+1} = \frac{R_{t+1}^e}{\pi} s_t + d\, w_{t+1}^e n_t .$$

Therefore, from the first order conditions we get the demand for children and the saving rate, respectively:

(6.1) $$n_t = \frac{\gamma w_t(1 - \pi d)}{(1 + \pi\beta + \gamma)\left(e - \pi d \frac{w_{t+1}^e}{R_{t+1}^e}\right)},$$

(6.2) $$s_t = \frac{w_t(1 - \pi d)}{(1 + \pi\beta + \gamma)\left(e - \pi d \frac{w_{t+1}^e}{R_{t+1}^e}\right)} \left[\pi\beta e - (\pi\beta + \gamma)\pi d \frac{w_{t+1}^e}{R_{t+1}^e}\right].$$

PAYG pensions. In the case of public pensions, the government accounting rule per pensioner at time t reads as follows:

(7) $$\pi p_t = \theta w_t n_t,$$

the left-hand side being the pension expenditure and the right-hand side the tax receipts, where $0 < \theta < 1$ is the fixed contribution rate levied on the young workers' income.

Therefore, in a PAYG-taxed economy the budget constraints of both the working period and retirement period of an individual of generation t read, respectively, as follows:

(8) $$c_{1,t} + s_t + e n_t = w_t(1 - \theta),$$

(9) $$c_{2,t+1} = \frac{R_{t+1}^e}{\pi} s_t + p_{t+1}^e .$$

By using the first-order conditions and the budget constraints, fertility and savings are respectively given by:

(10.1) $$n_t = \frac{\gamma w_t(1 - \theta)}{(1 + \pi\beta + \gamma)e - \gamma\theta \frac{w_{t+1}^e}{R_{t+1}^e}},$$

(10.2) $$s_t = \frac{w_t(1 - \theta)}{(1 + \pi\beta + \gamma)e - \gamma\theta \frac{w_{t+1}^e}{R_{t+1}^e}} \left(\pi\beta e - \gamma\theta \frac{w_{t+1}^e}{R_{t+1}^e}\right).$$

3. Equilibrium

Knowing that population evolves according to $N_{t+1} = n_t N_t$, equilibrium in the capital market is given by the equality between investments and savings, that can be written in per worker terms as follows:

(11) $$n_t k_{t+1} = s_t .$$

Since future expected values of factor prices affect the choice of fertility and saving in Eqs. (6.1), (6.2), (10.1) and (10.2), we have to specify the type of expectation formation mechanisms. The two polar cases generally used in the literature are: (*i*) myopic expectations, and (*ii*) rational expectations (de la Croix-Michel, 2002).

In this paper we exclusively focus on the case myopic expectation for the interesting dynamic events that the model shows in that case. The assumption of myopic expectations is rather usual in literature (Michel-de la Croix, 2000), and a certain level of myopia seems to be inherent with the life-cycle context when social security schemes exist. With this regard see, for instance, Samuelson (1975b) and Pecchenino-Pollard (2005). In particular, the words of Samuelson (1975b, 543) clarify the link between agents' myopia and public pensions: "Many social security systems, like the New Deal U.S. system, may be deemed most valuable precisely because the myopia ignored by the present models does in fact prevail. People live miserably in old age because they do not realize when young what are the consequences of their private saving habits. So by democratic fiat, they paternalistically impose on themselves a within-life pattern of consumption that favors old age at the expense of young. Precisely because of the myopia that makes paternalism optimal, once citizens are subject to social security taxation and benefits they do not see clearly how they can undo by private-saving offsets what the mandatory system is doing to them. So, both because of themselves and yet despite themselves, they contrive social security that makes them better off." (p. 543). In addition, Some authors also consider the myopic expectation rules to be preferred to the rational expectation rules for the sake of realism (Chen et al., 2008). In fact, they claimed: "In reality, however, the requirement of perfect expectation does not seem reasonable. Agents are more likely to construct their expectations based on current or past information, or simply nothing." (p. 48). In addition, as noted by Michel-de la Croix (2000, 40) "under certain conditions, the knowledge of the dynamics under myopic foresight (which are much simpler) allows us to know the dynamics under perfect foresight." However, we acknowledge that the myopic foresight assumption is introduced in a rather pragmatic way – as argued by an anonymous referee – as a special form of expectations able to generate interesting dynamical behaviours (e.g. cycles).

Therefore, under myopic expectations individuals expect that the values of both the interest factor and wage rate at time $t+1$ depend on the stock of capital at time t, that is:

(12) $$\begin{cases} R_{t+1}^e = \alpha A k_t^{\alpha-1} \\ w_{t+1}^e = (1-\alpha) A k_t^{\alpha} \end{cases}.$$

By combining Eqs. (6.1), (6.2), (11) and (12) in the case of intra-family gifts, and Eqs. (10.1), (10.2), (11) and (12), the dynamic path of capital accumulation is described by the following linear difference equations in OAS[13] and PAYG economies, respectively:

(13) $$k_{t+1} = \frac{\pi\beta e}{\gamma} - \pi d \frac{\pi\beta+\gamma}{\gamma}\frac{1-\alpha}{\alpha} k_t .$$

(14) $$k_{t+1} = \frac{\pi\beta e}{\gamma} - \theta \frac{1-\alpha}{\alpha} k_t .$$

Steady-state implies $k_{t+1} = k_t = k^*$.[14] Therefore, the long-term capital per worker in both the OAS and PAYG economies are respectively given by:

[13] The subscript OAS means "old-age support".

[14] Note that the steady state is the same irrespective of the expectation formation mechanisms (see Michel-de la Croix, 2000).

(15) $$k^*_{OAS} = \frac{\pi\beta e\alpha}{\alpha\gamma + (1-\alpha)\pi d(\pi\beta+\gamma)},$$

(16) $$k^*_{PAYG} = \frac{\pi\beta e\alpha}{\gamma[\alpha + \theta(1-\alpha)]}.$$

4. Dynamic analysis

It is easy to see that the second addendum at the right-hand side of Eqs. (13) and (14) represents a potential destabilising factor, and its size is different in the cases of intra-family gift and PAYG pensions.

From Eqs. (13) and (14), therefore, the following proposition holds:

Proposition 1. *(a) In an economy with intra-family gifts, the steady-state stock of capital is stable (resp. unstable) for* $d < (\text{resp. } >)\, \hat{d}$, *where*

(17) $$\hat{d} = \hat{d}(\alpha,\beta,\gamma,\pi) := \frac{\alpha}{1-\alpha}\cdot\frac{\gamma}{\pi(\pi\beta+\gamma)}.$$

(b) In an economy with PAYG pensions the steady-state stock of capital is stable (resp. unstable) for $\theta < (\text{resp. } >)\, \hat{\theta}$, *where*

(18) $$\hat{\theta} = \hat{\theta}(\alpha) := \frac{\alpha}{1-\alpha}.$$

Proof. The well-known local stability condition $\frac{\partial k_{t+1}}{\partial k_t} \gtrless -1$ implies:

(19) (a) $$\frac{\partial k_{t+1}}{\partial k_t} = -\pi d\,\frac{\pi\beta+\gamma}{\gamma}\,\frac{1-\alpha}{\alpha} \gtrless -1 \Rightarrow d \lessgtr \hat{d},$$

(20) (b) $$\frac{\partial k_{t+1}}{\partial k_t} = -\theta\,\frac{1-\alpha}{\alpha} \gtrless -1 \Rightarrow \theta \lessgtr \hat{\theta}.$$

Q.E.D.

Since the focus of the paper is on the role played by the rate of longevity, Proposition 1 can easily be interpreted by the following proposition:

Proposition 2. *(1) In an economy with intra-family gifts, the rate of longevity (*π*) acts as an economic de-stabiliser. In contrast, in an economy with PAYG pensions the rate of longevity does not affect stability outcomes. (2) The higher the subjective discount factor (*β*) (resp. the lower the output elasticity of capital (*α*) and the taste for children (*γ*)), the stronger (the weaker) the de-stabilising effect of the longevity rate.*

Proof. Since $\frac{\partial \hat{d}}{\partial \pi} = \frac{-\alpha\gamma(2\pi\beta+\gamma)}{(1-\alpha)\pi^2(\pi\beta+\gamma)^2} < 0$ and $\frac{\partial \hat{\theta}}{\partial \pi} = 0$. The proof of Point 2 follows from the derivatives of $\frac{\partial \hat{d}}{\partial \pi}$ with respect to β, α and γ, respectively. **Q.E.D.**

5. Static and dynamic effects of a change in longevity: private intra-family gifts versus public PAYG pensions

The aim of this section is to compare both the steady-state and dynamic adjustment processes under the two alternatives to transfer resources between generations. In particular, we find that a PAYG-taxed pensions is more prone to cyclical instability than an economy with a private intra-family gifts when the rate of longevity is relatively small.

With regard to the steady state, the analysis of Eqs. (15) and (16) gives the following proposition:

Proposition 3. *Let* $d = \theta$ *hold. If* $0 < \pi < \bar{\pi}$ $(\pi = \bar{\pi})$ $[\bar{\pi} < \pi < 1]$, *then* $k^*_{PAYG} < k^*_{OAS}$ $(k^*_{PAYG} = k^*_{OAS})$ $[k^*_{PAYG} > k^*_{OAS}]$, *where*

(21) $$\bar{\pi} := \frac{-\gamma + \sqrt{\gamma^2 + 4\beta\gamma}}{2\beta}, \quad 0 < \bar{\pi} < 1.$$

Proof. By comparing Eqs. (15) and (16) for any $d = \theta$ gives $k^*_{PAYG} = k^*_{OAS}$ if and only if

(22) $$-\pi^2\beta - \pi\gamma + \gamma = 0.$$

By solving Eq. (22) for π gives $\underline{\pi} := -\frac{\gamma + \sqrt{\gamma^2 + 4\beta\gamma}}{2\beta} < 0$ and $0 < \bar{\pi} < 1$ (see Eq. 21), which is the unique economically relevant solution. Therefore, it is straightforward to verify that $k^*_{PAYG} < k^*_{OAS}$ for any $0 < \pi < \bar{\pi}$ and $k^*_{PAYG} > k^*_{OAS}$ for any $\bar{\pi} < \pi < 1$. **Q.E.D.**

With regard to stability outcomes, we have the following proposition:

Proposition 4. *Let* $d = \theta$ *hold. If* $0 < \pi < \bar{\pi}$ *(resp.* $\bar{\pi} < \pi < 1$*), then cyclical instability occurs more likely in a PAYG-taxed economy (resp. in an economy with intra-family gifts).*

Proof. By comparing Eqs. (17) and (18) gives $\frac{\hat{d}}{\hat{\theta}} = \frac{\gamma}{\pi(\pi\beta + \gamma)} = 1$ if and only if Eq. (22) holds. Therefore, if $0 < \pi < \bar{\pi}$ (resp. $\bar{\pi} < \pi < 1$) then $\hat{\alpha} < 1/2$ and $\hat{d} > \hat{\theta}$ (resp. $\hat{\alpha} > 1/2$ and $\hat{d} < \hat{\theta}$), and the parametric region of cyclical instability in a PAYG-taxed economy is larger (resp. smaller) than the parametric region of cyclical instability in an economy with private intra-family gifts for any $d = \theta$. **Q.E.D.**

In an economy with private old-age support, the size of the inter-generational transfer is proportional to the number of surviving old parents, while in an economy with a public system of social security the amount of resources that the government collects to finance PAYG pensions is independent of whether parents of current young-workers are survived or deceased at the end of youth. This means that the disposable income of the young is higher when private transfers are in place, *ceteris paribus* with regard to both the wage rate and the contribution (gift) rate. In contrast, since in an economy with public pensions, the benefit received by the surviving old agents increases along with the rate of adult mortality, then the disposable income of old-age people is higher, *ceteris paribus*, with respect to that obtained in an economy with private transfers. Therefore, the change from a pure private system of old-age backing to a pure public system of social security implies that the higher is adult mortality, the higher is the amount of resources transferred from young people to the elderly as a means of old-age insurance, as can be ascertained by looking at the individual budget constraints in both economies.

With a private system of old-age backing, we expect therefore to observe that the higher adult mortality, the higher the disposable income of the young and the lower the benefit received when old. Both these factors contribute to make the saving rate in an economy with private old-age backing higher (lower) than that in a PAYG-taxed economy when longevity is low (high) enough, because the relative importance of the increased benefit to old (reduced disposable income when young) dominates in hat case. The fertility rate, instead, is always lower with PAYG pensions.[15]

Since capital accumulation is the ratio between savings and fertility, when adult mortality is low, an economy with public PAYG pensions is less prone to cyclical instability, and approaches a higher steady state stock of capital and GDP per worker in the long run, because the relative weight of the negative inter-generational transfer effect is lower than that exerted in an economy with private old-age backing, ceteris paribus as key parameters of the problem.

5.1. A numerical illustration

Proposition 4 shows that the region of cyclical instability in an economy private old-age backing is larger than that with PAYG pensions only when adult mortality is relatively high. Moreover from eq. (18) it is easy to observe that when there is a PAYG system the stability is always preserved provided that the output elasticity of capital is larger than one-half. Therefore, to the extent that capital shares and contribution rates are not too low, therefore, we may conclude that economies under PAYG systems enjoy good health as regards stability concerns, while economies under a private intra-family system of supporting the old may do not.

Below we present a numerical illustration to show the effects of introducing a public system of social security as an alternative to a private-based old-age support system when the rate of longevity is either relatively high or low (see Propositions 3 and 4). We take the following parameter set. As regards technology: $A = 10$ (simply a scale parameter) and $\alpha = 0.30$ (an average value of the capital share based on Table 1)[16]; as regards preferences: $\beta = 0.60$ (Žamac, 2007), $\gamma = 0.20$, which determine a threshold value of the rate of longevity of $\bar{\pi} = 0.4342$. Then we also choose $e = 1.50$ and $\theta = d = 0.16$ (Liikanen, 2007), through which we obtain the equilibrium values of the number of children around the replacement level (e.g. 2.1 children per couple) and a percentage cost of child rearing of about 25 per cent of the equilibrium competitive wage. With this parameter set we obtain $\hat{\theta} = 0.4285$. Now, assuming $\pi = 0.95$ (i.e. the case of low mortality $0 < \pi < \bar{\pi}$) we get $\hat{d} = 0.1171$. Therefore, it is easy to verify that the stability condition in an economy with private old-age backing is largely violated, while that of the PAYG-taxed economy is satisfied, that is the substitution of private old-age support with public PAYG pensions magnifies the stability properties generated by the PAYG system when the rate of longevity is high enough.

6. Conclusions

Using a neoclassical growth model with overlapping generations à la Diamond (1965) with endogenous fertility, we have analysed the long-term and dynamic effects of changes in adult

[15] The proof is not reported for economy of space. Of course, it is available on request.

[16] Table 1 shows the share of physical capital (α) for the G7 countries.

Table 1. Capital shares, α, for the G7 countries.

Canada	France	Germany	Italy	Japan	U.K.	U.S.
0.32	0.26	0.31	0.29	0.32	0.25	0.26

Source: Bernanke-Gürkaynak (2001), as reported in Pecchenino-Pollard (2005, p. 458).

mortality when two alternatives are considered to transfer resources between generations, namely (*i*) voluntary intra-family transfers from young to old people, and (*ii*) pay-as-you-go public pensions. As is known, two alternative theories – children as "consumption" versus "investment" good – attempt to inquire about the reasons why people have children in developing as well as in developed societies and thus in case (*i*) children are also desired for the old-age support reasons, while in the case (*ii*) only for the "consumption" motive. Interestingly, recent empirical work (e.g. Billari-Galasso, 2010) have shown that the old age insurance reason to have children is prevailing also in highly developed countries such as Italy.

In this paper we have shown that the rate of longevity affects the differently the long-run accumulation and GDP per worker depending on whether private intra-family transfers or and public pensions are used. In addition, the rate of longevity does not affect the stability outcomes in a PAYG-taxed economy while acting as a destabilising device private transfers are in existence.

In particular, a comparison of the two systems of inter-generational transfer has revealed that when adult mortality is small, an economy with public pensions is less prone to cyclical instability, and approaches a higher steady state stock of capital and GDP per worker in the long run. An implication is the transition from a private system of old-age backing to a public system of social security in countries where life expectancy is relatively high (e.g., developed countries) promotes capital accumulation in the long run (and, hence, the long-run GDP per worker), while also reducing the risk of cyclical instability, *ceteris paribus* as regards the other parameters of the problem.

On the contrary, our findings also represent a policy warning about the dramatic destabilising effects of adopting public pay-as-you-go pension systems as an alternative to voluntary intra-family transfers in societies where adult mortality is relatively large (e.g., developing or under-developed countries) due, for instance, to non-economic reasons such as endemic diseases and ethnical and civil wars. Moreover, our results have interesting implications not only for the comparative analysis of different historical epochs and different economies characterised by different longevity rates, but also for contemporary highly developed economies such as Italy where, on the one side, pension policies are high in the political agenda and on the other side both the old age support motive seems to be prominent and longevity is fairly high and increasing.

The suggested interpretation is that shifting from private intra-family transfers to public transfers to old individuals seems to be valuable at a certain stage of the process of adult mortality reduction, since it promotes a higher GDP per worker and works for economic stability. Therefore, the fact that "it has been observed that at certain stage of economic growth and development a nation starts to consider the introduction of social security programs." (Zhang-Zhang, 1995, p. 441), could have other motivations, in addition to those so far pointed out by the previous literature, such as for instance the equity reasons by a paternalistic government (Samuelson, 1975b) or the purpose of reducing fertility (Zhang-Zhang, 1995).[17]

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[17] "In developing nations, there has been increasing discussion in popular press of introducing social security systems, apparently for the purpose of reducing fertility… Many people believe that social security has contributed to fertility decline… The presence of the pay-as-you-go system may be a reason why people want fewer children, (Zhang-Zhang, 1995, p. 440)".

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Summary: Adult mortality drops and the effects of the evolution from private intra-family gifts to public pensions (J.E.L. C62; H55; J14; J18; J26)

Since increasing attention is paid to consider the macroeconomic effects of the increasing longevity, we study an overlapping-generations model with endogenous fertility to investigate the steady-state and dynamic effects (under myopic expectations) of two historical alternatives as a means of old-age insurance: voluntary intra-family transfers from young to old members versus pay-as-you-go public pensions. We show that the change from a private system of old-age insurance to a public system of social security has favoured the rise in capital accumulation while also reducing cyclical instability in countries where longevity is large enough. In contrast, when adult mortality is high such a change makes an economy with

public pensions more prone to cyclical instability, while also reducing the steady-state stock of capital and GDP per worker. In addition, since the old-age insurance motive seems to prevail also in developed countries with long-lived individuals such as Italy, our results may also be of interest for pension policies.