Increasing PAYG pension benefits and reducing contribution rates°

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Abstract

We analyse how a reduced contribution rate affects the balanced pay-as-you-go pension budget in the basic overlapping generations model of neoclassical growth (Diamond, 1965). It is shown that PAYG pensions can be increased by reducing the payroll tax paid by the young contributors.

Keywords PAYG pension; OLG model

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1. Introduction

Population in developed countries has dramatically aged in recent years as longevity has risen and birth rates have fallen firmly, and, unfortunately, this downward trend seems to be accelerating in the near future.[1] Since in many developed countries, especially in Europe, old-age public pensions are financed on a pay-as-you-go (PAYG) basis, the viability of the PAYG system facing with population ageing seems to be in peril.[2] Moreover, a widespread belief, shared both by economists and policymakers, retains that in order to keep balanced future PAYG pension budgets, the contribution rate (*ceteris paribus* as regards both the productivity growth and the length of the retirement period) will have to rise.[3] This common wisdom leads governments to pose a dismal trade-off: the viability of the widespread PAYG systems needs either a reduction in pension payments received by the elderly (alternatively, a lengthening of the working life) or a rise in the payroll tax paid by young workers, or both.

In this paper, by using deliberately for its simplicity[4] the textbook Diamond-style OLG model – where all retirees receive PAYG pension benefits –, we address the following question: is the inter-generational conflict implied by the common wisdom inspiring policymakers always a theoretical hard fact? The answer is no. We show in fact that a "virtuous" relationship rather than a disagreeable trade-off between the benefit received during the retirement period and the contribution rate paid during the working life may exist whenever the size of the latter is sufficiently high: PAYG pensions in fact can be increased by decreasing the contribution rate paid by the currently active generation. The reason is straightforward: the level of the benefit financeable from a given payroll tax depends on the level of the wage rate (tax base). The latter in turn depends on the capital stock per person. A reduction in the contribution rate increases the incentive to save and thus implies a higher pace of accumulation of capital while uplifting the tax base. Therefore, in the long-run, the general equilibrium effect due to higher wages may more than counterbalance the direct impact effect which – by contrast – tends to reduce PAYG pensions. Definitely, the effect of a reduction in the contribution rate can be a pension enhancement.

Moreover, we show that the existence of either a trade-off or a virtuous relationship between pensions and contribution rates depends exclusively on the relative size of both technology and preference parameters, i.e., the distributive capital share and the inter-temporal subjective discount factor. In particular, numerical simulations revealed that the higher is the weight of capital in production (i.e., the technology is relatively capital-oriented rather than labour-oriented) and the lower is the subjective discount factor (i.e., individuals are relatively impatient and prefer to smooth their consumption between the two periods of life at a sufficiently high rate), the more likely a reduction in the contribution rate increases PAYG pensions.

The paper is organised as follows. In Section 2 we develop the model. In Section 3 the main steady-state results are analysed and discussed. Section 4 concludes.

2. The model

2.1. Individuals

Young population N_t grows at a constant rate n and agents are assumed to belong to an overlapping generations structure with finite lifetimes. Life is divided into two periods: youth (working period) and old-age (retirement period). Individuals belonging to generation t have a homothetic and separable utility function (U_t) defined over young-aged and old-aged consumption, $c_{1,t}$ and $c_{2,t+1}$, respectively. Each young individual is endowed with one unit of time which is supplied inelastically to the labour market, while receiving wage income at the competitive rate w_t. This income is used to consume during youth, to save and to support material consumption of the elderly. During old-age agents are

[1] For example, while the age dependency ratio (over 65 in total population) was for most countries around 20 per cent in 1995, it will be around 67 per cent in Italy and around 57 per cent in Japan, and by considering the whole Western Europe around 49 per cent in 2040 (see United Nations, 1998).

[2] The reason is that "the public pension systems of the G7 countries were established in an era when the number of contributors to the pay-as-you-go schemes far outweighed the number of beneficiaries… Now, for each beneficiary there are fewer contributors, and this downward trend is projected to accelerate." (Pecchenino and Pollard, 2005, p. 449–450).

[3] As Pecchenino and Pollard (2005, p. 450) claimed: "to maintain benefit levels, tax rates and/or productivity growth will have to rise."

[4] We retain that embarking in a more fashioned endogenous growth model may obscure, for instance owing to the working of the implied externalities, rather than clarify the essential message of this paper which focuses exclusively on the relationship between PAYG pensions and the contribution rate. Although some papers (e.g. Pecchenino and Pollard, 2002, 2005; Zhang et al., 2001) addressed – in an endogenous growth context with unfunded social security – the issue of the relationship between longevity and economic growth, arguing that such a relationship might be positive, the result of this paper, showing that even in the textbook OLG model pension benefits may be increased by reducing the payroll tax paid by current workers, has not been so far, to the best of our knowledge, pointed up.

retired and live on the proceeds of their savings (s_t) plus the accrued interest at the rate r_{t+1} as well as on the publicly provided pension benefit, p_{t+1} .

The representative individual born at time t solves the following problem:

$$\max_{\{s_t\}} U_t = \ln(c_{1,t}) + \gamma \ln(c_{2,t+1}), \tag{1}$$

subject to

$$\begin{aligned} c_{1,t} + s_t &= w_t(1-\theta) \\ c_{2,t+1} &= (1+r_{t+1})s_t + p_{t+1} \end{aligned},$$

where $0 < \theta < 1$ is the contribution rate to the PAYG scheme paid by current workers and $0 < \gamma < 1$ is the subjective discount factor, i.e., γ represents the household's ability to smooth consumption over time.

Maximisation of (1) thus gives the following saving function:

$$s_t = \frac{\gamma w_t(1-\theta)}{1+\gamma} - \frac{p_{t+1}}{(1+\gamma)(1+r_{t+1})}. \tag{2}$$

2.2. Government

The government balances in every period the PAYG social security budget according to the following formula:

$$p_t = \theta w_t(1+n), \tag{3}$$

where the left-hand side represents the social security expenditure and the right-hand side the tax receipts.

Now, inserting the one-period-forward pension accounting rule (3) into (2) to eliminate p, the saving function becomes:

$$s_t = \frac{\gamma w_t(1-\theta)}{1+\gamma} - \frac{(1+n)\theta}{1+\gamma}\frac{w_{t+1}}{1+r_{t+1}}. \tag{4}$$

2.3. Firms

We assume a competitive market with identical firms. The (aggregate) constant returns to scale Cobb-Douglas technology is $Y_t = AK_t^{\alpha}L_t^{1-\alpha}$, where Y_t, K_t and $L_t = N_t$ are output, capital and the time-t labour input respectively, $A > 0$ represents a scale parameter and $0 < \alpha < 1$ is the distributive capital share. Defining $k_t := K_t / N_t$ and $y_t := Y_t / N_t$ as capital and output per-capita respectively, the intensive form production function may be written as $y_t = Ak_t^{\alpha}$. The capital stock is assumed to depreciate fully in one generation and the price of final output is normalised to unity. Therefore, profit maximisation leads to the following marginal conditions for capital and labour, respectively:

$$r_t = \alpha A k_t^{\alpha-1} - 1, \tag{5}$$

$$w_t = (1-\alpha)Ak_t^{\alpha}. \tag{6}$$

2.4. Equilibrium

Given the government budget (3) and knowing that $N_{t+1} = (1+n)N_t$, equilibrium implies $(1+n)k_{t+1} = s_t$. Substituting out for s_t from (4), exploiting (5) and (6), and assuming individuals are perfect foresighted, the dynamic equilibrium sequence of capital reads as:

$$k_{t+1} = \frac{\gamma(1-\theta)\alpha(1-\alpha)A}{(1+n)[\alpha(1+\gamma)+\theta(1-\alpha)]}k_t^{\alpha}. \tag{7}$$

Steady-state implies $k_{t+1} = k_t = k^*$. Therefore:[5]

$$k^*(\theta) = \left\{\frac{\gamma(1-\theta)\alpha(1-\alpha)A}{(1+n)[\alpha(1+\gamma)+\theta(1-\alpha)]}\right\}^{\frac{1}{1-\alpha}}. \tag{8}$$

[5] Using (7) and (8) it can easily be shown that the steady-state is always stable.

From (8) it can easily be seen that a rise in the contribution rate reduces the long-run stock of capital per person, that is:

$$\frac{\partial k^*(\theta)}{\partial \theta}=\frac{-k^*(\theta)(1+\alpha\gamma)}{(1-\theta)(1-\alpha)[\alpha(1+\gamma)+\theta(1-\alpha)]}<0. \tag{9}$$

3. PAYG pensions and the contribution rate in the long-run

In this section we study how a reduction in the payroll tax paid by current workers affects long-run PAYG pensions. In particular, we are wondering about the possibility to increase the benefit entitled to retired (old-aged) people by reducing the contribution rate paid by (young-aged) workers. The common belief suggests that if the government wishes to guarantee an adequate benefit to current pensioners it must increase the payroll tax paid by the young contributors. May the policymaker increase PAYG pensions by decreasing the contribution rate? In what follows we give an answer to this simple but important question finding that such a common belief may be totally reversed. It is shown, in fact, that there is room for a pension enlargement by cutting down the contribution rate.

Given Eq. (3), the long-run PAYG pension formula as a generic function of θ may be written as:

$$p^*=p^*\{\theta,w^*[k^*(\theta)]\}. \tag{10}$$

Totally differentiating (10) with respect to θ gives:[6]

$$\frac{dp^*}{d\theta}=\overbrace{\frac{\partial p^*}{\partial \theta}}^{+}+\underbrace{\overbrace{\frac{\partial p^*}{\partial w^*}}^{+}\cdot\overbrace{\frac{\partial w^*}{\partial k^*}}^{+}\cdot\overbrace{\frac{\partial k^*}{\partial \theta}}^{-}}_{-}. \tag{11}$$

Eq. (11) reveals that the final effect of a marginal reduction in the contribution rate on the steady-state pension benefit depends on two counterbalancing forces, and thus it appears to be ambiguous: (i) a direct straightforward effect which tends to reduce pensions owing to the lower contribution of each single worker to the PAYG system, and (ii) an indirect general equilibrium feedback effect which – by contrast – increases pensions through the higher tax base owing to an increased stock of capital per person. Given the positive relationship between PAYG pensions and wages, the higher is the wage earned by the young workers (contributors) the higher is the benefit received by old age pensioners.

To analyse ultimately which of the two forces dominates we now collect (3), (6) and (8) to obtain:

$$p^*(\theta)=\theta(1+n)(1-\alpha)A\cdot\left\{\frac{\gamma(1-\theta)\alpha(1-\alpha)A}{(1+n)[\alpha(1+\gamma)+\theta(1-\alpha)]}\right\}^{\frac{\alpha}{1-\alpha}}. \tag{12}$$

From Eq. (12), therefore, the following proposition holds:

Proposition 1. *An increase in the contribution rate increases PAYG pensions if and only if* $\theta<\overline{\theta}$.

Proof. The proof uses the following derivative:

$$\frac{\partial p^*(\theta)}{\partial \theta}=\frac{A\cdot[k^*(\theta)]^{\alpha}(1+n)\cdot[-\mathrm{H}_1\theta^2+\mathrm{H}_2\theta+\mathrm{H}_3]}{(1-\theta)[\alpha(1+\gamma)+\theta(1-\alpha)]},$$

where $\mathrm{H}_1:=(1-\alpha)^2>0$, $\mathrm{H}_2:=2\alpha^2-(4+\gamma)\alpha+1$ and $\mathrm{H}_3:=\alpha(1-\alpha)(1+\gamma)>0$. Thus,

$$\operatorname{sgn}\left\{\frac{\partial p^*(\theta)}{\partial \theta}\right\}=\operatorname{sgn}\{-\mathrm{H}_1\theta^2+\mathrm{H}_2\theta+\mathrm{H}_3\}.$$

Since $\Delta:=\mathrm{H}_2^{\,2}+4\mathrm{H}_1\mathrm{H}_3>0$, then by applying the Descartes' rule of sign we find that, independently of the sing of H_2, there always exist two real roots $\underline{\theta}\equiv\frac{\mathrm{H}_2-\sqrt{\Delta}}{2\mathrm{H}_1}<0$ and $\overline{\theta}\equiv\frac{\mathrm{H}_2+\sqrt{\Delta}}{2\mathrm{H}_1}>0$ such that $\frac{\partial p^*(\theta)}{\partial \theta}=0$.[7] Since $\underline{\theta}<0$ it is automatically ruled out.

Therefore,

[6] Details are given in the Appendix.

[7] Notice that $\overline{\theta}<1$ always holds. This inequality in fact implies $\sqrt{\Delta}<2\mathrm{H}_1-\mathrm{H}_2$ where $2\mathrm{H}_1>\mathrm{H}_2$. Therefore, $\Delta<(2\mathrm{H}_1-\mathrm{H}_2)^2$. Exploiting Δ and rearranging terms we get $\alpha(1+\alpha\gamma)>0$ always holds.

$$\frac{\partial p^*(\theta)}{\partial \theta} \gtrless 0 \Leftrightarrow \theta \lessgtr \bar{\theta},$$

with $\theta = \bar{\theta}$ being an interior global maximum. **Q.E.D.**

Proposition 1 states that the benefit received by the elderly increases along with the contribution rate paid by the young workers if and only if the latter is not fixed at too high a level, that is $\theta < \bar{\theta}$. Beyond such a level, in fact, there exists a negative relationship between PAYG pensions and payroll taxes. Proposition 1 therefore suggests that there is room for an increase in future pensions along with a reduction in the tax burden, meaning that (for a sufficiently high value of θ) the general equilibrium effect due to the increased tax base more than counterbalances the direct positive impact effect – which by contrast reduces PAYG pensions – in such a way that final effect of a reduced contribution rate is a pension enhancement. This result reverses the idea that a higher payroll tax is always beneficial for future pensions. Moreover, a pension-maximising value of θ exists independently of the relative size of the basic parameters of the model, i.e., technology and preference parameters and the level of the population growth rate.

In the next section we will investigate – through a numerical simulation – how the value of the payroll tax that marks the start of the pension-declining region ($\bar{\theta}$) changes along with the size of both the distributive capital share and the individual subjective discount factor.

3.1. A numerical illustration

An example – chosen only for illustrative purposes – of the result stated in Proposition 1 is summarised in the following Table 1, which illustrates how both the benefit maximising value of the tax rate and the ratio of the benefit with a 90 per cent tax rate to that with the benefit-maximising one vary along with the distributive capital share and the degree of individual impatience to consume over the life cycle, *ceteris paribus* as regards the other parameters of the model, i.e., the scale parameter in the Cobb-Douglas production function ($A = 10$) and the rate of population growth ($n = 0$).

Table 1 shows that: (1) for any given value of the subjective discount factor, a relatively capital-oriented rather than labour-oriented technology promotes the beneficial effect of a payroll tax reduction on PAYG pensions; and (2) for any given value of the distributive capital share, a decreasing subjective discount factor (i.e., a lower degree of individual "thriftiness") reduces the value of the benefit-maximising tax rate. Thus, for any γ, the higher is α (alternatively, for any α, the lower is γ) the lower is the value of the tax rate that marks the start of the pension-declining region ($\bar{\theta}$). As a consequence, the higher are both the distributive capital share and the degree of individual impatience to consume over the life cycle, the more likely a decrease in the contribution rate increases PAYG pensions.

Table 1 also shows the beneficial effect on PAYG pensions of a payroll tax reduction by illustrating – for increasing (decreasing) values of the distributive capital share (the subjective discount factor) – the declining ratio of the pension benefit with a 90 per cent tax rate to that with the benefit-maximising one.[8]

Table 1. The pension-maximising contribution rate and the ratio of the benefit with a 90 per cent tax rate to that with the benefit-maximising one for different values of both the distributive capital share and the subjective discount factor.

	$\alpha = 0.20$	$\alpha = 0.30$	$\alpha = 0.33$	$\alpha = 0.45$	$\alpha = 0.55$	$\alpha = 0.60$
$\gamma = 0.05$	$\bar{\theta} = 0.7650$	0.6360	0.5989	0.4654	0.3723	0.3303
	$\frac{p^*(0.90)}{p^*(\bar{\theta})} = 0.9213$	0.7409	0.6740	0.3889	0.1825	0.1061
$\gamma = 0.10$	0.7654	0.6374	0.6007	0.4681	0.3750	0.3328
	0.9216	0.7421	0.6755	0.3913	0.1845	0.1075
$\gamma = 0.20$	0.7664	0.6401	0.6039	0.4728	0.3798	0.3372
	0.9222	0.7444	0.6783	0.3956	0.1880	0.1101
$\gamma = 0.30$	0.7673	0.6425	0.6068	0.4771	0.3840	0.3411
	0.9228	0.7465	0.6810	0.3996	0.1912	0.1124
$\gamma = 0.50$	0.7689	0.6468	0.6119	0.4843	0.3911	0.3476
	0.9239	0.7503	0.6856	0.4064	0.1966	0.1163
$\gamma = 0.70$	0.7704	0.6505	0.6163	0.4902	0.3968	0.3528
	0.9249	0.7535	0.6897	0.4121	0.2011	0.1195

[8] We thank an anonymous referee for this suggestion.

Table 1 reveals that the pension-maximising tax rate is largely affected by a change in the distributive capital share, while it is relatively less sensitive to a change in the thriftiness parameter. As a policy implication, this means that countries with a relatively high capital share may incur to be "excessively" taxed in the sense that a contribution rate fixed beyond $\bar{\theta}$ would be harmful for both the level of income per capita and the pension benefit. As an illustrative example we may see, using capital share estimates recently provided by Jones (2003), that countries such as the US and Sweden (which show a distributive capital share between 0.3 and 0.4, depending on how α is measured) [9] could fix a contribution rate higher than those of countries such as the UK (showing both measures of the capital share around 0.50), or such as Italy and Ireland (both having the highest estimates close to 0.60), without risking to burden "excessively" the economy.

4. Conclusions

Motivated by the common assertion that in the near future many overly generous PAYG pension systems will be challenged both by an increased number of pensioners and a reduced number of contributors and thus will become unsustainable, we investigate how a decreasing contribution rate affects the balanced pay-as-you-go pension budget in a fairly standard overlapping generations model of neoclassical growth (Diamond, 1965). It is shown, rather unexpectedly, that it is possible to increase PAYG pensions by decreasing the contribution rate paid by current workers. This result occurs more likely the higher is the capital's share in production and the lower is the individual subjective discount factor, and therefore it may be relevant for some developed countries, as shown by an illustrative numerical example. The policy implication is that the unpleasant trade-off, often asserted for tackling the population ageing problem, between either a reduction in pension payments (alternatively, a lengthening of the retirement age) or an increase in the contribution rate to the PAYG scheme paid by the currently active people, and considered as a necessary policy measure to guarantee the sustainability of the widespread PAYG pension systems, does not hold in many realistic cases. Therefore, there is room for a pension enhancement along with a reduction in the contribution rate.

Appendix

Effects of a reduction in the contribution rate on long-run PAYG pension payments.

$$\frac{\partial p^*}{\partial \theta} = (1+n)w^* > 0, \tag{A1}$$

$$\frac{\partial p^*}{\partial w^*} = \theta(1+n) > 0, \tag{A2}$$

$$\frac{\partial w^*}{\partial k^*} = \alpha(1-\alpha)A \cdot (k^*)^{\alpha-1} > 0. \tag{A3}$$

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[9] As regards the case with self-employment correction, Jones (2003) calculated the labour share "by dividing by the employee share of total employment". By contrast, in the absence of self-employment correction, the labour share is "employee compensation as a share of GDP." (see Jones, 2003 p. 8).