Indeterminacy and nonlinear dynamics in an OLG growth model with endogenous labour supply and inherited tastes

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Abstract This study analyses the dynamics of a two-dimensional overlapping generations economy with endogenous labour supply à la Reichlin (1986) and aspirations. We show that the degree of nonlinearity of consumption externality in individual utility is responsible for the existence of either one steady state or two steady states. In addition, some interesting global dynamic properties, such as cyclical behaviour and/or global indeterminacy, emerge depending on the relative importance of aspirations in utility.

Keywords Aspirations; Indeterminacy; Labour supply; OLG model; Nonlinear dynamics

JEL Classification C61; C62; C68; J22; O41

1. Introduction

The study of economic models with habit and aspiration formation (i.e., past actions) has received in depth attention in both the theoretical and empirical literature in recent decades. A pioneering paper with regard to this issue is Abel (1990), which argues that habits can contribute to explain the equity premium puzzle.

Habits are defined as the case in which individual preferences depend on current consumption as well as on a benchmark level that weights the consumer's past consumption experience (these are known as internal habits or catching-up-with-the-Joneses). In contrast, aspirations represent the case in which preferences of an individual are affected by both his/her own consumption and a benchmark level that weights the consumption experience of others, e.g. parents (these are known as external habits or keeping-up-with-the-Joneses), see Carroll et al. (1997, 2000).[1]

In a context with overlapping generations (OLG), the existence of habits implies that preferences over old age consumption by individuals of the current generation are evaluated in comparison with their own consumption when young. In contrast, the existence of aspirations implies that preferences over consumption bundles by the current generation are affected by the standard of living based on the consumption experience of past generations

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[1] As specified by Carroll et al. (1997), when a consumer's utility depends on his/her own consumption relative to a benchmark consumption, two different specifications of utility functions are defined: outward-looking utility and inward-looking utility. The former (resp. latter) represents the case of external (resp. internal) habits.

(parents), which represents a reference to contrast the level of current consumption.

On theoretical grounds, several papers have contributed to highlight the importance of consumption externalities on the macroeconomics. In this regard, studies are especially focused on: (*i*) how consumption externalities affect saving and growth (Carroll et al., 1997, 2000); (*ii*) the role that habits and aspirations can play in affecting the way intergenerational transfers operate on the mechanics of capital accumulation and wealth inequality (de la Croix and Michel, 2001; Alonso-Carrera et al., 2007); (*iii*) the effects of habits and aspirations on economic growth and stability of long-term equilibria (de la Croix, 1996; de la Croix and Michel, 1999; de la Croix, 2001; Wendner, 2002; Artige et al., 2004; Alonso-Carrera et al., 2005); (*iv*) how consumption externalities can be a source of local indeterminacy in a continuous time infinite horizon growth model with endogenous labour supply (Alonso-Carrera et al., 2008). In particular, Alonso-Carrera et al. (2008) show that if the marginal rate of substitution between an agent's own consumption and consumption spillovers is constant (resp. not constant), the equilibrium path does not exhibit (resp. may exhibit) indeterminacy. With regard to consumption externalities as a source of indeterminacy and sunspots, two other contributions are Weder (2000) and Chen and Hsu (2007). Other papers have dealt with: (*v*) the dynamic and steady-state effects of assuming additive or multiplicative habits (Carroll, 2000; Wendner, 2003; Ikefuji and Mino, 2009); (*vi*) the effects of consumption externalities in an economy with asset bubbles (Mino, 2008); (*vii*) Boldrin et al. (2001) have introduced habit preferences in a business cycle model with asset returns, while Boldrin et al. (1997) and Lahiri and Puhakka (1998) have studied the effects of habit persistence in pure exchange economy. (*viii*) Other two interesting papers are Hodgson (2010) and Hiraguchi (2011). The former takes a view of habits from an evolutionary perspective. It explains the differences between habit and instinct, and argues that habits occur but are not programmed. This actually occurs in our model, in accordance with de la Croix (1996) and de la Croix and Michel (1999), and different from Alonso-Carrera et al. (2007), where individual of every generations care about the utility of children and leave bequests. The latter one builds on a two-sector endogenous growth model and finds that habit formation contributes to reduce the speed of convergence of the (uniquely determined) optimal growth path towards the balanced growth path.

On empirical grounds, there exists evidence on the effects of habit and aspiration formation on consumers' tastes over time (e.g. Ferson and Constantinides, 1991; de la Croix and Urbain, 1998; Carrasco et al., 2005). The first paper finds that habit persistence has a significant effect in the consumption of durable goods. The second paper shows that habits are an important determinant of import demand. The third one finds evidence in favour of time non-separabilities in inter-temporal household consumption In addition, McBride (2001) shows that there exists micro-evidence that the subjective well-being is affected by relative income, while Stutzer (2004) finds that income aspiration negatively affects individual utility.

Different from Alonso-Carrera et al. (2008), who introduce consumption externality in a continuous time model with endogenous labour supply, the aim of the present paper is to analyse the role played by inherited tastes (aspirations) on the dynamics of a two-dimensional OLG economy where individuals work when they are young and consume only when they are old (Reichlin, 1986). Analysing growth models that generate endogenous deterministic fluctuations that resemble random ones dates back to Grandmont (1985), Farmer (1986), Reichlin (1986) and Azariadis (1993). Subsequently, several authors have dealt with this topic in OLG models either with exogenous (Yokoo, 2000) or endogenous (Nourry, 2001; Nourry and Venditti, 2006) labour supply.

The papers most closely related to ours, especially with regard to the structure of the model-economy, are de la Croix (1996) and de la Croix and Michel (1999). However, both papers deal with an OLG model where the labour supply is inelastic and individuals consume in both the first period and second period of life (as in Diamond, 1965). In such a case the existence of inherited tasted makes the dynamics of the economy be characterised by a two-dimensional system because of the accumulation of the stock of physical capital and the accumulation of the stock of aspirations. In particular, de la Croix (1996) shows that when the intensity of aspirations in utility is large, individuals want to increase consumption because the standard of living of their parents is high and then savings becomes low. This can generate a Neimark-Sacker bifurcation and endogenous cycles when savings experience too high a contraction because of the importance of past consumption levels. De la Croix and Michel (1999), instead, concentrates on the optimality issue in a growth model with aspirations that generate a negative consumption externality (when individual are selfish) that can be corrected by using investment subsidies and lump-sum transfers. Moreover, the authors also show that: (*i*) when the taste externality is large, the competitive equilibrium can be destabilised through a Neimark-Sacker bifurcation and endogenous fluctuations can occur, and (*ii*) the planner solution can experience damped oscillations even under the case of no discounting of utilities of future generations.

The novelty of this study is the introduction of aspirations in an OLG growth model with endogenous labour supply à la Reichlin (1986), i.e. individuals work when they are young and consume when they are old. First, we show that the relative importance of aspirations in utility is responsible for the existence of either one (normalised) steady state (which can be determinate or indeterminate) or two steady states. Second, some interesting local and global dynamic properties of the two-dimensional decentralised economy emerge: when the relative importance of aspirations in utility is large enough cyclical behaviour and/or coexistence of attractors may occur. In addition, some global properties of the map may cause global indeterminacy to the model, while the stationary equilibria are locally determinate.

It is now useful to mention the differences between local and global indeterminacy. We say that a fixed point is locally indeterminate if for every arbitrarily small neighbourhood of it, and for a given value of the state variable (the stock of capital) close enough to its coordinate at the stationary state, there exist a continuum of values of the control variable (the labour supply) for which the equilibrium trajectories converge towards the fixed point. Differently, we say that the system is globally indeterminate when there exist values of the state variable such that different choices on the control variable lead to different invariant sets. In this case, the initial condition of the stock of capital is not sufficient to define the long-run dynamics of the system.

The paper is organised as follows. Section 2 outlines the model. Section 3 studies the conditions for the existence of steady states, and analyses both the local and global properties of the two-dimensional dynamic system. Conclusions are drawn in Section 4.

2. The model

2.1. Individuals

We consider an OLG closed economy populated by a continuum of perfectly rational and identical two-period lived (selfish) individuals of measure one per generation (Diamond, 1965). A new generation is born in every period. In the first period of life (youth), the individual of generation t supplies $\ell_t > 0$ units of labour to firms and receives the wage w_t per unit of labour. Individuals consume only in the second period of life (Reichlin, 1986; Galor and Weil, 1996; Grandmont et al., 1998; Antoci and Sodini, 2009; Gori and Sodini, 2011).

The budget constraint of an individual of generation t is $s_t = w_t \ell_t$, that is the (labour) income earned in the first period of life ($w_t \ell_t$) is entirely saved (s_t) for consumption purposes in the second period of life (C_{t+1}). When old, individuals retire and consumption is constrained by the amount of resources saved when young plus the interest accrued from time t to time $t+1$ at the rate r^e_{t+1}, that is $C_{t+1} = R^e_{t+1} s_t$, where $R^e_{t+1} := 1 + r^e_{t+1}$ is the (expected) interest factor (which will become the realised interest factor at time $t+1$).[2]

Therefore, the lifetime budget constraint of an individual of generation t is expressed as follows:

$$C_{t+1} = R^e_{t+1} w_t \ell_t . \quad (1)$$

Individuals draw utility from material consumption when old and suffer from the disutility of labour when young. In addition, individual preferences by the current generation are negatively affected by the level of consumption of the old (parents) that belong to the previous generation (c_t).[3] The existence of inherited tastes represents an inter-generational (negative) consumption externality (de la Croix, 1996; de la Croix and Michel, 1999), because it gives rise to a reference against which consumption when old is compared, and an increase in it increases the need for an individual of the current generation to raise his/her own consumption bundles to keep utility unaffected. Effective consumption when old, therefore, depends on the consumption experience of old parents, and this gives rise to a form of aspirations in our model (external habits).[4]

We assume that the lifetime utility index of generation t is described by a twice continuously differentiable utility function $U_t(\hat{C}_{t+1}, \ell_t)$, where $\hat{C}_{t+1}$ represents effective consumption when old. We now assume that the individual that belongs to the current generation "inherits standard-of-living aspirations [x_t] from… parents" (de la Croix and Michel, 1999, p. 521). Habits and aspirations can take either a multiplicative form (Galí, 1994; Bunzel, 2006), $\hat{C}_{t+1} = C_{t+1}/(x_{t+1})^a$, or additive form (de la Croix, 1996; Alonso-Carrera et al., 2007), $\hat{C}_{t+1} = C_{t+1} - a x_{t+1}$, where x_{t+1} represents some consumption reference level and a is a non-negative parameter that weights the intensity of the effect of the intergenerational spill-over (de la Croix, 1996). In addition, a flow- or stock-concept of aspirations can be taken into account. In this paper, we restrict attention to the case of *external habits* by using the *flow-concept*. This because in our model, individuals consume only when they are old and live for two periods only. Then, by assuming no population growth and a reference level that is given by the consumption of the individual of the past generation (parent), c_t, we define $x_{t+1} = c_t$ for every t.[5] This amounts to assume that the consumption reference when old is determined by the aspirations inherited from the parent. It also implies that aspirations are not forgotten at the end of every period, that is the rate of depreciation of aspirations is zero and thus they affect consumption in the second period of life.

By assuming that aspirations take the following additive form $\hat{C}_{t+1} = C_{t+1} - a c_t^{\rho}$, where $\rho > 0$ governs the curvature of the negative consumption externality and represents the degree of nonlinearity of aspirations in utility, we specify the lifetime utility index as follows (see Benhabib and Farmer, 1994):

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[2] The existence of inter-generational transfers, e.g., intentional bequests (Chakraborty and Das, 2005; Alonso-Carrera et al., 2007) is avoided in the present study. The reasons why people make intergenerational transfers can be different. However, the empirical literature has found evidence about intergenerational altruism as one of the most important reasons for the existence of these transfers (Hurd, 1987; Gale and Scholz, 1994).

[3] Note that every agent of previous generations is the parent of an agent of subsequent generations.

[4] Alternatively, this means that the standard of living of an individual born at time t is determined by the standard of living of an individual born at time $t-1$.

[5] Differently, by assuming *average consumption* of the past generation as a reference against which consumption of the current generation is compared, one finds that $x_{t+1} = c_t / 2$.

$$U_t(C_{t+1}, \ell_t, c_t) = B\ln(C_{t+1} - ac_t^{\rho}) - \frac{\ell_t^{\gamma}}{\gamma}, \tag{2}$$

where $\gamma > 1$ is the constant elasticity of utility with respect to labour and $B > 0$ is a scale parameter. It is important to note that by relaxing the hypothesis of linearity of the externality ($\rho = 1$), as usually specified in literature (Alonso-Carrera et al., 2007), we have dramatic changes in the long-term results of the model, as we can see later in this paper. One can also wonder about whether it is better to get utility from leisure today than utility from consumption tomorrow and then attach a factor to discount utility from consumption tomorrow in (2). However, since lifetime utility takes the usual additively separable specification, then the difference of evaluating utilities from heterogeneous goods over time can be captured by the different functional form of intra-temporal utilities.[6]

We are aware that the hypothesis that young individuals do not consume material goods is a strong assumption when analysing the impact of (external) habit formation.[7] However, by assuming that at time t the parent consumes only a fraction b of his/her available resources ($w_{t-1}R_t\ell_{t-1}$), with the remaining fraction $1-b$ being devoted to the consumption of his/her child, we can introduce the following alternative formulation for the utility function

$$\widetilde{U}_t = \ln((1-b)w_{t-1}R_t\ell_{t-1}) + B\ln(bw_tR_{t+1}^e\ell_t - d(1-b)(w_{t-1}R_t\ell_{t-1})^{\rho}) - \frac{\ell_t^{\gamma}}{\gamma}, \tag{2.1}$$

where $d \geq 0$ and $0 < b < 1$, that measure the degree of "altruism" towards children of every generation. This generates the same optimal allocation of consumption and labour/leisure for an individual than (2) by defining $a := d(1-b)/b$. For this reason and to simplify notation, we use utility function (2) later in this paper.

By the study of the Hessian matrix associated with (2) (see Appendix A), we find that if $\rho > 1$ then the utility function is strictly concave with respect to the three variables C_{t+1}, ℓ_t and c_t. In addition, we note that several models with externalities violate the hypothesis on the concavity of the utility function. From a theoretical point of view, we note that the results on nonlinear dynamics and indeterminacy in models with externalities are usually related to the violation of the hypothesis of concavity of utility functions (Chen, 2007; Antoci and Sodini, 2009) and production functions (Benhabib and Farmer, 1994; Boldrin and Rustichini, 1994; Cazzavillan, 2001). In contrast, our findings *are not* related to the violation of this hypothesis. We note that with this specification of aspiration formation, the utility function (2) would not be well-defined. To avoid this problem, in what follows we impose some restrictions on parameters in order to ensure a positive value of $C_{t+1} - ac_t^{\rho}$ (Carroll, 2000; Alonso-Carrera et al., 2004). Furthermore, from (2) we find that if $a = 0$ then the allocation of the supply of labour is constant over time (Benhabib and Farmer, 1994).

By taking factor prices and the consumption reference of parents (C_t) as given, the individual representative of generation t chooses ℓ_t to maximise (2) subject to (1). Therefore, the first order conditions for an interior solution are given by:

$$-\ell_t^{\gamma-1} + \frac{Bw_tR_{t+1}^e}{w_tR_{t+1}^e\ell_t - ac_t^{\rho}} = 0. \tag{3}$$

2.2. Production and equilibrium

[6] See concluding comments for a further discussion on this topic.

[7] This is a highly stylised model and then we do not have the aim of suggesting practical policy recipes. The study of OLG models with habits, consumption when young, consumption when old and leisure/labour decisions is left for future research.

Since the focus of this study is on the dynamic effects of the existence of a negative externality on the consumers side, then in contrast to, e.g. Grandmont et al. (1998) and Cazzavillan (2001), which adopt a Constant Elasticity of Substitution (CES) technology or consider externality in the production sector, we assume that at time t identical and competitive firms produce a homogeneous good, Y_t, by combining capital and labour, K_t and L_t, respectively, through the *constant returns to scale Cobb-Douglas technology* $Y_t = A \cdot F(K_t, L_t) = AK_t^{\alpha} L_t^{1-\alpha}$, where $A > 0$ and $0 < \alpha < 1$. The equilibrium supply of labour at time t is given by $L_t = \ell_t$. Then, by assuming that capital fully depreciates at the end of every period and output is sold at the unit price, profit maximisation implies that the marginal productivities of capital and labour are equal to the interest factor and wage rate, respectively, that is:

$$R_t = \alpha A K_t^{\alpha-1} \ell_t^{1-\alpha}, \tag{4}$$

$$w_t = (1-\alpha) A K_t^{\alpha} \ell_t^{-\alpha}. \tag{5}$$

The market-clearing condition in the capital market can be expressed as follows:

$$K_{t+1} = s_t = w_t \ell_t. \tag{6}$$

By using Eqs. (3)-(6) and knowing that: (*i*) individuals have perfect foresight, that is the expected interest factor R_{t+1}^e is a function of the capital stock at time $t+1$, and (*ii*) the consumption reference can be expressed as $x_t = c_t = \alpha A K_t^{\alpha} \ell_t^{1-\alpha}$, equilibrium implies:

$$-\ell_t^{\gamma-1} + \frac{A^2 B \alpha (1-\alpha) K_t^{\alpha} \ell_t^{-\alpha} K_{t+1}^{\alpha-1} \ell_{t+1}^{1-\alpha}}{A^2 \alpha (1-\alpha) K_t^{\alpha} \ell_t^{1-\alpha} K_{t+1}^{\alpha-1} \ell_{t+1}^{1-\alpha} - a(\alpha A K_t^{\alpha} \ell_t^{1-\alpha})^{\rho}} = 0, \tag{7}$$

$$K_{t+1} = (1-\alpha) A K_t^{\alpha} \ell_t^{1-\alpha}. \tag{8}$$

3. Local and global dynamics

3.1. Existence of steady states

The dynamic system expressed by (7) and (8) defines the variables K_{t+1} and ℓ_{t+1} as functions of K_t and ℓ_t. In this section, we study the stability of steady states of that system. To simplify the analysis, we now impose some restrictions on the parameters for which a steady state (K_{ss}, ℓ_{ss}) with $K_{ss} = \ell_{ss} = 1$ exists. This allows us to analyse the effects on stability due to changes in some parameter values, while being sure that the steady state does not vanish. Therefore, by setting $K_{ss} = \ell_{ss} = 1$ and using (7) and (8), we get:

$$A = A^* := \frac{1}{1-\alpha}, \tag{9}$$

$$B = B^* := 1 - a\left(\frac{\alpha}{1-\alpha}\right)^{\rho-1}. \tag{10}$$

In order to ensure that $C_{t+1} - ac_t^{\rho}$ evaluated at the fixed point $(1,1)$ is positive and $B^* > 0$, we set:

$$a < \hat{a} := \left(\frac{\alpha}{1-\alpha}\right)^{1-\rho}. \tag{10.1}$$

Furthermore, we will impose the following restriction on the value of the output elasticity of capital α (which is in line with empirical estimates for developed countries, see Gollin, 2002).

Assumption 1. $\alpha < 1/2$.

Under Assumption 1, we find that: if $\rho \in (0,1)$ then $\hat{a} < 1$; if $\rho \geq 1$ then $\hat{a} \geq 1$ and a can be larger than one.

By using (7)-(10), the system that characterises the dynamics of the economy can explicitly be expressed by map M comprised of the following two equations:

$$\ell_{t+1} = V(K_t, \ell_t) := \left(\frac{a K_t^{\alpha(\rho-\alpha)} \ell_t^{(1-\alpha)(\rho-\alpha)+\gamma}}{a + \alpha^{1-\rho}(1-\alpha)^{\rho-1}(\ell_t^{\gamma} - 1)} \right)^{\frac{1}{1-\alpha}}, \tag{11}$$

$$K_{t+1} = Z(K_t, \ell_t) := K_t^{\alpha} \ell_t^{1-\alpha}. \tag{12}$$

Now, let $\ell_{\min} := \left(1 - \frac{a}{\hat{a}}\right)^{\frac{1}{\gamma}}$. Then, we note that given (K_t, ℓ_t) it is possible to compute its subsequent iterate if and only if we start by a point in the set $D := \{(K_t, \ell_t) \in R^2 \ : \ K_t > 0, \ \ell_t > \ell_{\min}\}$, where the last inequality in brace guarantees that both the denominator of V and the difference $C_{t+1} - ac_t^{\rho}$ are positive.[8] However, feasible trajectories lie in a set smaller than D since by starting from an initial condition in D it is possible to have an iterate from which the existence of the subsequent one is not guaranteed.

Then, the set of feasible trajectories is defined as follows:

$$F := \{(K_t, \ell_t) \in R^2 \ : \ K_t > 0, \ \ell_t > \ell_{\min}, \forall t > 0\}.$$

For trajectories that belong to the set $D \setminus F$, there exists a threshold value t^* for which it not possible to compute the subsequent iterate (unfeasible trajectories), see Agliari and Vachadze (2011).

From (12) it follows that $K = \ell$ holds at the steady state. Then, by substituting it into (11) we find that fixed points of ℓ are determined as solutions of the following equation:

$$g(\ell) := \frac{a \ell^{-1+\rho+\gamma}}{a + \alpha^{1-\rho}(1-\alpha)^{\rho-1}(\ell^{\gamma} - 1)} = 1, \tag{13}$$

in the interval $(\ell_{\min}, +\infty)$.

We are now in a position to state the following proposition with regard to the existence of fixed points of map M.

Proposition 1. *[Existence of steady states]. If* $\rho \leq 1$ *then* $(1,1)$ *is the unique fixed point of map* M. *If* $\rho > 1$ *and* $a < \frac{\gamma \hat{a}}{\rho + \gamma - 1}$ *(resp.* $a > \frac{\gamma \hat{a}}{\rho + \gamma - 1}$*) then another fixed point exists, with* $\ell < 1$ *(resp.* $\ell > 1$*).*

Proof. If $\rho \leq 1$ then g is a monotonic decreasing function from which $(1,1)$ is the unique fixed point. If $\rho > 1$ then by the study of its first derivative we find that g is unimodal. It admits a minimum at $\hat{\ell} = \left[\frac{\rho + \gamma - 1}{\rho - 1} \left(1 - a \left(\frac{\alpha}{1-\alpha} \right)^{\rho-1} \right) \right]^{\frac{1}{\gamma}}$, and $\lim_{\ell \to \ell_{\min}^+} g(\ell) = +\infty$, $\lim_{\ell \to +\infty} g(\ell) = +\infty$. By computing $g'(1)$ we get the result. **Q.E.D.**

From now we will identify the coordinates of the non-normalised fixed point with (K^{**}, ℓ^{**}).

[8] We note that from the assumption $\alpha < 1/2$ and $B^* > 0$, it follows that $(1,1)$ is always feasible (see Appendix B).

3.2. Local bifurcation and stability

The aim of this section is to analyse the dynamics around the normalised fixed point. In the present model, the stock of capital K_t is a state variable, so that its initial value K_0 is given, while the supply of labour ℓ_t is a control variable. It follows that individuals of the first generation ($t = 0$) *choose* the initial value ℓ_0.

If the normalised steady state is a saddle (that is, there exist an eigenvalue, λ_1, smaller than 1 and an eigenvalue, λ_2, greater than 1), and K_0 is close enough to 1, there exists a unique initial value of ℓ_t (ℓ_0) and, consequently, a unique value of the expectation on the interest rate, such that the orbit that passes through (K_0, ℓ_0) approaches the steady state.

If the normalised steady state is a sink (that is, both eigenvalues are in modulus smaller than 1), then given K_0 and the expectations on the interest rate, there exists a continuum of initial values ℓ_0 such that the orbit that passes through (K_0, ℓ_0) approaches the steady state. As a consequence, the orbit that the economy will follow is "locally indeterminate" because it depends on the choice of ℓ_0.

If the steady state is a source (that is, both eigenvalues are in modulus greater than 1), then trajectories that start out in its neighbourhood do not converge to the same steady state: they are attracted by another attractor or, alternatively, they can lie in the unfeasible region.

We now compute the Jacobian matrix associated with map M and evaluated at $(1,1)$. This is given by:

$$J = \begin{pmatrix} \alpha & 1-\alpha \\ \dfrac{\alpha(\rho-\alpha)}{1-\alpha} & \dfrac{(1-\alpha)(\alpha-\rho)-\gamma}{\alpha-1} + \dfrac{\gamma}{a(\alpha-1)}\left(\dfrac{\alpha}{1-\alpha}\right)^{1-\rho} \end{pmatrix}. \tag{14}$$

The trace and determinant associated with (14) are the following:

$$Tr(J) = \rho + \frac{\gamma}{1-\alpha} - \frac{\gamma}{a(1-\alpha)}\left(\frac{\alpha}{1-\alpha}\right)^{1-\rho}, \tag{15}$$

$$Det(J) = \frac{\alpha\gamma}{a(1-\alpha)}\left(a - \left(\frac{\alpha}{1-\alpha}\right)^{1-\rho}\right). \tag{16}$$

In order to characterise the local dynamics and local bifurcations around the normalized steady state, we now apply the geometrical-graphical method developed by Grandmont et al. (1998). In particular, we study the variation of the trace and the determinant when a varies, while keeping the other parameters unchanged. We recall that the trace and determinant associated with the Jacobian matrix are respectively the sum and product of the roots of the characteristic polynomial $Q(\lambda) = \lambda^2 - Tr(J)\lambda + Det(J)$ evaluated at $(1,1)$. From straightforward calculation, it is simple to verify that: along the line Π_1: $1 - Tr(J) + Det(J) = 0$, one eigenvalue is equal to 1; along the line Π_2: $1 + Tr(J) + Det(J) = 0$, one eigenvalue is equal to -1; along the segment-line Π_3: $1 - Det(J) = 0,\ |T| < 2$, two complex conjugated eigenvalues exist. This means that within the triangle of vertices ABC in Figures 1 and 2, the steady state is a sink (indeterminate equilibrium); in the open regions above and below the triangle ABC, the steady state is a source; in the remaining open regions the steady state is a saddle (determinate equilibrium). In addition, when we let a vary in the open interval $(0,\hat{a})$, the point $(Tr(J),Det(J))$ defines a half-line with slope α (that belongs to the line $Det(J)-\alpha Tr(J)+\alpha\rho=0$) that starts from $(-\infty,-\infty)$ for $a\to 0$ and approaches to the point $(\rho,0)$ for $a\to\hat{a}$. Then, when the curve $(Tr(J),Det(J))$ crosses Π_1, a transcritical bifurcation occurs and we have an exchange of stability between (1,1) and (K^{**},ℓ^{**}) (see Proposition 1), similar to Cazzavillan et al. (1998). When the curve $(Tr(J),Det(J))$ crosses Π_2, the equilibrium undergoes a flip bifurcation and a cycle of period 2 appears after the bifurcation. When the curve crosses Π_3, the normalised steady state loses its stability through a Neimark Sacker bifurcation. This last phenomenon, however, can never be observed in our model.

With regard to local bifurcations, the following proposition holds.

Proposition 2. *[Local bifurcation]. Let* $a_{fl}:=\dfrac{\gamma(\alpha+1)\hat{a}}{(1-\alpha)(\gamma+\rho+1)+2\gamma\alpha}$ *and* $a_{tc}:=\dfrac{\gamma\hat{a}}{\rho+\gamma-1}$ *hold. Then, (1) if* $\rho\in(0,1]$ *then the normalised (unique) steady state is a saddle (locally determinate) for* $a<a_{fl}$*, undergoes a flip bifurcation at* $a=a_{fl}$*, is indeterminate for* $a_{fl}<a<\hat{a}$*; (2) if* $1<\rho<1/\alpha$ *then the normalised steady state is a saddle (locally determinate) for* $a<a_{fl}$*, undergoes a flip bifurcation at* $a=a_{fl}$*, is indeterminate for* $a_{fl}<a<a_{tc}$*, undergoes a transcritical bifurcation at* $a=a_{tc}$*, is a saddle (locally determinate) for* $a_{tc}<a<\hat{a}$*; (3) if* $\rho>1/\alpha$ *then the normalised steady state is a saddle (locally determinate) for* $a<a_{tc}$*, undergoes a transcritical bifurcation at* $a=a_{tc}$*, is a source for* $a_{tc}<a<a_{fl}$*, undergoes a flip bifurcation at* $a=a_{fl}$*, is a saddle (locally determinate) for* $a_{fl}<a<\hat{a}$*.*

Proof. We identify Cases 1-3 in Proposition 2 by considering the relative position of the point $(\rho,0)$ and the slope α with respect to the stability triangle ABC. In particular, for a value of a close enough to zero, the point $(Tr(J),Det(J))$ lies outside the triangle (we recall that $(Tr(J),Det(J))\to(-\infty,-\infty)$). Thus, if $\rho\in(0,1]$ then there exists a value $a=a_{fl}$ such that the point $(Tr(J),Det(J))$ enters the triangle. For larger values of a, the point $(Tr(J),Det(J))$ will remain within the triangle. If $\rho>1$, the final point (for $a\to\hat{a}$) lies outside the triangle. Therefore, since the slope of the half-line $(Tr(J),Det(J))$ is positive and smaller than 1, then two cases are possible depending on the value of the determinant when the trace is equal to zero. If $Det(J)\in(-1,0]$, i.e. $-\alpha\rho>-1$, the point $(Tr(J),Det(J))$ crosses the triangle two times, first at $a=a_{fl}$ and then at $a=a_{tc}$. If $Det(J)<-1$, i.e. $-\alpha\rho<-1$, the point $(Tr(J),Det(J))$ never crosses the triangle. The bifurcation values a_{fl} and a_{tc} derive from straightforward algebra. **Q.E.D.**

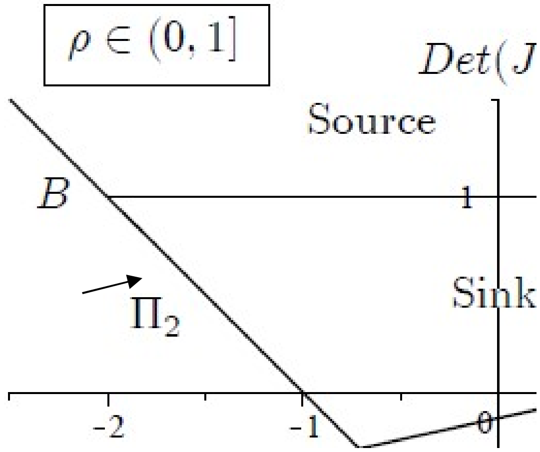

Figure 1. Stability triangle when $\rho \in (0,1]$.

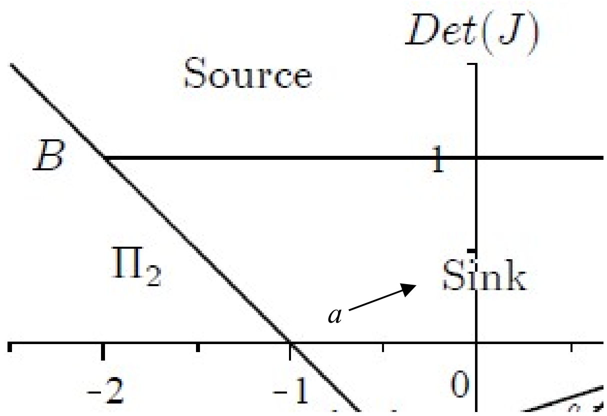

Figure 2. Stability triangle when $\rho \in (1, 1/\alpha)$ and when $\rho > 1/\alpha$.

3.3. Global analysis

In the previous section we have characterised the local properties of the dynamic system in the neighbourhood of the normalised steady state. The aim of this section is to study: (*i*) the dynamics around the non-normalised steady state, and (*ii*) the global structure of map M. This permits us to explain interesting phenomena that cannot be observed with the local analysis (Pintus et al., 2000, Agliari and Vachadze, 2011). We start the global analysis by showing that map M is invertible. The invertibility of a map is an important result when the global properties of a dynamic system are studied. For instance, it implies that the basins of attraction of any attracting set of a map are connected sets. Furthermore, by making use of the inverse map, we can obtain the boundary of the attracting sets and, more generally, the stable manifolds of saddle points.

With regard to map M, we have the following result.

Result 1. *The map M is invertible on the set D and its inverse is given by:*

$$M^{-1}:\begin{cases} \ell_t=\left(\dfrac{\ell_{t+1}^{1-\alpha}[\alpha^{1-\rho}(1-\alpha)^{\rho-1}-a]}{\ell_{t+1}^{1-\alpha}\alpha^{1-\rho}(1-\alpha)^{\rho-1}-aK_{t+1}^{\rho-\alpha}}\right)^{\frac{1}{\gamma}} \\ K_t=\left[K_{t+1}\left(\dfrac{\ell_{t+1}^{1-\alpha}[\alpha^{1-\rho}(1-\alpha)^{\rho-1}-a]}{\ell_{t+1}^{1-\alpha}\alpha^{1-\rho}(1-\alpha)^{\rho-1}-aK_{t+1}^{\rho-\alpha}}\right)^{\frac{\alpha-1}{\gamma}}\right]^{\frac{1}{\alpha}} \end{cases}. \tag{17}$$

Before performing the global analysis, we recall the definitions of both the stable manifold

$$W^s(p)=\{x:M^{zn}(x)\to p\ as\ n\to+\infty\}, \tag{18}$$

and unstable manifold

$$W^u(p)=\{x:M^{zn}(x)\to p\ as\ n\to-\infty\}, \tag{19}$$

of a periodic point p of period z. If the periodic point $p\in R^2$ is a saddle, then the stable (resp. unstable) manifold is a smooth curve through p, tangent at p to the eigenvector of the Jacobian matrix evaluated at p corresponding to the eigenvalue λ with $|\lambda|<1$ (resp. $|\lambda|>1$), see Guckenheimer and Holmes (1983). Outside the neighbourhood of p, the stable and unstable manifolds may even intersect each other with dramatic consequences on the global dynamics of the model (Guckenheimer and Holmes, 1983, p. 22).

Non-trivial intersection points of stable and unstable manifolds of a unique saddle cycle are known as homoclinic points. However, when multiple saddle cycles exist, heteroclinic bifurcations may also occur. We recall that given two saddle cycles h_1 and h_2, a heteroclinic bifurcation is defined as the birth of a non trivial point E of intersection between the stable manifold of one cycle and the unstable manifold of the other cycle. Starting from this new configuration, it is possible to find a path on the two manifolds that connects the cycles. This phenomenon is interesting from an economic point of view because is related to global indeterminacy. We recall that global indeterminacy occurs when, starting from the same initial condition K_0 of the state variable K_t, different fixed points or other ω-limit sets can be reached according to the initial value ℓ_0 of the jumping variable ℓ_t chosen by individuals of the first generation (Agliari and Vachadze, 2011; Gori and Sodini, 2011).

In order to look at and discuss global phenomena, we assume the following parameter constellation: $\alpha=0.32$, $\rho=1.42$, $\gamma=2.2$ and let a vary (from high to low values).

(1) For $a=0.89$ the normalised fixed point is locally indeterminate, in accordance with the study of the previous section, while the non-normalised fixed point $(K^{**},\ell^{**})=(2.503,2.503)$ is a saddle (locally determinate) and its stable manifold defines the border of the basin of attraction of the normalised one. Thus, by starting from values of K_t close enough to K^{**}, agents can: (*i*) coordinate themselves on initial values ℓ_{low} or ℓ_{high} (see Figure 3) so that the system lie on the path convergent to (K^{**},ℓ^{**}); (*ii*) choose any initial condition $\ell_0\in(\ell_{low},\ell_{high})$ and let the economic system converge towards $(1,1)$. Global indeterminacy arises around the locally determinate fixed point (K^{**},ℓ^{**}). Other initial values of the choice variable ℓ_t define unfeasible trajectories.

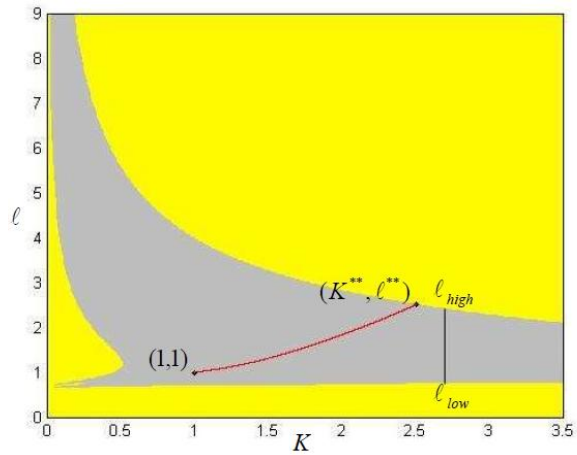

Figure 3. A saddle (locally determinate) equilibrium, (K^{**},ℓ^{**}), coexists with the normalised fixed point, $(1,1)$. The unstable manifold (the red-coloured line) of (K^{**},ℓ^{**}) creates a heteroclinic connection between the two fixed points. The gray-coloured region represents a portion of the basin of attraction of $(1,1)$. The yellow-coloured represents initial conditions that do not belong to F. Parameter set: $\alpha = 0.32$, $\rho = 1.42$, $\gamma = 2.2$ and $a = 0.89$.

(2) We now let a reduce to 0.75. The normalised fixed point has lost stability at $a = a_{fl} = 0.876$ through a flip bifurcation. It has become a saddle and a two-period attracting cycle has arisen around it, whose basin is bounded by the stable manifold of the saddle (K^{**},ℓ^{**}) and by the stable manifold of the saddle $(1,1)$. In order to inquire about the complicated nature of this situation, we concentrate on values of K_0 close enough to K^{**}. In Figure 4, the stable manifold of $(1,1)$ and the unstable manifold of (K^{**},ℓ^{**}) are shown together with the basin of attraction of the attracting cycle. The picture shows that three different long-term scenarios are possible: (*i*) the first scenario describes the case in which agents can coordinate themselves on the stable manifold of (K^{**},ℓ^{**}); (*ii*) in the second scenario, agents through their choices on ℓ_t make the economy lie on a trajectory convergent towards the two-period cycle (K_0 and ℓ_0 belong to the gray-coloured region in the figure); (*iii*) in the third scenario, the economy can be positioned on whatever point (K_0,ℓ_0) belonging to the stable manifold of $(1,1)$. In addition, since a heteroclinic orbit from (K^{**},ℓ^{**}) to $(1,1)$ exists, then ℓ_0 may be close to ℓ^{**}.

The dynamic complexity tends to increase even further by considering smaller values of a. The bifurcation diagram (Figure 5), in fact, shows the coexistence of three attractors. It is interesting to note that this last phenomenon is *not* related to the nonlinearity of the consumption externality ($\rho \neq 1$), as shown in Figure 6, where two attractors are portrayed when $\rho = 1$, which is the usual case of consumption externality (see Alonso-Carrera et al., 2007). Figure 5 depicts a portion of the phase plane where a cycle of period 4 coexists with a stable cycle of period 6. The former cycle is given by the classical period-doubling process that appears after the flip bifurcation. The latter indeterminate cycle has appeared through a saddle-node bifurcation together with a saddle cycle of the same period. The period-6 cycle is locally determinate, but global indeterminacy exists around it because of the existence of a heteroclinic connection.

The coexistence of attractors and saddles, the structure of the basin of attraction (as well as the structure of both the stable manifolds and saddles) make any prediction difficult about long-term dynamics. In this case, therefore, the animal spirits play a preeminent role in defining the long-term dynamics of the economy.

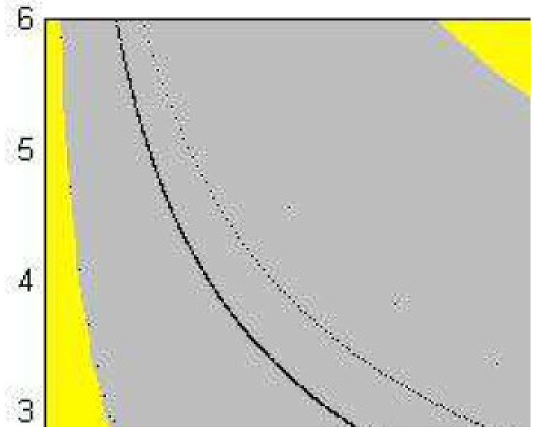

Figure 4. Basin of attraction, and stable and unstable manifolds. The gray-coloured region is the basin of attraction of $(1,1)$. The yellow-coloured represents initial conditions that do not belong to F. The red line is the unstable manifold of (K^{**},ℓ^{**}) that converges to the two-period cycle. The black curve is a portion of the stable manifold of $(1,1)$. The heteroclinic points in the figure prove the existence of a heteroclinic orbit starting from (K^{**},ℓ^{**}) and ending to $(1,1)$ and explains the winding behaviour of $W^s(1,1)$. The figure also shows the fixed points $(1,1)$, $(K^{**},\ell^{**})=(4.003,4.003)$ and the attracting two-period cycle, whose coordinates are $(1.356,0.889)$ and $(1.018,1.553)$. $\alpha=0.32$, $\rho=1.42$, $\gamma=2.2$ and $a=0.75$.

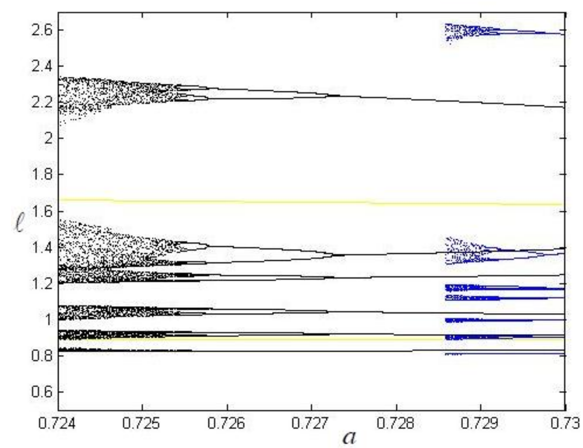

Figure 5. Bifurcation diagram for a. The picture shows the coexistence of attractors.

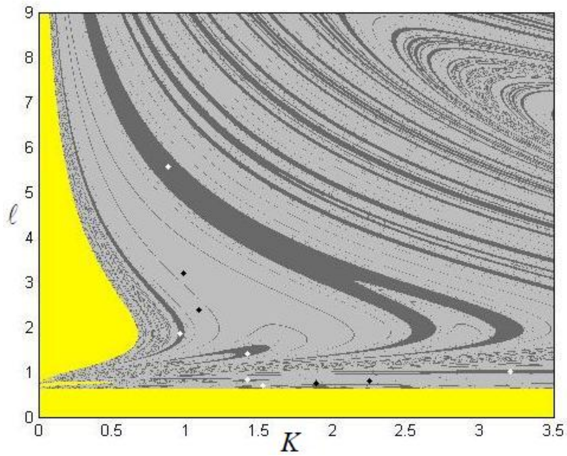

Figure 6. Coexistence of attractors. Parameter set: $\alpha = 0.3$, $\rho = 1$, $\gamma = 1.1$ and $a = 0.403$. The light (resp. dark) gray-coloured region represents a portion of the basin of attraction of the 4-period (6-period) cycle. The yellow-coloured represents initial conditions that do not belong to F.

(3) As the parameter a is further decreased, several windows of coexistence of attractors do exist. At the same time, we observe the classical period-doubling route to chaos that creates attractors that move more and more towards the boundary of their basins of attraction (see Figure 7 depicted for $a = 0.694$) and eventually reach it. We show the phase plane just after this global bifurcation, $a = 0.685$, and we note that a heteroclinic orbit that starts from (K^{**}, ℓ^{**}) and ends up to $(1,1)$ still exists (see Figure 8). Then, also in this case, by fixing the initial condition of the stock of capital, K_0, even close enough to K^{**}, several values of ℓ_0 exist such that the economic system converges towards the normalised fixed point: some of which can display a long time of convergence with oscillations in macroeconomic variables. It is important to note that for the chosen values of a, the stationary equilibria are locally determinate even if the system is globally indeterminate. In addition, other feasible trajectories exist because of the existence of an invariant set (chaotic repellor) which is made up by infinitely many (countable) repelling cycles and uncountable aperiodic trajectories (see Guckenheimer and Holmes, 1983 for details). This analysis shows the importance of the global analysis, whose conclusions can be dramatically different from those based on the local analysis (de Vilder, 1996; Agliari and Vachadze, 2011).

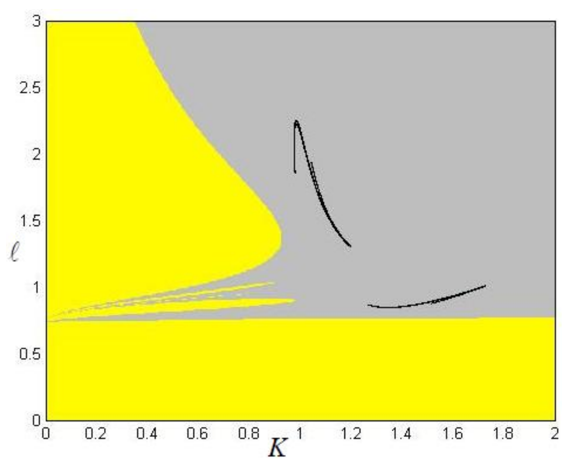

Figure 7. Parameter set $\alpha = 0.32$, $\rho = 1.42$, $\gamma = 2.2$ and $a = 0.694$. Hénon-like attractor.

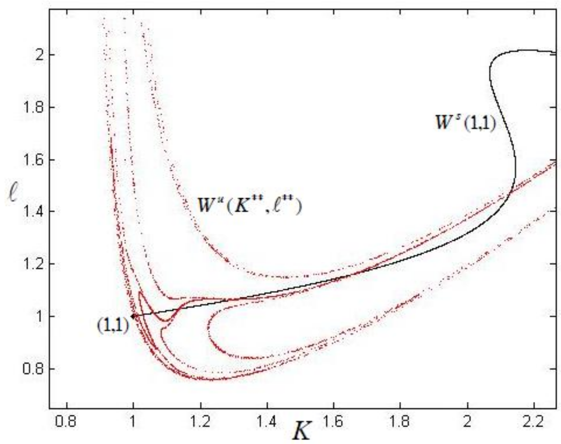

Figure 8. Parameter set $\alpha = 0.32$, $\rho = 1.42$, $\gamma = 2.2$ and $a = 0.685$. After the final bifurcation only saddles and a zero measure invariant set exist.

4. Conclusions

We have studied the dynamic properties of a two-dimensional overlapping generations growth model with endogenous labour supply (Reichlin, 1986) and inherited tastes (external habits): i.e., preferences of an individual depend on his/her own consumption experience as well as on a benchmark level that weights the consumption experience of others. In doing this, we have built on a model related to an established OLG literature with endogenous labour/leisure choice and consumption when old (heterogeneous goods), pioneered by Woodford (1984) and Reichlin (1986), and later followed by works on indeterminacy and sunspots (e.g., de Vilder, 1996; Cazzavillan, 2001; Gardini et al., 2009; Agliari and Vachadze, 2011 and so on), where there is no rate of discounting. The reason for this choice being merely economical: the arguments of the lifetime (additively separable) utility function are heterogeneous goods evaluated at different dates (i.e., leisure at time t and consumption at time $t+1$). An inter-temporal discount factor smaller than one is usually used in the traditional Diamond's (1965) model with both consumption when young and consumption when old, that is in a context where the arguments of the utility function are homogeneous goods evaluated at different dates. In addition, the use of a subjective discount factor included between zero and one in our context (as well as in all models of the above mentioned literature), would imply to discount utilities derived by heterogeneous goods for which empirical estimates are lacking (at the best of our knowledge). In fact, on the one hand the literature where our paper is framed is essentially a theoretical one, the aim of it being, amongst other things, to try to give an answer to the following question: are the coexistence of equilibria and nonlinear dynamics possible in a highly stylised OLG model where consumption when young is not included? On the other hand, this literature (except from the paper by Agliari and Vachadze, 2011) assumes the existence of a normalised equilibrium $(1,1)$ to analyse local dynamics when key parameters of the model are varied. The same procedure is also used to investigate global dynamics (Grandmont et al., 1998). Definitively, the multiplicative parameters of the utility function (such as the inter-temporal discount factor) or production function are appropriately fixed to make the existence of the normalised equilibrium possible when other parameters (e.g., the parameters that govern elasticity) vary. As a consequence, by including a subjective discount factor in our inter-temporal OLG model with heterogeneous goods would be neutral with regard to the main findings of the paper.

This because there exists at least another parameter that can adequately be varied to sterilise the explicit inclusion of it.[9]

By following de la Croix (1996), de la Croix and Michel (1999) and Alonso-Carrera et al. (2007), in this paper we have assumed that preferences of an individual of the current generation are affected by the consumption experience of his/her parent. In particular, different from de la Croix (1996) and de la Croix and Michel (1999), which have studied local dynamics in a general equilibrium OLG model à la Diamond (1965) with exogenous labour supply and inherited tastes by assuming a stock-concept of aspirations, in this paper we have assumed a flow-concept of aspirations and studied local and global dynamics of an economy described by a two-dimensional map because of the accumulation of capital and the time evolution of the individual supply of labour.

We have shown that the degree of nonlinearity of the consumption externality matters for the existence either of one (normalised) steady state or two steady states. In both cases, when the relative importance of aspirations in utility is included in an intermediate range of values, it is possible to observe interesting global phenomena such as global indeterminacy and chaotic dynamics.

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Appendix A. On the concavity of the utility function

The Hessian matrix associated with the utility function (2) is the following (we have omitted the time subscript for simplicity):

$$H(U(C,\ell,c))=\begin{pmatrix} -\frac{B}{(C-ac^{\rho})^2} & 0 & -\frac{Ba\rho c^{\rho-1}}{(C-ac^{\rho})^2} \\ 0 & -\ell^{\gamma-2}(\gamma-1) & 0 \\ -\frac{Ba\rho c^{\rho-1}}{(C-ac^{\rho})^2} & 0 & -Ba\rho\frac{c^{\rho-2}[(\rho-1)C+ac^{\rho}]}{(C-ac^{\rho})^2} \end{pmatrix}. \tag{A.1}$$

From (A.1) we have that:

$$\det H(U(C,\ell,c))=-B^2\ell^{\gamma-2}(\gamma-1)\frac{c^{\rho-2}a\rho(\rho-1)(C-ac^{\rho})}{(C-ac^{\rho})^4}\leq 0, \tag{A.2}$$

if and only if $\rho \geq 1$. The sign of the other principal minors follows immediately. When $\rho \in (0,1)$, the concavity of the utility function with respect to control variables (C,ℓ) continues to hold.

Appendix B. On the definiteness of the utility function

Let K_{t+1} and ℓ_{t+1} by defined by Eqs. (11) and (12). Then, the argument of the logarithm in (2) is positive if and only if

$$\left(\frac{\alpha K_t^{\alpha}\ell_t^{1-\alpha}}{1-\alpha}\right)^{1-\rho} K_{t+1}^{\alpha-1}\ell_{t+1}^{1-\alpha}-a>0. \tag{B.1}$$

[9] We would like to thank an anonymous reviewer and editors Gian Italo Bischi, Michael Kopel and Jose Cánovas for pointing this out as well as for constructive comments and stimulating discussions on this topic.

By direct substitution of K_{t+1} and ℓ_{t+1} in (B.1), we get:

$$a\frac{-a+\left(\frac{\alpha}{1-\alpha}\right)^{1-\rho}}{a+\alpha^{1-\rho}(1-\alpha)^{\rho-1}(\ell_t^{\gamma}-1)}>0\,. \tag{B.2}$$

From (10.1) and (B.2) we find that the inequality is fulfilled if and only if $\ell_t > \ell_{\min}$.

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