Family policies and the optimal population growth rate: closed and small open economies

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Abstract We re-examine the issue of optimal population in the basic overlapping generations model of neoclassical growth (Diamond, 1965) with endogenous fertility and time cost children. Under the closed economy hypothesis, it is shown that the golden rule of population growth can be realised by a benevolent government in the market who maximises the individuals' indirect utility index through either child subsidies or child taxes, and this result depends on the size of the distributive capital share relative to preference parameters. Under the small open economy hypothesis, it is shown that the command optimum can be perfectly replicated by merely adopting a single intra-generational instrument: if parents do (not) devote enough time to take care of their children, then a child subsidy (tax) can be used to effectively cancel out the external effects of children on society as a whole. Our theoretical model provides useful insights for government family policies.

Keywords Child policies; Fertility; Optimal population growth; Optimality; Welfare

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1. Introduction

Analysis of optimal population growth in Diamond's (1965) overlapping generations model dates back to the seminal paper by Samuelson (1975) who assumed an exogenous rate of fertility. Samuelson's analysis has been criticised by Deardorff (1976) pointing out interiority issues concerning the optimal population growth rate (OPGR henceforth) in the social planner's problem. The latter author showed that – in the case of Cobb-Douglas utility and technology – there would be no interior maximising solutions as regards the steady-state representative agent's utility. Assuming exogenous fertility, but considering CES-typed utility and production functions, Michel and Pestieau (1993) concluded that the interiority of the golden rule of population growth required complementarities either in consumption or in production.

Only recently has it been proved (see Abio, 2003) – contrary to Deardorff (1976) – that even with Cobb-Douglas preferences and technologies it is possible to obtain an interior OPGR in the planned economy when fertility rates are endogenously determined. However Abio (2003) analyses neither (1) the decentralised solution of the economy nor (2) the introduction of family policies.

As regards the latter point, some papers have investigated the effects of child allowances on individuals' fertility (amongst others, Walker, 1995; Cigno and Rosati, 1996; Cigno et al., 2000, the latter two suggesting that financial support based on childcare activities will have a limited impact on fertility), but considered only static contexts. Our approach also differs markedly from that used by Cigno at al. (2003) who constructed a partial equilibrium small open economy model where the relationship between government (principal) and households (agents) is of the principal-agent type and sought a socially optimal solution.

In this paper we focus on the role that a policymaker may play in a simple general equilibrium overlapping generations model where the government adopts a balanced budget (intra-generational) policy with only two instruments: a proportional-to-wage child subsidy (tax) financed by (used to fund) a lump-sum tax (subsidy) and looks at the effects of such a policy from both positive and normative perspectives. In particular, since raising children is supposed to be time-consuming, the child subsidy acts like a "parental leaves" instrument. In other words, the policymaker may subsidise or tax each child and simultaneously it may tax or rebate the same amount of money to the same child bearing generation (parents) in a lump sum fashion. Although this family policy seems to be income-neutral, it affects the individuals' behaviour in a very interesting way both in closed and small open economies.

The novelty of our results is that, although the government does abstain (for instance for ethical reasons) from pursuing – via centrally planned actions – the OPGR objective, then (1) if the economy is closed to international trade it may achieve the golden rule of population growth and (2) under the small open economy hypothesis it may replicate the first best (since the external effects of children can be effectively eliminated) by adopting a family policy based on a simple intra-generational redistributive tax-subsidy effect, thus avoiding any problems of inter-generational equity. This result contributes to shed new light on the extensively debated question of the existence and feasibility of an optimal population growth. Many economists, dating back to John Stuart Mill and following the Malthusian theory of population have advocated birth control policies,[1] but with the fundamental drawback involving major ethical concerns. Our findings avoid the need to introduce any normative birth regulation systems (e.g., a dictatorial imposition of the number of children raised by families), showing – in a closed economy – that a benevolent government (whose purpose is not to achieve the OPGR, but only to maximise the lifetime utility of individuals derived by their atomistic choices) is able to determine the golden rule of population growth either subsidising or taxing child-rearing of households to maximise individuals' welfare. In other words, once the proportional-to-wage child subsidy (tax) rate has been fixed such as to maximise the lifetime welfare of the representative generation, the number of children freely chosen by

[1] See also Coale and Hoover (1958), Enke (1960 and 1976) and, currently, the one-child per family policy forced by the Chinese government (e.g., Coale, 1981).

individuals automatically coincides with that which would be chosen by the planner. Moreover, considering an OLG small open economy it is shown that the first best allocation can be perfectly realised by adopting exclusively a single intra-generational policy instrument: either a child subsidy or a child tax.

The plan of the paper is as follows: in Section 2 we build up the decentralised closed economy model; in Section 3 (Section 4) we present the welfare analysis under the benevolent government (central planner) hypothesis; Section 5 illustrates the theoretical findings qualitatively; in Section 6 we modify the model and analyse how an intra-generational family policy can be used to realise the first best in an OLG small open economy; Section 7 draws some conclusions.

2. Decentralised economy

2.1. Individuals

Agents have identical preferences and are assumed to belong to an overlapping generations structure with finite lifetimes. Life is separated among three periods: childhood, young adulthood and old age, and the economy is closed to international trade. During childhood individuals do not make economic decisions and thus they consume a fixed fraction of the time endowment from their parents. Adult individuals belonging to generation t have a homothetic and separable utility function (U_t) defined over working period consumption of the adult ($c_{y,t}$), retirement period consumption of the adult ($c_{o,t+1}$) and the number of children raised (n_t).[2] Each adult agent is assumed to have an endowment of one unit of time. We assume that raising children requires a time-consuming technology which asks each couple to spend a (time) cost $0 < q < 1$ per child, with q being the exogenous fraction of the time endowment of one parent that must be spent raising a child.[3] This hypothesis results in an endogenous supply of labour: indeed, there is a trade-off between working in the labour market and raising children. Only young-adult individuals (N_t) join the labour force devoting a fraction $h_t(n_t) = 1 - qn_t$ of their time to work in the labour market with $q n_t$ being the share of time spent raising children. The perceived market-clearing wage (w_t) is used for consumption and saving purposes. Furthermore, parents are entitled to (must pay) a per child proportional-to-wage subsidy (tax) at the constant rate $0 < \theta < q$ $(-1 < \theta < 0)$, that is each child is subsidised (taxed) with θw_t wage-units. During old age agents are retired and live on the proceeds of their savings (s_t) plus the accrued interest at the rate r_{t+1}.

The representative individual born at time t is therefore faced with the problem of maximising the following logarithmic utility function:[4]

[2] The preference structure adopted here is similar to that used by Galor and Weil (1996), where the quantity of children raised is evaluated by the parents as a normal good. Moreover, in this paper we ignore the trade-off between child quantity and quality, and we do not look at the effects of child policies in a purely altruistic context where parents care about the utility of their offspring (e.g., amongst many others, Barro and Becker, 1989; Becker et al, 1990; Ehrlich and Lui, 1991).

[3] Women lose a large amount of their potential income as a consequence of leaving work to give birth and raise children (i.e., the female opportunity cost of bearing and raising an extra child is high, especially in developed countries). It could be interesting, therefore, in order to take into account the negative substitution effect – on fertility – of the female labour earnings due to the potential increase of women's labour force participation (Mincer, 1963 and 1966), to differentiate the child rearing cost on the basis of the different ability of male and female to raise children and thus imagine part of the child-rearing cost as being proportional to the female working income. Therefore, a natural extension of this model is to introduce a gender gap, assume the economy consists of male and female individuals and divide the household's membership on the basis of the ability of parents to earn wages and raise children (for instance, by introducing two different child rearing technologies). Such an extension is a promising direction of research.

[4] Since there exists a trade-off between working in the labour market and raising children, the Cobb-Douglas utility function we adopted here captures the individual's utility drawn from having children as well as from working period

$$\max_{\{c_{y,t},c_{o,t+1},n_t\}} U_t(c_{y,t},c_{o,t+1},n_t)=(1-\rho)\cdot\{\ln(c_{y,t})+\beta\ln(c_{o,t+1})\}+\rho\ln(n_t), \tag{P}$$

subject to the intra-temporal budget constraints[5]

$$\begin{aligned} c_{y,t}+s_t &= w_t[1-n_t(q-\theta)]-\tau_t \\ c_{o,t+1} &= (1+r_{t+1})s_t \end{aligned},$$

where $\tau_t>0$ ($\tau_t<0$) is a lump-sum tax (subsidy) levied on the young-adult generation, $0<\beta<1$ is the subjective discount factor and $0<\rho<1$ captures the importance in the welfare function of consuming over the life cycle relative to the utility of children. The higher is β (i.e., the lower the rate of time preference) the more individuals prefer to postpone consumption over time and the higher is ρ the more parents are children-interested.

The first order conditions for an interior solution are:

$$\frac{c_{t+1}^{o}}{c_t^{y}}\cdot\frac{1}{\beta}=1+r_{t+1}, \tag{1}$$

$$\frac{c_{y,t}}{n_t}\cdot\frac{\rho}{1-\rho}=w_t(q-\theta). \tag{2}$$

Eq. (1) equates the marginal rate of substitution between young and old age consumptions to the interest rate, whereas Eq. (2) equates the marginal rate of substitution between consuming during youth and having children to the marginal cost of rearing an additional child. The higher the child subsidy (tax) the lower (higher) is such a marginal cost.

Exploiting the first order conditions along with the individual's budget constraints, the demand for children and the saving function are respectively given by:

$$n_t=\frac{\rho}{1+\beta(1-\rho)}\cdot\frac{w_t-\tau_t}{(q-\theta)w_t}, \tag{3}$$

$$s_t=\frac{\beta(1-\rho)}{1+\beta(1-\rho)}(w_t-\tau_t). \tag{4}$$

2.2. Government

The child subsidy (tax) is supposed to be entirely funded by levying and adjusting over time (used for the financing of time varying) lump-sum taxes (subsidies) on the same child bearing generation. Therefore, the balanced per-capita time-t government budget is:

$$\theta w_t n_t=\tau_t. \tag{5}$$

Inserting (5) into (3) and (4) to eliminate τ_t and rearranging terms yields:[6]

$$n^*(\theta)=\frac{\rho}{q[1+\beta(1-\rho)]-\theta(1+\beta)(1-\rho)}, \tag{6}$$

$$s_t(\theta)=\frac{\beta(1-\rho)(q-\theta)\cdot w_t}{q[1+\beta(1-\rho)]-\theta(1+\beta)(1-\rho)}. \tag{7}$$

and retirement period consumptions of the adults. That is, once the number of children (and, thus, the share of time spent raising each child) has been chosen optimally by individuals, the hours devoted to the labour market are determined as well. Overall, our model endogenously determines consumption of both periods of adulthood, savings, the fertility rate and the time spent working in the labour market.

[5] Note that the condition $1-qn_t>0$ implies $n_t<1/q$, i.e., the higher is the number of children raised by parents the lower is the time spent raising each child as required by the child bearing technology. Given both the individual's and the government's budget constraints, it can be shown that the latter inequality is always fulfilled. Details are given in Appendix A.

[6] Notice that a necessary and sufficient condition to ensure that Eqs. (6) and (7) are positive is $\theta<q$.

2.3. Firms

Firms are identical and act competitively. The Cobb-Douglas (aggregate) constant returns to scale technology of production is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$,[7] where Y_t, K_t and $L_t = h_t(n_t) \cdot N_t$ are output, capital and the time-t labour input respectively, $A > 0$ is a scale parameter and $0 < \alpha < 1$ is the capital's share in technology. Therefore, the intensive form production function may be written as $y_t = Ak_t^{\alpha}[h_t(n_t)]^{1-\alpha}$, with $k_t := K_t / N_t$ and $y_t := Y_t / N_t$ being capital and output per capita respectively. Assuming total depreciation of capital at the end of each period and knowing that the price of output has been normalised to unity, profit maximisation leads to the following marginal conditions for capital and labour, respectively:

$$r_t = \alpha A \left[\frac{k_t}{h_t(n_t)} \right]^{\alpha - 1} - 1, \tag{8}$$

$$w_t = (1 - \alpha) A \left[\frac{k_t}{h_t(n_t)} \right]^{\alpha}. \tag{9}$$

2.4. Equilibrium

The equilibrium of the output good market is $y_t = c_{y,t} + c_{o,t} / n_{t-1} + n_t k_{t+1}$, which, by substituting the zero profit condition, the government budget (5) and the lifetime household's budget constraint, implies that the market-clearing condition in goods as well as in capital market is expressed by the equality between savings and investments. Therefore, knowing that population evolves according to $N_{t+1} = n_t N_t$ equilibrium implies $n_t k_{t+1} = s_t$, i.e., the stock of capital available in period $t+1$ equals the amount of resources saved in period t discounted by the number of individuals in the same period.

Substituting out for n and s from (6) and (7) respectively, and using (9) the dynamic equilibrium sequence of capital is given by:

$$k_{t+1} = \beta \left(\frac{1-\rho}{\rho} \right) (q - \theta)(1 - \alpha) A \left[\frac{k_t}{h_t(n_t)} \right]^{\alpha}. \tag{10}$$

Steady-state implies $k_{t+1} = k_t = k^*$. Hence, the long-run per-capita stock of capital is:[8]

$$k^*(\theta) = (q - \theta) \left(\frac{1-\rho}{\rho} \right) \cdot [\beta(1-\alpha)A]^{\frac{1}{1-\alpha}} \cdot [(1+\beta) n^*(\theta)]^{\frac{-\alpha}{1-\alpha}}. \tag{11}$$

3. The benevolent government

We now turn to the welfare analysis which was carried out by comparing steady-state paths of the lifetime welfare of the representative generation, following – amongst others – Samuelson (1975). The benevolent government is supposed to be a Stackelberg leader with respect to individuals and firms (Stackelberg followers). Given the followers' behaviour, the government chooses θ such as to maximise the steady-state indirect utility index of the representative generation, i.e.:

$$\max_{\{\theta\}} V^*(\theta) = (1 - \rho) \cdot \{\ln(c_y^*(\theta)) + \beta \ln(c_o^*(\theta))\} + \rho \ln(n^*(\theta)), \tag{12}$$

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[7] Adding exogenous growth in labour productivity does not alter any of the substantive conclusions of the model and hence it is not included here.

[8] Using (10) we can show that the steady-state is always stable. Details about derivation of Eq. (11) are reported in Appendix B.

subject to (6) and

$$c_y^*(\theta)=\frac{(1-\rho)(q-\theta)}{q[1+\beta(1-\rho)]-\theta(1-\rho)(1+\beta)}w^*(\theta),$$

$$c_o^*(\theta)=\frac{\beta(1-\rho)(q-\theta)}{q[1+\beta(1-\rho)]-\theta(1-\rho)(1+\beta)}[1+r^*(\theta)]w^*(\theta),$$

with $w^*(\theta)$ and $r^*(\theta)$ being the steady-state wage and interest rates, respectively. Differentiating (12) with respect to θ gives:

$$\frac{\partial V^*(\theta)}{\partial\theta}=\frac{1-\rho}{c_y^*(\theta)}\cdot\frac{\partial c_y^*(\theta)}{\partial\theta}+\frac{(1-\rho)\beta}{c_o^*(\theta)}\cdot\frac{\partial c_o^*(\theta)}{\partial\theta}+\frac{\rho}{n^*(\theta)}\cdot\frac{\partial n^*(\theta)}{\partial\theta},\tag{13}$$

or

$$\frac{\partial V^*(\theta)}{\partial\theta}=\frac{n^*(\theta)(1-\rho)(1+\beta)\{\theta[\alpha(1+2\beta(1-\rho))-\rho-\beta(1-\rho)]-q(1-\rho)[\alpha(1+2\beta)-\beta]\}}{\rho(1-\alpha)(q-\theta)}.\tag{14}$$

The sign of (14) crucially depends on the size α relative to those of β and ρ. Now, define $\alpha_1\equiv\frac{\beta}{1+2\beta}$ and $\alpha_2\equiv\frac{\rho+\beta(1-\rho)}{1+2\beta(1-\rho)}$ with $\alpha_2>\alpha_1$.

Case $\alpha>\alpha_2$. The Stackelberg game has no interior solutions for θ, that is, $\partial V^*(\theta)/\partial\theta<0$ for any $0<\theta<q$ and $\partial V^*(\theta)/\partial\theta>0$ for any $-1<\theta<0$. Thus, if the size of the capital's share on total output relative to those of intra- and inter-temporal preference parameters is high enough (or, alternatively, if both the preference for raising children and the subjective discount factor are low enough), introducing subsidies (taxes) on to incentive (disincentive) child rearing of households always worsens (improves) welfare.

Since there always exists a negative relationship between fertility and child taxes, the policy prescription is to deter fertility just by taxing each child at the highest possible rate, that is $\theta\to-1$, such that the fertility rate approaches zero and welfare is enhanced at the highest possible level.

Case $\alpha<\alpha_2$. The solution of the Stackelberg game is:

$$\frac{\partial V^*(\theta)}{\partial\theta}=0\Leftrightarrow\theta=\theta^*\equiv\frac{q(1-\rho)[\alpha(1+2\beta)-\beta]}{\alpha[1+2\beta(1-\rho)]-\rho-\beta(1-\rho)}.\tag{15}$$

In this latter case $V^*(\theta)$ is an inverted U-shaped function of θ with $\theta=\theta^*$ being an interior global maximum. It is important to note that θ^* could be either a subsidy ($0<\theta^*<q$) or a tax ($-1<\theta^*<0$)[9] depending on whether $\alpha<\alpha_1$ or $\alpha_1<\alpha<\alpha_3$, respectively. Thus, for low (moderate) values of the capital's share in technology, the policy rule is to subsidise (tax) each child at the rate θ^* to obtain a welfare maximum.

4. Centrally planned economy

As in Samuelson (1975), and following Abio (2003), the first best solution is defined as the allocation that maximises steady state individual's utility subject to the economy's resource constraint. In particular, the social planner is faced with the problem

[9] For ensuring $-1<\theta^*<0$ we need $\alpha_1<\alpha<\alpha_3$ with $\alpha_1<\alpha_3<\alpha_2$ for any q, where $\alpha_3\equiv\frac{\rho+\beta(1-\rho)(1+q)}{1+2\beta(1-\rho)+q(1-\rho)(1+2\beta)}$.

$$\max_{\{c_y, c_o, n, k\}} U(c_y, c_o, n) = (1-\rho)\cdot\{\ln(c_y) + \beta\ln(c_o)\} + \rho\ln(n),$$

subject to

$$Ak^{\alpha}(1-qn)^{1-\alpha} = c_y + \frac{c_o}{n} + nk.$$

The first order conditions for an interior solution are therefore given by:

$$\frac{c_o}{c_y}\cdot\frac{1}{\beta} = n, \tag{16}$$

$$\frac{c_y}{n}\cdot\frac{\rho}{1-\rho} + \frac{c_o}{n^2} = k + q(1-\alpha)A\left(\frac{k}{1-qn}\right)^{\alpha}, \tag{17}$$

$$\alpha A\left(\frac{k}{1-qn}\right)^{\alpha-1} = n. \tag{18}$$

Eq. (16) gives the optimal allocation between young-adult and old-age consumption. It equates the social marginal utility of current and future consumption in terms of current consumption. Eq. (17) determines the socially optimal number of children by equating benefits and costs of a marginal increase in the rate of fertility. In particular, the left-hand side of (17) captures the marginal benefit of an increase in the number of children, whereas the right-hand side says that the marginal cost of having an extra child may be split amongst two terms: the first term represents the capital that must be expanded in order for the capital-labour ratio to be kept constant when population grows over time; the second term captures the production loss due from having assumed a time-consuming technology of bearing children. Finally, Eq. (18) determines the golden-rule of capital accumulation.

Using the first order conditions together with the economy's resource constraint, the social planner obtains, respectively, the golden rule of population growth, the golden rule of capital accumulation and the optimal social values of both periods consumption, that is:

$$n_{GR} = \frac{\rho + \beta(1-\rho) - \alpha[1 + 2\beta(1-\rho)]}{q\{1 + 2\beta(1-\rho) - \alpha[1 + 2\beta(1-\rho) + (1-\rho)(1+\beta)]\}}, \tag{19}$$

$$k_{GR} = \left(\frac{\alpha A}{n}\right)^{\frac{1}{1-\alpha}}\cdot(1-qn), \tag{20}$$

$$c_{y,GR} = \frac{1-\alpha}{(1+\beta)\alpha}n\cdot k, \tag{21}$$

$$c_{o,GR} = \frac{\beta(1-\alpha)}{(1+\beta)\alpha}n^2\cdot k. \tag{22}$$

The existence of a golden rule of population growth has been widely debated in the economic literature, especially in the OLG framework, which is, as known, prone to inefficient outcomes even in the absence of endogenous fertility. Indeed, Samuelson (1975) showed in the Diamond style OLG model with exogenous fertility, that even if the economy lies on the golden rule path it might not reach the social optimum as long as the population growth rate differs from its optimal social value. Therefore in such a competitive equilibrium context there exist at least two potential sources of inefficiency that must be corrected: (1) the achievement of the golden rule of capital accumulation, and (2) the achievement of the optimal population growth. Moreover, it is known (see Samuelson, 1975; Deardorff, 1976) that in the OLG model with exogenous fertility and Cobb-Douglas preferences and technology an interior OPGR cannot exist and therefore individuals' utility increases as long as fertility falls until an infinite level of utility obtained when population tends to disappear. This traditional result may be considered rather "dismal" (sometimes defined even as "repugnant") from an ethical point of view. By contrast, when fertility is endogenously determined, an interior OPGR might exist even with Cobb-Douglas utility and production functions. In this section we show that an interior golden rule of population growth exists in an economy closed to international trade provided the distributive capital share is not too high, and we investigate the competitive equilibrium as well as the achievement of the golden rule of population growth by adopting an appropriate intra-generational child policy.

Let us better detail the reason why the competitive equilibrium fertility rate may differ from the optimal rate. In a nutshell, the decentralised solution is inefficient because individuals do not take into account both marginal benefits and marginal costs of raising an extra child. The marginal benefit is represented by the so-called *inter-generational transfer effect* which captures the benefit derived by the increased number of individuals in supporting retired people. By contrast, the marginal cost is captured by the *capital dilution effect* (i.e., the effect on the accumulation of capital of a higher fertility rate) and represents the reduction in the per capita stock of capital given by the higher number of heads in the whole economy (*ceteris paribus* as regards the level of the capital stock). The representative individual does not take into account both external effects and thus the market solution could be inefficient.

By contrast, the planner internalises both marginal benefits and marginal costs of raising an extra child (i.e., it maximises individuals welfare subject to the economy's resource constraint) and thus chooses optimally the golden rule of population growth as well as the golden-rule of capital accumulation.

We now show that the inefficient solution as regards the number of children raised by families in the market could be adjusted by using appropriately an intra-generational family policy.

Exploiting (20)-(21) and by using the properties of logarithms we may evaluate the representative generation's indirect utility as a function of the number of children, that is:

$$\max_{\{n\}} V(n) = \ln\left(\mathrm{H} \cdot (1-qn)^{(1-\rho)(1+\beta)} \cdot n^{\frac{\rho+\beta(1-\rho)-\alpha[1+2\beta(1-\rho)]}{1-\alpha}} \right), \tag{23}$$

where $\mathrm{H} \equiv \left[\frac{1-\alpha}{\alpha(1+\beta)} \right]^{(1-\rho)(1+\beta)} \cdot \beta^{\beta(1-\rho)} \cdot (\alpha A)^{\frac{(1-\rho)(1+\beta)}{1-\alpha}}$.

Differentiating (23) with respect to n yields:

$$\frac{\partial V(n)}{\partial n} = \frac{\rho+\beta(1-\rho)-\alpha[1+2\beta(1-\rho)] - qn\{1+2\beta(1-\rho)-\alpha[1+2\beta(1-\rho)+(1-\rho)(1+\beta)]\}}{n(1-qn)(1-\alpha)}. \tag{24}$$

The sign of Eq. (24) depends exclusively on both technology and preference parameters.

Case $\alpha > \alpha_2$.[10] In this case a strictly positive OPGR (i.e., an "interior" solution) does not exist and the highest possible utility is obtained when n approaches zero; this is the same traditional result found when fertility is exogenously given, see Samuelson, 1975; Deardorff, 1976, that is, $V(n)$ falls short monotonically as n raises ($\partial V(n)/\partial n < 0$ for any $n > 0$). This rather "dismal" result holds because the uplifting effect of the increased capital accumulation on working and retirement period consumptions more than counterbalances the negative effect of the reduced number of heads in the whole economy. Therefore,

$$\lim_{n\to 0} V(n) = \lim_{n\to 0} \ln\left(\mathrm{H} \cdot (1-qn)^{(1-\rho)(1+\beta)} \cdot n^{\frac{\rho+\beta(1-\rho)-\alpha[1+2\beta(1-\rho)]}{1-\alpha}} \right),$$

[10] Let $\alpha_4 \equiv \frac{1+2\beta(1-\rho)}{1+2\beta(1-\rho)+(1-\rho)(1+\beta)}$. Since $\alpha_4 > \alpha_2$, the case $\alpha > \alpha_4$ is uninteresting and it is automatically ruled out. Notice also that if both β and ρ approach zero, then $\alpha_4 \to 1/2$, that is $\alpha_4 > 1/2$ for any $\beta, \rho > 0$.

and thus in the case $\alpha > \alpha_2$ we get $\lim_{n \to 0} V(n) = +\infty$. Definitely, individuals' welfare increases as n shrinks.

Case $\alpha < \alpha_2$. $V(n)$ is an inverted U-shaped function with $n = n_{GR}$ being an interior global maximum, and the following proposition holds:

Proposition 1. *Let $\alpha < \alpha_1$ ($\alpha_1 < \alpha < \alpha_3$) hold. Then*
(1) the OPGR is determined by the optimal child subsidy (tax):

$$\theta_{GR} = \frac{q(1-\rho)[\alpha(1+2\beta)-\beta]}{\alpha[1+2\beta(1-\rho)]-\rho-\beta(1-\rho)}, \tag{25}$$

and

(2) the optimal child subsidy (tax) is exactly that one obtained by a benevolent government in the market (Stackelberg game), that is $\theta_{GR} = \theta^*$.

Proof. (1) The proof straightforwardly derives by subtracting $n^*(\theta)$ from n_{GR} and solving for θ. As regards point (2), it is obvious from (15) and (25). **Q.E.D.**

Eq. (25) determines the value of the child subsidy/tax such that the individuals' fertility in the market equals the golden rule of population growth. Thus, the welfare maximisation at θ^* also means that the number of children freely chosen by atomistic individuals automatically coincides with that chosen by the social planner. The remarkable result claimed in Proposition 1 is that a benevolent government (which does not pursue the OPGR) behaves just as a social planner and reaches the golden rule of population growth by implementing an intra-generational balanced budget child policy.

5. Qualitative analysis

A graphical illustration, for parametric configurations chosen only for illustrative purposes, may help us evaluate the working of the family policy presented both from positive and normative view points in the previous sections. Panels A of Figures 1-3 show the representative generation's lifetime indirect utility index (as a function of the child subsidy/tax) evaluated by the benevolent government in the market. By contrast, the corresponding panels B depict the indirect utility index (as a function of the number of children) evaluated by the social planner.

Figures 1-3 were plotted, respectively, for low ($\alpha < \alpha_1$), moderate ($\alpha_1 < \alpha < \alpha_3$) and high ($\alpha > \alpha_2$) values of the capital's share in production in order to clarify the three different government policy suggestions of our theoretical model. We take the following parameters values: $A = 10$ (simply a scale parameter when production takes place according to the usual Cobb-Douglas technology and preferences are represented by a log-linear utility function), $\beta = 0.23$ (the inter-temporal subjective discount factor), $\rho = 0.50$ (that is, parents evaluate one-half their own consumption over the life cycle and one-half to raise children). Finally we supposed $q = 0.30$, that is, 30 per cent of the whole time available to parents is devoted to care about children.

Figure 1.A shows that when the share of capital in production is sufficiently low, an optimal welfare-maximising child subsidy ($0 < \theta^* < q$) does exist, whereas introducing child taxes always reduces individuals' utility as compared with the case we had before applying the family policy (i.e., Diamond's (1965) economy with endogenous fertility, V^*_c in the figures). On the contrary, with moderate values of α, see Figure 2.A, an optimal child tax ($-1 < \theta^* < 0$) does exists, with

individuals' utility always being worsened by child subsidies. In both cases (low or moderate values of α), when $\theta = \theta^*$, the benevolent government replicates the OPGR, as clearly depicted in panel B of Figures 1 and 2.

Finally, Figure 3.A show – for a relatively high capital weight in technology – that a benevolent government is always able to increase (reduce) individuals' welfare just by taxing (subsidising) child-rearing of households, whilst from Figure 3.B it can be clearly seen that an interior golden rule of population growth does not exist, i.e., individuals' utility increases as long as fertility falls until an infinite level of utility obtained when population tends to disappear. Since there always exists a negative relationship between fertility and child taxes, in this latter case the government must deter child-rearin just by taxing at the highest possible rate each child, that is $\theta \to -1$, so as to reduce to the lowest possible rate population growth ($n \to 0$) and approach towards the goal of maximum welfare.

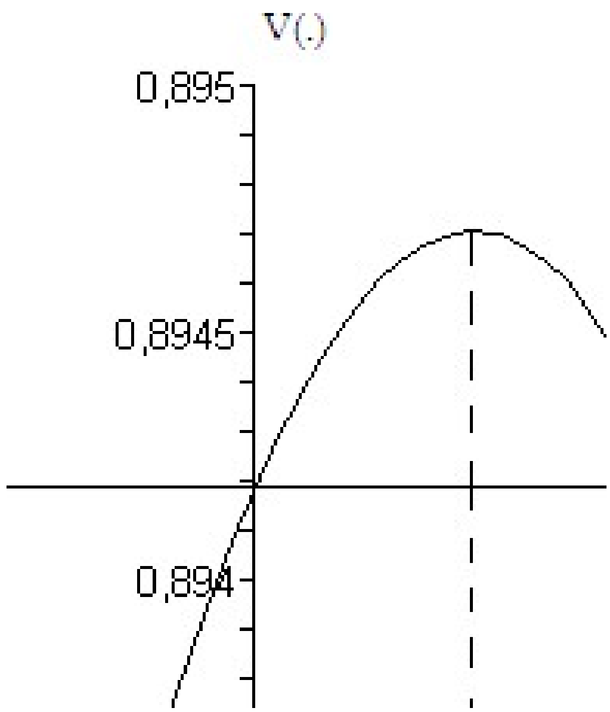

Figure 1.A. Case $\alpha < \alpha_1$: Decentralised Economy. Lifetime welfare as a function of the child subsidy (tax), $V^*(\theta)$, versus lifetime welfare in Diamond's economy with endogenous fertility ($\theta = 0$), V^*_c. Parameter values: $A = 10$, $\alpha = 0.12$, $\beta = 0.23$, $\rho = 0.50$ and $q = 0.30$. The welfare maximising child subsidy is $\theta^* = 0.017$.

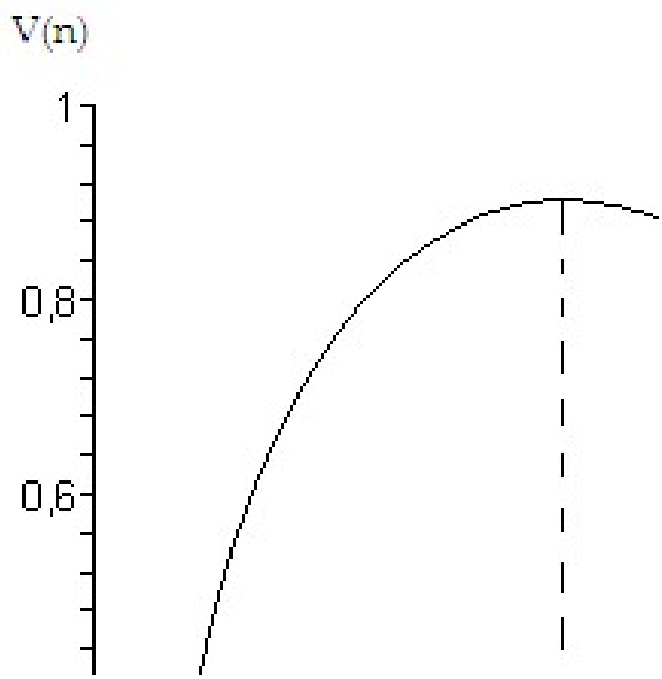

Figure 1.B. Case $\alpha < \alpha_1$: Centrally Planned Economy. Lifetime welfare as a function of the number of children. Parameter values: $A = 10$, $\alpha = 0.12$, $\beta = 0.23$, $\rho = 0.50$ and $q = 0.30$. The OPGR is $n_{GR} = n^*(\theta^*) = 1.544$ with the optimal child subsidy being $\theta_{GR} = \theta^* = 0.017$.

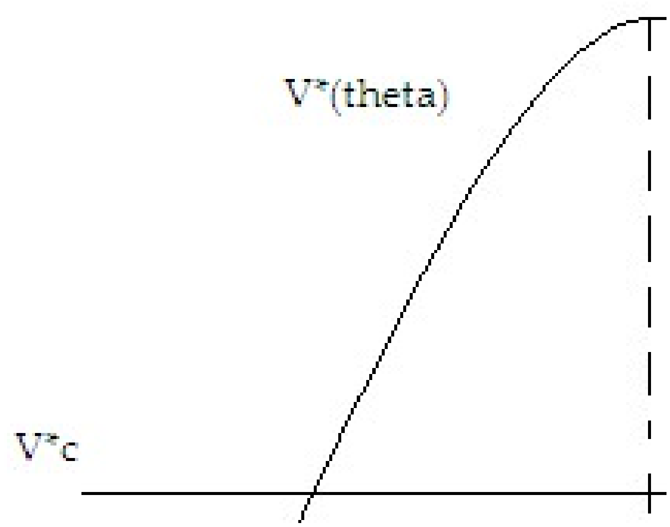

Figure 2.A. Case $\alpha_1 < \alpha < \alpha_3$: Decentralised Economy. Lifetime welfare as a function of the child subsidy (tax) versus lifetime welfare in Diamond's economy. Parameter values: $A = 10$, $\alpha = 0.33$, $\beta = 0.23$, $\rho = 0.50$ and $q = 0.30$. The welfare maximising child-rearing tax is $\theta^* = -0.18$.

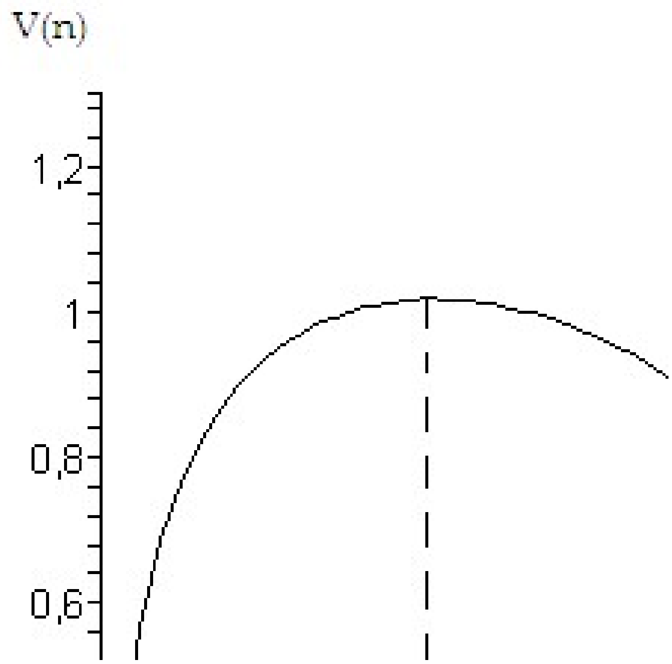

Figure 2.B. Case $\alpha_1 < \alpha < \alpha_3$: Centrally Planned Economy. Lifetime welfare as a function of the number of children. Parameter values: $A = 10$, $\alpha = 0.33$, $\beta = 0.23$, $\rho = 0.50$ and $q = 0.30$. The OPGR is $n_{GR} = n^*(\theta^*) = 1.122$ with the optimal child tax being $\theta_{GR} = \theta^* = -0.18$.

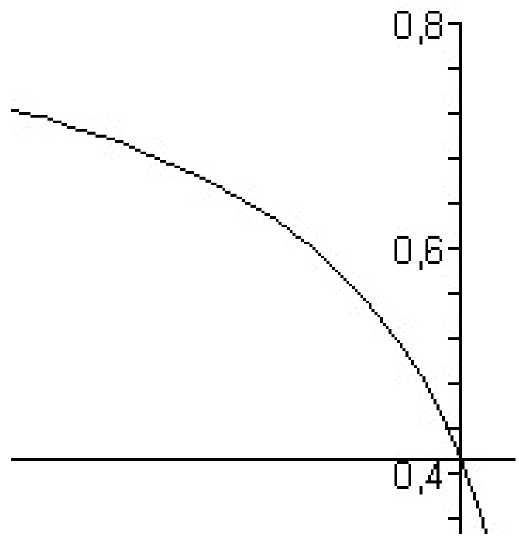

Figure 3.A. Case $\alpha > \alpha_2$: Decentralised Economy. Lifetime welfare as a function of the child subsidy (tax) versus lifetime welfare in Diamond's economy. Parameter values: $A = 10$, $\alpha = 0.52$, $\beta = 0.23$, $\rho = 0.50$ and $q = 0.30$.

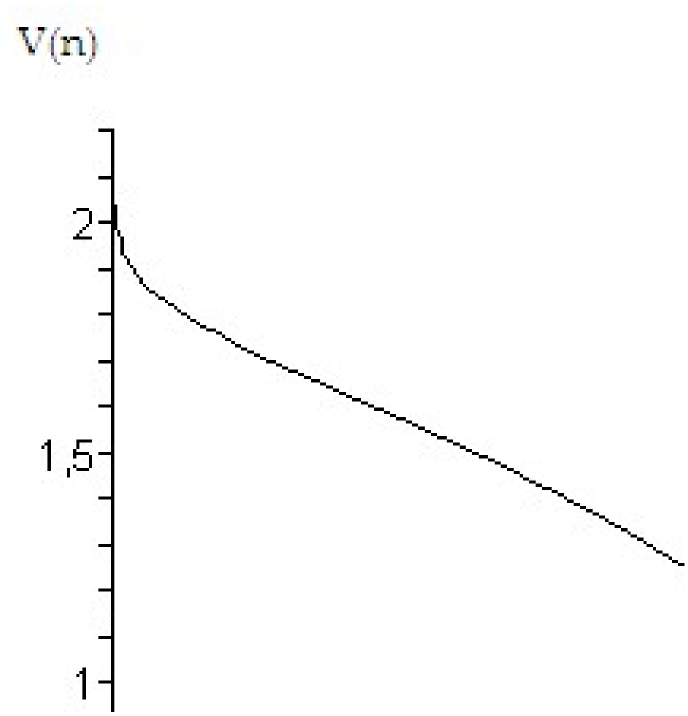

Figure 3.B. Case $\alpha > \alpha_2$: Centrally Planned Economy. Lifetime welfare as a function of the number of children. Parameter values: $A = 10$, $\alpha = 0.52$, $\beta = 0.23$, $\rho = 0.50$ and $q = 0.30$. Lifetime welfare is a negative monotonic function of the number of children.

To complete the analysis of the child policy described in the previous sections, we now compare two countries with a very different population size but with a similar population dynamics, i.e., China (the most populous country in the world) and Italy (a country plagued by population ageing due to the increased longevity and the reduced birth rates experienced in the last decades),[11] to look at the effectiveness of child subsidies and taxes in the achievement of the golden rule of population growth.

We took capital share estimates from Rodriguez and Ortega (2006) – who surveyed actual values for the distributive capital share in production for several developing and developed countries –, whilst looked at He and Cao (2007) and OECD (2008) as regards the Chinese and the Italian propensity to save, respectively. As regards the capital's share in production we found $\alpha = 0.486$ (China) and $\alpha = 0.477$ (Italy), whereas the propensity to save is approximately 25 per cent of disposable income in China and 10 per cent of disposable income in Italy with a number of children per woman assumed to be around 1.8 in China ($n = 0.92$) and 1.3 in Italy ($n = 0.65$). Therefore, we calibrated the preference parameters, i.e., the subjective discount factor and the parents' preference for children, to replicate such values in both countries, that is $\beta = 0.70$ and $\rho = 0.60$ (China),

[11] Since the 1980 China forced a one-child per family policy who effectively reduced the Total Fertility Rate (TFR), i.e., the number of children per woman, from 2.5 in the 1980 to 1.8 in recent years, whilst Italy experienced a process who transformed the structure of modern Italian families (increasing divorces, the postponement of the birth of the first child, delayed marriages, and a new trend in deciding when male and female want to become parents) accompanied by a dramatic drop in birth rates (the TFR in Italy shrank from 2.1 in 1981 to 1.3 in recent years). In any case, needless to say, both countries experienced a dramatic reduction in population growth over the last 25 years.

$\beta = 0.15$ and $\rho = 0.36$ (Italy).[12] This parameter sets generate $\alpha_2 = 0.564$ (China) and $\alpha_2 = 382$ (Italy).[13]

As regards China, we found that a positive OPGR $n_{GR,China} = 0.516$ exists; it can be replicated in the market by adopting an optimal child tax, that is, $\theta_{GR,China} = -0.765$. By contrast, as regards Italy a positive OPGR does not exist and thus the traditional "dismal" result is obtained, i.e., welfare is always increasing when fertility is falling. Therefore, rather unexpectedly, Italy should reduce as much as possible the rate of population growth by introducing a per child tax $\theta \to -1$ so as to increase to the highest possible value the lifetime individual welfare. In China, instead, the one-child per family policy forced by the Chinese government could be unnecessary to the extent that the golden rule of population growth can be obtained in the market by adopting an appropriate child tax policy; as a consequence, applying a too high child taxation could be welfare reducing.

6. Modification: small open economy, time cost of children, family policy and optimality

Abio et al. (2004) built up an OLG closed-economy model with endogenous fertility and heterogeneous agents (male and female) finding that it is sufficient to introduce a fertility-related pay-as-you-go (PAYG) pension system[14] to internalise both external effects of children on society as a whole, i.e., the capital dilution effect and the inter-generational transfer effect, and thus reproduce jointly the golden rule of population growth and the golden rule of capital accumulation with a single instrument. Interestingly, they obtained a closed form solution as regards the payroll tax adopted by the government to decentralise the first best under two rather special assumptions: (a) only female spend time to take care of children (with male being exempted to attend their babies); (b) only male contribute to fund the fertility-related PAYG budget (with female being untaxed).[15]

In this section we opted to keep the hypothesis introduced in Section 2 and thus consider identical parents being treated and taxed as a single homogeneous agent who must consume and spend time to care about children.[16] We will show that both externalities of having children can be eliminated, and thus the command optimum can be perfectly realised in the market, if the government has at its disposal a single (intra-generational) policy instrument in a small open economy. In fact, if parents do (not) devote enough time to take care of their children, then by merely adopting a system of child subsidies (taxes) the first best is replicated. This sharply differs from Abio et al. (2004), because (1) the command optimum is obtained exclusively with an *intra-generational* policy, rather than with inter-generational transfers (e.g., PAYG pensions), thus avoiding any problems of inter-generational equity, and (2) while fertility-related pensions, although theoretically deeply analysed (e.g., Kolmar, 1997; Abio et al., 2204; Fenge and Meier, 2005), are not implemented in the most part of countries, child policies are widespread adopted and in any case they seem to be easily available as an instrument.

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[12] Notice that we also assumed $A = 10$ (simply a scale parameter in the Cobb-Douglas production function) and $q = 0.50$ (the exogenous fraction of the time endowment of one parent that must be spent raising a child) in both countries.

[13] We recall that values of the distributive capital share lower (higher) than the threshold α_2 discriminate in favour of (against) the existence of a positive golden rule of population growth.

[14] In a nutshell, the fertility-related pension system links the benefit received by each pensioner to the number of children raised when young.

[15] Indeed, by relaxing the special hypotheses (a) and (b) above mentioned, closed form solutions as regards the payroll tax used to replicate the first best would be prevented in a closed economy.

[16] It could be interesting, especially as regards the analysis of individuals' fertility in developed countries, to extend the present paper by assuming a gender gap and allow both male and female to raise children by distinguishing them on the bases of their ability to raise offspring and earn wages on the labour market by introducing a differentiated time-technology of child rearing. Parents therefore should be either taxed or subsidised at different rates.

6.1. The market economy

6.1.1. Firms

Consider a small open economy with perfect capital mobility that faces an exogenously given (constant) interest rate r. Production takes place according to a standard neoclassical constant-returns-to-scale technology $f(k,h)$, where k is the amount of capital per-capita and h represents the hours supplied in the labour market by each young adult agent in each period. Since capital is perfectly mobile, both the capital-labour ratio and the wage rate w are fixed and constant.

6.1.2. Government

As we have assumed in Section 2, the government levies lump-sum taxes ($\tau_t > 0$) on the young to fund at any date a proportional-to-wage child subsidy ($0 < \theta < q$) balancing out its budget. Thus, the time-t per capita government constraint is:[17]

$$\theta w n_t = \tau_t , \tag{M1}$$

where the left-hand side represents the child expenditure and the right-hand side the tax receipts.

6.1.3. Individuals

Individuals behave just as we described in Section 2. Therefore, the representative individual born at time t is faced with the following program:

$$\max_{\{c_{y,t}, c_{o,t+1}, n_t\}} U_t\left(c_{y,t}, c_{o,t+1}, n_t\right) = (1-\rho)\cdot\left\{\ln\left(c_{y,t}\right) + \beta\ln\left(c_{o,t+1}\right)\right\} + \rho\ln\left(n_t\right), \tag{PP}$$

subject to the intra-temporal constraints

$$\begin{aligned} c_{y,t} + s_t &= w\left[1 - n_t(q-\theta)\right] - \tau_t \\ c_{o,t+1} &= (1+r)s_t \end{aligned} .$$

The maximisation of program (PP) gives the following first order conditions for an interior solution:

$$\frac{c_{o,t+1}}{c_{y,t}} \cdot \frac{1}{\beta} = 1 + r , \tag{M2}$$

$$\frac{c_{y,t}}{n_t} \cdot \frac{\rho}{1-\rho} = w(q-\theta). \tag{M3}$$

Eq. (M2) represents the condition to substitute out consumption bundles over youth and old-age by equating the marginal rate of substitution between time t and time $t+1$ consumption to the given world interest rate, whereas Eq. (M3) equates the marginal rate of substitution between consuming during youth and having children to the marginal cost of rearing an additional child.

Thus, exploiting (M1)-(M3) along with the individual's lifetime budget constraint we get:

$$n^* = \frac{\rho}{q\left[1 + \beta(1-\rho)\right] - \theta(1+\beta)(1-\rho)}, \tag{M4}$$

$$c_y^* = \frac{1-\rho}{1+\beta(1-\rho)} w\left(1 - \theta n^*\right), \tag{M5}$$

$$c_o^* = \beta(1+r)c_y^* . \tag{M6}$$

[17] Agents act in an atomistic way and thus do not take the government budgets (M1) into account when deciding on the desired number of children and the saving path. Notice also that we allow for the existence of a proportional-to-wage child tax ($-1 < \theta < 0$) used to fund a lump-sum subsidy ($\tau_t < 0$).

6.2. The first best solution

In a small open economy the social planner is faced with the following program:

$$\max_{\{c_y, c_o, n, d\}} U(c_y, c_o, n, d) = (1-\rho)\cdot\{\ln(c_y) + \beta\ln(c_o)\} + \rho\ln(n),$$

subject to the economy's resource constraint

$$f(k,h) = c_y + \frac{c_o}{n} + nk - nd + (1+r)d\ ,$$

where the variable d represents the amount of per-capita foreign debt.[18] Thus, the first order conditions for the command optimum are

$$n_{GR} = 1 + r\ , \tag{M7}$$

$$\frac{c_o}{c_y}\cdot\frac{1}{\beta} = n\ , \tag{M8}$$

$$\frac{c_y}{n}\cdot\frac{\rho + \beta(1-\rho)}{1-\rho} = q\,w + k - d\ . \tag{M9}$$

Eq. (M7) says that the socially optimal number of children is determined exclusively by the constant world interest rate. Eq. (M8) gives the optimal allocation between young-aged and old-aged consumptions in the social planner program by equating the marginal rate of substitution to the rate of fertility. Finally, Eq. (M9) determines the optimal allocation between consumption during youth and the number of offspring. By comparing (M3) and (M9) in steady-state it can be seen the reason why the private fertility rate may differ from the socially optimal one: the planner takes into account both the *inter-generational transfer effect* (captured by the subjective discount factor β in (M9)), representing the marginal benefit of raising an additional child in the planner solution, and the *capital dilution effect* (captured by the difference $k-d$, i.e., the effects of fertility on saving), representing the marginal cost of raising an additional child in the planner solution, which, instead, are not taken into account by households in the market.

Exploiting (M7)-(M9) along with the economy's resource constraint, we get:[19]

$$c_{y,GR} = \frac{w[1 - q(1+r)]}{1+\beta}, \tag{M10}$$

$$c_{o,GR} = \beta(1+r)c_{y,GR}\,. \tag{M11}$$

The first best outcome therefore is jointly determined by (M7), (M10) and (M11).

As it can easily be seen individuals' fertility may be higher or lower than the socially optimal population growth rate and thus the market solution may be inefficient due to the existence of external effects of children on society as a whole. In the following proposition we will show that the command optimum can be perfectly decentralised by a benevolent government in the market by adopting a single intra-generational instrument which permits to internalise both the capital dilution effect and the inter-generational transfer effect thus avoiding the need of using inter-generational transfers to effectively alter individuals' behaviour.

Define

$$\overline{q} \equiv \frac{\rho}{(1+r)[1 + \beta(1-\rho)]}, \tag{M12}$$

then the following proposition holds:

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[18] Recall that the capital stock totally depreciates over time.

[19] From (M9) it can easily be shown that a necessary and sufficient condition for the existence of a positive consumption bundle in the planned economy is $q < 1/(1+r)$, which is assumed to be always fulfilled.

Proposition 2. *(1) If the child rearing technology is such that parents devote exactly a fraction $q = \overline{q}$ of time to care about children, then the command optimum automatically coincides with the market solution and no government interventions are required.*

(2) If the child rearing technology is such that parents do (not) devote enough time to care about children, i.e., $q > \overline{q}$ ($q < \overline{q}$), then the command optimum may be realised by a benevolent government in the market with the following child subsidy (child tax):

$$\theta = \theta_{GR} \equiv \frac{q(1+r)[1+\beta(1-\rho)]-\rho}{(1+r)(1+\beta)(1-\rho)} . \tag{M13}$$

Proof. Let $q = \overline{q}$ hold. Substituting out $\theta = 0$ in (M4)-(M6) gives $n^* = n_{GR}$, $c_y^* = c_{y,GR}$ and $c_o^* = c_{o,GR}$. This proves point (1). As regards point (2), subtracting (M4) from (M7) and solving for θ yields (M13). Inserting (M4) and (M13) into (M5) and (M6) gives $c_y^* = c_{y,GR}$ and $c_o^* = c_{o,GR}$. This proves point (2). **Q.E.D.**

Point 1 in Proposition 2 defines the amount of time required by the child rearing technology with respect to which the command optimum can be perfectly realised without government interventions. But this represents only a special case and it is certainly not the end of the story. If the technology of raising children is such that parents devote too much time to take care of each child they have ($q > \overline{q}$), then the economy is dynamically efficient ($n < 1+r$), i.e., the privately chosen number of children is lower than the golden rule of population growth and the government can replicate the command optimum by subsidising each child at the rate $\theta_{GR} > 0$ to incentive private fertility.[20] By contrast, if the child rearing technology is such that parents do not devote enough time to take care of each child they have ($q < \overline{q}$), then the economy is dynamically inefficient ($n > 1+r$), i.e., the privately chosen number of children is higher than the corresponding optimal social value, and the government can let the market solution coincide with the first best by deterring fertility with a per child tax policy at the rate $-1 < \theta_{GR} < 0$.

Therefore, the first best can be effectively decentralised by merely adopting a single intra-generational instrument (e.g., either a child subsidy or a child tax); this avoids the need of using any form of transfers to redistribute between generations and thus the emergence of any problems of inter-generational equity.

7. Conclusions

We analysed the effects of an intra-generational family policy in the basic Diamond (1965) overlapping generations model of neoclassical growth extended to account for endogenous fertility decisions of households and a time-technology of child rearing from both positive and normative perspectives.[21] Under the closed economy hypothesis we find that societies may normatively have a

[20] Notice that $0 < \theta_{GR} < q$ for any $q < 1/(1+r)$.

[21] We note that normative criteria to evaluate policy changes commonly used in static problems are not well-defined in inter-temporal models of endogenous population: for example, the Pareto criterion requires that the number and the identity of individuals is unaffected by policy choices. Several authors (e.g. amongst others, Blackorby and Donaldson, 1984, and recently Golosov et al., 2007) developed different attempts to deal with this problem; they argued that the need to assign ethical rights to potential (unborn) persons whose preferences do not exist implies a reconsideration of the individual orderings of alternatives of current individuals, in that an evaluation of preferences of unborn persons is logically based on future interests of the generation currently alive. However, in this paper we evaluate the optimality exactly as Diamond, 1965 or Samuelson, 1975, that is, in terms of the steady state utility index of the representative individual: therefore, in such a frame and for our purposes the representative individual welfare level still remains valid

golden rule of population growth objective, confirming the results of Abio (2003), but we also proved that, depending on the size of the capital's share in production relative to those of the individuals' preference parameters, a benevolent government may achieve in the market both a welfare maximum and the Optimal Population Growth Rate either by taxing or subsidising child-rearing of households. Under the small open economy hypothesis, it is shown that the first best can be perfectly decentralised in the market by adopting exclusively an intra-generational family policy thus avoiding the need the redistribute between generations and the emergence of any problems of inter-generational equity.

Our findings contribute to clarify the role played by family policies in a dynamic overlapping generations macroeconomic context both in closed and small open economies. In particular, as regards the closed economy model, we suggest that countries which display a relatively high labour share in production – low values of α – (or, alternatively, when individuals prefer to postpone consumption in the future) should introduce policies to deter fertility by implementing an appropriate per child tax used to fund a lump-sum subsidy within the same child bearing generation at a balanced budget, so that the goals of maximum welfare and a golden rule of population growth are both met. If, on the contrary, the technology displays moderate values of α – (or, alternatively, if individuals are impatient enough, that is the subjective discount factor is low enough, and/or the relative weight of the parents' preference for children in welfare evaluation is not too high) there exists an optimal value of the proportional-to-wage subsidy on child-rearing (financed by a lump-sum tax on the younger generation) for which welfare is maximised and the number of children freely chosen by individual coincides with that chosen by the planner. Finally, if the capital share in production is high – high values of α – (or, alternatively, if the preference for children and the subjective discount factor are both sufficiently low) then our model suggests taxing each child at the highest possible rate to reduce birth rates and increase welfare. In the small open economy model, we suggest that in countries in which parents do (not) devote enough time to care about their children the government should adopt a child subsidy (tax) both to promote (deter) individuals' fertility and to realise the first best.

Our findings are particularly interesting as regards the vexed question of the existence and (material and moral) feasibility of an optimal population growth rate. As regards the existence, we showed that not only does the golden rule of population growth exist but it may also be achieved in a market context. Moreover, we showed that the material feasibility of a socially optimal fertility rate depends on a very simple family policy.

Finally, a conclusive comment as regards the moral feasibility of a golden rule of population growth: although government interference in the form of population control policies has often been advocated on the basis of the existence of the "normative" result (an extreme example of contemporary population control policy is China's policy of one child per family since 1980 (see Coale, 1981),[22] we recall that such policies are not exempt from ethical concerns.[23] It should be

(see, for example, Abio, 2003 who extends – by endogenising the fertility rate – Samuelson, 1975). We thank an anonymous referee for having suggested to clarify this point.

[22] As the most populous country in the world, China needs to consider the carrying capacity of its land and, for instance, water resources. Since after the World War II the health care system was improved, as well as the ability to supply enough food, the population exploded between 1950 and 1980. For this, and other reasons, China has held a tight family planning programme over the past years. Due to the huge population base and large annual increase central government needs to establish a national target and policy. At present, Chinese Family Planning Policy guidelines may be summarised by (1) controlling the rapid population growth and reducing birth defects, and (2) encouraging couples to have only one child. It took time for the Chinese government to recognise the population problem. In the 1950s the Chinese government preferred a large population. The central government paid no attention to population growth. Rapid population increase and fertility remained at very high levels in the late 1960s, leading the Chinese government to reduce the population growth rate. Despite the introduction of the planning programme, in the 1970s the large population base and annual net increase still placed tremendous pressure on China's economic development, resources and environment. Therefore, while adhering to the policy of reform and opening up the outside world so as to sustain a rapid and healthy development of the national economy, China was forced to adopt a strategy of sustainable development, further promoting its family planning programme and providing quality reproductive health service to

pointed out that ethical issues involved with birth regulation systems do not arise here, since the golden rule of population growth can be freely chosen by individuals in the market (closed economy) and the first best can be perfectly realised (small open economy) by adopting a simple balanced budget family policy.

Some possible extensions of the present analysis should also be mentioned. First, the type of the family policy adopted might be extended, for instance by considering (1) the public provision of child-care facilities or, alternatively, a per child direct monetary transfers, rather than the parental-leaves-type policy investigated here,[24] and (2) the existence of child costs in terms of consumption goods. Secondly, the robustness of our results may be checked under different taxation systems unrelated to fertility.

Appendix A

The individual's inter-temporal budget constraint is the following:

$$c_{y,t} + \frac{c_{o,t+1}}{1+r_{t+1}} = w_t\left[1 - n_t(q-\theta)\right] - \tau_t .$$

For the right-hand side (the income of the young-aged individuals) to be positive we need

$$w_t\left[1 - n_t(q-\theta)\right] - \tau_t > 0 ,$$

or, alternatively, solving for n_t

$$n_t < \frac{w_t - \tau_t}{w_t(q-\theta)}. \tag{A1}$$

Knowing that at balanced budget the government must verify:

$$\theta w_t n_t = \tau_t ,$$

substituting out the latter equation into (A1) for τ_t we have:

$$n_t < \frac{1-\theta n_t}{q-\theta},$$

and solving for n_t

$$n_t + \frac{\theta n_t}{q-\theta} < \frac{1}{q-\theta},$$

$$n_t\left(\frac{q}{q-\theta}\right) < \frac{1}{q-\theta},$$

$$n_t q < 1,$$

$$n_t < \frac{1}{q}. \tag{A2}$$

This implies that (A2) is satisfied whatever the value of θ.

Appendix B

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eligible couples. In the early 1980s central government thus advocated a one-child per family policy in urban areas. In this paper we showed that such a policy could be unnecessary since the golden rule of population growth in China could be obtained along with an appropriate child tax system.

[23] Also Samuelson (1976) is cautious about the "normative" policy saying that "an important purpose of the original analysis was not so much to enable society to identify n^* and normatively to move to it, as to learn what is implied for society's net welfare potentialities by the post-1957 drop in birth rates".

[24] *En passant*, we note that family policies based either on childcare facilities or parental leaves are widespread in Scandinavian countries, while family policies based essentially on direct monetary transfers are adopted in countries such as Italy and Poland.

We report here technical details of the derivation of Eq. (11) in the main text.

Eq. (10) can be read as

$$k_{t+1} = \beta\left(\frac{1-\rho}{\rho}\right)(q-\theta)(1-\alpha)Ak_t^{\alpha}\left[1-q\cdot n^*(\theta)\right]^{-\alpha}. \tag{B1}$$

Inserting (6) into the steady-state of (B1) and rearranging terms we get

$$k^* = \left[\beta\left(\frac{1-\rho}{\rho}\right)(q-\theta)(1-\alpha)A\right]^{\frac{1}{1-\alpha}} \cdot \left[\frac{(q-\theta)(1+\beta)(1-\rho)}{q[1+\beta(1-\rho)]-\theta(1+\beta)(1-\rho)}\right]^{\frac{-\alpha}{1-\alpha}}. \tag{B2}$$

Therefore,

$$k^* = (\beta(1-\alpha)A)^{\frac{1}{1-\alpha}}(1-\rho)^{\frac{1}{1-\alpha}}(1-\rho)^{\frac{-\alpha}{1-\alpha}}(q-\theta)^{\frac{1}{1-\alpha}}(q-\theta)^{\frac{-\alpha}{1-\alpha}}\left(\frac{1}{\rho}\right)^{\frac{1}{1-\alpha}} \times \left[\frac{1+\beta}{q[1+\beta(1-\rho)]-\theta(1+\beta)(1-\rho)}\right]^{\frac{-\alpha}{1-\alpha}}. \tag{B3}$$

Multiplying and dividing the term in square brackets of (B3) by ρ yields

$$k^* = [\beta(1-\alpha)A]^{\frac{1}{1-\alpha}}(1-\rho)(q-\theta)\left(\frac{1}{\rho}\right)^{\frac{1}{1-\alpha}}\left(\frac{1}{\rho}\right)^{\frac{-\alpha}{1-\alpha}}\left[\frac{(1+\beta)\rho}{q[1+\beta(1-\rho)]-\theta(1+\beta)(1-\rho)}\right]^{\frac{-\alpha}{1-\alpha}}. \tag{B4}$$

Definitely

$$k^*(\theta) = (q-\theta)\left(\frac{1-\rho}{\rho}\right)\cdot[\beta(1-\alpha)A]^{\frac{1}{1-\alpha}}\cdot\left[(1+\beta)n^*(\theta)\right]^{\frac{-\alpha}{1-\alpha}}.$$

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