Endogenous Lifetime in an Overlapping Generations Small Open Economy

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Using an overlapping generations small open economy with endogenous lifetime à la Chakraborty (2004), we show that an increase in public investments in health is beneficial to life expectancy but can actually reduce both saving and domestic income per worker. Moreover, using the notion of $\mathcal{A}$–efficiency introduced by Golosov et al. (2007) in a context of endogenous population, i.e., Pareto efficiency applied to people alive in every state, which contrasts the $\mathcal{P}$–efficiency criterion with respect to which the preference profiles of both born and unborn agents are evaluated in every state, some normative conclusions can be drawn: although the financing of health investments reduces both the disposable income and marginal propensity to consume of young individuals through an increased labour income taxation, it also increases the length of life and can represent an $\mathcal{A}$–Pareto improvement when the world interest rate is fairly high, because consumption when old becomes more attractive in such a case. Indeed, there exist: (*i*) a whole range of health tax rates that can effectively be used to increase social welfare, and (*ii*) an $\mathcal{A}$–Pareto efficient allocation. Moreover, the numerical simulations presented for some actual small open economies reveal that raising public health spending over the existing level causes both saving and domestic income to fall in all countries, while also finding that developed and underdeveloped (transition) economies are currently under-investing (over-investing) in health.

Keywords: $\mathcal{A}$–efficiency, health spending, life expectancy, OLG model, small open economy

JEL Classification: I18, O41

1. Introduction

In the past few decades dramatic increases in life expectancy in several countries in the world have been observed (see, e.g., Livi-Bacci, 2006; Acemoglu and Johnson, 2007), essentially because of the decline in mortality in the later period of life that in the course of the past century resulted, especially in developed economies, in a doubling of the ratio of life cycle

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years lived after 65 to years lived 20 to 64.[1] Indeed, as well evidenced by some demographers (see e.g., Lee, 1994; Lee and Tuljapurkar, 1997), while in early stages of economic development the mortality decline brought upon an even more rapid growth in population, because children and young people in reproductive age were strongly affected by such a reduction, in the recent stage most years of life have gained at the older post-retirement ages thus causing (together with the steadily reducing fertility) the phenomenon known as population ageing. Therefore, the economic consequences of a further mortality decline mainly involve the response of individuals and governments, the latter facing with the need to fund consumption (through, e.g., public pensions) and health care services for the increasingly numerous elderly which are expected to live a longer and healthy lifetime.

In the present study we address the following question: which are the long-run effects of an increase in public health spending on income and welfare in an overlapping generations (OLG) small open economy model with endogenous lifetime and domestic annuities market? In a dynamic life cycle context a rise in life expectancy produces its main effect on savings and the process of economic growth,[2] as well evidenced by Zhang et al. (2001, p. 486) “longer life may motivate increased life cycle saving and thereby stimulate growth”, and Chakraborty (2004) “health plays a role quite unlike any other human capital: by increasing lifespans it

[1] The level of adult mortality is an important indicator for the comprehensive assessment of the mortality pattern in a population which has dramatic interactions with several macroeconomic variables. For instance, Barro and Sala-i-Martin (2004) stress the importance of longevity as a determinant to a more rapid economic growth. See also Mason (1988), Fogel (1994; 2004), de la Croix and Licandro (1999), Elmendorf and Sheiner (2000) and Weil (2007). Recently, de la Croix et al., forthcoming, revisit the Serendipity Theorem by Samuelson (1975) under risky lifetime in the simple and intuitive overlapping generations model à la Diamond (1965).

[2] Of course, other effects can exist: for instance, bequests are strongly affected, as Zhang et al. (2003, p. 84) maintain: “Lengthening life also means that bequests may be received later in life by the children of the elderly, and the amount of bequests may be diminished by longer consumption on the part of the elderly”.

makes individuals effectively more patient and willing to invest, and by reducing mortality risks, it raises the return on investment…, [while] in poorer societies, when life expectancy is low, individuals discount the future more heavily and are less inclined to save and invest." (p. 120).

Other papers have instead focused on the role that the mortality decline can play in the early stages of life: in their pioneering study on longevity in an endogenous growth model with positive old-age support from children to old parents, Ehrlich and Lui (1991) show that a decline in mortality reduces fertility, raises the human capital investment in children and thereby stimulates growth, while in a neoclassical growth setting Mateos-Planas (2003) finds that declines in child mortality can explain the non-monotonic path of fertility observed in the demographic transition.

All the above mentioned papers, however, treat longevity as an exogenous variable. Indeed, there exists a burgeoning theoretical literature that consider the individual lifetime being endogenously determined in models with either exogenous or endogenous fertility,[3] such as the recent papers by Lagerlöf (2003), Blackburn and Cipriani (2002) and Chakraborty (2004). Nevertheless, the first two papers do neither analyse savings nor public health spending.[4] The third one makes the length of life of individuals being dependent of their health status, which

[3] On empirical grounds, Li et al. (2007) analyses how the main macroeconomic variables react to a shock in longevity. In particular, they find that a rise in longevity positively affects savings while also showing that "differences in the demographic variables across countries or over time can well explain the differences in aggregate savings rates." (p. 138).

[4] In particular, the first paper is concerned with explaining the transition from a Malthusian Regime of development (positive relationship between per capita income and population growth) to a Modern Growth one (negative relationship between per capita income and population growth) in presence of mortality shocks (i.e., epidemics). The second one instead focuses on the study of the effects of an increasing longevity on the timing of childbearing and educational investment. The third paper is the most similar in spirit to the model presented here. Both the second and third ones are framed in the well-know overlapping generations growth model.

is, in turn, augmented by public investments in health, and focuses mainly on the study of the transitional dynamics and long-run demo-economic outcomes in a general equilibrium closed economy. He finds that development traps due to scarce investments in health can exist when the level of technological development is fairly low, and shows, in particular, that raising the health tax rate may lead individuals to live longer (i.e., it increases the quantity rather than the quality of life) and this, in turn, provides an incentive to increase savings, physical capital accumulation and higher life expectancy as well.

Moreover, there are recent important papers that deal with the relationship amongst endogenous survival, savings and capital accumulation in a OLG closed economy, from either a positive (Zhang et al., 2006; Bhattacharya and Qiao, 2007; Pestieau et al., 2008; Leung and Wang, 2010) or normative (de la Croix and Ponthière, 2010) point of view. As regards the former point of view, the above cited authors crucially focus on private health decisions. In particular, Zhang et al. (2006) and Pestieau et al. (2008) build on an OLG growth model with private health investments and public pay-as-you-go (PAYG) pensions to study how the financing of pensions affects life expectancy, economic growth and welfare by distinguishing between spending on health at different dates (i.e., when young and old), Zhang et al., or only when young, Pestieau et al. They find, respectively, that subsidising private health and increasing public pensions can improve welfare, and private health expenditure should be subsidised or not as a second-best policy depending on the relative size of the pension replacement rate, i.e. the ratio between pension benefits and wages. By using a model similar to that of Zhang et al. and Pestieau et al., but without assuming the existence of public pensions, Leung and Wang (2010) show that although an increase in health spending directly reduces resources from the production of goods, it raises life span, savings and capital accumulation. Moreover, they also find that savings and health care are complements in equilibrium. Bhattacharya and Qiao (2007) assume that individual lifetime is dependent of the health status which is, in turn, augmented by private health investments accompanied by a complementary tax-financed public health program, and show that the economy may be exposed to aggregate endogenous fluctuations and even chaotic motions when the private and public inputs in the longevity function are fairly complementary. As regards the latter point of view, de la Croix and Ponthière (2010) show that when raising health purely increases longevity, the steady-state Golden Rule of capital accumulation in an economy with endogenous lifetime is lower than that of the standard Diamond's (1965) one.

Unlike previous studies, in this paper we use the Diamond's OLG model in the particular case of the small open economy, i.e. the interest rate in the domestic country is constant and fixed at the level that prevails in the world capital market, to investigate whether the belief concerning the positive effect of longevity on savings holds in such a case. This issue is relevant and deserves attention because: (*i*) the final result of a change in longevity in a closed economy is crucially driven by a general equilibrium feedback effect on savings and capital accumulation, which is absent in a small open economy environment, as the interest rate, the capital-labour ratio and the wage are fixed and constant. A change in longevity, through a change in wage income taxation to finance additional investments in health, causes a direct income effect by reducing the disposable income of the young workers, as well as an indirect effect because the marginal propensity to save changes, so that the transmission mechanism on macroeconomic variables is different than in a closed economy; (*ii*) several actual (developed and developing) economies are small and open to international trade. Then, an analysis of longevity and public health spending may be highly valuable in such a context; (*iii*) there exists a burgeoning literature based on the overlapping generations growth model à la Diamond with endogenous fertility that deals with several questions about public economics, especially pensions, children-related pensions and child policies (see, e.g., Kolmar, 1997; van Groezen et al., 2003; Fenge and Meier, 2005, 2009; Fenge and von Weizsäcker, 2010).

Therefore, in order to fill a gap in the existing literature, in this paper we build on a typical OLG small open economy model that also incorporates endogenous lifetime through public expenditure on health.[5]

As in Chakraborty (2004), the novelty of our paper is the treatment of longevity as an endogenous variable determined by the individual health status, which is, in turn, augmented by the public provision of specific investments in health financed by labour income taxation at balanced budget. Our findings are clear cut: although a rise in the health tax rate increases both health investments and the individual length of life, and this, in turn, provides an incentive to save, whether savings definitely increases or decreases depends on the relative importance between two opposite forces, namely the positive effect due to the increased marginal propensity to save and the negative income effect due the reduced disposable income. If the former (latter) dominates, then a rise in longevity following a rise in public health spending increases (reduces) savings and domestic income. Indeed, in a small open economy the negative correlation between savings and longevity is the likelier event to occur because, unlike the case with a closed economy, the interest rate is fixed at the level that prevails in the world capital market, i.e. it does not react to a change in life expectancy, and, thus, the incentive effect to invest more due to the rise in it vanishes in such a case. In fact, in a closed economy the equilibrium interest rate negatively depends on capital accumulation:

[5] It should be noted that a burgeoning literature exists that deals with problems of population ageing and public policies in dynastic small open economies overlapping generations models with continuous time à la Yaari (1965) and Blanchard (1985): see, amongst others, Bettendorf and Heijdra (2006), Heijdra and Romp (2008; 2009). In particular, the first paper studies demographic and pension shocks in a model with non-traded goods. The second and third papers, instead, focus on the study of how demography (i.e. the mortality) is important in determining macroeconomic variables as well as retirement behaviours of individuals and pension policies. Moreover, recently Hong and Ríos-Rull (2007) analysed an OLG small open economy to study the welfare effects of social security, life insurance and annuities for families.

since a rise in the health tax rate increases the probability of surviving from youth to old-age and this, in turn, increases the equilibrium stock of capital, then the rate of interest reduces when the tax rate raises. Therefore, different effects of the health policy exist depending on whether the economy is closed or small and open to international trade.

Since when a domestic market for annuities is in existence, individuals do not take the beneficial effects of public health spending on longevity into account, the welfare analysis shows that: (*i*) the public provision of health services represents an $\mathcal{A}$–Pareto improvement when the world interest rate is fairly high, because the value in terms of utility of both lengthening the lifetime and smoothing consumption over the retirement period is high in such a case, (*ii*) a whole range of health tax rates can effectively be used to increase social welfare, and (*iii*) an $\mathcal{A}$–Pareto efficient allocation exists.

Some clarifications on the concept of $\mathcal{A}$–efficiency are now in order. In their pioneering work, Golosov et al. (2007) have generalised the notion of Pareto efficiency to compare alternatives in environments with endogenous population, i.e. to compare either the welfare of the same groups of people with respect to different allocations or the welfare of different groups of people, i.e. different generations at different dates, with respect to one allocation, because the standard concept of Pareto efficiency requires constant population to be properly used.[6] In that paper, a distinction is made between $\mathcal{A}$–efficiency (i.e., Pareto efficiency applied to people *alive* in each state) and $\mathcal{P}$–efficiency (i.e., Pareto efficiency applied to agents who are alive in each state and to *potential* agents too, namely those who are not yet born). In the former case, therefore, efficiency is defined by comparing utilities only amongst born agents at different states, and thus it is unnecessary to specify utility functions for the unborn. In the latter, unborn descendants are treated symmetrically with those who are already born, and a

[6] Indeed, the Pareto criterion, usually applied to compare feasible allocations when population is exogenous, is too weak to compare alternatives when population is endogenous (see Dasgupta, 1969).

utility function for them exists in turn.[7] In particular, these two concepts of Pareto efficiency lead to different specifications about preferences (of consumption plans which may also include population variables) of born and unborn agents over allocations in which they are not already born. If A –efficiency is taken into account, then the utility of an additional child is valued on the basis of the relative degree of (backward or forward) altruism towards children by parents (or, alternatively, the utility of lengthening the lifetime), but preference profiles of the unborn in each state are not modelled. In contrast, if P –efficiency is considered preferences are well defined and also capture the utility of the potential unborn in every state.

Since in the present paper population is endogenous because different levels of health spending affect the lifetime of people and, hence, the population size, the standard notion of Pareto efficiency cannot be applied as a guide for optimal policy making as well as to draw valid normative conclusions. Though the treatment of the unborn as economic agents is a relevant issue, we follow Conde-Ruiz et al. (2010) and exclusively evaluate the preference profiles of agents who are alive in every period and, hence, we use the concept of A –efficiency to compare alternatives throughout, because we are considering the case in which "utility functions are not defined for unborn agents over allocations in which they are not born", but this does not mean that a "child [or, alternatively the rate of longevity] is not considered… [indeed he/she can] enter only through the utility of other, alive, agents through familial external effects (e.g., parental altruism, and so forth)." (Golosov et al., 2007, p. 1046).

It should be stressed that conclusions of the present paper are based on a simple and stylised double Cobb-Douglas economy, the main objective of which being to fill a gap in the existing theoretical economic growth literature with endogenous lifetime that has remained,

[7] Other two important papers that deal with the modified notion of Pareto efficiency with endogenous population in models with overlapping generations are Michel and Wigniolle (2007), that compares utilities generation by generation, and Conde-Ruiz et al. (2010), that adopts the notion of Millian efficiency and evaluate welfare of symmetric allocations.

at the best of our knowledge, so far unexplored, especially in the small open economy environment. However, we also think the simplest Cobb-Douglas case could aid researchers to better understand the key economic forces at work, while also representing a useful abstraction and a good starting point for future theoretical analyses, especially in the treatment of the positive externalities induced by health investments on both the quantity and quality of life.

The present paper can be extended in several directions: for instance, utility and production functions can be generalised; fertility can be added to the analysis and considered as an endogenous determined variable; other balanced budget policies can be assumed, e.g. pension and family policies, following an interesting recent literature by van Groezen et al. (2003), van Groezen and Fenge and Meier (2005; 2009), where the pension benefits received when old are contingent on the number of children raised when young.

The rest of the paper is organised as follows. Section 2 introduces the model. Section 3 analyses and discusses the effects of longevity on savings and the per worker steady-state domestic income. Section 4 extends the analysis to account for the welfare effects of public investments in health. Section 5 presents numerical exercises for some actual small open economies to test for the robustness of the theoretical findings and discusses some policy implications; in particular, we select three groups of small open economies (namely, European developed economies, European economies in transition and African underdeveloped economies) with different characteristics amongst them as regards per capita GDP, adult mortality and the ratio of health spending on per capita GDP. Taking data from World Bank (2009) and World Health Statistics (2010), we provide examples of whether governments should either increase or decrease the public health spending to achieve the social optimum. Section 6 concludes.

2. The model

Consider an OLG small open economy with perfect capital mobility that faces an exogenously given constant interest rate, r. Production takes place according to a standard neoclassical constant-returns-to-scale technology $f(k)$, where k is the (per worker) stock of capital. Since capital is perfectly mobile, both capital-labour ratio k and wage rate w are fixed and constant.

The economy is populated by a continuum of perfectly rational and identical two-period lived individuals of measure one per generation: indeed, in every period two generations are alive (see Diamond, 1965). When young, an individual is endowed with one unit of labour inelastically supplied to firms and earns wage w. When old, she is retired. Moreover, the probability of surviving from youth to old age is endogenous and determined by the individual state of health, which is, in turn, augmented by the public provision of health investments such as, for instance, hospitals, vaccination programmes, scientific research so on (see Chakraborty, 2004). Therefore, the survival probability at the end of youth of an agent born at time t, π_t, depends upon her health capital, h_t, and it is characterised by a non-decreasing concave function $\pi_t = \pi(h_t)$. We specialise this relationship with the following function[8] of health investments (see Blackburn and Cipriani, 2002; de la Croix and Ponthière, 2010; Fioroni, 2010a; Fanti and Gori, 2011):

[8] This functional form shows that longevity monotonically increases with decreasing returns when health investments start increasing, that is they are very effective when longevity is relatively low, while becoming scarcely effective when it is close to its saturating value (see, e.g., World Health Statistics, 2010, Figure 13, p. 128, which relates per capita total expenditure on health to life expectancy at birth, and Fioroni, 2010b, Figures 1 and 2, p. 357). Though health investments can have a more intense effect in reducing adult mortality often when a certain threshold of health capital is achieved (i.e., the functional form between health investments and longevity may be S-shaped), in this paper the analysis is limited to a concave saturating function. Our results are confirmed

$$\pi_t = \pi(h_t) = \frac{\pi_0 + \pi_1 h_t}{1 + h_t}, \tag{1}$$

which satisfies the following properties: $0 < \pi_1 \leq 1$, $0 \leq \pi_0 < \pi_1$, $\pi(0) = \pi_0 > 0$, $\pi'_h(h) = \frac{\pi_1 - \pi_0}{(1 + h_t)^2} > 0$, $\pi''_{hh}(h) = \frac{-2(\pi_1 - \pi_0)}{(1 + h_t)^3} < 0$ and $\lim_{h \to \infty} \pi(h) = \pi_1 \leq 1$.

We define the demographic parameter π_0 as the "natural" or "basic" individual rate of longevity (see, e.g., Ehrlich, 2000; Leung and Wang, 2010), that can be affected by both economic and non-economic elements. For instance, π_0 may be thought to be higher: the better the lifestyle (e.g., healthy life), the higher the relative importance ascribed to the protection of workers' rights due to, e.g., cultural, economic and historical reasons, the higher the (public and/or private) investment in social infrastructures and education. However, it can also be affected by climate conditions, the existence of ethnical and civil wars, endemic diseases and so on. Moreover, some underdeveloped and developing countries stuck in destitution because of weak institutions put in place by colonial overlords, or because of climates that foster disease, or geographies that limit access to global markets, or simply by the fact that poverty is overwhelmingly self-perpetuating. For these and other reasons, therefore, we may realistically expect to observe that π_0 is higher in developed than in developing or under-developed countries.[9]

a fortiori by employing a S-shaped function (as used by Blackburn and Cipriani, 2002 in their numerical example, and Fanti and Gori, 2011 in a model where public health investments affect the old-age labour productivity through an improvement in the individual state of health).

[9] For instance, Mirowsky and Ross (1998) suggest that both higher economic growth and standards of living encourage, especially through an increasing education, the adoption of more healthy lifestyles on the part of individuals for several socio-economic reasons. Indeed, Blackburn and Cipriani (2002, p. 188) argues that "personal education improves personal health primarily because it improves personal effective agency: that is,

The demographic parameter π_1 measures the effectiveness of an additional increase in health investments as a means to longer lifetime, and a rise in it can be interpreted as an exogenous medical advance due to, e.g., scientific research. In other words, π_1 tells us to what extent a rise in public health spending is transformed into a higher life span, and it is retained that it is higher in rich than in poor societies. In fact, as claimed by Chakraborty (2004, p. 126) "… [the parameter π_1] depends on medical advances as also on their provision, and the latter has been lacking in poor countries. Indeed, … [there exists] extensive evidence [on] how publicly provided health services are often ineffective in poor countries because they bypass rural areas and the urban poor who would benefit from them the most."

Of course, *ceteris paribus* as regards π_1, the intensity of the effectiveness of an additional increase in health investments on longevity is lower in economies where individuals naturally live longer (developed countries) than in economies where they naturally live shorter (developing or under-developed countries), because the adult mortality rate is already low in the latter case.

We assume that per young health expenditure at time t (h_t) is provided by the government and constrained by the following identity:

$$h_t = \tau w, \tag{2}$$

where $0 < \tau < 1$ is a constant wage income tax rate (see Chakraborty, 2004; Bhattacharya and Qiao, 2007).

At time t, therefore, the first-period budget constraint of the typical agent reads as follows:

$$c_{1,t} + s_t = w(1-\tau), \tag{3.1}$$

education allows people to develop knowledge, skills and abilities that make them better equipped to create a way of living that is conducive to their welfare and that is not mediated by economic status."

i.e. wage income – net of the contribution to finance health care services – is divided into material consumption when young, $c_{1,t}$, and savings, s_t.

When old, an individual retires and lives with the amount of resources saved when young plus the interest accrued from period t to period $t+1$ at the rate that prevails in the world capital market, r. Indeed, the existence of a *domestic* or *national* perfect annuity market also implies that old survivors will benefit not only from their own past saving plus interest, but also from the saving plus interest of those who have deceased, and savings are intermediated through mutual funds, which invest these savings and guarantee a return factor ($\widetilde{R}_t = 1+\widetilde{r}_t$), determined by the international interest factor ($R = 1+r$), earned by the fund on its investments, divided by the national longevity rate (π_t), to the surviving old insured inhabitants of the small open economy. Moreover, since perfect competition in the market for annuities exists, then $\widetilde{R}_t = R/\pi_t$. In other words, in this model we assume the existence of a national annuity market and all inhabitants of the small open economy are annuitized. At the beginning of old-age, each annuitant deposits her saving with a mutual fund. The fund invests these savings and receives the constant interest factor determined on the world capital market. Then, if an old annuitant is alive she obtains the return $\widetilde{R}_t$. However, if an annuitant dies at the onset of old-age, the contract with the mutual fund will terminate and her wealth is entirely forfeited by the fund (if no heirs exist) and then redistributed to the surviving old annuitants within the small open economy. Loosely speaking, the domestic insurance determines the payment streams to each individual by considering the internal mortality rates. Of course, the hypothesis of a national annuity market is the most realistic one. Indeed, if a world annuity market existed, then there would be no link between the longevity rate of the domestic country and the return on investments earned by the fund on the world capital market.

This being said, the budget constraint at time $t+1$ of an old person born at time t can therefore be expressed as

$$c_{2,t+1} = \frac{1+r}{\pi_t} s_t, \tag{3.2}$$

where $c_{2,t+1}$ is old-age consumption.

The individual representative of generation t derives utility from material consumption when young and when old. By taking the wage rate, the interest rate and the probability of surviving from youth to old age as given, she chooses how much to save out of her disposable income to maximise the following expected lifetime utility index (in which preferences of the unborn in every state are not defined), where the death-contingent utility index is normalised to zero: [10]

$$U_t = \ln(c_{1,t}) + \pi_t \ln(c_{2,t+1}), \tag{4}$$

subject to Eqs. (3.1) and (3.2). The constrained maximisation of Eq. (4) with respect to consumption when young and when old gives the following first order condition:

$$\frac{c_{2,t+1}}{c_{1,t}} = 1 + r. \tag{4.1}$$

Eq. (4.1) says that the marginal rate of substitution between young- and old-age consumption equals the constant world interest rate, while being independent of the probability of surviving. This means that when a domestic annuities market exists, each individual does not take into account the (social) benefits of an increase in public healthcare investments on (individual) health and longevity, that is so that the benefit of the increased savings is too small to be taken into account by each single individual in the market.

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[10] The formulation for expected utility Eq. (4) is usual in literature (see, e.g., Abel, 1985; Chakraborty, 2004; Chakraborty and Das, 2005; Pestieau et al., 2008; Chakraborty et al., 2010). However, as stressed by Abel (1985, p. 779): "It is not necessary that the utility index is equal to zero in the case of death. All that is required is that utility in the state of death does not depend on the level of wealth."

Combining the first order condition Eq. (4.1) with the budget constraints Eqs. (3.1) and (3.2) yields the saving function, that is:

$$s_t(\tau) = s(\tau) = \frac{\pi(\tau) w(1-\tau)}{1+\pi(\tau)}, \tag{5.1}$$

where

$$\pi_t(\tau) = \pi(\tau) = \frac{\pi_0 + \pi_1 \tau w}{1+\tau w}, \tag{6}$$

is jointly determined by Eqs. (1) and (2). It should be noted that in this simple small open economy model, once public health spending is set out through Eq. (2), the economy faces a constant survival rate as well as constant saving and consumption functions from period t onwards, because the interest rate is fixed at the world-wide constant rate r and then both the capital-labour ratio and wage rate are fixed and constant.

From Eq. (5.1) we note a crucial difference between the closed economy and the small open economy. Of course, in both cases the interest rate does not affect saving when preferences are logarithmic,[11] while being dependent on the wage rate. Now, in a small open economy the wage rate is fixed and constant, while in closed economy it depends on the capital stock through the marginal productivity of labour. Then, to the extent that a change in the health tax rate τ affects the steady state stock of capital in a closed economy (see Chakraborty, 2004), it affects savings in equilibrium through a change in the wage rate. This indirect feedback effect of τ on saving is indeed absent in a small open economy, where an endogenous dynamics of capital does not exist. Therefore, the interest to complement the paper by Chakraborty (2004) and study the long-run effects of public health spending in a small open economy with fixed prices and a domestic annuity market, may be highly valuable. In particular, in this paper we are essentially interested to determine both the long-run outcomes and differences in terms

[11] This because a rise in the return on savings implies both a substitution and income effects which exactly cancel each other out in the case of Cobb-Douglas preferences (see de la Croix and Michel, 2002, p., 14).

of income, longevity and health spending amongst countries when the constant tax rate τ varies (as can also be ascertained in the numerical simulations presented in Section 5 for some actual small open economies).[12]

Now, combining Eqs. (3.1), (3.2), (4.1) and (5.1), young-age consumption and old-age consumption are respectively given by:

$$c_{1,t}(\tau)=c_1(\tau)=\frac{w(1-\tau)}{1+\pi(\tau)}, \tag{5.2}$$

$$c_{2,t+1}(\tau)=c_2(\tau)=(1+r)c_1(\tau). \tag{5.3}$$

At time t, the per worker domestic income is determined as $y_t=w+r\,s_{t-1}$, that can also be rewritten at the steady state,[13] upon substitution of Eq. (5.1), as:

$$y(\tau)=w+rs(\tau). \tag{7}$$

As can easily be seen from Eq. (7), the public health expenditure affects per worker domestic income y uniquely through saving. It is therefore sufficient to analyse how a rise in the health tax rate affects saving to capture the transmission channels on domestic income, as described in the next section.[14]

3. Health policy and macroeconomic variables

[12] It is important to note that this is a standard OLG small open economy model with a dynamic inter-temporal optimisation process of the consumer. Of course, there is no endogenous dynamics of capital but there exists a dynamics of the balance of trade determined according to the difference between the capital stock (which does not depend on the choices of consumers and firms in the domestic country) and saving.

[13] Indeed, saving and income in the initial period do not respond to a once-and-for-all change in the health tax rate τ in such a period.

[14] Note that in a small open economy there also exists an amount of per capita foreign debt determined as $d_{t+1}=k_{t+1}-s_t$. Since the focus of the present paper is on saving and welfare effects of a change in public health spending through a change in the health tax rate, we do not pursue the analysis of debt further on.

We now analyse how a change in the health tax rate affects saving and domestic income.

Let the saving function be generically rewritten as $s = s\{\tau, \pi[h(\tau)]\}$. Then, the total derivative of it with respect to τ gives:

$$\frac{ds}{d\tau} = \overbrace{\frac{\partial s}{\partial \tau}}^{-} + \overbrace{\frac{\partial s}{\partial \pi}}^{+} \cdot \overbrace{\frac{\partial \pi}{\partial h}}^{+} \cdot \overbrace{\frac{\partial h}{\partial \tau}}^{+} \gtrless 0. \tag{8}$$

Eq. (8) reveals that a rise in the health tax rate ambiguously affects savings through two channels: a negative direct effect due to the reduced disposable income when young, and a positive indirect effect due to the increased health investments following the rise in longevity, i.e. individuals live longer and thus discount the future at a lower rate to save more and smooth consumption over the retirement period.

The following proposition shows that depending on the relative size of π_0, saving and domestic income can either be increased or decreased when the health tax is raised.

Proposition 1. *(Effects of public health spending on both saving and steady-state per worker domestic income). (1) The financing of public investments in health (evaluated at $\tau = 0$) increases $s(\tau)$ and $y(\tau)$ if and only if $0 < \pi_0 < \hat{\pi}_0$, where*

$$\hat{\pi}_0 := \frac{1}{2}\left[-(1+w) + \sqrt{(1+w)^2 + 4\pi_1 w}\right], \quad 0 < \hat{\pi}_0 < 1. \tag{9}$$

(2.1) Let $0 < \pi_0 < \hat{\pi}_0$ hold. Then $s(\tau)$ and $y(\tau)$ are inverted U-shaped with $\tau = \hat{\tau}(\pi_0)$ being the saving- and income-maximising health tax rate, where

$$\hat{\tau}(\pi_0) := \frac{-(1+\pi_0)\pi_1 + \sqrt{(1+\pi_0)^2 \pi_1^{\,2} + \pi_1(1+\pi_1)\Pi_0}}{\pi_1(1+\pi_1)w}, \quad 0 < \hat{\tau}(\pi_0) < 1, \tag{10}$$

and $\Pi_0 := -\pi_0^{\,2} - (1+w)\pi_0 + \pi_1 w$.

(2.2) Let $\hat{\pi}_0 < \pi_0 < \pi_1$ hold. Then $s(\tau)$ and $y(\tau)$ monotonically decrease with τ for every $0 < \tau < 1$.

Proof. Differentiating Eq. (5.1) with respect to τ and evaluating it at $\tau = 0$ gives:

$$\frac{\partial s(\tau)}{\partial \tau}\Big|_{\tau=0} = \frac{w\Pi_0}{(1+\pi_0)^2}. \tag{11.1}$$

Applying the Descartes' rule of sign we find that two real roots exist, namely $\underline{\pi}_0 < 0$ and $0 < \hat{\pi}_0 < \pi_1$ (see Eq. 9), such that $\Pi_0 = 0$. Since $\underline{\pi}_0 < 0$ is not economically relevant, it can be ruled out. Then, $\Pi_0 > 0$ (< 0) and from Eq. (11.1) $\frac{\partial s(\tau)}{\partial \tau}\Big|_{\tau=0} > 0$ (< 0) for every $0 < \pi_0 < \hat{\pi}_0$ ($\hat{\pi}_0 < \pi_0 < \pi_1$). This proves Point 1 of Proposition 1.

Now, differentiating Eq. (5.1) with respect to τ gives:

$$\frac{\partial s(\tau)}{\partial \tau} = \frac{w_t\left\{-\pi_1(1+\pi_1)w^2\tau^2 - 2(1+\pi_0)\pi_1 w\tau + \Pi_0\right\}}{\left[1+\pi_0+(1+\pi_1)\tau w\right]^2}. \tag{11.2}$$

Applying the Descartes' rule of signs we find that two real roots of Eq. (11.2) for τ exist, namely $\underline{\tau}(\pi_0) < 0$ and $\hat{\tau}(\pi_0) > 0$ (see Eq. 10), the latter one being the unique economically relevant solution. Therefore, if $0 < \pi_0 < \hat{\pi}_0$, then $\frac{\partial s(\tau)}{\partial \tau} \gtrless 0$ for every $\tau \lesseqgtr \hat{\tau}(\pi_0)$, where $0 < \hat{\tau}(\pi_0) < \hat{\tau}(0)$ is an interior global maximum ($\frac{\partial \hat{\tau}(\pi_0)}{\partial \pi_0} < 0$ for every $0 < \pi_0 < \hat{\pi}_0$ and $0 < \hat{\tau}(0) < 1$). In contrast, when $\hat{\pi}_0 < \pi_0 < \pi_1$ two negative real roots of Eq. (11.2) exist and then $\frac{\partial s(\tau)}{\partial \tau} < 0$ for every $0 < \tau < 1$. In this case, therefore, raising the health tax rate monotonically reduces savings. Moreover, from Eqs. (7) it is easy to verify that $\frac{\partial y(\tau)}{\partial \tau} = r \cdot \frac{\partial s(\tau)}{\partial \tau}$, i.e., a change in τ affects $y(\tau)$ in the same way as $s(\tau)$. This proves Points (2.1) and (2.2) of Proposition 1.

Q.E.D.

Although a rise in public health investments increases the individual length of life, Proposition 1 shows that it may reduce savings and domestic income per young person. The analysis of how a change in the health tax rate affects savings has already shown the existence of two counterbalancing forces at work: a negative direct effect through a reduced disposable income, and a positive indirect effect due to the increased propensity to save. Indeed, in this simple life cycle model, a higher survival probability increases the fraction of income for savings and reduces the fraction of income for consumption.

Proposition 1 distinguishes between two cases: $0 < \pi_0 < \hat{\pi}_0$ (the natural rate of longevity is fairly low: for instance, the case of underdeveloped or developing countries) and $\hat{\pi}_0 < \pi_0 < \pi_1$ (the natural rate of longevity is fairly high: for instance, the case of developed countries). The final effect on savings of a change in the tax rate τ is different depending on the relative position of π_0 with respect to the threshold $\hat{\pi}_0$.

Case $0 < \pi_0 < \hat{\pi}_0$. When the natural rate of longevity is fairly low, individuals naturally live shorter and thus discount the future heavily because they want to consume more today than tomorrow. Since, at least for small values of τ, the efficiency of an increase in health investments in reducing adult mortality is higher the lower is the natural rate of longevity (see Eq. 1), then when π_0 is fairly low the weight of the positive indirect effect on savings of a rise in the tax rate is higher than the negative one because longevity sharply increases in such a case. Of course, when τ increases (and, in particular, it goes beyond the threshold $\hat{\tau}(\pi_0)$), the weight of the positive longevity effect in Eq. (8) becomes smaller than the negative income effect, because life expectancy is already high and cannot be efficiently increased by raising health investments further. As a consequence, savings can actually be reduced when the health tax is raised (at least for relatively high values of τ).

Case $\hat{\pi}_0 < \pi_0 < \pi_1$. When the natural rate of longevity is fairly high, the efficiency of a rise in health investment as an inducement to a higher life span is low, and this effect is stronger the more π_0 is close its saturating value, π_1 (see Eq. 1). In such a case, individuals naturally live longer and thus do not discount the future too much because they want to smooth consumption over the retirement period. Since longevity is already high, raising public health investments does not efficiently increase longevity, while reducing the disposable income of the young workers. As a consequence, the positive longevity effect on saving is offset by the negative one, i.e. a reduced disposable income, and thus saving monotonically reduces as the health tax rate increases in such a case.

4. Health policy and welfare

While we have shown that both saving and steady-state domestic income can actually be reduced by the health policy, another crucial aspect that deserves attention is represented by the welfare effects of it. This section deals with this subject and contrasts welfare levels in an economy where the government finances health investments at balanced budget with that would be obtained in the absence of public intervention.

In our model, public health spending is exclusively financed by labour income taxation. Thus, only individuals of the younger (working-age) generation, whose survival probability at the end of youth depends on their health status, benefit from the advantages of an increase in health care services, while also bearing the cost of funding it. The health tax, therefore, does not hurt the old people living at the moment of the introduction of the health policy, as the interest rate is fixed at the level that prevails in the world capital market.

Therefore, assume that at time t the government introduces a balanced-budget health policy financed with labour income taxes at the rate $0 < \tau < 1$. Since in every period the interest rate is constant at the world-wide level r, the welfare of the current elderly (V_{t-1}) is kept unaltered (i.e., it does not depend on τ). Moreover, individuals are identical and both young-age consumption and old-age consumption are constant from time t onwards (see Eqs. 5.2 and 5.3). Therefore, through Eq. (4) the lifetime indirect utility index of the individual of generation t ($V_t(\tau)$) as well as that of individuals belonging to all the infinite future generations ($V_{t+1}(\tau)=V(\tau)$ for every t) are identical and can uniquely be described, after straightforward algebra, by the following (expected) lifetime indirect utility function, defined only for born agents in every state:[15]

$$V(\tau)=\ln\left(c_1(\tau)^{1+\pi(\tau)}(1+r)^{\pi(\tau)}\right), \tag{12}$$

where $V_{t+1}(\tau)=V_t(\tau)=V(\tau)$ and $c_1(\tau)$, $c_2(\tau)$ and $\pi(\tau)$ are determined by Eqs. (5.2), (5.3) and (6), respectively. Therefore, the expected social welfare function $W_t=\sum_{i=t}^{+\infty}\delta^{i-t}\cdot U(c_{1,i-1},c_{2,i})$, where $0<\delta<1$ is the social discount factor, is maximised if the level of the health tax rate maximises the lifetime utility index (12) at every date (see van Groezen et al., 2003; Fanti and Gori, 2012).

Before performing the welfare analysis, it is important to note that although Eq. (12) represents an index of the expected lifetime welfare of every generation at every date, discussions about P–efficiency or A–efficiency are not irrelevant. Note that in this model fertility is exogenous and longevity is endogenous. Therefore, population is not constant because the individual length of life varies as the rate of longevity changes. Since a change in τ modifies longevity, and since we adopt an ex-post welfare criterion to compare alternatives, then the use either of the P–efficiency or A–efficiency criterion matters for the

[15] Eq. (12) has been obtained by inserting Eq. (5.2) and (5.3) into Eq. (4). Since we assumed a national annuity market, the interest rate factor is $1+r$, determined in the world capital market, and the savings of the dead of the domestic country are redistributed by the national insurer in such a country.

results. Of course, when the social objective is an index of individual expected welfare for every generation, the use of an ex-ante or ex-post welfare viewpoint would bring to significant differences as regards what is optimal ex-ante and what is optimal ex-post the introduction of public policies. However, one can adopt the P–efficiency criterion only whether utility functions for the unborn in every state are specified. Therefore, since in the present study we do not specify any utility functions for the potential unborn over allocations in which they are not born, one can adopt: (*i*) A–efficiency if, and only if, a social objective function is specified and the welfare effects of public health spending are evaluated from an ex-post point of view (because it is only from an ex-post point of view that population depends on the level of health expenditure, i.e. population is constant ex-ante), or (*ii*) neither P–efficiency nor A–efficiency if the social objective is average lifetime welfare or, alternatively, the expected lifetime welfare.

Indeed, Eq. (12) describes how utility of every generation (i.e., the current old, the current generation and all the infinite future generations) varies when the health tax rate τ continuously changes. In particular, an increase in τ keeps the welfare of the current old unchanged, while changing in the same way the utility indexes of the current as well as of the infinite future generations, which are exactly the same and not defined for unborn agents over allocations in which they are not born. Therefore, the notion of A–efficiency can actually be used to compare alternatives in such a case, because the maximisation of Eq. (12) with respect to τ implies the maximisation of the expected social welfare function W_t. In our context, the notion of P–efficiency cannot be used because utilities of the unborn in every state are not defined. Of course, the use of P–efficiency and A–efficiency criteria to compare alternatives bring to different results because utility functions for either born and unborn agents (P–efficiency) or only for born agents (A–efficiency) in every state should be specified, and then different levels of τ should affect in a different way the surviving population depending on which criterion is applied, because different preference profiles are specified.

Therefore, if for some $0<\tau<1$ the utility index Eq. (12) goes beyond the value of it without health investments, i.e. $V(\tau)>V(0)$, then the introduction of public health spending represents an A –Pareto improvement because the social welfare index W_t increases and the current as well as all subsequent generations would be better off without making any of the born agents in every state worse off. This can be observed because with endogenous longevity the planner chooses consumption and longevity. Therefore, at the optimum a (necessary) condition with respect to which the marginal welfare gain from a higher longevity should be equal to the marginal cost of it should hold, in order for an interior optimal rate of longevity to properly exist, other than the usual optimality condition on the inter-temporal allocation of consumption (see de la Croix et al., forthcoming, for an analysis on the existence of an interior optimal rate of longevity in a closed economy). Therefore, when a perfect market for annuities do exist, individuals do not take the benefit of an increase in their life span into account but the planner chooses optimally the longevity rate and then takes both advantages and disadvantages of longevity into account.

Since in the present study population is endogenous because changes in health spending affect (from an ex-post point of view) the length of life of people and, hence, the population size, the standard notion of Pareto efficiency cannot be applied to draw policy conclusions. Therefore, we follow Conde-Ruiz et al. (2010) and evaluate only utilities of agents who are alive in every period and then we use the notion of A –efficiency (i.e., Pareto efficiency applied to people alive in each state), introduced by Golosov et al. (2007) and discussed in Section 1 in the present paper, in order to compare welfare levels of the representative individual of every generation at every dates with respect to one allocation when τ varies.

We are now going to study whether A –Pareto improvements, i.e. $V(\tau) > V(0)$, can be obtained when the government changes the health tax rate τ. In order to disentangle the welfare effects of a change in the health tax rate, we differentiate Eq. (12) with respect to τ to obtain:[16]

$$\frac{\partial V(\tau)}{\partial \tau} = \frac{1+\pi(\tau)}{c_1(\tau)} \cdot \overbrace{\frac{\partial c_1(\tau)}{\partial \tau}}^{-} + \ln((1+r)c_1(\tau)) \cdot \overbrace{\frac{\partial \pi(\tau)}{\partial \tau}}^{+}, \tag{13}$$

Eq. (13) reveals that a rise in the health tax rate ambiguously affects social welfare because two opposite forces are involved: a negative consumption effect and a positive longevity effect. The former is due to a reduction in both the disposable income and marginal propensity to consume. The latter, instead, implies a positive effect on social welfare because

[16] Note that both the longevity and consumption functions can be rewritten as $\pi = \pi[h(\tau)]$ and $c_1 = c_1\{\tau, \pi[h(\tau)]\}$, respectively. Totally differentiating π and c_1 with respect to τ gives:

$$\frac{d\pi}{d\tau} = \overbrace{\frac{\partial \pi}{\partial h}}^{+} \cdot \overbrace{\frac{\partial h}{\partial \tau}}^{+} > 0,$$

i.e., a rise in τ increases health investments and this, in turn, prolongs the individual lifetime, and

$$\frac{dc_1}{d\tau} = \overbrace{\frac{\partial c_1}{\partial \tau}}^{-} + \overbrace{\frac{\partial c_1}{\partial \pi}}^{-} \cdot \overbrace{\frac{\partial \pi}{\partial h}}^{+} \cdot \overbrace{\frac{\partial h}{\partial \tau}}^{+} < 0,$$

i.e., a rise in τ negatively acts on consumption through a twofold channel: (*i*) the reduced disposable income, and (*ii*) the reduced marginal propensity to consume. Indeed, the rise in life expectancy makes saving more attractive than young-age consumption because individuals prefer to postpone consumption in the retirement period due to the lengthening in lifetime. Defining the social welfare function as $V = V\{\pi[h(\tau)], c_1\{\tau, \pi[h(\tau)]\}\}$ and taking the total derivative of it with respect to τ gives:

$$\frac{dV}{d\tau} = \overbrace{\frac{\partial V}{\partial \pi}}^{+} \cdot \overbrace{\frac{\partial \pi}{\partial h}}^{+} \cdot \overbrace{\frac{\partial h}{\partial \tau}}^{+} + \overbrace{\frac{\partial V}{\partial \pi}}^{+} \cdot \left(\overbrace{\frac{\partial c_1}{\partial \tau}}^{-} + \overbrace{\frac{\partial c_1}{\partial \pi}}^{-} \cdot \overbrace{\frac{\partial \pi}{\partial h}}^{+} \cdot \overbrace{\frac{\partial h}{\partial \tau}}^{+} \right) \gtrless 0,$$

the sign of which is clearly ambiguous.

individuals live longer and, hence, discount the future at a lower rate, i.e. they prefer to smooth consumption over the retirement period; this effect can also be interpreted as a direct increase in utility due to the fact that people expect to live longer.[17]

The welfare effect of a rise in τ, therefore, depends on whether the advantages of the increased life expectancy are higher or lower than the disadvantages of the reduced consumption, and this, in turn, depends on the mutual relationship between the demographic parameters π_0 and π_1 as well as the wage and interest rates, w and r, respectively. In particular, from Eq. (13) it is evident that a rise in the world interest rate causes, *ceteris paribus*, an increase in the relative weight of the positive longevity effect (the second addendum on the right-hand side of Eq. 13), because consumption in the second period of life becomes more attractive in such a case, while leaving the negative consumption effect unchanged (the first addendum on the right-hand side of Eq. 13). Therefore, the higher r, the likely a rise in public health investments increases social welfare.

To ultimately analyse which of these two opposite forces dominates, we use Eqs. (5.2) and (5.3) to rewrite Eq. (13) as:

$$V(\tau) = \ln\left(\left[\frac{w(1-\tau)}{1+\pi(\tau)}\right]^{1+\pi(\tau)} \cdot (1+r)^{\pi(\tau)}\right), \tag{14}$$

where $\pi(\tau)$ is determined by Eq. (6).

Let

$$\hat{r} := \frac{1+\pi_0}{w} \cdot e^{1+\frac{1+\pi_0}{w(\pi_1-\pi_0)}} - 1 \tag{15}$$

be a threshold value of the world interest rate. Then, the following proposition holds:

[17] In fact, if we assume that every period consists of 30 years and the retirement period starts at 60 years, then the expected length of life of the young (that can be evaluated when they start making economic decisions) is equal to $60 + 30\pi$.

Proposition 2. *(Welfare effects of the health tax rate). Let* $r > \hat{r}$ *[*$r < \hat{r}$*] hold. Then, the financing of public investments in health (evaluated at* $\tau = 0$*) represents an* A*–Pareto improvement [*A*–Pareto worsening].*

Proof. Differentiating Eq. (14) and evaluating it at $\tau = 0$ gives:

$$\frac{\partial V(\tau)}{\partial \tau}\Big|_{\tau=0} = -(1+\pi_0) - w(\pi_1 - \pi_0)\left[1 - \ln\left(\frac{w(1+r)}{1+\pi_0}\right)\right]. \tag{16}$$

Substituting Eq. (15) into Eq. (16) to eliminate r proves Proposition 2 since $\frac{\partial V(\tau)}{\partial \tau}\Big|_{\tau=0} > 0$ [≤ 0] if and only if $r > \hat{r}$ [$r \leq \hat{r}$]. **Q.E.D.**

Proposition 2 shows that when the market interest rate is fairly high (i.e., the investment in annuities becomes more attractive) the positive longevity effect dominates and, hence, the introduction of a public health policy represents an A–Pareto improvement, because it makes the current as well as all subsequent generations better off without hurting the current old. This result more likely occurs the higher are both the wage w and efficiency of the health technology π_1. In such a case, in fact, the advantages in terms of utility of the increased lifetime more than counterbalances the disadvantages of the reduced consumption. In fact, the lower longevity, the higher the level of the lifetime consumption profile. Since a rise in π increases the population size, it contributes to reduce consumption per capita at every date. Indeed, when mortality rates are low, the cost in terms of utility to raise the wage tax τ helps more than it costs and then increases social welfare.

Moreover, the following proposition shows that when $r > \hat{r}$ a whole range of health tax rates can be used to effectively improve social welfare.

Proposition 3. *(Welfare effects of the health tax rate). Let* $r > \hat{r}$ *hold. Then,* $0 < \tau < \tau^{\circ}$ *is* A*–Pareto-improving,* $\tau = \tau^{*}$ *is* A*–Pareto efficient and* $\tau^{\circ} < \tau < 1$ *is* A*–Pareto worsening.*

Proof. From Proposition 2 we know that the introduction of a public health programme (evaluated at $\tau = 0$) represents an A–Pareto improvement for every $r > \hat{r}$. Since utilities from generation t onwards are identical, the maximisation of $V(\tau)$ with respect to τ gives the maximum social welfare. Therefore, differentiating Eq. (14) with respect to τ yields:

$$\frac{\partial V(\tau)}{\partial \tau} = \frac{\mathrm{B}_1(\tau) - \mathrm{B}_2(\tau)}{(1-\tau)(1+\tau w)^2}, \tag{17}$$

where

$$\mathrm{B}_1(\tau) := w(1-\tau)(\pi_1 - \pi_0)\ln\left(\frac{w(1-\tau)(1+\tau w)(1+r)}{1+\pi_0+(1+\pi_1)\tau w}\right) > 0, \tag{18}$$

$$\mathrm{B}_2(\tau) := (1+\pi_1)w^2\tau^2 + 2(1+\pi_0)w\tau + 1 + \pi_0 + w(\pi_1 - \pi_0) > 0. \tag{19}$$

Moreover,

$$\frac{\partial \mathrm{B}_1(\tau)}{\partial \tau} = -\frac{\mathrm{B}_1(\tau)}{1-\tau} - \frac{w(1-\tau)(\pi_1-\pi_0)}{\ln\left(\frac{w(1-\tau)(1+\tau w)(1+r)}{1+\pi_0+(1+\pi_1)\tau w}\right)} \cdot \frac{w(1+r)\mathrm{B}_2(\tau)}{\left[1+\pi_0+(1+\pi_1)\tau w\right]^2} < 0, \tag{20}$$

and

$$\frac{\partial \mathrm{B}_2(\tau)}{\partial \tau} = 2w\left[1+\pi_0+(1+\pi_1)w\tau\right] > 0. \tag{21}$$

Since $\mathrm{B}_1(\tau)$ ($\mathrm{B}_2(\tau)$) is a negative (positive) monotonic function of τ for every $0 < \tau < 1$ and $\mathrm{B}_1(0) > \mathrm{B}_2(0) > 0$ for every $r > \hat{r}$, then: (*i*) $\mathrm{B}_1(\tau) = \mathrm{B}_2(\tau)$ only *once* at $\tau = \tau^{*}$, and (*ii*) $\frac{\partial V(\tau)}{\partial \tau} \underset{>}{\overset{<}{-}} 0 \Leftrightarrow \tau \underset{>}{\overset{<}{-}} \tau^{*}$ with $\tau = \tau^{*}$ being the *unique* interior global maximum of $V(\tau)$ for every $0 < \tau < 1$. Moreover, since $V(\tau)$ is a positive (negative) monotonic function for every $0 < \tau < \tau^{*}$ ($\tau^{*} < \tau < 1$) and $\lim_{\tau \to 1} V(\tau) = -\infty$, then there exists one and only one threshold value of the health tax rate $\tau^\circ \in (\tau^*, 1)$ such that $V(\tau^\circ) = V(0)$. Therefore, Proposition 3 follows because $V(\tau) > V(0)$ ($V(\tau) < V(0)$) for every $0 < \tau < \tau^\circ$ ($\tau^\circ < \tau < 1$). **Q.E.D.**

Proposition 3 shows that when $r > \hat{r}$, the financing of public investments in health can represent an A –Pareto improvement depending on the relative size of τ. Indeed, there exist: (*i*) a whole range of health tax rates that allows social welfare to go beyond the value without public health investments ($0 < \tau < \tau^\circ$), and (*ii*) an A –Pareto efficient allocation ($\tau = \tau^*$).[18]

When the tax rate is fairly low, in fact, the relative weight of the reduction in both the disposable income and marginal propensity to consume due to a rise in τ is low, but the gain in longevity is high because raising health investments is very effective as a means to a higher lifetime, i.e., the positive longevity effect on welfare weights more that the negative consumption one. By contrast, when the health tax rate is fairly high, a reduction in τ slightly reduces longevity (because it is already high) and, hence, the positive welfare effects due to both the increased disposable income and propensity to consume more than offset the negative effect of the reduced lifetime in such a case. For these reasons, social welfare increases.

Figure 1 depicts in a stylised way the welfare gain that all generations can obtain in a small open economy with a domestic market for annuities when a public health policy is implemented (solid line) and the health tax rate is fixed at a not too high level ($0 < \tau < \tau^\circ$), in contrast with the welfare level obtained in the absence of it (dashed line). When the health tax rate tax becomes larger ($\tau^\circ < \tau < 1$), however, i.e. the government finances public health

[18] We note that a political process of voting would be redundant in this model because the government maximises the social welfare W_t, that is, the lifetime indirect utility index (12) of every generation, with respect to constant the health tax rate τ, and no generation would disagree on the introduction of public health spending when $r > \hat{r}$.

investments at too high a rate, social welfare becomes lower than the suboptimal case of absence of health policy, because the negative effects of the reduced disposable income of the young workers dominates in such a case.

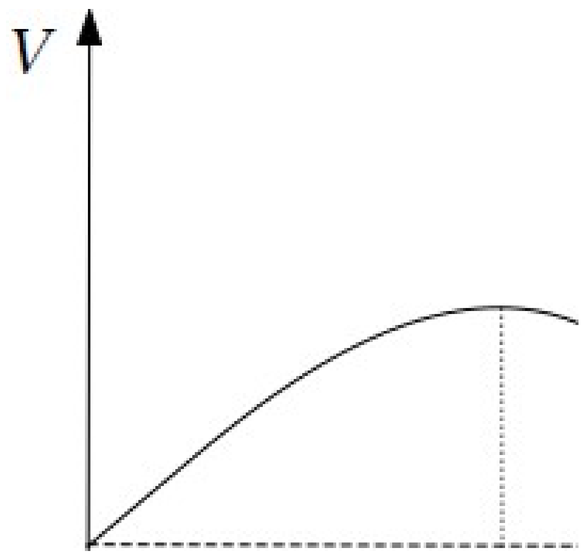

Figure 1. Social welfare as a function of τ in a small open economy with a domestic market for annuities.

5. Numerical exercises

In order to give to readers some clues about the policy conclusions that can effectively be drawn from the theoretical findings, and to discuss the normative issue in greater depth, we now present some numerical experiments to show how saving and welfare in several actual small open economies react to a change in the health tax rate τ.

We select the following three groups of countries with similar (different) characteristics within (among) them regarding per capita GDP, adult mortality and public health spending relative to GDP.

Countries of group [*i*] – European developed economies: Austria, Belgium, Denmark, Iceland, The Netherlands and Sweden, where per capita GDP is high, adult mortality is low and the ratio of per capita health spending on per capita GDP is fairly high.

Countries of group [*ii*] – European economies in transition: Croatia, Czech Republic, Lithuania, Poland, Romania and Slovenia, where per capita GDP and total expenditure on health as percentage of per capita GDP are lower, and adult mortality higher, than the corresponding values in countries of group (*i*).

Countries of group [*iii*] – African underdeveloped economies: Angola, The Republic of the Congo and Madagascar, where per capita GDP and total expenditure on health relative to per capita GDP are low, while adult mortality is fairly high.

We use data from World Bank (2009) and World Health Statistics (2010), as regards, respectively, per capita GDP at purchasing power parity (PPP), assumed as a proxy of per young domestic income (y), and the probability of dying from youth to old-age,[19] assumed as a proxy of adult mortality ($1-\pi$). In addition, as in our model public health spending represents the unique determinant of life expectancy, we have selected countries where the general government expenditure on health relative to the total expenditure on health is fairly high in order to properly consider the former as a reasonable approximation of the latter. Then, we assume the ratio of per capita health spending to per capita GDP (taken from World Health Statistics, 2010) as a proxy of the health tax rate τ .

[19] In particular, we use data on the probability of dying between 15 and 60 years or, alternatively, the probability that a 15 year old individual will die before reaching 60 years of life.

Table 1 reports a summary statistics of the observed values of the per capita GDP (Column 2), the ratio of total expenditure on health to per capita GDP (Column 3), the ratio of government expenditure on health to total expenditure on health (Column 4), and the adult longevity rate (Column 5), for the selected groups of countries.

Table 1. Summary statistics.

	Per capita GDP $ (PPP)[a]	Total expenditure on health as % of per capita GDP[b]	Government expenditure on health as % of total expenditure on health[b,20]	Adult longevity rate (probability of a 15 year old person of being alive at the age of 60)[b]
Countries of group [*i*]				
Austria	38,748	10.1	76.4	0.925
Belgium	36,048	9.4	74.1	0.914
Denmark	36,762	9.8	84.5	0.91
Iceland	37,595	9.3	82.5	0.944
The Netherlands	40,715	8.9	82.0	0.932
Sweden	37,905	9.1	81.7	0.938
Countries of group [*ii*]				
Croatia	19,803	7.6	87.0	0.885
Czech Republic	25,232	6.8	85.2	0.895
Lithuania	16,745	6.2	73.0	0.785
Poland	19,059	6.4	70.9	0.858
Romania	14,198	4.7	80.3	0.844
Slovenia	27,004	7.8	71.5	0.905
Countries of group [*iii*]				
Angola	5,789	2.5	80.3	0.579
Congo (Republic of the)	4,248	2.4	70.4	0.619
Madagascar	1,050	4.1	66.2	0.737

Source: our elaboration from [a]World Bank (2009) and [b]World Health Statistics (2010).

Based on our theoretical model, the questions we now pose are the following.

[20] It should be noted that countries of group [*iii*] benefit from some external resources for health in addition to the internal ones. In particular, the foreign aid for health in Angola (Congo) [Madagascar] amounts to 3.7 (5.4) [17.8] per cent of total health spending. These resources include "all grants and loans whether passing through governments or private entities for health goods and services, in cash or in kind." (see World Health Statistics, 2010).

(1) Can different countries with different characteristics as regards per capita income and longevity raise savings and social welfare either by increasing or decreasing the (public) financing of health care services?

(2) Does a country spend too much or too little on health with respect to the social optimum?

Answer to question (1). By taking from Table 1 the observed values of per capita GDP (Column 2), total expenditure on health relative to per capita GDP (Column 3) and adult longevity rate (Column 5) for each country, and considering them as proxies of (the observed values of), respectively, domestic income ($\bar{y}$), life expectancy ($\bar{\pi}$) and health tax rate ($\bar{\tau}$), we "calibrate" both the wage[21] w and demographic parameter π_1 to fit the two key variables of our model, namely, y and π.[22]

[21] Notice that choosing the wage as a "calibrated" variable to match data on per capita income in a small open economy is equivalent to calibrate the technology index of the production function (e.g., the index A in the Cobb-Douglas function) in a closed economy (see, e.g., Barro and Sala-i-Martin, 2004, p. 433; Kraay and Raddatz, 2007, p. 321).

[22] It should be noted that the "calibrated" values of w and π_1, i.e.

$$w^C = \frac{\bar{y}(1+\bar{\pi})}{1+\bar{\pi}[1+r(1-\bar{\tau})]},$$

and

$$\pi_1{}^C = \frac{\bar{\pi}(1+\Phi\,\bar{\tau}\,w^C)-\pi_0}{\Phi\,\tau\,w^C},$$

are obtained by solving the system of Eqs. (6) and (7), where we have used the function $\pi = \frac{\pi_0+\pi_1\Phi\,\tau\,w}{1+\Phi\,\tau\,w}$, with $\Phi > 0$ being a scale parameter introduced to properly match the data, i.e. to smooth the effectiveness of a rise in health investments on longevity (see Blackburn and Cipriani, 2002, p. 198; de la Croix and Ponthière, The demographic parameter π_0 is assumed to be fixed at: 0.5 for countries of groups [*i*] and [*ii*], and 0.3 for countries of group [*iii*]. This to account for the different influence of both economic and non-economic factors as well as to reflect differences in the length of life of people in developed and underdeveloped nations regardless of the existence of the public expenditure on health.[23] Finally, the world interest rate is set at $r=1$, which corresponds to a real interest rate around 2.3 per cent per annum by assuming that a generation consists of 30 years (see de la Croix and Michel, 2002); this is in line with the observed values of the real rate of interest (see, e.g., Del Negro and Schorfheide, 2009).

Given (*a*) the observed values of $\bar{y}$, $\bar{\tau}$ and $\bar{\pi}$ (reported in Columns 2, 3 and 5 of Table 1, respectively), (*b*) the chosen values of both the natural rate of longevity and rate of interest, and (*c*) the corresponding "calibrated" values of w^C and $\pi_1{}^C$, Tables 2-4 show, for countries of groups [*i*]-[*iii*], how health investment, longevity, saving and welfare react to a small percentage increase in τ.

Table 2. Countries of group [*i*]. (5 percent increase in τ, $\pi_0=0.5$ and $\Phi=0.004$).

	$\bar{\tau}$	$\bar{\pi}$	$h(\bar{\tau})$	$s(\bar{\tau})$	$V(\bar{\tau})$	$\bar{\tau}+\Delta\tau$	$\Delta h/h$ ($\times 100$)	$\Delta\pi/\pi$ ($\times 100$)	$\Delta s/s$ ($\times 100$)	ΔV
Austria $w^C=27{,}058$ $\pi_1{}^C=0.963$	0.101	0.925	2,732	11,689	18.82	0.10605	5.014	0.108	-0.47	+
Belgium $w^C=25{,}162$ $\pi_1{}^C=0.957$	0.094	0.914	2,365	10,886	18.6	0.0987	4.99	0.197	-0.413	+
Denmark $w^C=25{,}712$	0.098	0.91	2,519	11,049	18.59	0.1029	5.00	0.198	-0.443	+

2010, p. 232). Of course, the existence of $\Phi>0$ does not qualitatively modify any of the results of Propositions 1-3 of the present paper.

[23] For instance, we may realistically assume that for several reasons the quality of life, institutions, infrastructures and so on, and, hence, the length of adult lifetime, still remain higher in developed European countries than underdeveloped African ones even in the absence of any public health care provision.

$\pi_1^C = 0.9506$										
Iceland $w^C = 26{,}099$ $\pi_1^C = 0.989$	0.093	0.944	2,427	11,495	18.94	0.0976	4.94	0.211	-0.4	+
The Netherlands $w^C = 28{,}284$ $\pi_1^C = 0.975$	0.089	0.932	2,517	12,430	19.0	0.0934	4.92	0.214	-0.386 -	+
Sweden $w^C = 26{,}323$ $\pi_1^C = 0.983$	0.091	0.938	2,395	11,581	18.9	0.0955	4.92	0.213	-0.388	+

Table 3. Countries of group [*ii*]. (5 percent increase in τ, $\pi_0 = 0.5$ and $\Phi = 0.04$).

	$\bar{\tau}$	$\bar{\pi}$	$h(\bar{\tau})$	$s(\bar{\tau})$	$V(\bar{\tau})$	$\bar{\tau} + \Delta\tau$	$\Delta h / h$ $(\times 100)$	$\Delta \pi / \pi$ $(\times 100)$	$\Delta s / s$ $(\times 100)$	ΔV
Croatia $w^C = 13{,}811$ $\pi_1^C = 0.894$	0.076	0.885	1,049	5,991	17.23	0.0798	5.05	0.045	-0.383	-
Czech Republic $w^C = 17{,}520$ $\pi_1^C = 0.903$	0.068	0.895	1,191	7,711	17.78	0.0714	4.95	0.033	-0.337	-
Lithuania $w^C = 11{,}854$ $\pi_1^C = 0.794$	0.062	0.785	734	4,890	16.13	0.0651	5.04	0.05	-0.306	-
Poland $w^C = 13{,}307$ $\pi_1^C = 0.868$	0.064	0.858	751	5,751	16.96	0.0672	5.05	0.046	-0.312	-
Romania $w^C = 9{,}885$ $\pi_1^C = 0.862$	0.047	0.844	464	4,312	16.33	0.04935	4.95	0.118	-0.208	+
Slovenia $w^C = 18{,}778$ $\pi_1^C = 0.911$	0.078	0.905	1,464	8,225	17.99	0.0819	4.98	0.033	-0.401	-

Table 4. Countries of group [*iii*]. (5 percent increase in τ, $\pi_0 = 0.3$ and $\Phi = 0.4$).

	$\bar{\tau}$	$\bar{\pi}$	$h(\bar{\tau})$	$s(\bar{\tau})$	$V(\bar{\tau})$	$\bar{\tau} + \Delta\tau$	$\Delta h / h$ $(\times 100)$	$\Delta \pi / \pi$ $(\times 100)$	$\Delta s / s$ $(\times 100)$	ΔV
Angola $w^C = 4{,}264$ $\pi_1^C = 0.585$	0.025	0.579	106	1,524	12.83	0.02625	4.71	0.051	-0.065	+
Congo (Republic of the) $w^C = 3{,}093$ $\pi_1^C = 0.629$	0.024	0.619	74	1,154	12.62	0.0252	5.40	0.161	-0.086	+
Madagascar	0.041	0.737	30	303	10.96	0.05145	26.66	0.814	-0.66	+

$w^C = 746$ $\pi_1^C = 0.772$										

What can be observed from Tables 2-4 is the following. In all countries within each group saving slightly reduces when the health tax rate raises of five percentage points with respect to the observed values (as Columns 10 of Tables 2-4 reveal). This means, regardless of whether a country is either rich or poor, that the increased marginal propensity to save due to the higher life expectancy is not sufficient to offset the negative income effect because of the increased labour income taxation.

In high-income countries (see Table 2), both health investments relative to per capita income and longevity are already high. Therefore, even if the health technology is relatively efficient in such a case (i.e., the demographic parameter π_1 is high), a rise in the health tax rate cannot be expected to greatly increase life expectancy and the marginal propensity to save, while reducing the disposable income sensibly because labour income taxation is fairly high. As a consequence, the positive longevity effect on saving is more that counterbalanced by the negative income effect. However, the advantages in terms of utility of lengthening the individual lifetime (i.e., discounting future at a lower rate) are relatively higher than the disadvantages of the reduced consumption, and thus social welfare increases.

In average-income countries (see Table 3), the existing level of taxation to finance health investments is lower than in high-income countries, but the effects on savings of increasing health investments are still negative. However, we can observe that the rise experienced in Croatia and Czech Republic is really low. This implies that the value in terms of utility of an increase in the length of individual lifetime is lower than the negative effect due to the reduction in both the disposable income and marginal propensity to consume, following the rise in labour income taxation, i.e. the advantages in terms of utility of increasing health investment are lower than the disadvantages of financing it.

In low-income countries (see Table 4), both health investments relative to per capita income and longevity are fairly low. Then a rise in τ causes a slight increase in longevity (but in Madagascar where life expectancy is relatively high, because the ratio of health spending to per capita income is higher than in the other two African countries), but the negative effect of the reduced disposable income outweighs the beneficial effect of the increased marginal propensity to save. However, in this group of countries, the value in terms of utility of the increased lifetime weights more than the value of the reduced consumption. To sum up a rise in the public health expenditure is harmful to GDP in all countries, while it may be beneficial to social welfare in several countries.

Answer to question (2). The previous numerical analysis has revealed the direction of the change in health investments that should be undertaken in order to raise social welfare. Now, to know to what extent should a government either increase or decrease the public health expenditure (with respect to the observed values reported in Table 1) to achieve the social optimum, we compute the optimal health tax rate τ^* (which is implicitly defined by Eq. 17) for the chosen countries. The results are presented in Table 5.

Our exercise reveals that all economies of groups [*i*] and [*iii*] but Angola are currently under-investing on health, and they should therefore increase health spending further to achieve the social optimum. By contrast, all economies of group [*ii*] but Romania seem to currently over-invest on health and should therefore reduce health spending. In particular, the reduction in Croatia and Czech Republic should be of almost 30 per cent, while in Lithuania, Poland and Slovenia it should be around 19, 17 and 38 per cent, respectively.

Of course, our rough numerical experiments only represent an example of the theoretical findings presented in this paper and thus do not have the ambition of suggesting specific policy recipes and remedies as regards the provision of health care services for the chosen countries.

To sum up, this illustrative and non-exhaustive numerical exercises reveal that different economies as regards several interacting features such as technical progress (i.e. wages and output), efficacy and size of health spending, basic and observed longevity, may require different public health programmes.[24] In particular, from a normative point of view it is shown that: (1) while developed economies have still room for an increase in health spending, several transition economies are currently overspending on health;[25] (2) the richest economies should devote more resources on health than the poorest. Of course, from a positive point of view, comparison of the reasons why we observe different health tax rates around the world is difficult, and may require an *ad hoc* empirical analysis. For instance, differences in observed taxation may be due to either specific tax laws or mechanisms for health financing and service provision. Indeed, the public plans in some countries provide basic coverage only, so that their citizens can purchase supplemental insurance for additional coverage, or universal coverage (such as, for instance, all countries of group [*i*]).

Table 5. Observed versus optimal health tax rates.

	$\bar{\tau}$	τ^*
Countries of group [*i*]:		
Austria	0.101	0.123
Belgium	0.094	0.126
Denmark	0.098	0.124
Iceland	0.093	0.128
The Netherlands	0.089	0.122
Sweden	0.091	0.127

[24] This is also in accord with the conclusions by Pestieau et al. (2008, p. 16) which, however, are focused on the role of the PAYG system: "Given that different economies may not be characterised by the same PAYG system, preferences, and survival process, the optimal public policy under a PAYG system should not be uniform across countries, but should reflect the specificities of each economy."

[25] This result complements and partially contrasts the results by Pestieau et al. (2008, p. 13) who, in a model with both private health spending partially subsidised by the government and PAYG pensions, find that "… actual economies with PAYG systems tend, by oversubsidizing health expenditures, to overspend on health."

Countries of group [*ii*]:		
Croatia	0.076	0.053
Czech Republic	0.068	0.048
Lithuania	0.062	0.050
Poland	0.064	0.053
Romania	0.047	0.059
Slovenia	0.078	0.048
Countries of group [*iii*]:		
Angola	0.025	0.027
Congo (Republic of the)	0.024	0.033
Madagascar	0.041	0.065

6. Conclusions

The demographic variables, i.e. fertility and longevity, have been recognised by the economic literature to be an important determinant of the growth process (Becker and Barro, 1988; Barro and Becker, 1989; Galor and Weil, 1999, 2000; Barro and Sala-i-Martin, 2004; Fogel, 2004). Using a simple small open economy model with overlapping generations and endogenous lifetime à la Chakraborty (2004), in this paper we showed that a rise in public investments in health can actually reduce both saving and domestic income per worker. Indeed, a rise in the health tax rate: (*i*) reduces (increases) the disposable income of the young (the survival probability) and this, in turn, negatively (positively) affects saving; (*ii*) if the natural length of life is high, then the gain in longevity due to an additional increase in health spending is low and thus the positive saving effect due to the gain in life expectancy tends to become negligible.

Moreover, since the financing of public investments in health is beneficial to life expectancy, then although raising the health tax rate reduces consumption because it decreases both the disposable income and marginal propensity to consume, when the world interest rate if fairly high the advantages in terms of utility of lengthening the lifetime are higher then the disadvantages of the reduced consumption. Therefore, the public provision of health services may represent a A –Pareto improvement, because population is endogenous since longevity is not constant and only preference profiles of born agents are evaluated at every state of the world. Indeed, this result holds because each individual does not take into account the advantages on an increase in (public) health investments on longevity.

Our numerical simulations obtained for three different groups of countries, namely European developed and transition economies and African underdeveloped economies, revealed that while the rise in public health spending over the existing level reduces savings and domestic income in all countries, it ambiguously affects social welfare: indeed, the developed economies should actually increase the public investment in health, while several economies in transition are found to currently overspend on it.

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