**Are the Regulation of Wages and Unemployment always Detrimental for Economic Growth?**

by Luciano Fanti and Luca Gori

*We would like to thank two anonymous referees for very helpful comments and suggestions. Usual disclaimers apply.*

## 1. Introduction

The conventional wisdom – originated in a static context by the seminal Stigler (1946) – claims that the regulation of wages in a simple competitive economy reduces the level of income per person due to the unemployment occurrence. In this paper, by adopting the conventional overlapping generations (OLG) model of neoclassical growth (Diamond, 1965) with endogenous fertility of households, we argue that this belief could not be warranted. In particular, we show that the long run effects of the introduction of wage minima[1] in a dynamical context (i.e., a simple OLG frame) could have a favourable impact on the long-run economic growth. The value added of this paper grounds on three novel results, so far escaped closer scrutiny: (i) under suitable conditions – say, a sufficiently high capital's share in technology as well as a generous unemployment benefit system – a regulated-wage economy with unemployment may perform better than a market-wage economy with full employment; (ii) the correlation between unemployment and economic growth may be positive; and (iii) the minimum wage might also be seen as a policy parameter to affect population growth. Moreover, we show that such results hold even with exogenous fertility and a lump-sum tax system on old people used to fund the unemployment benefit system.

As to the first result, we note that only few papers (e.g., Cahuc-Michel, 1996) have investigated the possibility of positive long run macroeconomic effects of minimum wages in an inter-temporal OLG context with exogenous fertility. Cahuc-Michel (1996)[2] considered mainly a two-sector economy with a Lucas-type externality; however they initially also studied the neoclassical

[1] It is worth noting that in this model where, for simplicity, there exists only one type of labour, a binding minimum wage simply indicates a regulated wage fixed by law over the prevailing market-clearing wage. In the case of more than one type of labour with uniformly distributed wages, this assumption would simply mean a regulated wage fixed over the average market wage.

[2] This paper may be considered the benchmark model as to this issue, in that other subsequent papers such as Ravn-Sorensen (1999) and Askenazy (2003) do not depart from its line of reasoning.

Diamond-style growth model (that is, the framework adopted in this paper) where minimum wages are assumed to cause a positive external effect by inducing workers to accumulate human capital in order to avoid unemployment, that is, it would create an incentive for unskilled workers to educate themselves in order to become skilled: notwithstanding, Cahuc-Michel (1996, p. 1470) concluded their analysis claiming that "minimum wage legislation has negative aggregate effects in an exogenous growth model." The previous literature up to now[3] therefore seems to retain, even with the assumption of positive external effects, that introducing minimum wages in the standard labour market context of supply and demand in a dynamic neoclassical OLG growth model, is always harmful for the long-run per capita income. In contrast with the past literature, we argue that this belief may be reversed even in the absence of any kind of positive external effects generated by minimum wage legislation.

As to the second point, we note that the result which establishes a positive relationship between unemployment and the long-run income per capita reverts a widespread belief according to which the unemployment occurrence is always detrimental for growth. The relationship between unemployment and economic growth is very controversial, both on theoretical and empirical grounds. On the theoretical side, the bulk of the existing theoretical literature[4] has mainly emphasised the negative effect of unemployment on per capita income growth (i) by comparing only different partial equilibrium static contexts, see Rowthorn, 1999; Pissarides, 2000); and (ii) by developing a fully dynamic general equilibrium model, where the link between unemployment and growth may be driven either by a monopolistic union who sets wages above the market-clearing level (Daveri-Tabellini, 2000) or by specific search costs (Bean-Pissarides, 1993).

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[3] For instance, in a recent paper Cardona-Sánchez Losada (2006), state that "Cahuc and Michel (1996) show, in an endogenous growth through human capital accumulation economy, that an increase in the minimum wage raises both the human capital accumulation and the endogenous growth rate, while the minimum wage has a negative effect when the growth rate is exogenous." (p. 53).

[4] For a survey on this literature see Pugno (1998).

However, in contrast with the previously cited papers, some authors have shown the possibility of a theoretical *negative* relationship between employment and growth. Among these, two important papers are: (1) Aghion-Howitt (1994), which argued that an application of the Schumpeterian idea of 'creative disruption' may lead to a negative relation between employment and growth; and (2) Mortensen-Pissarides (1998), which, instead, showed that the (theoretical) relationship between economic growth and the employment rate in matching models of unemployment is either negative or positive depending on the assumption of embodied or disembodied technical progress. Another example can be found in the paper by Fanti-Manfredi (2003) who found that unemployment can promote or discourage long-run economic growth, depending on the level of the rate of fertility of the unemployed people.

In this paper, therefore, we establish another theoretical channel – so far overlooked and based on a combination of wage minima and unemployment benefits – through which unemployment and (neoclassical) economic growth may be positively linked in the long-run.

On the empirical side, there is no consensus regarding the sign of the correlation between growth and unemployment either across countries or across longer periods of time in the same country. In particular the empirical literature found a threefold different result: (1) the correlation is essentially zero, e.g., Aghion-Howitt (1992), Bean-Pissarides (1993) [5]; (2) the correlation is negative, e.g. Hoon-Phelps (1997), Muscatelli-Tirelli (2001); (3) the correlation is positive, e.g. Caballero (1993)[6]. However we note, following the intuition by Mortensen (2004), that the diffuse empirical result which emphasises no correlation between unemployment and growth is perhaps due to the

[5] Aghion-Howitt (1992) reported that both high and low growth countries experience lower unemployment rate relative to those with intermediate rates of productivity growth among the 20 OECD countries included in their study. Bean-Pissarides (1993), instead, found no correlation between the unemployment rate and the productivity growth across OECD economies.

[6] Caballero (1993) found a positive time series relationship between growth and unemployment in the UK and US between 1966 and 1989 while Muscatelli-Tirelli (2001) found negative correlations for the five G7 economies but UK and US.

fact that both unemployment and growth rates are simultaneous determined in a market context, and thus changing different common determinants in different countries and time periods can induce uncorrelated co-movements on average. In this sense, since our model determines endogenously and simultaneously both the long-run income per capita and the unemployment rate,[7] time-series and cross-country empirical analyses could also find zero correlation.

The plan of the paper is as follows. In section 2 we build up the model. In section 3 the main steady state results are analysed discussed by comparing competitive-wage and regulated wage economies. In section 4 we present another theoretical channel (i.e., an OLG exogenous fertility economy where lump-sum taxes on the elderly are used to fund the unemployment benefit system) through a combination of minimum wages and unemployment insurance benefits may be beneficial for the (neoclassical) economic growth, in spite of a reduced employment rate. Section 5 concludes.

## 2. The model

We consider a conventional dynamic general equilibrium OLG closed economy (Diamond, 1965) with endogenous fertility and regulated wages (the complete list of symbols used in this paper is summarised in the Appendix).

The distinctive features of the present paper are resumed as follows: (1) following a standard way to endogenise fertility in OLG frameworks, by assuming for simplicity that every single young adult agent can have children, life is divided into three periods: childhood, young adulthood, and old-age. During childhood, individuals make no economic decisions. As an adult each agent derives utility from (young and old age) material consumption and the number of children raised, as in

[7] Differently from the previous literature in which the long run sign is based on exogenous growth and endogenous unemployment.

Galor-Weil (1996);[8] (2) child rearing activities require a (variable) cost – indexed with the total income of the young $W_t$ –, that is, the cost of raising each child is simply $qW_t$ with $0 < q < 1$.

To begin with, we briefly discuss assumptions (1) and (2). (i) The way in which fertility has been endogenised is in line with the empirical findings by Cigno-Rosati (1996), who found that parents are self-interested and thus both savings and fertility are chosen without regard to their offspring. Moreover, we note that our model further differs from the so-called dynastic models (e.g., Barro, 1974; Ehrlich-Lui, 1991), where parents internalise the lifetime utility of their children, because such models are not suited to investigate inter-generational policy effects such as those due to the introduction of minimum wages.[9] (ii) Modelling child-rearing costs as a percentage of the total income of the young-workers is rather usual in literature (e.g., Wigger, 1999; Strulik, 1999, 2004; Boldrin-Jones, 2002; Fanti-Manfredi, 2003), and it is coherent with the microeconomic dependence of such a cost on the opportunity cost of the parents' home time which is increasing in their working income (see Cigno, 1991). Besides, in order to take into account the negative substitution effect (on fertility) of the female wage due to the potential increase of women's labour force participation (Mincer, 1963, 1966), this cost, as known, should be proportional to the (female) labour income. Likewise, it is natural to conjecture that even the component of the cost of child rearing due to expenditure for the material consumption of children is positively linked with the working income as well.

Moreover, (3) we suppose that the unemployment insurance benefit received by the young for each hour left unemployed by the regulation of wages ($\gamma \underline{w}$) is a fraction $0 < \gamma < 1$ of the (constant)

[8] Note that the variable $n_t$ represents the number of children with $n_t - 1$ being the population growth rate (for simplicity, the mortality rate has not been included in the analysis). Some authors, including Samuelson (1975), used $N_{t+1} / N_t = 1 + n$ with $n$ being the rate of population growth. Our approach is used in most papers with endogenous fertility.

[9] In fact, as noted by Pecchenino-Pollard (2002, p. 149) "…In these models the effects of changes in taxes are negated via changes in bequests, and so are ill suited to analyzing social security or publicly funded education."

minimum wage $\underline{w}$; this assumption is rather usual in the economic literature and resembles the hypothesis made by Cahuc-Michel (1996); (4) we have deliberately chosen to fund the unemployment benefit system with a lump-sum tax on the young-adult generation. In this way the nature of unemployment benefits is purely redistributive, that is income taxed away from the young rebated to the same individuals as a benefit for the unemployment time. This feature is important because in OLG models, as known dating back to Bertola (1996) and Uhlig-Yanagawa (1996), taxing the income from capital could lead to faster economic growth since all savings are performed by the young (savers). Then, in the present model, the taxation policy does not cause any transfers from the old to the young (as, instead, it would have been the case with capital income taxes); hence, the effects on both demographic and macroeconomic variables should be entirely ascribed to the regulation of wages (and, thus, to the unemployment occurrence) rather than to the inter-generational tax transfer channel.

*2.1. Government*

The unemployment benefit system is entirely financed by levying and adjusting over time a lump-sum tax on the income of the young. Therefore, per capita time-$t$ (balanced) budget is simply:

(1) $$\gamma \underline{w} u_t = \tau_t \, .$$

*2.2. Individuals*

Individuals behave just like in the standard OLG model with endogenous fertility, and each young agent is endowed with one unit of time which is supplied inelastically to the labour market. The only departure is that the wage perceived by the young-adult workers is regulated with an appropriate legislation and thus fixed over the market-clearing wage by the government. Therefore, in every period, involuntary unemployment occurs. Each young adult individual earns a regulated wage ($\underline{w}$) for the employment time, while receiving an unemployment insurance benefit for the unemployment time.[10] The aggregate unemployment rate (defined in terms of hours not worked) is $u_t = (N_t - L_t)/N_t$, where $L_t$ is the labour demand.[11] Thus, the representative individual's total income – as given by the sum of the working income plus the unemployment benefit – is simply $W_t(\underline{w}) := \underline{w}(1 - u_t) + \gamma \underline{w} u_t$.

The problem faced by the representative individual is the following:

(2) $$\max_{\{c_t^y, c_{t+1}^o, n_t\}} U_t(c_t^y, c_{t+1}^o, n_t) = (1-\phi)\ln(c_t^y) + \phi \ln(c_{t+1}^o) + \rho \ln(n_t),$$

subject to the lifetime budget constraint

(3) $$c_t^y + \frac{c_{t+1}^o}{1 + r_{t+1}} = W_t(\underline{w})(1 - q n_t) - \tau_t .$$

Therefore, exploiting the first order conditions together with (1) and (3), the demand for children and the saving function are respectively given by:

(4) $$n_t(\underline{w}) = \frac{\rho}{1+\rho} \cdot \frac{1 - u_t}{q[1 - u_t(1-\gamma)]},$$

(5) $$s_t(\underline{w}) = \frac{\phi}{1+\rho} \underline{w}(1 - u_t).$$

*2.3. Firms*

[10] This is, for instance, the typical case of the Italian unemployment insurance system (i.e., *Cassa Integrazione Guadagni*) which pays benefits for the hours of unemployment due to temporary and partial layoffs.

[11] Note that in this model there is no uncertainty. Thus, each young-adult agent will be employed for $1 - u_t$ hours and unemployed for $u_t$ hours.

All the firms in the economy are identical and own a constant returns to scale Cobb-Douglas technology $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$.[12] Given the labour demand $L_t = (1-u_t)N_t$, the per-capita production function is:

(6) $$y_t = A(1-u_t)\left(\frac{k_t}{1-u_t}\right)^{\alpha},$$

where $k_t := K_t / N_t$ and $y_t := Y_t / N_t$.

Assuming that capital totally depreciates at the end of each period[13] and also that the final output is treaded at unit price, profit maximisation leads to the following marginal conditions for capital and labour, respectively:

(7) $$r_t = \alpha A\left(\frac{k_t}{1-u_t}\right)^{\alpha-1} - 1.$$

(8) $$\underline{w} = (1-\alpha)A\left(\frac{k_t}{1-u_t}\right)^{\alpha}.$$

As far as labour is concerned, the marginal product will adjust to meet the fixed real wage. From (8) the endogenous (current) rate of unemployment is:

(9) $$u_t(k_t, \underline{w}) = 1 - \left[\frac{(1-\alpha)A}{\underline{w}}\right]^{\frac{1}{\alpha}} \cdot k_t,$$

which is positively related with the minimum wage and strictly decreasing in the stock of capital per person. Note that once the wage has been fixed the real interest rate is exogenous (that is, capital returns are independent of the capital stock). A binding minimum wage, in fact, necessarily causes

[12] Adding exogenous growth in labour productivity does not alter any of the substantive conclusions of the model and, hence, it is not included here.

[13] This assumption is not unrealistic in the present context, because as noticed by de la Croix-Michel (2002, p. 338) "even if one assumes a rather low annual depreciation rate of 5 per cent, 79 per cent of the stock of capital is depreciated after 30 years."

any increase of the capital stock to be matched by an identical increase of the employment level, keeping the capital-labour ratio constant. Indeed, substitution of (9) into (7) yields:

$$r(\underline{w}) = \alpha A \left[ \frac{(1-\alpha)A}{\underline{w}} \right]^{\frac{1-\alpha}{\alpha}} - 1 . \tag{10}$$

As it can easily be seen from (10) introducing a minimum wage pushes down the real interest rate below the competitive level.

*2.4. Equilibrium*

The model is closed with the analysis of the long-run equilibrium. The market-clearing condition is given by the equality between savings and investments, that is:

$$n_t k_{t+1} = s_t . \tag{11}$$

Substituting out for $n_t$ and $s_t$ according to (4) and (5) respectively and using (9), the dynamics of capital is driven by the following first order linear difference equation:[14]

$$k_{t+1} = \mu \gamma \underline{w} + \mu (1-\gamma) [(1-\alpha)A]^{\frac{1}{\alpha}} \underline{w}^{-\left(\frac{1-\alpha}{\alpha}\right)} k_t , \tag{12}$$

where $\mu := \phi q / \rho$ .

## 3. The steady-state results

In the following tables we resume the main analytical steady-state outcomes of both competitive-wage and regulated-wage economies with respect to:[15] (1) the stock of capital per capita ($k^*$); (2)

[14] Note that the steady-state equilibrium is globally stable whatever the value of the regulated wage. The proof is of course available on request.

[15] Note that, as regards the steady-state unemployment rate, $0 < u^*(\underline{w}) < 1$ always holds.

the income per capita ($y^*$); and (3) the fertility rate ($n^*$). From Eq. (12) the steady state results of the models are straightforwardly derived and presented in Tables 1.A and 1.B.[16]

**Table 1.A**. The regulated-wage economy ($w_c < \underline{w} < +\infty$).

| $k^*(\underline{w})$ | $y^*(\underline{w})$ | $n^*(\underline{w})$ |
|---|---|---|
| $\dfrac{\mu \gamma \underline{w}^{\frac{1}{\alpha}}}{\underline{w}^{\frac{1-\alpha}{\alpha}} - \mu(1-\gamma)[(1-\alpha)A]^{\frac{1}{\alpha}}}$ | $\dfrac{A[(1-\alpha)A]^{\frac{1-\alpha}{\alpha}} \mu \gamma \underline{w}}{\underline{w}^{\frac{1-\alpha}{\alpha}} - \mu(1-\gamma)[(1-\alpha)A]^{\frac{1}{\alpha}}}$ | $\dfrac{\phi}{1+\rho} \cdot \dfrac{[(1-\alpha)A]^{\frac{1}{\alpha}}}{\underline{w}^{\frac{1-\alpha}{\alpha}}}$ |

**Table 1.B**. The competitive-wage economy ($w_c$).

| $k^*_c$ | $y^*_c$ | $n^*_c$ |
|---|---|---|
| $[\mu(1-\alpha)A]^{\frac{1}{1-\alpha}}$ | $A[\mu(1-\alpha)A]^{\frac{\alpha}{1-\alpha}}$ | $\dfrac{\rho}{(1+\rho)q}$ |

**Table 2**. Critical values of the capital's share in technology ($\alpha$) beyond which the regulated wage economy performs better than the market wage economy as regards both capital and output per capita.

| Condition for a higher accumulation of capital | Condition for a higher income per capita |
|---|---|
| $\alpha > \alpha_k = 1-\gamma$ | $\alpha > \alpha_y = \dfrac{1}{1+\gamma}$ |

Therefore, the following propositions hold:[17]

[16] The building of the Diamond OLG model with competitive wage and endogenous fertility as well as the derivation of its steady state outcomes in terms of capital stock, output and the rate of fertility are rather conventional and, thus, not reported here.

**Proposition 1**. *The long-run per-capita stock of capital is:*

*(1) an increasing function of the regulated wage provided that* $\alpha > \alpha_k$*;*

*(2) a U-shaped function under the condition* $0 < \alpha < \alpha_k$*.*

*In both cases there always exists a value of the regulated wage which ensures a higher capital stock than in the market-wage economy.*

**Proof**. The proof straightforwardly derives by differentiating $k^*(\underline{w})$ with respect to $\underline{w}$. In particular,

$$\operatorname{sgn}\left\{\frac{\partial k^*(\underline{w})}{\partial \underline{w}}\right\} = \operatorname{sgn}\left\{\underline{w}^{\frac{1-\alpha}{\alpha}} - \mu\left(\frac{1-\gamma}{\alpha}\right)[(1-\alpha)A]^{\frac{1}{\alpha}}\right\},$$

so that

$$\frac{\partial k^*(\underline{w})}{\partial \underline{w}} \substack{<\\>} 0 \Leftrightarrow \underline{w} \substack{<\\>} w_k := \left(\frac{1-\gamma}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} \cdot w_c .$$

Since $\frac{\partial k^*(\underline{w})}{\partial \underline{w}} > 0$ for any $\underline{w} > w_k$ and $w_k < w_c$ for any $\alpha > \alpha_k$, then knowing that $k^*(\underline{w}) = k^*{}_c$ if and only if $\underline{w} = w_c$, the latter inequality implies $k^*(\underline{w}) > k^*{}_c$ for any $\underline{w} > w_c$. **Q.E.D.**

**Proposition 2**. *The long-run income per-capita is:*

*(1) an increasing function of the regulated wage provided that* $\alpha > \alpha_y$*;*

[17] Notice that Table 2 and the proof of Propositions 1 and 2 are straightforwardly derived from the analyses of both $k^*(\underline{w})$ and $y^*(\underline{w})$. Details are here omitted for economy of space and are of course available on request.

*(2) a U-shaped function under the condition* $1/2 < \alpha < \alpha_y$*;*

*(3) a decreasing function if* $\alpha < 1/2$.

*In the first two cases there always exists a level of the regulated wage which ensures a higher income per person than in the market-wage economy. In the third case the regulation of wages always causes an output loss.*

**Proof**. The proof straightforwardly derives by differentiating $y^*(\underline{w})$ with respect to $\underline{w}$. In particular,

$$\mathrm{sgn}\left\{\frac{\partial y^*(\underline{w})}{\partial \underline{w}}\right\} = \mathrm{sgn}\left\{\frac{2\alpha - 1}{\alpha}\underline{w}^{\frac{1-\alpha}{\alpha}} - \mu(1-\gamma)[(1-\alpha)A]^{\frac{1}{\alpha}}\right\},$$

If $0 < \alpha < 1/2$, then $\frac{\partial y^*(\underline{w})}{\partial \underline{w}} < 0$ for any $\underline{w} > w_c$, that is, $y^*(\underline{w})$ is a negative monotonic function of the minimum wage. Since $y^*(\underline{w}) = y^*{}_c$ if and only if $\underline{w} = w_c$, then $y^*(\underline{w}) < y^*{}_c$ for any $\underline{w} > w_c$.

If $1/2 < \alpha < 1$, then

$$\frac{\partial y^*(\underline{w})}{\partial \underline{w}} \stackrel{<}{>} 0 \Leftrightarrow \underline{w} \stackrel{<}{>} w_y := \left[\frac{\alpha(1-\gamma)}{2\alpha - 1}\right]^{\frac{\alpha}{1-\alpha}} \cdot w^*{}_c.$$

Since $\frac{\partial y^*(\underline{w})}{\partial \underline{w}} > 0$ for any $\underline{w} > w_y$ and $w_y < w_c$ for any $\alpha > \alpha_y$, then knowing that $y^*(\underline{w}) = y^*{}_c$ if and only if $\underline{w} = w_c$, the latter inequality implies $y^*(\underline{w}) > y^*{}_c$ for any $\underline{w} > w_c$. **Q.E.D.**

Propositions 1 and 2 imply that while the minimum may always promote the accumulation of capital, a necessary condition for the long-run income per capita to be enhanced by the regulation of wages is $\alpha > 1/2$. Therefore, provided the capital's share in technology and the replacement rate (as part of the unemployment benefit system) are both high enough, then, in spite of a positive unemployment rate, the long run (neoclassical) economic growth can be higher in a regulated-wage economy than in a market-wage setting. Whether policymaker should introduce or not a minimum wage law is ultimately an empirical issue.

In the following remark we show the effects of the regulation of wages on the rate of fertility.

**Remark 1**. *In a regulated-wage economy the long-run fertility rate is always lower than in the market-wage economy, that is,* $n^*(\underline{w}) < n_c^*$ *for any* $\underline{w} > w_c$.

Remark 1 stems from the role played by the unemployment rate on the long-run demand for children of households: indeed wages and unemployment play two opposite effects on fertility but, since the unemployment depends negatively on the wage, the overall wage-effect is negative. Interestingly, the latter remark shows that the value of the regulated wage may also be treated as a policy instrument for the control of individual fertility.[18]

A simple numerical simulation, for a parametric configuration chosen only for illustrative purposes, may help us in evaluating how the capital stock, the income per capita and the fertility rate change along with the level of the regulated wage. Figure 1 shows that both the long-run stock of capital stock and the long-run income per capita are increasing with the regulated wage. The policymaker therefore should fix a regulated wage as high as possible. Figure 2, instead, depicts the negative response of the fertility rate to a rise in the minimum wage; due to this "Modern" fertility behaviour, when the regulated wage is fixed at too high a level, population becomes stationary or it may even decrease.

[FIGURES 1 AND 2 ABOUT HERE]

[18] Remark 1 shows that, while in the competitive-wage economy the rate of fertility is constant, in the regulated-wage economy the number of children raised depends (negatively) on the wage rate, showing the feature of the so called "Modern" fertility.

The reason why ultimately unemployment may be positively linked with the level of the income per capita is based on the following economic mechanism. To begin with, minimum wages cause unemployment. A higher unemployment rate implies both a reduction in the labour demand by firms and, provided that unemployment benefits are sufficiently high, it may be associated with a higher saving function of individuals. Therefore, with a Cobb-Douglas technology when the output elasticity of capital (increased by wage minima) is larger than the output elasticity of labour (reduced by wage minima), then minimum wage legislation may enhance the long-run (neoclassical) economic growth. However, is the endogenous fertility hypothesis crucial for obtaining the this result? The answer is positive because we assumed that a tax system completely retrieves the unemployment benefits so that there is effectively no support for unemployment, and income is smaller because of unemployment.[19] In this case, only the depressing effect of the unemployment on fertility may cause an increase in the stock of capital. In fact, the reduced saving function (owing to minimum wages) may generate a higher stock of capital per capita if and only if the unemployment rate causes a reduction in fertility larger than that experienced in individual savings. However, it is worth to be noted that if taxation does not retrieve the unemployment benefits, allowing for the total income of individuals (the minimum wage for the employment time plus the unemployment benefit for the unemployment time) to be sustained, then also the standard OLG model with exogenous fertility may produce the same results. For showing this, in the following section we develop the same model of the previous section by allowing for a different way of financing the unemployment benefit system: we will introduce a lump-sum tax on old rather than a lump-sum tax on young.

## 4. Exogenous fertility and lump-sum taxes on the old people

[19] We thank an anonymous referee for having raised this question.

In this section we present the minimum wage model described in the previous section with the only departures that (1) the rate of fertility is now assumed to be exogenously given rather than endogenously determined by households, that is young population $N_t$ grows at the constant rate $n$, and (2) the unemployment benefit system is entirely funded with lump-sum taxes levied only on the elderly. The structure of the production side remains exactly the same than that presented in the previous section.

*4.1. Individuals*

The maximisation problem faced by agents born at time $t$ becomes the following:

(13) $$\max_{\{c_t^{\ y}, c_{t+1}^{\ o}\}} U\left(c_t^{\ y}, c_{t+1}^{\ o}\right) = (1-\phi)\ln\left(c_t^{\ y}\right) + \phi \ln\left(c_{t+1}^{\ o}\right),$$

subject to

(14) $$\begin{aligned} c_t^{\ y} + s_t &= W_t(\underline{w}) \\ c_{t+1}^{\ o} &= (1+r_{t+1})s_t - \tau_{t+1} \end{aligned},$$

Using the first order conditions and the lifetime individual's budget constraint, young-aged and old-aged consumptions are, respectively:

(15) $$c_t^{\ y}(\underline{w}, \tau_{t+1}) = (1-\phi)\left[\underline{w}(1-u_t) + \gamma \underline{w} u_t - \frac{\tau_{t+1}}{1+r_{t+1}}\right],$$ (

(16) $$c_{t+1}^{\ o}(\underline{w}, \tau_{t+1}) = \phi(1+r_{t+1})\left[\underline{w}(1-u_t) + \gamma \underline{w} u_t - \frac{\tau_{t+1}}{1+r_{t+1}}\right],$$

whereas the saving function is:

(17) $$s_t(\underline{w}, \tau_{t+1}) = \phi[\underline{w}(1-u_t) + \gamma \underline{w} u_t] + (1-\phi)\frac{\tau_{t+1}}{1+r_{t+1}}.$$

*4.2. Government*

The government balances the unemployment benefit expenditure in each period by levying and adjusting over time a lump-sum tax on the elderly. Thus, the per-capita time-$t$ government constraint is:

(18) $$\frac{\tau_t}{1+n} = \gamma \underline{w} u_t \Rightarrow \tau_t = (1+n)\gamma \underline{w} u_t .$$

*4.3. Equilibrium*

Given (18), the market clearing condition in goods as well as in capital markets is given by the equality between savings and investment, that is the capital stock in period $t+1$ is equal to the amount of resources saved in period $t$ discounted by the number of individuals, that is $(1+n)k_{t+1} = s_t(\underline{w}, \tau_{t+1})$. Combining the latter equation with (17) yields:

(19) $$k_{t+1} = \frac{\phi}{1+n}\underline{w}\left[1 - u_t(k_t, \underline{w}) \cdot (1-\gamma)\right] + \frac{1-\phi}{1+n} \cdot \frac{\tau_{t+1}}{1+r(\underline{w})} .$$

Substituting out from (9) for the current rate of unemployment, $u_t(k_t, \underline{w})$, and using the one-period in advance government constraint from (18), the dynamic equilibrium sequence of capital is represented by the following equation:

(20) $$k_{t+1} = \frac{\gamma \underline{w}^{\frac{1}{\alpha}}\{1+n+\phi[r(\underline{w})-n]\} + \phi(1-\gamma)[(1-\alpha)A]^{\frac{1}{\alpha}}[1+r(\underline{w})]k_t}{(1+n)\left\{\underline{w}^{\frac{1-\alpha}{\alpha}}[1+r(\underline{w})] + \gamma(1-\phi)[(1-\alpha)A]^{\frac{1}{\alpha}}\right\}} .$$

Steady-state implies $k_{t+1} = k_t = k^*$. Therefore, the long-run unemployment rate, capital stock and income per capita become the following:[20]

(21) $$u^*(\underline{w}) = \frac{\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n) - \phi[(1-\alpha)A]^{\frac{1}{\alpha}}}{\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n) + \left[(1-\phi)\gamma\frac{1+n}{1+r(\underline{w})} - \phi(1-\gamma)\right][(1-\alpha)A]^{\frac{1}{\alpha}}} ,$$

[20] Note that $u^*(\underline{w}) = 0$ if and only if $\underline{w} = w_c$ .

$$(22) \qquad k^*(\underline{w}) = \frac{\gamma \underline{w}^{\frac{1}{\alpha}}\{1+n+\phi[r(\underline{w})-n]\}}{[1+r(\underline{w})]\left\{\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n)+\left[(1-\phi)\gamma\frac{1+n}{1+r(\underline{w})}-\phi(1-\gamma)\right][(1-\alpha)A]^{\frac{1}{\alpha}}\right\}},$$

$$(23) \qquad y^*(\underline{w}) = \frac{A((1-\alpha)A)^{\frac{1-\alpha}{\alpha}}\gamma \underline{w}[(1+n)+\phi(r(\underline{w})-n)]}{[1+r(\underline{w})]\left\{\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n)+\left[(1-\phi)\gamma\frac{1+n}{1+r(\underline{w})}-\phi(1-\gamma)\right][(1-\alpha)A]^{\frac{1}{\alpha}}\right\}}.$$

The steady-state balanced-budget lump-sum tax as a function of the regulated wage is

$$(24) \qquad \tau^*(\underline{w}) = \frac{\gamma \underline{w}(1+n)\left[\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n)-\phi\psi^{\frac{1}{\alpha}}\right]}{\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n)+\left[(1-\phi)\gamma\frac{1+n}{1+r(\underline{w})}-\phi(1-\gamma)\right][(1-\alpha)A]^{\frac{1}{\alpha}}}.$$

The following Figure 3 compares the pace of accumulation of capital in both regulated-wage and competitive-wage economies, i.e. $k_{t+1} = f(\underline{w}, k_t)$ versus $k_{t+1} = f(k_t)$. The figure clearly shows, for a parametric set chosen only for illustrative purposes, that the locus of accumulation of capital in the regulated-wage economy lies always above the locus of accumulation of capital in the competitive-wage economy, resulting in a higher steady-state stock of capital per person.

[FIGURE 3 ABOUT HERE]

Since Eqs. (22) and (23) are difficult to handle analytically, we resort to numerical simulation to show that the minimum-wage economy performs better than the market-wage economy in terms of long-run stock of capital per-capita and income per-capita, as shown in the following Figure 4. This means that if the production is sufficiently capital oriented and the unemployment ratio is high enough the positive effect created by the minimum wage on the accumulation of capital always dominates the negative effect due to employment reduction, in spite of a positive rate of unemployment. Therefore, the final result of the introduction of minimum wages in this simple one-sector OLG economy is to increase the long-run output per-capita. Notice that the higher is the minimum wage the higher are both unemployment and output per capita. To sum up we may conclude that the positive effect of the minimum wage on capital and income may occur even in the conventional OLG model with exogenous fertility, provided that the unemployment benefits system is financed by a lump-sum tax levied on the elderly rather than on the young.

[FIGURE 4 ABOUT HERE]

## 5. Concluding comments

In this paper we have focused on the long-run effects on both macroeconomic and demographic variables of the regulation of wages in the conventional OLG model of neoclassical growth, extended to account for endogenous fertility. Our results differ markedly from the conventional wisdom – originated mainly in static partial equilibrium models with exogenous fertility – which argues that the regulation of wages always causes an output loss. Indeed we have shown that in a dynamic overlapping generations framework, where capital accumulation depends on wages and the demand for children is endogenous, the introduction of minimum wages could increase the long run income per capita and reduce population growth.[21] The former result may also occur when fertility is exogenous, provided that the unemployment benefits system is financed through a lump-sum tax levied on the elderly rather than on the young people, showing that the positive effect of minimum wages on the (neoclassical) economic growth might be considered as a robust feature of OLG economies.

Furthermore, in contrast with the prevailing past literature, we have shown that the regulation of wages in an otherwise competitive labour market within the OLG framework may reverse the correlation between unemployment and economic growth.[22]

## Appendix

[21] In a nutshell, to better understand the reason why the introduction of minimum wages may favour the long run (neoclassical) economic growth, it is sufficient to say that it acts – although only indirectly – as a reversed social security scheme: that is, in principle, it transfers resources from the old to the young raising the labour income and decreasing the interest rate.

[22] It is worth to be noted that this new perspective as regards the regulation of wages has been obtained within the standard OLG model with both exogenous and endogenous fertility, and without any additional "ingredient" (the only departures being, of course, an appropriate minimum wage law as well as the payment of unemployment benefit financed at balanced budget).

List of symbols

| | |
|---|---|
| $N$ | young adult population |
| $n$ | number of children |
| $\underline{w}$ | regulated wage |
| $w_c$ | competitive wage |
| $W$ | total income of the young adult agent |
| $r$ | rate of return on savings |
| $s$ | Savings |
| $c^y$ | young-age consumption |
| $c^o$ | old-age consumption |
| $\phi$ | consumption preference parameter |
| $\rho$ | preference for children parameter |
| $q$ | fraction of the young adult income for rearing one child |
| $K$, $k = K / N$ | total and per-capital stock of capital |
| $L$ | employed young population |
| $Y$, $y = Y / N$ | total and per-capital output |
| $A$ | technology index |
| $\alpha$ | capital share in the production function |
| $u$ | unemployment rate |
| $\gamma$ | replacement ratio |
| $\tau$ | lump-sum tax on young adult |

| | |
|---|---|
| $\mu$ | $\phi q / \rho$ |

**References**


Aghion P. – Howitt P. (1992), Growth and Unemployment, University of Western Ontario, *mimeo*.

Aghion P. – Howitt P. (1994), Growth and Unemployment, *Review of Economic Studies*, vol. 61, no. 3, pp- 477–494.

Askenazy P. (2003), Minimum Wage, Exports and Growth, *European Economic Review*, vol. 47, no.1, pp. 147–164.

Barro R.J. (1974), Are Government Bonds Net Wealth?, *Journal of Political Economy*, vol. 82, no. 6, pp. 1095–1117.

Barro R.J. – Sala-i-Martin X. (2003), *Economic Growth*, *2nd ed.*, MIT Press, Cambridge, Massachusetts.

Bean C. – Pissarides C. (1993), Unemployment, Consumption and Growth, *European Economic Review*, vol. 37, no. 4, pp. 837–854.

Bertola G. (1996), Factor Shares in OLG Models of Growth, *European Economic Review*, vol. 40, no. 8, pp. 1541–1560.

Boldrin M. – Jones L.E. (2002), Mortality, Fertility and Saving in a Malthusian Economy, *Review of Economic Dynamics*, vol. 5, no. 4, pp. 775–814.

Caballero R. (1993), Comment on Bean and Pissarides, *European Economic Review*, vol. 37, no. 4, pp. 855–859.

Cahuc P. – Michel P. (1996), Minimum Wage, Unemployment and Growth, *European Economic Review*, vol. 40, no. 7, pp. 1463–1482.

Cardona D. – Sánchez-Losada F. (2006), Unions, Qualification Choice, and Output. *Oxford Economic Papers*, vol. 58, no. 1, pp. 50–76.

Cigno A. (1991), *Economics of the Family*. Oxford University Press, Oxford.

Cigno A. – Rosati F. (1996), Jointly Determined Saving and Fertility Behaviour: Theory, and Estimates for Germany, Italy, UK and USA, *European Economic Review*, vol. 40, no. 8, pp. 1561–1589.

Daveri F. – Tabellini G. (2000). Unemployment, Growth and Taxation in Industrial Countries, *Economic Policy*, vol. 15, no. 30, pp. 49–104.

de la Croix D. – Michel P. (2002), *A Theory of Economic Growth. Dynamics and Policy in Overlapping Generations*, Cambridge University Press, Cambridge.

Diamond P.A. (1965), National Debt in a Neoclassical Growth Model, *American Economic Review*, vol. 55, no. 5, pp. 1126–1150.

Ehrlich I. – Lui F.T. (1991), Intergenerational Trade, Longevity, and Economic Growth, *Journal of Political Economy*, vol. 99, no. 5, pp. 1029–1059.

Fanti L. – Manfredi P. (2003), Population, Unemployment and Economic Growth Cycles: A Further Explanatory Perspective, *Metroeconomica*, vol. 54, no. 2-3, pp. 179–207.

Galor O. – Weil D.N. (1996), The Gender Gap, Fertility, and Growth, *American Economic Review*, vol. 86 no. 3, pp. 374–387.

Mankiw N.G. – Romer D. – Weil D.N. (1992). A Contribution to the Empirics of Economic Growth. *Quarterly Journal of Economics*, vol. 107, no. 2, pp. 407–437.

Mincer J. (1963), Market Prices, Opportunity Costs, and Income Effects. In Carl F. Christ (Ed.), *Measurement in Economics: Studies in Mathematical Economics and Econometrics in Memory of Yehuda Grunfeld*, Stanford, (CA), Stanford University Press, pp. 67–82.

Mincer J. (1966), Labor-Force Participation and Unemployment: A Review of Recent Evidence. In Gordon, R.A. and Gordon, H.S. (Eds.), *Prosperity and Unemployment*. Wiley, New York.

Mortensen D.T. (2004), Innovation, Growth, and Labor Market Policy, 2004 Meeting Papers no. 77, Society for Economic Dynamics, *mimeo*.

Mortensen D.T. – Pissarides C.A. (1998), Technological Progress, Job Creation, and Job Destruction, *Review of Economic Dynamics*, vol. 1, no. 4, pp. 733–753.

Muscatelli V.A. – Tirelli P. (2001), Unemployment and Growth: Some Empirical Evidence from Structural Time Series Models, *Applied Economics*, vol. 33, no. 8, pp. 1083–1088.

Pecchenino R.A. – Pollard P.S. (2002), Dependent Children and Aged Parents: Funding Education and Social Security in an Aging Economy, *Journal of Macroeconomics*, vol. 24, no. 2, pp. 145–169.

Pissarides C. (2000), *Equilibrium Unemployment Theory 2nd ed.*, Blackwell, Oxford.

Pugno M. (1998), Crescita Economica e Disoccupazione: Recenti Sviluppi Teorici, *Economia Politica*, vol. 15, n. 1, pp. 119–165.

Rowthorn R. (1999). Unemployment, Wage Bargaining and Capital-Labour Substitution, *Cambridge Journal of Economics*, vol. 23, no. 4, pp. 413–425.

Samuelson P.A. (1975), The Optimum Growth Rate of Population, *International Economic Review*, vol. 16, no. 3, pp. 531–538.

Stigler G.J. (1946), The Economics of Minimum Wage Legislation, *American Economic Review*, vol. 36 no. 3, pp. 358–365.

Strulik H. (1999). Demographic Transition, Stagnation, and Demoeconomic Cycles in a Model for the Less Developed Economy, *Journal of Macroeconomics*, vol. 21, no. 2, pp. 397–413.

Strulik H. (2004). Economic Growth and Stagnation with Endogenous Health and Fertility, *Journal of Population Economics*, vol. 17, no. 3, pp. 433–453.

Uhlig H. – Yanagawa N. (1996), Increasing the Capital Income Tax May Lead to Faster Growth, *European Economic Review*, vol. 40, no. 8, pp. 1521–1540.

Wigger B. (1999), Pay-as-you-go Financed Public Pensions in a Model of Endogenous Growth and Fertility, *Journal of Population Economics*, vol. 12, no. 4, pp. 625–640.

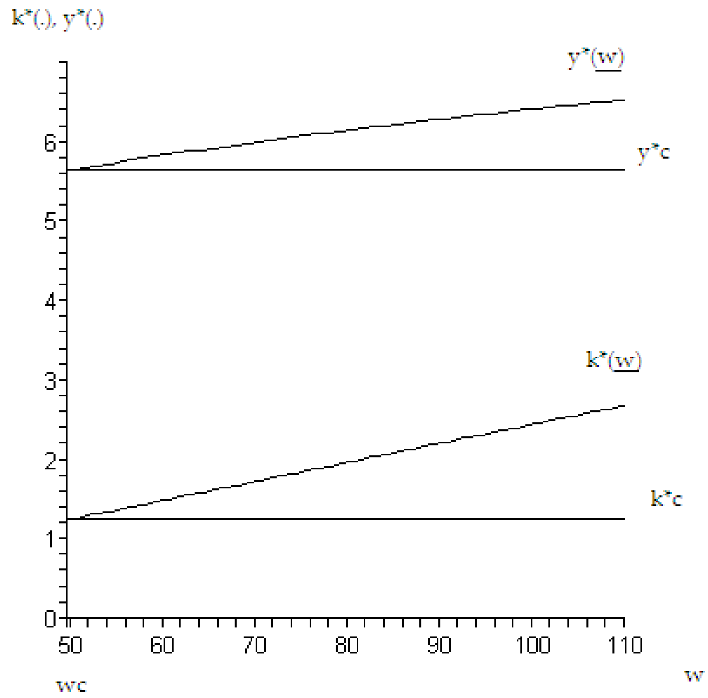


**Figure 1**. The long-run stock of capital and the long-run income in both the market-wage ($w_c$) and regulated-wage ($\underline{w}$) economies. $y^*(\underline{w})$ is scaled 1:10. The starting point of the horizontal axis is the market-clearing wage, that is $w_c = 49.67$. Parameter set: $A = 100$, $\alpha = 0.56$,[23] $\phi = 0.05$, $\gamma = 0.95$, $\rho = 0.10$ and $q = 0.05$.

[23] In order to better clarify the meaning of the coefficient $\alpha$ (the capital share in technology), it is worth noting that a possible interpretation is that the capital stock may be thought in its broad concept, including physical and human components and that the labour input only includes non-specialised labour. In fact, as argued by Mankiw et al. (1992, p. 417), the minimum wage may be thought to be a proxy of the return to labour without human capital; they suggest that since the minimum wage has averaged about 30 to 50 percent of the average wage in manufacturing, then 50 to 70 percent of total labour income represents the return to human capital, so that if the physical capital's share of income is expected to be about 1/3, the human capital's share of income should be between 1/3 and one half. In sum, with the broad view of capital the coefficient $\alpha$ may be fairly about 0.6 and 0.8. Indeed, for instance, Barro-Sala-i-Martin (2003, p. 110) used $\alpha = 0.75$ saying that: "Values in the neighbourhood of 0.75 accord better with the empirical evidence, and these high values of $\alpha$ are reasonable if we take a broad view of capital to include human components".

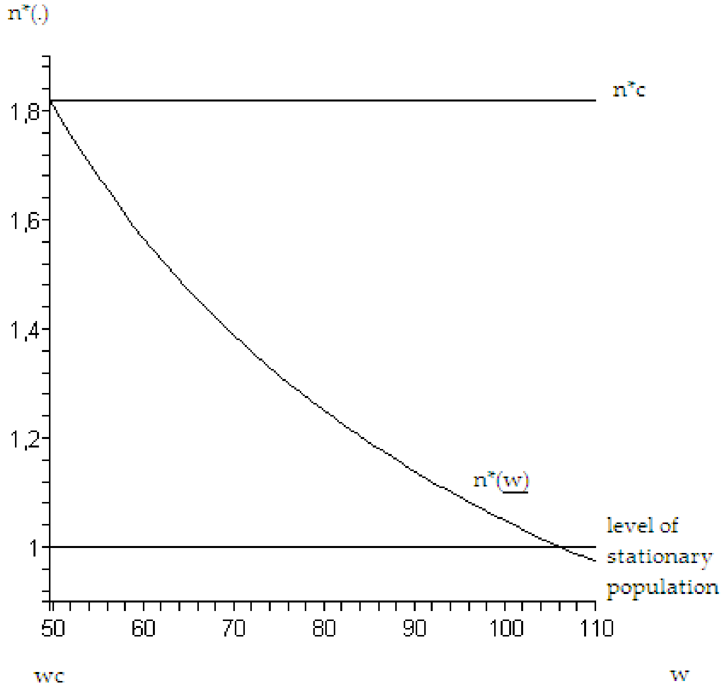


**Figure 2**. The long-run fertility rate in both the market-wage ($w_c$) and regulated-wage ($\underline{w}$) economies. The starting point of the horizontal axis is the market-clearing wage, that is $w_c = 49.67$. Parameter set: $A = 100$, $\alpha = 0.56$, $\phi = 0.05$, $\gamma = 0.95$, $\rho = 0.10$ and $q = 0.05$.

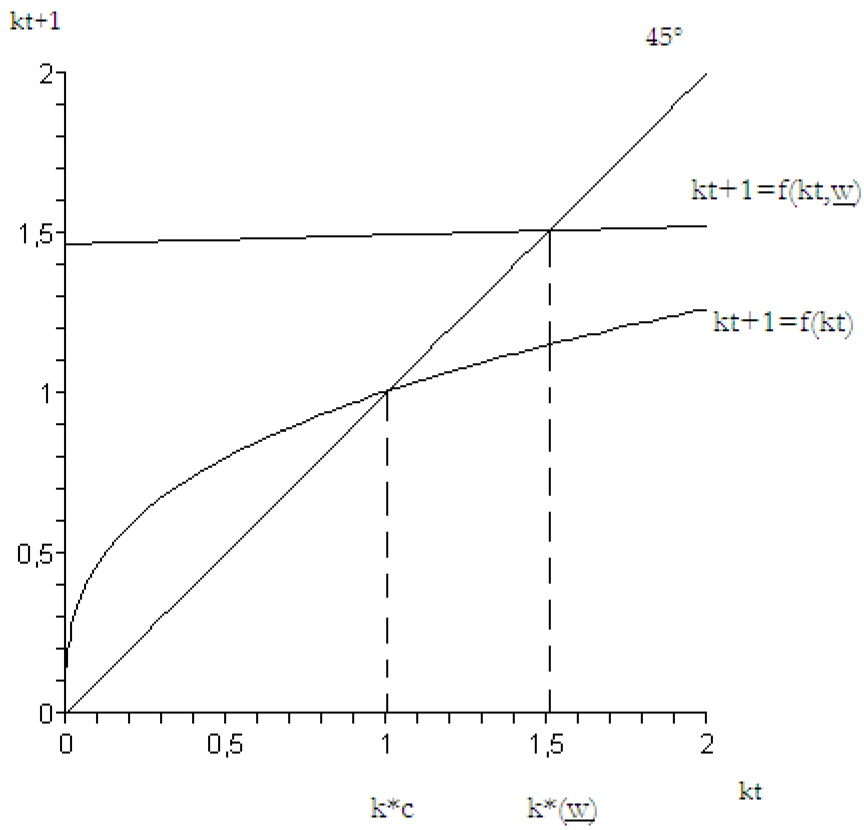


**Figure 3**. The capital accumulation path in the case of both competitive-wage and regulated-wage economies, $k_{t+1} = f(k_t)$ and $k_{t+1} = f(\underline{w}, k_t)$ respectively. The steady-state competitive wage is $w_c = 6.71$ while the minimum wage is $\underline{w} = 8$. Parameter set: $A = 10$, $\alpha = 0.33$, $\phi = 0.15$, $\gamma = 0.90$ and $n = 0$.

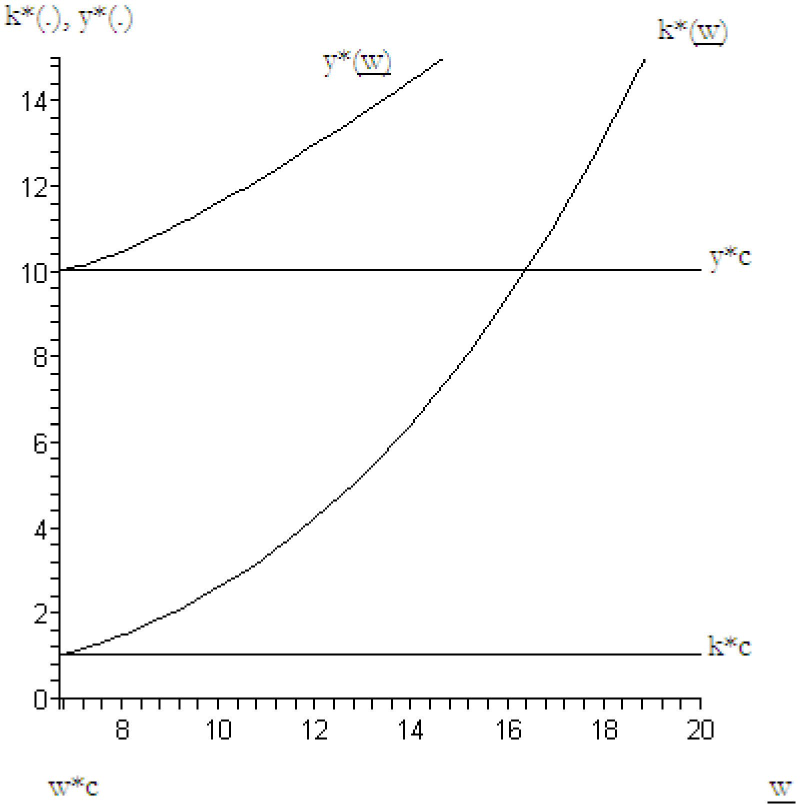


**Figure 4**. The long-run per-capita stock of capital and the long-run per-capita income in both minimum-wage ($k^*(\underline{w})$ and $y^*(\underline{w})$) and competitive-wage ($k^*_c$ and $y^*_c$) economies. The starting point of the horizontal axis is the steady-state competitive wage $w_c = 6.71$. Parameter set: $A = 10$, $\alpha = 0.33$, $\phi = 0.15$, $\gamma = 0.90$ and $n = 0$.

*Summary*: Are the Regulation of Wages and Unemployment always Detrimental for Economic Growth? (J.E.L. E24; H24; J13; O41)

Although the debate about the effects of the regulation of wages is long lasting, little attention has been paid to the role played by minimum wages in inter-temporal contexts with endogenous fertility. This paper investigates such effects within a standard OLG model of neoclassical growth. Some new results, so far escaped closer scrutiny, emerge: introducing a regulated wage may, despite the unemployment occurrence, (i) have a favourable impact on both capital accumulation and output per capita; (ii) reduce the population growth rate. This occurs more likely when a sufficiently high capital share as well as significant unemployment benefits do coexist. Moreover we show that such results also hold even with exogenous fertility and lump-sum taxation on the elderly. Therefore, we conclude that under suitable conditions the Stigler's (1946) result that the regulation of wages always causes a production loss may be violated, that is, in a dynamical context a regulated-wage economy may perform better than a market-wage economy, and the higher is the unemployment rate the higher is the (neoclassical) economic growth. Furthermore, we argue that the minimum wage may also be treated as a policy parameter to control population growth.