The dynamics of bequeathed tastes with endogenous fertility

Abstract

This article shows that endogenous fluctuations are prevented in a general equilibrium economy with overlapping generations and endogenous fertility.

Keywords Aspirations; Fertility; Overlapping generations; Global stability

JEL Classification C61; C62; C68; E32; J22; O41

1 Introduction

In two influential works, de la Croix (1996) and de la Croix and Michel (1999) show that standard-of-living aspirations may be responsible for endogenous and persistent fluctuations in income in a general equilibrium economy with overlapping generations (OLG) and finitely lived agents. Aspirations, in fact, are a negative externality affecting savings and consumption of the working generation. In the words of de la Croix (1996, p. 89): "If children inherit life standard aspirations from their parents, then their savings are affected and cycles may appear in OLG models with production. At some point of an expansion, aspirations grow faster than wages, savings decrease, and a contraction begins." However, de la Croix (1996) and de la Croix and Michel (1999) do not include endogenous fertility in their models. According to the most part of the recent literature stimulated by the new home economics, the number of children comes from a rational choice based on a comparison between economic constraints and incentives. In a world where tastes are inherited the issue of endogenous fertility becomes of particular importance given the existing influence on consumption decisions of children caused by the behaviour of parents (external habits). This article shows that by accounting for endogenous fertility (weak altruism towards children), life standard aspirations are no more a source of instability and fluctuations as the stationary equilibrium of the resulting two-dimensional discrete time map is *globally stable* (i.e., there are no converging or ever lasting cycles). This is because aspirations reduce saving and fertility in the same way, so that capital accumulation is independent of the intensity of inherited tastes. In addition, standard-of-living aspirations may represent an alternative explanation for the declining fertility observed in developed countries in the last decades, as they are an element that favours the substitution of fertility for the consumption of material goods. This is in line with the results recently obtained by Kaneko et al. (2016).

The rest of the paper proceeds as follows. Section 2 (resp. 3) characterises a model with inherited tastes, endogenous fertility and logarithmic (resp. CIES) preferences. Section 4 concludes.

2 The model

The model is an extension of de la Croix (1996) augmented with endogenous fertility, as in Galor and Weil (1996). The OLG closed economy is populated by rational and identical individuals of measure N_t per generation ($t = 0, 1, 2, ...$). Life of the typical agent is divided into childhood and adulthood. An individual makes economic decisions only when he is adult. Labour supply is inelastic and normalised to one. The lifetime budget constraint of an individual belonging to generation t is:

$$c_{1,t} + \frac{c_{2,t+1}}{R^e_{t+1}} = w_t(1 - qn_t), \tag{1}$$

where $c_{1,t}$ and $c_{2,t+1}$ are material consumption when young and when old, R^e_{t+1} is the expected interest factor, $0 < q < 1$ is the fraction of income required to care about one child and n_t is the number of children. Generation t inherits life standard aspirations (h_t) from the previous generation (parent). By assuming logarithmic preferences, the lifetime utility function of the individual representative of generation t is given by:

$$U_t = \ln(c_{1,t} - \gamma h_t) + \beta \ln(c_{2,t+1}) + \phi \ln(n_t), \tag{2}$$

where $0 < \gamma < 1$ is the intensity of aspirations, $0 < \beta < 1$ is the subjective discount factor and $\phi > 0$ is the relative taste for (the quantity of) children. Maximisation of (2) subject to (1) yields:

$$n_t = \frac{\phi(w_t - \gamma h_t)}{qw_t(1 + \beta + \phi)}, \tag{3}$$

and

$$s_t = \frac{\beta(w_t - \gamma h_t)}{1 + \beta + \phi}. \tag{4}$$

By looking at (3) and (4), it is clear that aspirations negatively affect saving and fertility. Consumption when young is:

$$c_{1,t} = \frac{w_t + \gamma h_t(\beta + \phi)}{1 + \beta + \phi}. \tag{5}$$

As material consumption and fertility are normal goods, the presence of inherited tastes makes the achievement of high standards in consumption important. This favours the substitution of fertility for consumption to get higher utility. This result is in line with Kaneko et al. (2016), who study an OLG model of endogenous growth with transitional dynamics, aspirations and endogenous fertility.

Identical and competitive firms produce output Q_t by employing a technology encompassing the case of constant-returns-to-scale in production and the AK set up. For doing this, we adopt a kind of production function formerly introduced by Jones and Manuelli (1990) and subsequently used, amongst others, by Rebelo (1991). By following the formulation used by Barro and Sala-i-Martin (2003), the production function is:

$$Q_t = AK_t^{\alpha} L_t^{1-\alpha} + BK_t, \quad A > 0, \quad B \geq 0, \quad 0 < \alpha < 1, \tag{6}$$

where K_t and $L_t = N_t$ are capital and labour inputs. Notice that when $B = 0$ then (6) boils down to the standard Cobb-Douglas technology with constant-returns-to-scale adopted by de la Croix (1996). Production function (6) allows to get endogenous growth with transitional dynamics, as the growth rate diminishes as the economy develops. Profit maximisation allows to get the usual equilibrium conditions:

$$w_t = (1-\alpha)(Ak_t^\alpha + Bk_t), \tag{7}$$

$$R_t = \alpha(Ak_t^{\alpha-1} + B), \tag{8}$$

where $k_t = K_t/N_t$ is capital per worker.

Aspirations of generation t are $h_t = c_{1,t-1}$, reflecting "the idea that children become habituated to a certain life standard when they still live with their parents" (de la Croix, 1996, p. 91). This statement makes the difference depending on whether one assumes exogenous fertility or endogenous fertility. In fact, given the market clearing condition in the capital market $n_t k_{t+1} = s_t$ $(N_{t+1} = n_t N_t)$, endogenous fertility implies that capital accumulation is independent of aspirations. By taking the couple of non-negative initial endowments (k_0, h_0) as given, equilibrium implies

$$T = \begin{cases} k_{t+1} = \frac{\beta q}{\phi} w_t = \frac{\beta q}{\phi}(1-\alpha)[Ak_t^\alpha + Bk_t] \\ h_{t+1} = c_{1,t} = \frac{w_t + \gamma h_t(\beta+\phi)}{1+\beta+\phi} = \frac{(1-\alpha)[Ak_t^\alpha + Bk_t] + \gamma h_t(\beta+\phi)}{1+\beta+\phi} \end{cases}. \tag{9}$$

The most important difference between a model with endogenous fertility and a model with exogenous fertility is that in the former case aspirations do not affect the accumulation of capital, whereas in the latter case they do. This is because aspirations reduce saving and fertility and promote an increase in consumption to maintain high standards. The reduction in saving reduces capital accumulation, whereas the reduction in fertility increases capital accumulation. These two opposite forces cancel exactly each other out so that aspirations are neutral on capital accumulation.

2.1 Global convergence

Let us rewrite the continuous and differentiable map $T : \mathbb{R}_+^2 \to \mathbb{R}_+^2$ as follows:

$$T = \begin{cases} x' = f(x) = m_0 x^\alpha + m_1 x \\ y' = g(x,y) = \frac{m_2 x^\alpha + m_3 x + \gamma(\beta+\phi) y}{1+\beta+\phi} \end{cases}, \tag{10}$$

where $x' = k_{t+1}$, $x = k_t$, $y' = h_{t+1}$, $y = h_t$, $m_0 = \frac{\beta q}{\phi}(1-\alpha)A > 0$, $m_1 = \frac{\beta q}{\phi}(1-\alpha)B \geq 0$, $m_2 = (1-\alpha)A > 0$ and $m_3 = (1-\alpha)B \geq 0$.

Proposition 1. *Let T be given by (10) and define $x^* = \left(\frac{m_0}{1-m_1}\right)^{\frac{1}{1-\alpha}}$. Set $I_0 = \{(x,y) \in \mathbb{R}_+^2 : x = 0\}$ is invariant for T; if $B < \frac{\phi}{\beta q(1-\alpha)} = \bar{B}$ also set $I_{x^*} = \{(x,y) \in \mathbb{R}_+^2 : x = x^*\}$ is invariant for T.*

Proof. The proof simply follows from by considering that T is triangular and that $x = 0$ is a fixed point of map f for all parameter values while $x = x^*$ is the unique positive fixed point of f iff $B < \bar{B}$. Hence the restriction of system T to the vertical lines $x = 0$ and $x = x^*$ is trapping on that lines. □

Let us assume that $B < \bar{B}$ (i.e. B is fixed at not too high a level, as in the case of Cobb-Douglas production function). Proposition 1 states that if at $t = 0$ capital per worker $x(0)$ is equal to zero or equal to x^* then it will not change over time. Therefore, long-term dynamics of T on the two sets I_0 and I_{x^*} are fully obtained by the one-dimensional maps $g_0 = g(0, y)$ and $g_{x^*} = g(x^*, y)$ describing the evolution of aspirations. Since $x = 0$ and $x = x^*$ are fixed points of f, if T admits steady states, then they must belong to I_0 or to I_{x^*}. One can verify that g_0 admits a unique fixed point given by $y = 0$, whereas g_{x^*} admits a unique fixed point

$$y^* = \frac{m_2(x^*)^\alpha + m_3 x^*}{1 + (1-\gamma)(\beta + \phi)}.$$

Proposition 2. *The origin $E_0 = (0, 0)$ is a fixed point of T for all parameter values. T admits a unique interior fixed point $E^* = (x^*, y^*)$ iff $B < \bar{B}$.*

About the interior equilibrium the following remark holds.

Remark 3.
- *The intensity of aspirations (γ) does not affect capital per worker at the equilibrium, but it positively affects the equilibrium level of aspirations.*
- *The cost of children q (resp. the relative taste for children ϕ) positively affects (resp. negatively affects) both capital and aspirations at the equilibrium.*

The dynamics of T along the invariant lines I_0 and I_{x^*} (when $B < \bar{B}$) are described by the linear and strictly increasing maps g_0 and g_{x^*} whose slopes are smaller than one. Then, E_0 (resp. E^*) attracts all trajectories starting from I_0 (resp. I_{x^*}) whatever the initial value of aspirations. Notice that the speed of convergence increases as γ decreases. In order to discuss the global dynamics of system $(T, \mathbb{R}^2_+)$, we consider the Jacobian matrix $J(x, y)$ associated to T and we recall that, since T is triangular, the eigenvalues of $J(x, y)$ are real and given by:

$$\lambda_1(x) = f'(x) = \frac{\alpha m_0}{x^{1-\alpha}} + m_1 \ \text{ and } \ \lambda_2(x, y) = \frac{\partial g}{\partial y}(x, y) = \frac{\gamma(\beta + \phi)}{1 + \beta + \phi}.$$

Proposition 4. *The invariant set I_0 is repelling.*

Proof. Since $\lim_{x \to 0^+} \lambda_1(x) = +\infty$ then I_0 is repelling and consequently initial conditions with positive capital per worked cannot converge to set I_0. □

Since economic meaningful initial states are those characterised by $x(0) > 0$ we also observe that as long as $x(0) > 0$ then $T(x(0), 0) = (x(1), y(1))$ with $x(1) > 0$ and $y(1) > 0$, that is an initial condition taken on the x-axis exits the x-axis at the first iteration. Furthermore, no interior points can be mapped on the x-axis as equation $g(x, y) = 0$ has a unique solution given by E_0. Then, we can restrict the study to system (T, D), where T is given by (10) and $D = (0, +\infty) \times (0, +\infty)$.

Proposition 5. *If $B < \bar{B}$ then E^* is globally stable for system (T, D) while $\forall B \geq \bar{B}$ system (T, D) does not admit bounded trajectories.*

Proof. If $B < \bar{B}$ the fixed point x^* is globally attracting for map f defined in $(0, +\infty)$ so that the attractor of (T, D) must belong to set I_{x^*}. The dynamics of (T, D) on the set I_{x^*} are described by the one-dimensional map g_{x^*} having a unique fixed point y^* which is globally stable. If $B \geq \bar{B}$ all initial conditions $x(0) > 0$ produce trajectories diverging to $+\infty$. □

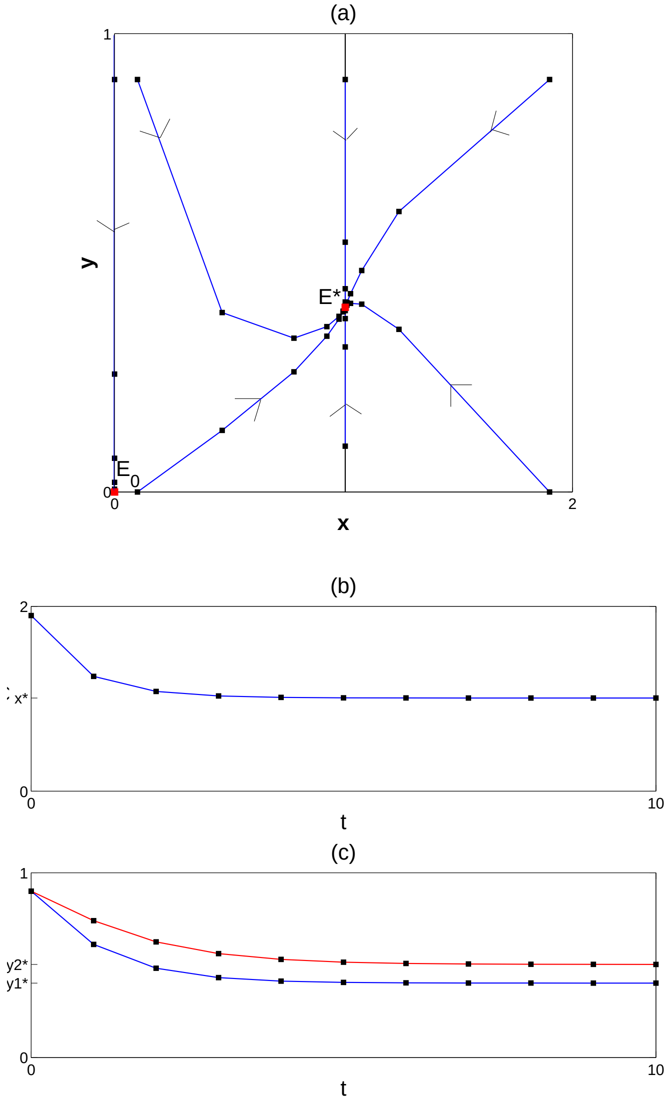

Figure 1: (a) Phase portrait of T for $B < \bar{B}$, the two fixed points are depicted in red. (b) Capital per capita versus time. (c) Aspiration versus time for a given γ in blue and for a greater γ in red.

Notice that in our model fluctuations are avoided, while with Cobb-Douglas technology the interior equilibrium attracts all economically meaningful trajectories (i.e., $x(0) > 0$). Unbounded growth is obtained with larger values of B.

The qualitative dynamics of T on $\mathbb{R}^2_+$ are summarised in Figure 1 (a). Figures 1 (b) and (c) depict the sequence of capital per worker and aspiration levels over time for a given initial condition. When γ increases the trajectory in (b) does not change whereas the one related to aspirations do.

3 CIES preferences and Cobb-Douglas technology

This section considers the maximisation problem of a representative individual with a Constant Inter-temporal Elasticity of Substitution (CIES) utility function (describing preferences with respect to material consumption over the life cycle and fertility), and a representative firm producing with a Cobb-Douglas technology ($B = 0$).[1] Therefore,

$$\max_{c_{1,t},c_{2,t+1},n_t} U_t = \left[1-\frac{1}{\sigma}\right]^{-1}(c_{1,t}-\gamma h_t)^{1-\frac{1}{\sigma}} + \beta\left[1-\frac{1}{\sigma}\right]^{-1}(c_{2,t+1})^{1-\frac{1}{\sigma}} + \phi\left[1-\frac{1}{\sigma}\right]^{-1}(n_t)^{1-\frac{1}{\sigma}}, \tag{11}$$

subject to (1), where $\sigma > 0$ ($\sigma \neq 1$). Then,

$$s_t = \frac{\beta^\sigma (R^e_{t+1})^{\sigma-1}(w_t - \gamma h_t)}{1+\beta^\sigma (R^e_{t+1})^{\sigma-1} + \phi^\sigma (q w_t)^{1-\sigma}}, \tag{12}$$

$$n_t = \frac{\phi^\sigma (w_t - \gamma h_t)}{(q w_t)^\sigma \left[1+\beta^\sigma (R^e_{t+1})^{\sigma-1} + \phi^\sigma (q w_t)^{1-\sigma}\right]}, \tag{13}$$

$$c_{1,t} = \frac{w_t + \gamma h_t \left[\beta^\sigma (R^e_{t+1})^{\sigma-1} + \phi^\sigma (q w_t)^{1-\sigma}\right]}{1+\beta^\sigma (R^e_{t+1})^{\sigma-1} + \phi^\sigma (q w_t)^{1-\sigma}}, \tag{14}$$

$$c_{2,t+1} = R^e_{t+1} s_t. \tag{15}$$

The dynamic system becomes:

$$T_{CIES} = \begin{cases} k_{t+1} = \frac{s_t}{n_t} = \left(\frac{\beta}{\phi}\right)^\sigma (R^e_{t+1})^{\sigma-1} (q w_t)^\sigma \\ h_{t+1} = c_{1,t} = \frac{w_t + \gamma h_t \left[\beta^\sigma (R^e_{t+1})^{\sigma-1} + \phi^\sigma (q w_t)^{1-\sigma}\right]}{1+\beta^\sigma (R^e_{t+1})^{\sigma-1} + \phi^\sigma (q w_t)^{1-\sigma}} \end{cases}, \tag{16}$$

where $w_t = (1-\alpha)A k_t^\alpha$ in the Cobb-Douglas case. Perfect foresight implies that $R^e_{t+1} = \alpha A k_{t+1}^{\alpha-1}$. Then,

$$k_{t+1} = \left(\frac{\beta q}{\phi}\right)^{\frac{\sigma}{\sigma(1-\alpha)+\alpha}} \alpha^{\frac{\sigma-1}{\sigma(1-\alpha)+\alpha}} (1-\alpha)^{\frac{\sigma}{\sigma(1-\alpha)+\alpha}} A^{\frac{2\sigma-1}{\sigma(1-\alpha)+\alpha}} k_t^{\frac{\alpha\sigma}{\sigma(1-\alpha)+\alpha}} = f_1(k_t). \tag{17}$$

Define

$$V := \left(\frac{\beta q}{\phi}\right)^{\frac{\sigma}{\sigma(1-\alpha)+\alpha}} \alpha^{\frac{\sigma-1}{\sigma(1-\alpha)+\alpha}} (1-\alpha)^{\frac{\sigma}{\sigma(1-\alpha)+\alpha}} A^{\frac{2\sigma-1}{\sigma(1-\alpha)+\alpha}}.$$

Then, the evolution of aspirations is described by

[1] In this section we do not consider positive values of B as it is not possible to obtain a closed-form expression for the dynamics of capital, $k_{t+1} = f(k_t)$, in such a case. An analysis of more general production functions may be the subject of future research.

$$h_{t+1} = g_1(k_t, h_t) = \frac{(1-\alpha)Ak_t^\alpha + \gamma h_t \psi(k_t)}{1+\psi(k_t)}$$

where

$$\psi(k_t) = \beta^\sigma(\alpha A V^{\alpha-1} k_t^{a(\alpha-1)})^{\sigma-1} + \phi^\sigma(q(1-\alpha)Ak_t^\alpha)^{1-\sigma}.$$

By using the same notation introduced in the previous section one yields:

$$T_1 = \begin{cases} x' = f_1(x) = Vx^a \\ y' = g_1(x,y) = \frac{(1-\alpha)Ax^\alpha + \gamma y \psi(x)}{1+\psi(x)} \end{cases}. \tag{18}$$

Observe that $a = \frac{\alpha\sigma}{\sigma(1-\alpha)+\alpha} \in (0,+\infty)$ and that $\psi(x) > 0$, $\forall x > 0$. We analyse the global dynamics of system (18) restricted to $D = (0,+\infty) \times (0,+\infty)$.

Proposition 6. *If $\alpha \leq 1/2$ or $\alpha > 1/2$ whenever $\sigma < \frac{\alpha}{2\alpha-1}$ then system (T_1, D) admits a unique globally stable fixed point.*

Proof. Consider (T_1, D) and assume that $(\alpha, \sigma) \in \Omega$ where

$$\Omega = \left\{ (\alpha,\sigma) \in (0,1) \times (0,+\infty) : (\alpha \leq 1/2) \text{ or } \left(\alpha > 1/2 \text{ and } \sigma < \frac{\alpha}{2\alpha - 1} \right) \right\}.$$

Then $a \in (0,1)$ and $x = f_1(x)$ admits a unique positive solution x_1^*. Being $\psi(x_1^*) > 0$, then $y = g_1(x_1^*, y)$ admits a unique positive solution y_1^*. The point $E_1^* = (x_1^*, y_1^*)$ is the unique fixed point of (T_1, D). To prove the global stability of E_1^* the same arguments used to prove Proposition 5 can be considered. □

According to Proposition 6,

(i) if $\alpha \leq 1/2$ for any given values of σ, or

(ii) if $\sigma < 1 + \frac{1-\alpha}{2\alpha-1}$ for any $\alpha > 1/2$,

all economic meaningful trajectories converge to the unique interior steady state. When the conditions on parameters stated in Proposition 6 are not fulfilled, the dynamics of (T_1, D) have less economic interest as the interior equilibrium disappears or divergent patterns are produced.

4 Conclusions

This article has shown that an OLG economy with aspirations augmented with endogenous fertility is globally stable. This is in sharp contrast with the exogenous fertility model developed by de la Croix (1996) and de la Croix and Michel (1999).

Conflict of Interest The authors declare that they have no conflict of interest.

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