

## A note on endogenous fertility, child allowances and poverty traps

Luciano Fanti\* and Luca Gori\*\*

*Department of Economics, University of Pisa, Via Cosimo Ridolfi, 10, I-56124 Pisa (PI), Italy*

*Department of Law and Economics "G.L.M. Casaregi", University of Genoa, Via Balbi, 30/19, I-16126 Genoa (GE), Italy*

**Abstract** This paper analyses how capital accumulation and fertility react to a child allowance policy in an overlapping generations model of growth with endogenous fertility. Multiple equilibria are shown to exist depending on the size of the child allowance.

**Keywords** Child allowance; Endogenous fertility; OLG model; Poverty trap

**JEL Classification** H24; J13; J18

### 1. Introduction

In the past few decades, several developed countries experienced a sharp decrease in fertility. This fact has contributed to raise debates between economists and politicians on the

---

\* E-mail addresses: [lfanti@ec.unipi.it](mailto:lfanti@ec.unipi.it) or [fanti.luciano@gmail.com](mailto:fanti.luciano@gmail.com); tel.: +39 050 22 16 369; fax: +39 050 22 16 384.

\*\* Corresponding author. E-mail addresses: [luca.gori@unige.it](mailto:luca.gori@unige.it) or [dr.luca.gori@gmail.com](mailto:dr.luca.gori@gmail.com); tel.: +39 010 209 95 03; fax: +39 010 209 55 36.

effectiveness of family policies to promote population growth. Indeed, there seems to be a general scepticism in economic literature as to whether such policies can effectively affect the fertility rate. For instance, the final effect can be different depending on whether childcare facilities (e.g., investments in infrastructure for day-care centres, schools and so on) or child allowances are used (see Apps and Rees, 2004).

Although population policies are currently high on the political agenda in several Western countries, few theoretical contributions have dealt with this topic in a dynamic general equilibrium framework (see, e.g., Momota, 2000; van Groezen et al., 2008; Fanti and Gori, 2009, 2010).

In this note we consider a general equilibrium model of neoclassical growth with overlapping generations (OLG) and endogenous fertility (see Galor and Weil, 1996), to analyse the effects of the public provision of child allowances on capital accumulation and the long-run demand for children.

The model by Galor and Weil (1996) allows for the possibility of multiple equilibria because a production function with constant elasticity of substitution (CES) is assumed. Conversely, in this paper a Cobb-Douglas technology is considered and multiple regimes of development are possible depending on the relative size of the child allowance. Indeed, if the child allowance is fixed at too high a level, an economy can permanently be entrapped into poverty. Moreover, such a policy instrument can be responsible either of a Malthusian or Modern Fertility Regime and, rather interestingly, the effect of raising the child allowance on the long-run demand for children may be ambiguous when, in particular, it is smaller than the size of the fixed cost of children.

The rest of the paper is organised as follows. Section 2 builds on the model and discusses the main results. Section 3 concludes.

## 2. The model

Consider an OLG closed economy populated by perfectly rational and identical individuals that have preferences towards material consumption and the number of children. Life is divided between childhood and adulthood. Economic decisions are taken in the latter period of life, which is in turn divided between youth (working period) and old age (retirement period). Each young adult individual is endowed with one unit of labour inelastically supplied to firms and earns wage  $w$ . The budget constraint of the young of generation  $t$  ( $N_t$ ) reads as:

$$c_{1,t} + s_t + (\eta_t - \beta)n_t = w_t - \tau_t, \quad (1.1)$$

that is, the disposable income is divided among consumption ( $c_{1,t}$ ), saving ( $s_t$ ) and the net cost of raising  $n_t$  descendants, with  $\tau_t > 0$  being a lump-sum tax to finance the fixed per child allowance  $\beta > 0$ . The cost of raising a child ( $\eta_t$ ) is  $\eta_t := m + qw_t$ , to capture both the consumption and time needed to care for children (see Boldrin and Jones, 2002).

When old, individuals retire and consume ( $c_{2,t+1}$ ) the resources saved when young plus the expected interest accrued from time  $t$  to time  $t+1$  at rate  $r^e_{t+1}$ , i.e.:

$$c_{2,t+1} = (1 + r^e_{t+1})s_t. \quad (1.2)$$

The individual representative of generation  $t$  chooses fertility and saving to maximise the lifetime utility function

$$U_t = (1 - \phi)\ln(c_{1,t}) + \gamma\ln(c_{2,t+1}) + \phi\ln(n_t), \quad (2)$$

subject to Eqs. (1.1) and (1.2), where  $0 < \gamma < 1$  and  $0 < \phi < 1$ . The first order conditions for an interior solution are:

$$\frac{c_{2,t+1}}{c_{1,t}} \cdot \frac{1-\phi}{\gamma} = 1 + r^e_{t+1}, \quad (3.1)$$

$$\frac{c_{1,t}}{n_t} \cdot \frac{\phi}{1-\phi} = q w_t + m - \beta. \quad (3.2)$$

Eq. (3.1) equates the marginal rate of substitution between young-age consumption and old-age consumption to the expected factor of interest. Eq. (3.2) equates the marginal rate of substitution between young-age consumption and the number of children to the marginal cost of raising an additional child. It is important to note that Eq. (3.2) requires that the net cost of children must be positive to guarantee a finite positive solution for  $n_t$ .

Combining Eqs. (1.1), (1.2), (3.1) and (3.2) gives:

$$n_t = \frac{\phi}{1+\gamma} \cdot \frac{w_t - \tau_t}{q w_t + m - \beta}, \quad (4.1)$$

$$s_t = \frac{\gamma}{1+\gamma} (w_t - \tau_t). \quad (4.2)$$

In every period the government runs a balanced-budget child allowance policy. The total childcare expenditure at time  $t$ ,  $\beta n_t N_t$ , is constrained by the amount of tax revenues  $\tau_t N_t$ . Therefore the government budget identity in per worker terms can be expressed as follows:

$$\beta n_t = \tau_t. \quad (5)$$

Substituting out Eq. (5) into Eqs. (4.1), (4.2) for  $\tau_t$  gives the following demand for children and saving, respectively:

$$n_t = \frac{\phi w_t}{(1+\gamma)(q w_t + m - \beta) + \phi \beta}, \quad (6)$$

$$s_t = \frac{\gamma(q w_t + m - \beta) w_t}{(1+\gamma)(q w_t + m - \beta) + \phi \beta}. \quad (7)$$

Since Eq. (7) shows that saving does not depend on the interest rate, the assumption on the type of expectation formation (i.e., myopic or rational) of individuals does not matter.

Firms are identical and markets are competitive. The (aggregate) constant returns to scale Cobb-Douglas technology is  $Y_t = A K_t^\alpha L_t^{1-\alpha}$ , where  $Y_t$ ,  $K_t$  and  $L_t = N_t$  are output, capital and the time- $t$  labour input, respectively,  $A > 0$  and  $0 < \alpha < 1$ . Assuming that capital fully depreciates at the end of every period and output is sold at unit price, profit maximisation implies:

$$r_t = \alpha A k_t^{\alpha-1} - 1, \quad (8)$$

$$w_t = (1-\alpha) A k_t^\alpha, \quad (9)$$

where  $k_t := K_t / N_t$  is the per worker stock of capital.

Given the government budget Eq. (5) and knowing that  $N_{t+1} = n_t N_t$ , the equilibrium on the capital market is:

$$n_t k_{t+1} = s_t. \quad (10.1)$$

Now, using Eqs. (6), (7), (9) and (10.2) we get:

$$k_{t+1} = f(k_t, \beta) = \frac{\gamma}{\phi} \left[ (m - \beta) + q(1 - \alpha) A k_t^\alpha \right]. \quad (10.2)$$

Steady-states of the time map Eq. (10.2) are defined as  $k_{t+1} = k_t = k^*$ . We now examine how an increase in the child allowance affects both capital accumulation and long-run fertility by

distinguishing the cases when  $\beta$  is higher and when it is lower than the fixed cost of children,  $m$ .

### 2.1. Case $m - \beta > 0$

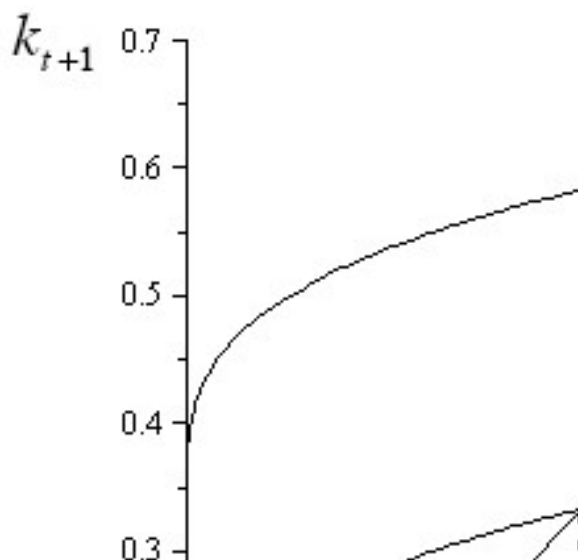
**Proposition 1.** *Let  $\beta < m$  hold. Then, there exists one and only one locally asymptotically stable steady-state ( $k^* > 0$ ).*

**Proof.** If  $\beta < m$ , then the intercept of the function  $f(k_t, \beta)$  is strictly positive. Since  $f(k_t, \beta)$  is a concave monotonic increasing function of  $k_t$ , then there exists one and only one (locally asymptotically stable) fixed point. **Q.E.D.**

**Proposition 2.** *Let  $\beta < m$  hold. Then, a rise in the child allowance reduces  $k^*$ .*

**Proof.** Since the intercept of the function  $f(k_t, \beta)$  negatively depends on the child allowance, then a rise in  $\beta$  shifts the capital accumulation locus downward, while keeping its slope unchanged. **Q.E.D.**

Using the following constellation of parameter:  $A = 10$ ,  $\alpha = 0.33$ ,  $\phi = 0.3$ ,  $\gamma = 0.15$ ,  $m = 1$ ,<sup>1</sup> Figure 1 shows that if  $\beta$  increases from  $\beta' = 0.3$  to  $\beta'' = 0.8$ , the capital accumulation locus Eq. (10.2) shifts downward and  $k^*$  reduces from  $k^* = 0.6389$  ( $\partial k_{t+1} / \partial k_t \big|_{k_t = k^*} = 0.1492$ ) to  $k^{**} = 0.333$  ( $\partial k_{t+1} / \partial k_t \big|_{k_t = k^{**}} = 0.2309$ ).



**Figure 1.** Case  $\beta < m$ . Phase map  $f(k_t, \beta)$  when  $\beta$  varies ( $q = 0.1$ ).

Now, let the demand for children Eq. (6) be expressed by the following generic function of  $\beta$ :

<sup>1</sup> Note that this parameter set is also subsequently in the paper (Figure 2 and Tables 1 and 2).

$$n^* = n^* \{ \beta, w^* [k^*(\beta)] \} = \frac{\phi w^* [k^*(\beta)]}{(1 + \gamma) \{ q w^* [k^*(\beta)] + m - \beta \} + \phi \beta}. \quad (11)$$

Then, we have the following proposition.

**Proposition 3.** *Let  $\beta < m$  hold. Then a rise in the child allowance ambiguously affects the long-run fertility rate.*

**Proof.** Consider the total derivative of Eq. (11) with respect to  $\beta$ :

$$\frac{dn^*}{d\beta} = \underbrace{\frac{\overset{+}{\partial n^*}}{\partial \beta} + \frac{\overset{+}{\partial n^*}}{\partial w^*} \cdot \frac{\overset{+}{\partial w^*}}{\partial k^*} \cdot \frac{\overset{-}{\partial k^*}}{\partial \beta}}_{+/-}$$

Then, Proposition 3 follows. **Q.E.D.**

Proposition 3 reveals that the relationship between fertility and the child allowance is ambiguous in the long run. The economic intuition is as follows: a rise in  $\beta$  directly reduces the marginal cost of children and then increases fertility through this channel, while also causing a reduction in saving (because of the increased resources devoted to child-bearing purposes) and thus indirectly reduces capital accumulation and wages. Since fertility is positively related with the wage when  $\beta < m$  (Malthusian Fertility),<sup>2</sup> then it tends to be reduced through such an indirect channel.

To sum up, given the above two counterbalancing effects, the long-run fertility rate may be a positive monotonic, hump-shaped or negative monotonic function of the child allowance.

Table 1 illustrates Proposition 3 for two different values of the percentage of child-rearing cost on working income ( $q$ ). If  $q$  is fairly high (Table 1.A), the positive effect on fertility of the reduced cost of children overcompensates the negative one due to the reduced capital accumulation and wages. In contrast, if  $q$  is fairly low (Table 1.B), fertility is hump-shaped in  $\beta$ .<sup>3</sup>

**Table 1.A.** Case  $\beta < m$ . Steady-state stock of capital and fertility when  $\beta$  varies ( $q = 0.1$ ).

$\beta$	0	0.30	0.6	0.7	0.8	0.85	0.9	0.95	0.97	0.99
$k^*$	0.8128	0.6389	0.4591	0.3969	0.333	0.3002	0.2665	0.2317	0.2174	0.2029
$n^*$	1.004	1.1116	1.2578	1.3194	1.3899	1.4291	1.4713	1.5165	1.5355	1.5548

**Table 1.B.** Case  $\beta < m$ . Steady-state stock of capital and fertility when  $\beta$  varies ( $q = 0.03$ ).

$\beta$	0	0.3	0.6	0.7	0.8	0.85	0.9	0.95	0.97	0.99
$k^*$	0.5841	0.4258	0.2648	0.21	0.1542	0.1256	0.0964	0.0659	0.0531	0.0396

<sup>2</sup> See the Appendix for details on the relationship between fertility and income, and where it is also shown that while when  $\beta < m$  fertility is always of the Malthusian type, when  $\beta > m$  the relationship between fertility and income may also be negative, i.e., Modern Fertility.

<sup>3</sup> Note that we have chosen the value  $\alpha = 0.33$ , as is usual in literature (see, e.g., Gollin, 2002). In such a case, the negative monotonic relationship between fertility and the child allowance do not appear when  $\beta < m$ , while being observed when  $\beta > m$  (see Table 2.C). Nevertheless, when  $\alpha$  is fairly high (as in developing countries, see, e.g., Kraay and Raddatz, 2007) and  $q$  is low, then a negative monotonic relationship between fertility and the child allowance can be observed even when  $\beta < m$ .

$n^*$	1.2528	1.4180	1.643	1.7328	1.8237	1.8633	1.8888	1.8765	1.8467	1.7846
-------	--------	--------	-------	--------	--------	--------	--------	--------	--------	--------

## 2.2. Case $m - \beta < 0$

**Proposition 4.** Let  $\beta > m$  hold. Then, (1) if  $\beta < \beta_T$  there exist two positive steady states,  $k^*_L < k^*_H$ , the former is locally unstable and the latter is locally asymptotically stable; (2) if  $\beta = \beta_T$  a tangent bifurcation emerges and the unique steady state  $\bar{k} > 0$  is neither an attractor nor a repellor; (3) if  $\beta > \beta_T$  no positive steady states exist.

**Proof.** If  $\beta > m$ , then the intercept of the function  $f(k_t, \beta)$  is negative. Now, differentiating Eq. (10.2) with respect to  $k_t$  gives:

$$\frac{\partial k_{t+1}}{\partial k_t} = \frac{\gamma}{\phi} q \alpha (1 - \alpha) A k_t^{\alpha-1}. \quad (12)$$

By equating the right-hand side of Eq. (12) to unity and solving for  $k_t$ , one obtains:

$$\bar{k} = \left[ \alpha (1 - \alpha) A q \frac{\gamma}{\phi} \right]^{\frac{1}{1-\alpha}}, \quad (13)$$

so that  $\frac{\partial k_{t+1}}{\partial k_t} > 1$  ( $< 1$ ) for any  $k_t < \bar{k}$  ( $k_t > \bar{k}$ ). Combining Eqs. (10.2) and (13) yields:

$$\bar{k} = \frac{\gamma}{\phi} \left[ (m - \beta) + q(1 - \alpha) A (\bar{k})^\alpha \right], \quad (14)$$

which is solved for  $\beta$  to obtain:

$$\beta_T = m - \frac{\phi}{\gamma} \bar{k} + q(1 - \alpha) A (\bar{k})^\alpha, \quad (15)$$

where  $\beta_T > m$ . Exploiting Eqs. (10.2) and (15) gives:

$$k_{t+1} = f(k_t, \beta_T) = \frac{\gamma}{\phi} \left[ (m - \beta_T) + q(1 - \alpha) A k_t^\alpha \right], \quad (16)$$

which defines the tangent locus to the 45° line at point  $\bar{k}$ , where  $\frac{\partial k_{t+1}}{\partial k_t} \Big|_{k_t=\bar{k}} = 1$ . Since  $\frac{\partial k_{t+1}}{\partial \beta} < 0$ , then: (1) if  $\beta < \beta_T$ ,  $f(k_t, \beta)$  lies above  $f(k_t, \beta_T)$  for any  $k_t$ . Since the  $f(k_t, \beta)$  is a concave monotonic increasing function of  $k_t$ , then two steady states  $k^*_L < \bar{k}$  and  $k^*_H > \bar{k}$  do exist, where  $\frac{\partial k_{t+1}}{\partial k_t} \Big|_{k_t=k^*_L} > 1$  and  $\frac{\partial k_{t+1}}{\partial k_t} \Big|_{k_t=k^*_H} < 1$ . Therefore, the low equilibrium is locally unstable and the high equilibrium is locally asymptotically stable; (2) if  $\beta = \beta_T$ , then a tangent bifurcation emerges and the unique positive steady state  $\bar{k}$  is neither an attractor nor a repellor as  $\frac{\partial k_{t+1}}{\partial k_t} \Big|_{k_t=\bar{k}} = 1$ ; (3) if  $\beta > \beta_T$  then  $f(k_t, \beta)$  lies below  $f(k_t, \beta_T)$  for any  $k_t$ , and thus no positive steady state can exist in such a case. **Q.E.D.**

Figure 2 illustrates Proposition 4 and shows that an increase in the child allowance shifts the capital accumulation locus downward, while leaving its slope unchanged.

(i) If  $\beta < \beta_T$  ( $\beta = \beta' = 1.1$ ), a fold bifurcation emerges and two steady states exist ( $k^*_L = 0.0039$  with  $\frac{\partial k_{t+1}}{\partial k_t} \Big|_{k_t=k^*_L} = 4.5046$  and  $k^*_H = 0.1132$  with  $\frac{\partial k_{t+1}}{\partial k_t} \Big|_{k_t=k^*_H} = 0.4756$ ). An

economy whose initial condition is beyond (below) the unstable equilibrium  $k_L^*$  will converge towards the high equilibrium  $k_H^*$  (will be entrapped into poverty, where the capital stock is zero). A rise in the child allowance reduces the steady state in the high regime of development and increases the basin of attraction towards the poverty trap.

(ii) if  $\beta = \beta_T = 1.1517$ , a tangent bifurcation exists, i.e., the phase map is tangent to the 45° line at the point in which its slope is equal to 1 ( $\bar{k} = 0.0373$  with  $\partial k_{t+1} / \partial k_t \big|_{k_t = \bar{k}} = 1$ ).

(iii) if  $\beta > \beta_T$  ( $\beta = \beta'' = 1.2$ ), no positive steady states exist and an economy is permanently stuck into poverty.

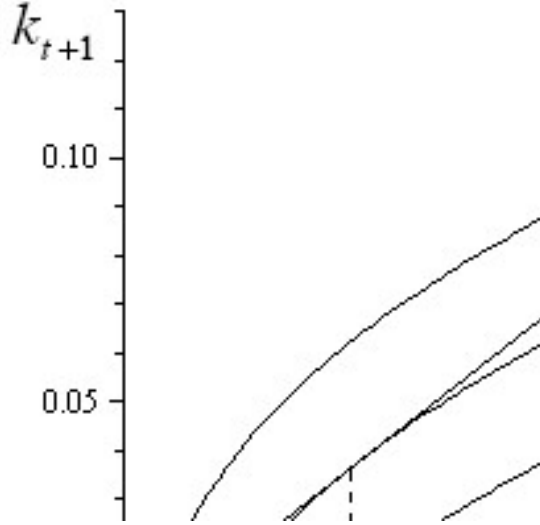


Figure 2. Case  $\beta > m$ . Phase map when  $\beta$  varies ( $q = 0.1$ ).

Therefore, different consequences in terms of long-run economic development turn out to be possible in pursuing the implementation of the child policy.

As regards fertility, the analysis of Eq. (11) gives the following proposition:

**Proposition 5.** *Let  $m < \beta < \beta_T$  hold. Then the effect of a rise in the child allowance on fertility is ambiguous if, and only if,  $\beta < \beta_C$ , where  $\beta_C := \frac{m(1+\gamma)}{1+\gamma-\phi} > m$ .*

**Proof.** Consider the total derivative of Eq. (11) with respect to  $\beta$  (see the Appendix for the study of the sign of  $\partial n^* / \partial w^*$ ). If either  $\beta < \beta_C < \beta_T$  or  $\beta < \beta_T < \beta_C$ , then

$$\frac{dn^*}{d\beta} = \underbrace{\frac{\partial n^*}{\partial \beta} + \frac{\partial n^*}{\partial w^*} \cdot \frac{\partial w^*}{\partial k^*} \cdot \frac{\partial k_H^*}{\partial \beta}}_{+/-}, \quad (17)$$

where  $\frac{\partial n^*}{\partial w^*} > 0$  for any  $m < \beta < \beta_T$  (Malthusian Fertility Regime). If  $\beta_C < \beta < \beta_T$  then

$$\frac{dn^*}{d\beta} = \frac{\partial n^*}{\partial \beta} + \underbrace{\frac{\partial n^*}{\partial w^*} \cdot \frac{\partial w^*}{\partial k^*} \cdot \frac{\partial k_H^*}{\partial \beta}}_{+} > 0, \quad (18)$$

where  $\frac{\partial n^*}{\partial w^*} < 0$  for any  $m < \beta < \beta_T$  (Modern Fertility Regime). **Q.E.D.**

Proposition 5 shows the existence of a critical value  $\beta_C$  beyond which the expected positive effect of the child allowance on fertility is always restored (i.e., in contrast to the case  $\beta < m$ , when  $\beta > \beta_C > m$  no negative effects of the child allowance on fertility exist).

Tables 2.A-2.C (Table 2.D) show(s) how fertility reacts when  $\beta$  varies and  $\beta < \beta_C$  ( $\beta > \beta_C$ ).

**Table 2.A.** Case  $m < \beta < \beta_C < \beta_T$  (Malthusian Fertility). Steady-state stock of capital and fertility when  $\beta$  varies ( $q = 0.2$ ,  $\beta_T = 1.4269$  and  $\beta_C = 1.3529$ ).

$\beta$	1.01	1.05	1.10	1.15	1.20	1.25	1.30	1.33	1.34	1.35
$k_H^*$	0.542 5	0.512 3	0.473 5	0.433 4	0.391 7	0.347 8	0.300 6	0.269 8	0.259 0	0.247 7
$n^*$	1.059 1	1.079 4	1.106 7	1.136 6	1.169 8	1.207 2	1.250 0	1.279 4	1.288 9	1.301 0

**Table 2.B.** Case  $m < \beta < \beta_T < \beta_C$  (Malthusian Fertility). Steady-state stock of capital and fertility when  $\beta$  varies ( $q = 0.1$ ,  $\beta_T = 1.1517$  and  $\beta_C = 1.3529$ ).

$\beta$	1.01	1.05	1.07	1.08	1.10	1.11	1.12	1.13	1.14	1.15
$k_H^*$	0.187 9	0.156 7	0.140 1	0.131 5	0.113 2	0.103 4	0.0929	0.0814	0.067 8	0.047 9
$n^*$	1.574 5	1.614 2	1.633 5	1.642 6	1.658 8	1.665 2	1.6692 5	1.6692 9	1.660 8	1.620 0

**Table 2.C.** Case  $m < \beta < \beta_T < \beta_C$  (Malthusian Fertility). Steady-state stock of capital and fertility when  $\beta$  varies ( $q = 0.03$ ,  $\beta_T = 1.0251$  and  $\beta_C = 1.3529$ ).

$\beta$	1.010	1.013	1.015	1.019	1.020	1.021	1.022	1.023	1.024	1.025
$k_H^*$	0.024 5	0.022 0	0.020 2	0.016 3	0.015 2	0.014 1	0.012 9	0.011 5	0.009 9	0.007 4
$n^*$	1.645 7	1.609 5	1.580 9	1.506 9	1.483 0	1.455 5	1.423 3	1.383 6	1.330 0	1.229 7

Malthusian Fertility Regime: for high (average) [low] values of  $q$ , Table 2.A (2.B) [2.C] shows that in the high regime of development  $n^*$  is a positive monotonic (hump-shaped) [negative monotonic] function of  $\beta$ .

**Table 2.D.** Case  $m < \beta_C < \beta < \beta_T$  (Modern Fertility). Steady-state stock of capital and fertility when  $\beta$  varies ( $q = 0.2$ ,  $\beta_T = 1.4269$  and  $\beta_C = 1.3529$ ).

$\beta$	1.36	1.37	1.38	1.39	1.40	1.405	1.408	1.41	1.42	1.425
$k_H^*$	0.236 0	0.223 8	0.210 8	0.196 9	0.181 5	0.173 0	0.167 6	0.163 8	0.141 0	0.123 3
$n^*$	1.312 5	1.324 7	1.337 7	1.351 5	1.366 6	1.374 7	1.379 9	1.383 4	1.403 4	1.416 6



Modern Fertility Regime: if  $\beta$  is larger than critical level  $\beta_C$  but small enough to guarantee the existence of multiple steady states, then – regardless of the value of  $q$  – the long-run fertility rate in the high regime of development monotonically increases as the child allowance raises.

### 3. Conclusions

We studied how a balanced-budget child allowance policy affects capital accumulation and fertility in a general equilibrium OLG closed economy. We showed either a unique regime or multiple regimes of development can exist depending on whether the child allowance is higher or lower than the fixed cost of children. In the case of multiple regimes of development, an economy can permanently be stuck into poverty if the child allowance is fairly high. Moreover, the demand for children can either be of the Malthusian or Modern type depending on the size of the child allowance. We also showed that the effects of the child policy on fertility are ambiguous in the long run.

### Appendix

From Eq. (11) we have  $\frac{\partial n^*}{\partial w^*} = \frac{\phi[(1+\gamma)m - \beta(1+\gamma - \phi)]}{[(1+\gamma)(qw^* + m - \beta) + \phi\beta]^2}$ , so that  $\partial n^* / \partial w^* > 0$  for any  $\beta < \beta_C$ ,

and  $\partial n^* / \partial w^* < 0$  for any  $\beta > \beta_C$ , where  $\beta_C := \frac{m(1+\gamma)}{1+\gamma-\phi} > m$ . If  $\beta < m$ , then  $\partial n^* / \partial w^* > 0$ . If

$m < \beta < \beta_T$ , then  $\partial n^* / \partial w^* > 0$  ( $< 0$ ) for any  $m < \beta < \beta_C$  ( $m < \beta_C < \beta < \beta_T$ ). If  $\beta_C > \beta_T$ , then  $\partial n^* / \partial w^* > 0$  for any  $m < \beta < \beta_T$ .

### Acknowledgments

The authors gratefully acknowledge an anonymous referee for valuable comments. The usual disclaimer applies.

### References

- Apps, P., Rees R., 2004. Fertility, taxation and family policy. *Scandinavian Journal of Economics* 106, 745–763.
- Boldrin, M., Jones, L.E., 2002. Mortality, fertility and saving in a Malthusian economy. *Review of Economic Dynamics* 5, 775–814.
- Diamond, P.A., 1965. National debt in a neoclassical growth model. *American Economic Review* 55, 1126–1150.
- Fanti, L., Gori, L., 2009. Population and neoclassical economic growth: a new child policy perspective. *Economics Letters* 104, 27–30.
- Fanti, L., Gori, L., 2010. Child policy solutions for the unemployment problem. *Economics Letters* 109, 147–149.
- Galor, O., Weil, D.N., 1996. The gender gap, fertility, and growth. *American Economic Review* 86, 374–387.
- Gollin, D., 2002. Getting Income Shares Right. *Journal of Political Economy* 110, 458–474.
- Groezen, B. van, Meijdam, L., 2008. Growing old and staying young: population policy in an ageing closed economy. *Journal of Population Economics* 21:573–588.

Kraay, A., Raddatz, C., 2007. Poverty traps, aid, and growth. *Journal of Development Economics* 82, 315–347.

Momota, M., 2000. The gender gap, fertility, subsidies and growth. *Economics Letters* 69, 401–405.