Efficient bargaining versus right to manage: a stability analysis in a Cournot duopoly with trade unions

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Abstract The present study considers a unionised (nonlinear) duopoly with two different labour market institutions, i.e. efficient bargaining (EB) and right to manage (RTM), to analyse product market stability under quantity competition with trade unions. We show that when the preference of unions towards wages is small, (*i*) the parametric stability region under RTM is higher than under EB, and (*ii*) a rise in the union power in the Nash bargaining played between firms and unions monotonically increases (resp. reduces) the parametric stability region under RTM (resp. EB). In contrast, when the preference of unions becomes larger, an increase in the union's bargaining power acts: (1) as an economic stabiliser when the union power is small; (2) as an economic de-stabiliser when the union power is high. In addition to established results with regard to equilibrium outcomes, our findings shed some light on the effects of how the labour market regulation affects out-of-equilibrium behaviours in a Cournot duopoly.

Keywords Bifurcation; Cournot; Duopoly; Efficient bargaining; Right to manage

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1. Introduction

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The existence of trade unions represents a hard stylised fact in several developed countries, especially in Europe, and empirical evidence of a positive correlation between high rates of unemployment and trade union behaviours exists in the long term (Layard et al., 2005), even if such a relationship can actually depend on the way unions operate.

As is known, wage and employment bargaining can be modelled in different ways: the "efficient bargaining" (EB) model and "right to manage" (RTM) model represent two standard examples. The key feature of the former is that both the wage and employment are chosen according to a bargaining process played by firms and employees' representatives (McDonald and Solow, 1981). In contrast, with the latter approach only the wage is subject to negotiation and firms are free to unilaterally choose employment (Oswald, 1982; Pencavel, 1984, 1985).[1]

The relative importance of wages and employment in the union's preferences may be different in the sense that trade unions can be either wage-oriented or employment-oriented. Furthermore, firm-specific (decentralised) and industry-wide (centralised) unions can also be distinguished. If unions are decentralised, the wage is bargained by potentially competitive unions at the firm level. If unions are centralised, the wage is bargained at the industry-wide level and all workers are covered by the unionised wage. The EB and RTM models represent the two most popular alternatives of wage-employment outcomes of collective bargaining. The trade union literature (Booth, 1995; Layard et al., 2005) has established some clear normative implications arising from the two alternatives.

(1) The RTM bargaining brings upon inefficiently low (high) levels of employment (wage), implying that unions may be viewed as socially inefficient institutions so that a weakening of union power would likely enhance social welfare.

(2) The EB bargaining causes either an efficient employment level or, at least, even in those cases in which employment will either be too high or too low for social efficiency, it causes a social inefficiency lower than that caused by the RTM outcome.

While static outcomes in a duopoly with different typologies of unions and bargaining structures have deeply been explored (e.g. Dowrick, 1989; Bughin, 1995; Kraft, 1998; Petrakis and Vlassis, 2000; Correa-López and Naylor, 2004; Pal and Saha, 2008; Fanti and Meccheri, 2011), less attention has been paid to stability outcomes in a nonlinear duopoly with quantity competition (e.g. Puu, 1991, 1998; Kopel, 1996; Bischi and Kopel, 2001; Bischi et al., 2010; Wu et al., 2010) and trade unions, with some exceptions (Fanti and Gori, 2012a). The aim of this paper is to fill this gap by explicitly taking into account both the EB and RTM bargaining between firms and unions, and by comparing the stability outcomes of the two models when players have either limited information with regard to their objective functions (they use "local" estimation – where local means at the current state of production – of the marginal value of the objective function in order to follow the steepest local slope of that function), or complete information with static (naïve) expectations with regard to output decisions of the rival. Indeed, according to several scholars (Dixit, 1986; Bischi et al., 1998, 1999; Bischi and Naimzada, 2000; Agliari at al., 2006), adaptive or static expectations may well represent the context of partially "bounded" rationality in which oligopolistic firms operate, while also serving the purpose of allowing for complex dynamics. Since the economies in several European economies are characterised by large companies and unionised workers, the study of labour market institutions different from the competitive market in a nonlinear duopoly is relevant. With regard to this issue, the paper by Fanti and Gori (2012a) represents a first attempt in this direction and studies the effects of co-determination laws on local stability of the Nash equilibrium, by extending the paper by Kraft (1998) to a nonlinear framework. Co-

[1] A special case of the RTM model is the so-called "monopoly union" model, where unions hold the whole bargaining power when it takes place.

determination rules are applied especially in Germany. They imply that unionised workers employed in large companies have almost the same decision rights as capital owners *with regard to employment bargaining* at the firm-level. The wage is outside the field of application of such laws: it is bargained at the industry-level and it is taken as given by every firm in the bargaining game with firm-specific unions to determine employment. The aim of that paper was to contrast the effects on local stability of an exogenous shock in wages in the cases of both co-determination and profit-maximising firms. The results are that under co-determination (resp. profit-maximising firms), an increase in the wage is ambiguous on stability depending on the relative size of the union bargaining power (resp. acts as an economic stabiliser). Therefore, the rules of application of the co-determination model are different from the rule of application of the EB model. At most, it may be viewed an efficient bargaining constrained by the fact that the wage is fixed at the centralised level.

Different from Fanti and Gori (2012a), in this paper we inquire about stability outcomes in a duopoly with two of the most important alternatives of *both employment and wage* determination at the firm-level, and to contrast them in the cases of heterogeneous and homogeneous players with regard to the information set about the objective functions. Therefore, this paper extends Fanti and Gori (2012a) and shows that EB and RTM have different effects not only on equilibrium outcomes, as established by the existing static literature, but also on market stability. In particular, four clear-cut results emerge.

The first result concerns the unambiguous role played by the relative degree of "wage-aggressiveness" in the union preferences in both cases of EB and RTM: the higher the relative importance of wages in the union's objective, the more likely the Cournot-Nash equilibrium of the duopoly game is stable.

The second and third results are claimed by separately considering the two typologies of bargaining: under RTM, we find that the lower the union bargaining power, the more likely the loss of market stability, whereas under EB the union power brings upon either an opposite effect (when both the union's power in the bargaining game and the preference towards wage in the union's objective are small) with respect to the case of RTM, or an ambiguous effect on stability: when the union power in the Nash bargaining is small and/or the union's preference towards wages is small, it still remains true that a rise in the union power works for instability, while when both the union power and the preference of unions towards wages are large, a further increase in the power of unions in the Nash bargaining acts as a stabilising device. However, when unions are employment-oriented, higher levels of union power, including the case of monopoly union, always work for market stability.

The fourth result concerns the comparison with regard to local stability of both EB and RTM and states that when the union's power is large and/or unions are employment-oriented, the RTM institution is neatly more favourable for market stability. In contrast, when the power of unions in the bargaining process is small and unions are wage-oriented, the equilibrium is more likely to be stable under EB.

Moreover, it must also be noted that in any case when the union power is approximately less than one half, the parametric stability region under RTM is larger than under EB, irrespective of the relative size of union's preferences.

The paper is organised as follows. Section 2 builds on the Cournot duopoly under RTM and EB assumptions. Sections 3 analyses dynamics and local stability. Section 4 compares the stability/instability regions under EB and RTM. Section 5 shows that the results are similar with heterogeneous and homogeneous players. Section 6 presents the conclusion.

2. A Cournot duopoly with unions: efficient bargaining versus right to manage

We consider a normalised Cournot duopoly for a single homogeneous product with a negatively sloped inverse demand given by $p = 1 - q_1 - q_2$, where $p > 0$ denotes the price and $q_1 \geq 0$ (resp. $q_2 \geq 0$) is the output produced by firm 1 (resp. firm 2). The average and marginal costs for firm $i = \{1, 2\}$ to provide an additional unit of output are given by $0 < w_i < 1$, which represents the wage negotiated by unions at the firm-specific level. This implies that production takes place by using a constant (marginal) returns to labour technology, that is $q_i = L_i$ (e.g. Dowrick, 1989; Bughin, 1995; Correa-López and Naylor, 2004), where L_i is the labour force employed by firm i.

Efficient bargaining. Under EB (McDonald and Solow, 1981) firms and unions bargain over employment and wages. The objective of firms (resp. unions) is to maximise profits $\Pi_i(w_i, L_i) = pq_i - w_i L_i$ (resp. utility $U_i(w_i, L_i) = (w_i - w°)^{\theta} L_i$) with respect to employment and wages, where $\theta > 0$ is the relative weight attached by unions to wages[2] and $w°$ is the reservation or competitive wage, which is set to zero without loss of generality. Since $q_i = L_i$, the Nash bargaining between firms and decentralised unions is summarised by the following equation:

$$V_i = [(1 - q_i - q_j - w_i) q_i]^{\beta} (w_i^{\theta} q_i)^{1-\beta}, \tag{1}$$

where the control variables are q_i and w_i, and $0 \leq \beta \leq 1$ (resp. $1 - \beta$) is the relative bargaining power of firms (resp. unions). The best reply functions of output and wages for the ith player are simultaneously determined by maximising Eq. (1) with respect to q_i and w_i, that is:

$$\frac{\partial V_i}{\partial q_i} = \frac{V_i [1 - q_i (1 + \beta) - q_j - w_i]}{(1 - q_i - q_j - w_i) q_i} = 0 \Leftrightarrow q_i(q_j, w_i) = \frac{1 - q_j - w_i}{1 + \beta}, \tag{2.1}$$

$$\frac{\partial V_i}{\partial w_i} = \frac{V_i (\theta (1 - \beta)(1 - q_i - q_j) - w_i [\beta + \theta (1 - \beta)])}{(1 - q_i - q_j - w_i) w_i} = 0 \Leftrightarrow w_i(q_i, q_j) = \frac{\theta (1 - \beta)(1 - q_i - q_j)}{\beta + \theta (1 - \beta)} < 1. \tag{2.2}$$

By substituting Eq. (2.2) into Eq. (2.1) to eliminate w_i, we obtain the firm i's output best-reply function as follows:

$$q_i = r_i(q_j) = \frac{1 - q_j}{1 + \beta + \theta (1 - \beta)}. \tag{3}$$

Right to manage. Under RTM firms maximise profits with respect to employment and wages, and unions maximise utility with respect to wages (Booth, 1995). Therefore, firms and decentralised unions bargain over wages, and firms unilaterally choose employment. The Nash product Eq. (1) is therefore maximised by player i with respect to w_i alone to get the optimal wage as determined by Eq. (2.2). With regard to employment (i.e., output), firm i's profit maximisation gives the following output best-reply function:

$$\frac{\partial \Pi_i}{\partial q_i} = 1 - w_i - 2q_i - q_j = 0 \Leftrightarrow q_i(q_j, w_i) = \frac{1 - q_j - w_i}{2}. \tag{4}$$

We now combine Eqs. (2.2) and (4) to definitely obtain the following output reaction function of firm i:

$$q_i = r_i(q_j) = \frac{\beta (1 - q_j)}{2\beta + \theta (1 - \beta)}. \tag{5}$$

[2] Values of θ smaller (higher) than 1 imply that the union is less (more) concerned about wages and more (less) concerned about jobs (e.g. Mezzetti and Dinopoulos, 1991; Fanti and Gori, 2011).

By comparing Eqs. (3) and (5) it is clear that output (i.e., employment) under EB is higher than under RTM.

3. Dynamics and local stability

We now consider a dynamic version of the model presented in Section 2. Time is discrete and indexed by $t = 0,1,2,\ldots$. Let q_i' be the unit-time advancement of variable q_i. We assume that players are heterogeneous with regard to the information set about objective functions (Tramontana, 2010; Fanti and Gori, 2012a, 2012b). In particular, player 1 has limited information regarding the Nash product under EB and profits under RTM (no knowledge of the market), however he/she follows an adjustment mechanism based on the local estimate of the marginal value of the Nash product ($\partial V_1 / \partial q_1$) under EB and marginal profits ($\partial \Pi_2 / \partial q_2$) under RTM in the current period. In particular, by following Bischi and Naimzada (2000) the adjustment mechanism of quantities over time of player 1 is the following:

$$\text{EB: } q_1' = q_1 + \alpha q_1 \frac{\partial V_1}{\partial q_1}, \tag{6.1}$$

$$\text{RTM: } q_1' = q_1 + \alpha q_1 \frac{\partial \Pi_1}{\partial q_1}, \tag{6.2}$$

where

$$\frac{\partial V_1}{\partial q_1} = \frac{V_1[1 - q_1(1+\beta) - q_2 - w_1]}{(1 - q_1 - q_2 - w_1)q_1}, \tag{6.3}$$

$$\frac{\partial \Pi_1}{\partial q_1} = 1 - w_1 - 2q_1 - q_2, \tag{6.4}$$

and the term αq_1 (with $\alpha > 0$) measures the intensity of the reaction of player 1 with respect to a marginal change in the value of the objective function when q_1 varies at time t. Therefore, player 1 increases or decreases production at time $t+1$ depending on whether the marginal value of the Nash product under EB (resp. of the profit function under RTM) is positive or negative.

In contrast, player 2 has a complete knowledge of both the profit function of firms and the utility functions of unions under EB (resp. the profit functions under RTM). Then he/she solves the maximization problem:

$$\text{EB: } q_2' = \arg\max_{q_2} V_2(q_1, q_2'^e), \tag{6.5}$$

$$\text{RTM: } q_2' = \arg\max_{q_2} \Pi_2(q_1, q_2'^e), \tag{6.6}$$

by using static (naïve) expectations about rival's output decision (Puu, 1991).

The two-dimensional systems that characterise the dynamics of a Cournot duopoly under EB and RTM are therefore the following:

$$\text{EB: } \begin{cases} q_1' = q_1 + \alpha V_1 \dfrac{1 - q_1(1+\beta) - q_2 - w_1}{1 - q_1 - q_2 - w_1} \\ q_2' = r_2(q_1) \end{cases}, \tag{7.1}$$

and

$$\text{RTM: } \begin{cases} q_1' = q_1 + \alpha q_1 (1 - w_1 - 2q_1 - q_2) \\ q_2' = r_2(q_1) \end{cases}, \tag{7.2}$$

Efficient bargaining. By using Eq. (2.2) to substitute for w_1 into Eq. (6.1) and Eq. (3), map (7.1) that characterises the dynamics of the economy under EB can be rewritten as follows:

$$\text{EB:} \quad \begin{cases} q_1' = q_1 + \alpha q_1 \dfrac{\beta^\beta [\theta(1-\beta)]^{\theta(1-\beta)}}{[\beta+\theta(1-\beta)]^{\beta+\theta(1-\beta)}} (1-q_1-q_2)^{(1-\beta)(\theta-1)} [1-q_1[1+\beta+\theta(1-\beta)]-q_2] \\ q_2' = \dfrac{1-q_1}{1+\beta+\theta(1-\beta)} \end{cases} . \tag{8}$$

Map (8) has the following unique interior fixed point:

$$E_{EB} = \left(\frac{1}{2+\beta+\theta(1-\beta)}, \frac{1}{2+\beta+\theta(1-\beta)} \right). \tag{9}$$

In order to investigate the local stability properties of the fixed point Eq. (9), we compute the Jacobian matrix of partial derivatives evaluated at E_{EB}, that is:

$$J(E_{EB}) = \begin{pmatrix} 1 - \dfrac{\alpha \mathrm{A} [\theta^2 (1-\beta)^2 - \theta(2\beta^2+\beta-3) + (1+\beta)(2+\beta)]}{[\beta+\theta(1-\beta)][2+\beta+\theta(1-\beta)]} & \dfrac{-\alpha \mathrm{A}}{\beta+\theta(1-\beta)} \\ \dfrac{-1}{1+\beta+\theta(1-\beta)} & 0 \end{pmatrix}, \tag{10}$$

where $\mathrm{A} := \dfrac{\beta^\beta [\theta(1-\beta)]^{\theta(1-\beta)}}{[2+\beta+\theta(1-\beta)]^{\beta+\theta(1-\beta)}}$. Trace and determinant of $J(E_{EB})$ are respectively given by:

$$T_{EB} = 1 - \frac{\alpha \mathrm{A} [\theta^2 (1-\beta)^2 - \theta(2\beta^2+\beta-3) + (1+\beta)(2+\beta)]}{[\beta+\theta(1-\beta)][2+\beta+\theta(1-\beta)]}, \tag{11}$$

$$D_{EB} = \frac{-\alpha \mathrm{A}}{[\beta+\theta(1-\beta)][1+\beta+\theta(1-\beta)]} < 0. \tag{12}$$

Therefore, the characteristic polynomial of map (8) is the following:

$$F_{EB}(\lambda) = \lambda^2 - T_{EB}\lambda + D_{EB}, \tag{13}$$

With regard to map (8), the stability conditions that ensure that both eigenvalues λ_a and λ_b, which are the roots of Eq. (13), remain within the unit circle are the following:

$$\begin{cases} (i) \quad F_{EB} = \dfrac{2[\theta^2(1-\beta)^2 + \theta[1-\beta^2+\beta(1-\beta)] + \beta(1+\beta)]}{[\beta+\theta(1-\beta)][1+\beta+\theta(1-\beta)][2+\beta+\theta(1-\beta)]} \\ -\dfrac{\alpha \mathrm{A}[(1-\beta)^2\theta^2 + 2(1-\beta^2)\theta + 1 + (1+\beta)^2]}{[\beta+\theta(1-\beta)][1+\beta+\theta(1-\beta)][2+\beta+\theta(1-\beta)]} > 0 \\ (ii) \quad TC_{EB} = \dfrac{\alpha \mathrm{A}[2+\beta+\theta(1-\beta)]}{1+\beta+\theta(1-\beta)} > 0 \\ (iii) \quad H_{EB} = \dfrac{\alpha \mathrm{A} + \theta^2(1-\beta)^2 + \theta[1-\beta^2+\beta(1-\beta)] + \beta(1+\beta)}{1+\beta+\theta(1-\beta)} > 0 \end{cases} . \tag{14}$$

From Eq. (14) it is clear that conditions (*ii*) and (*iii*) are always fulfilled, while condition (*i*) can be violated. We now develop the usual one-parameter bifurcation analysis for studying the stability properties of the fixed point E_{EB}. Let $\mathrm{P}_{EB}(\alpha,\beta,\theta)$ represent a boundary at which the Nash equilibrium Eq. (9) undergoes a flip bifurcation ($F_{EB} = 0$) when:

$$\begin{aligned} \mathrm{P}_{EB}(\alpha,\beta,\theta) &:= 2[\theta^2(1-\beta)^2 + \theta[1-\beta^2+\beta(1-\beta)] + \beta(1+\beta)] \\ &- \alpha \mathrm{A}[2+\beta+\theta(1-\beta)][(1-\beta)^2\theta^2 + 2(1-\beta^2)\theta + 1 + (1+\beta)^2] = 0 \end{aligned} . \tag{15}$$

Now, define

$$\alpha_{EB}^F(\beta,\theta) = \frac{2[(1-\beta)^2\theta^2 + [1-\beta^2+\beta(1-\beta)]\theta + \beta(1+\beta)]}{\mathrm{A}[(1-\beta)^2\theta^2 + 2(1-\beta^2)\theta + 1 + (1+\beta)^2]}, \tag{16}$$

as the (unique) flip bifurcation value of α. Then, the following proposition holds.

Proposition 1. *The fixed point E_{EB} of map (8) is locally asymptotically stable for any $0<\alpha<\alpha_{EB}^{F}(\beta,\theta)$. A flip bifurcation occurs if, and only if, $\alpha=\alpha_{EB}^{F}(\beta,\theta)$. The fixed point E_{EB} is locally unstable for any $\alpha>\alpha_{EB}^{F}(\beta,\theta)$.*

Proof. Since $\mathrm{P}_{EB}(\alpha,\beta,\theta)>0$ for any $0<\alpha<\alpha_{EB}^{F}(\beta,\theta)$, $\mathrm{P}_{EB}(\alpha,\beta,\theta)=0$ if, and only if, $\alpha=\alpha_{EB}^{F}(\beta,\theta)$ and $\mathrm{P}_{EB}(\alpha,\beta,\theta)<0$ for any $\alpha>\alpha_{EB}^{F}(\beta,\theta)$, then Proposition 1 follows. **Q.E.D.**

Right to manage. By using Eq. (2.2) to substitute for w_1 into Eq. (6.4) and Eq. (5), map (7.2) that characterises the dynamics of the economy under RTM can be rewritten as follows:

$$\text{RTM:}\ \begin{cases} q_1'=q_1+\alpha\, q_1\dfrac{\beta(1-q_2)-[2\beta+\theta(1-\beta)]q_1}{\beta+\theta(1-\beta)} \\ q_2'=\dfrac{\beta(1-q_1)}{2\beta+\theta(1-\beta)} \end{cases}. \tag{17}$$

Map (17) has the following unique interior fixed point:

$$E_{RTM}=\left(\frac{\beta}{3\beta+\theta(1-\beta)},\frac{\beta}{3\beta+\theta(1-\beta)}\right). \tag{18}$$

With regard to map (17), the stability conditions are the following:

$$\begin{cases} (i)\quad F_{RTM}=2-\dfrac{\alpha\beta[(1-\beta)^2\theta^2+4\beta(1-\beta)\theta+5\beta^2]}{[\beta+\theta(1-\beta)][2\beta+\theta(1-\beta)][3\beta+\theta(1-\beta)]}>0 \\ (ii)\quad TC_{RTM}=\dfrac{\alpha\beta}{2\beta+\theta(1-\beta)}>0 \\ (iii)\quad H_{RTM}=1+\dfrac{\alpha\beta^3}{[\beta+\theta(1-\beta)][2\beta+\theta(1-\beta)][3\beta+\theta(1-\beta)]}>0 \end{cases}. \tag{19}$$

From Eq. (19) it is clear that only condition (*i*) can be violated. Let $\mathrm{P}_{RTM}(\alpha,\beta,\theta)$ represent a boundary at which the Nash equilibrium Eq. (18) undergoes a flip bifurcation ($F_{RTM}=0$) when:

$$\begin{aligned}\mathrm{P}_{RTM}(\alpha,\beta,\theta)&:=2[\beta+\theta(1-\beta)][2\beta+\theta(1-\beta)][3\beta+\theta(1-\beta)]\\ &-\alpha\beta[(1-\beta)^2\theta^2+4\beta(1-\beta)\theta+5\beta^2]=0\end{aligned}. \tag{20}$$

Now, define

$$\alpha_{RTM}^{F}(\beta,\theta)=\frac{2[\beta+\theta(1-\beta)][2\beta+\theta(1-\beta)][3\beta+\theta(1-\beta)]}{\beta[(1-\beta)^2\theta^2+4\beta(1-\beta)\theta+5\beta^2]}, \tag{21}$$

as a threshold value α. Then, the following proposition holds.

Proposition 2. *The fixed point E_{RTM} of map (17) is locally asymptotically stable for any $0<\alpha<\alpha_{RTM}^{F}(\beta,\theta)$. A flip bifurcation emerges if, and only if, $\alpha=\alpha_{RTM}^{F}(\beta,\theta)$. The fixed point E_{RTM} is locally unstable for any $\alpha>\alpha_{RTM}^{F}(\beta,\theta)$.*

Proof. Since $\mathrm{P}_{RTM}(\alpha,\beta,\theta)>0$ for any $0<\alpha<\alpha_{RTM}^{F}(\beta,\theta)$, $\mathrm{P}_{RTM}(\alpha,\beta,\theta)=0$ if, and only if, $\alpha=\alpha_{RTM}^{F}(\beta,\theta)$ and $\mathrm{P}_{RTM}(\alpha,\beta,\theta)<0$ for any $\alpha>\alpha_{RTM}^{F}(\beta,\theta)$, then Proposition 2 follows. **Q.E.D.**

4. Efficient bargaining versus right to manage

In this section we compare the parametric stability/instability regions under EB and RTM. Since Eqs. (16) and (21) cannot be dealt with in a neat analytical form, we resort to numerical simulations (see Figure 1.a-1.d), which exhaustively summarise the behaviour of the EB and RTM oligopoly models when the key parameters vary. In particular, we let boundaries $\alpha_{EB}^{F}(\beta,\theta)$ and $\alpha_{RTM}^{F}(\beta,\theta)$ vary in the (α,β) plane for different values of θ.

Starting from the case $\theta=0$ and $\beta=1$, which replicates the case of profit-maximising firms with competitive labour markets, Figures 1.a-1.d show that the behaviour of the flip bifurcation boundaries in the (α,β) plane under EB and RTM unions is different, and θ plays a crucial role on stability.[3] In particular, under RTM, the relationship between the flip bifurcation boundary of α and β is monotonically negative, and the curve $\alpha_{RTM}^{F}(\beta,\theta)$ shifts upwards and to the right as long as θ increases, while under EB, the relationship between the flip bifurcation boundary of α and β is monotonically positive when θ is close to zero, and the curve $\alpha_{EB}^{F}(\beta,\theta)$ becomes hump-shaped as θ increases.

Therefore, the following results can be established.

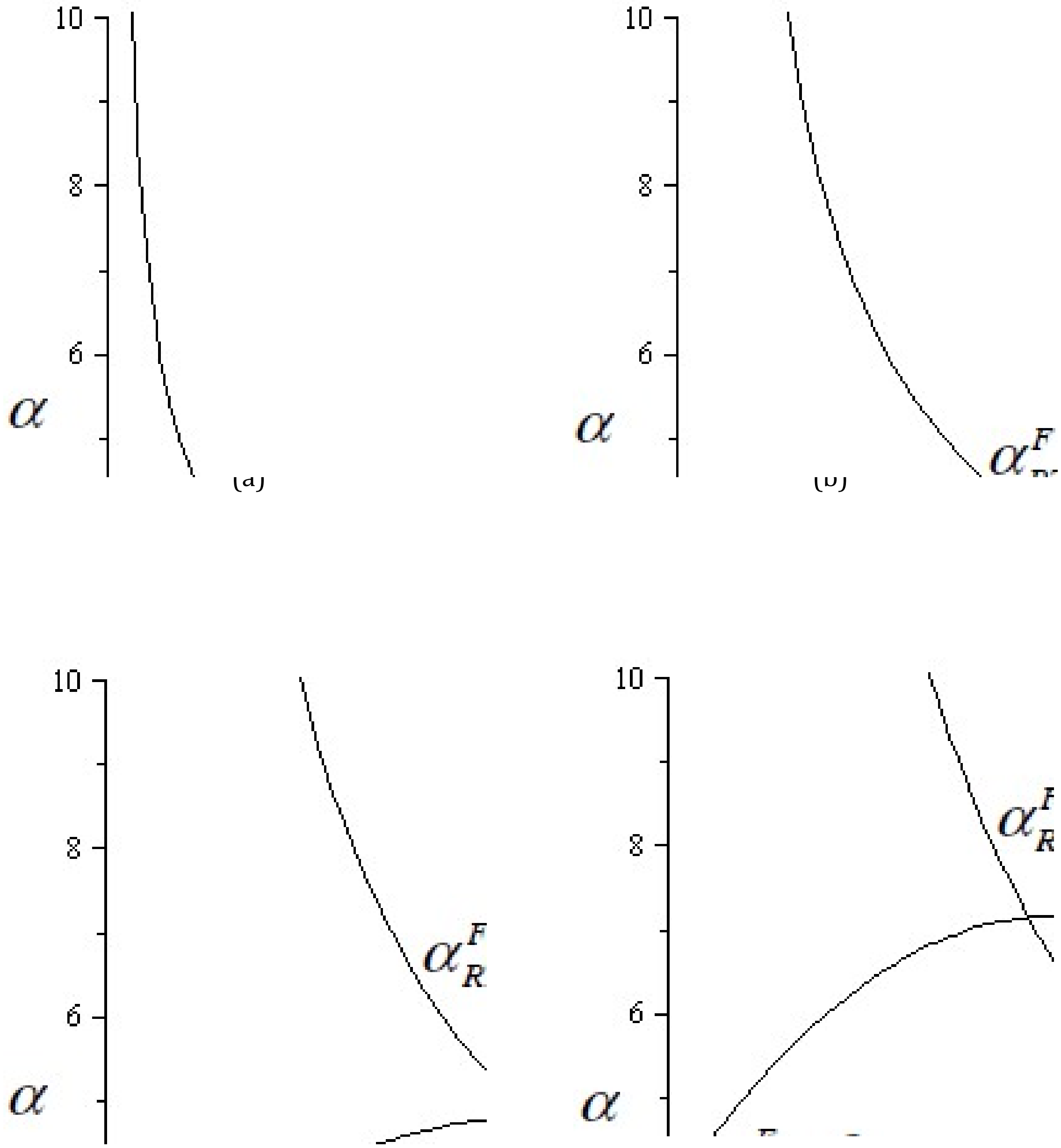
Result 1. *[EB and RTM]. An increase in the relative importance of wages in the union's objective (θ) acts as an economic stabiliser under RTM and EB. An increase in θ under both EB and RTM shifts the flip bifurcation loci $\alpha_{RTM}^{F}(\beta,\theta)$ and $\alpha_{EB}^{F}(\beta,\theta)$ upwards in the (α,β) plane [Figures 1a-1.d].*

Result 2. *[RTM]. Under RTM, an increase in the union's bargaining power, i.e. β moves from 1 to 0, unambiguously acts as an economic stabiliser for any $\theta>0$ [Figures 1.a-1.d].*

Result 3. *[EB]. Under EB, the relationship between α and β is ambiguous and depends on the relative size of θ. When θ is small [Figure 1.a], an increase in the union's bargaining power unambiguously acts an economic de-stabiliser. When θ is large [Figures 1.b-1.d], an increase in the union's bargaining power acts: (1) as an economic stabiliser when the union power is still small (i.e., large values of β); (2) as an economic de-stabiliser when the union power is already large (i.e., small values of β).*

Result 4. *[EB versus RTM]. When θ is small, the stability region in the (α,β) plane under RTM is larger than under EB irrespective of the relative importance of the union power in the Nash bargaining [Figure 1.a]. When θ is large, the stability region under RTM is lower (higher) than under EB when the union power is small (resp. large), i.e. for large (small) values of β.*

[3] Note that the stability (resp. instability) regions in the (α,β) plane are those below (resp. above) the flip bifurcation boundaries $\alpha_{EB}^{F}(\beta,\theta)$ and $\alpha_{RTM}^{F}(\beta,\theta)$.



(c) (d)

Figure 1. Heterogeneous players. Flip bifurcation boundaries in the (α,β) plane: (a) $\theta = 0.1$; (b) $\theta = 0.5$; (c) $\theta = 1$; (d) $\theta = 1.5$.

5. Homogeneous players

In this section we assume that both players have limited information about the objective functions (i.e., the Nash product under EB and the profit function under RTM). The two-dimensional systems that characterise the dynamics of the economy under EB and RTM are therefore the following:

$$\text{EB:}\ \begin{cases} q_1' = q_1 + \alpha V_1 \dfrac{1 - q_1(1+\beta) - q_2 - w_1}{1 - q_1 - q_2 - w_1} \\ q_2' = q_2 + \alpha V_2 \dfrac{1 - q_2(1+\beta) - q_1 - w_2}{1 - q_1 - q_2 - w_2} \end{cases}, \tag{22}$$

and

$$\text{RTM:}\ \begin{cases} q_1' = q_1 + \alpha q_1 (1 - w_1 - 2q_1 - q_2) \\ q_2' = q_2 + \alpha q_2 (1 - w_2 - 2q_2 - q_1) \end{cases}. \tag{23}$$

Knowing that $w_1 = w_2 = \dfrac{\theta(1-\beta)(1-q_1-q_2)}{\beta + \theta(1-\beta)}$, Eqs. (22) and (23) can alternatively be written as follows:

$$\text{EB:}\ \begin{cases} q_1' = q_1 + \alpha q_1 \dfrac{\beta^\beta [\theta(1-\beta)]^{\theta(1-\beta)}}{[\beta + \theta(1-\beta)]^{\beta+\theta(1-\beta)}} (1 - q_1 - q_2)^{(1-\beta)(\theta-1)} [1 - q_1[1 + \beta + \theta(1-\beta)] - q_2] \\ q_2' = q_2 + \alpha q_2 \dfrac{\beta^\beta [\theta(1-\beta)]^{\theta(1-\beta)}}{[\beta + \theta(1-\beta)]^{\beta+\theta(1-\beta)}} (1 - q_1 - q_2)^{(1-\beta)(\theta-1)} [1 - q_2[1 + \beta + \theta(1-\beta)] - q_1] \end{cases}, \tag{24}$$

and

$$\text{RTM:}\ \begin{cases} q_1' = q_1 + \alpha\, q_1 \dfrac{\beta(1-q_2) - [2\beta + \theta(1-\beta)]q_1}{\beta + \theta(1-\beta)} \\ q_2' = q_2 + \alpha\, q_2 \dfrac{\beta(1-q_1) - [2\beta + \theta(1-\beta)]q_2}{\beta + \theta(1-\beta)} \end{cases}. \tag{25}$$

The Nash equilibria of the EB and RTM economies are respectively given by Eq. (9) and Eq. (18). Let

$$\alpha'^{F}_{EB}(\beta,\theta) = \frac{2}{\mathrm{A}}, \tag{26}$$

and

$$\alpha'^{F}_{RTM}(\beta,\theta) = \frac{2[\beta + \theta(1-\beta)]}{\beta}, \tag{27}$$

be two threshold values of α under EB and RTM, respectively, when players are homogeneous with regard to the objective function. Then, by using similar arguments as in the previous sections, the following propositions hold.

Proposition 3. *[EB – homogeneous players]. The fixed point E_{EB} of map (24) is locally asymptotically stable for any $0 < \alpha < \alpha'^{F}_{EB}(\beta,\theta)$. A flip bifurcation emerges if, and only if, $\alpha = \alpha'^{F}_{EB}(\beta,\theta)$. The fixed point E_{EB} is locally unstable for any $\alpha > \alpha'^{F}_{EB}(\beta,\theta)$.*

Proposition 4. *[RTM – homogeneous players]. The fixed point E_{RTM} of map (25) is locally asymptotically stable for any $0 < \alpha < \alpha'^{F}_{RTM}(\beta,\theta)$. A flip bifurcation emerges if, and only if, $\alpha = \alpha'^{F}_{RTM}(\beta,\theta)$. The fixed point E_{RTM} is locally unstable for any $\alpha > \alpha'^{F}_{RTM}(\beta,\theta)$.*

Proof. The proof of Propositions 3 and 4 uses arguments similar to that used to show Propositions 1 and 2. **Q.E.D.**

The dynamic behaviour of the EB and RTM nonlinear duopolies when both players have no knowledge of the market (and then use local estimates of the objective functions) is qualitatively similar to the case in which one of the two players has complete information and static (naïve) expectations with regard to output decisions of the rival. Therefore, Results 1-4 can also be extended to the case of homogeneous (bounded rational) players. This is shown in Figure 2, which depicts the graph of $\alpha'^{F}_{EB}(\beta,\theta)$ and $\alpha'^{F}_{RTM}(\beta,\theta)$ in the (α,β) plane when $\theta=1$.

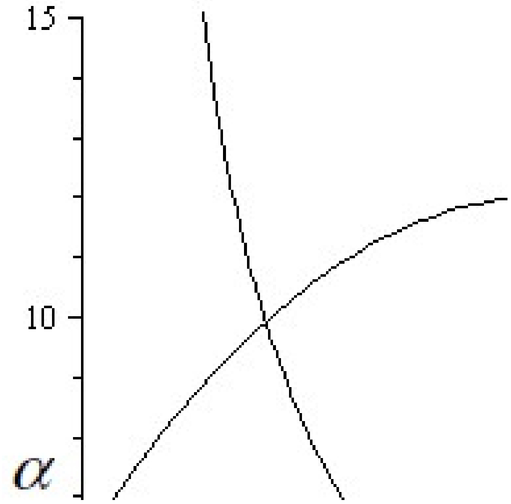


Figure 2. Homogeneous players. Flip bifurcation boundaries in the (α,β) plane: $\theta=1$.

6. Conclusions

We have analysed local stability in a unionised nonlinear duopoly with quantity competition, by comparing economies with trade unions under efficient bargaining (EB) and right to manage (RTM). Under EB, both the wage and employment are bargained by employees and employers representatives. Under RTM, only the wage is bargained by both parties, and firms choose employment according to their labour demand. We have shown that EB and RTM have different effects on local stability of the Nash equilibrium, which depends on the relative preference of unions towards wages. In particular, when the union preference for higher wages is small: (*i*) the stability region under RTM is larger than under EB, and (*ii*) a rise in the union power in the Nash bargaining monotonically increases (resp. reduces) the parametric stability regions under RTM (resp. EB). In contrast, when the preference of unions towards wages is large, an increase in the union's bargaining power acts: (1) as an economic stabiliser when the union power is still small; (2) as an economic de-stabiliser when the union power is already large.

Therefore, provided that we have shown that the unions' "wage-aggressiveness" works for stability under both institutions, the RTM is the one that should actually be preferred when the union power is large and/or unions are wage-oriented. By contrast, under EB the region of stability is the largest when firm and union power are near-parity than when one significantly prevails on the other. Of course, our aim is not to give insights about normative implications of possible unpredictable fluctuations on social welfare, and we are not considering undisputed the fact that a stable scenario is better than an unstable one for firms, workers and trade unions. With regard to this issue, it is important to mention the works by Matsumoto (2003) and Huang (2008), where it is shown that chaotic dynamics can be desirable with respect to convergent trajectories as regards long-run social welfare.

Our findings clarify how different labour market institutions affect out-of-equilibrium behaviours in a duopoly market.

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Appendix. Complete information with naïve expectations

The aim of this appendix is to stress the importance of assuming limited information with regard to the objective function of every player (which uses the local estimates of the objective function in order to properly choose the quantity to be produced in the future period) for stability outcomes. To this purpose we now show that when both players have a complete knowledge about demand and cost functions with naïve expectations, the Nash equilibrium is always locally stable under both kinds of labour market regulations, i.e. EB and RTM.[4]

When both players have complete knowledge of the market, the use of Eqs. (3) and (5) allows us to write the systems that characterises the dynamics of EB and RTM economies in the following way:

$$\text{EB:} \begin{cases} q_1' = \dfrac{1-q_2}{1+\beta+\theta(1-\beta)} \\ q_2' = \dfrac{1-q_1}{1+\beta+\theta(1-\beta)} \end{cases}, \tag{A.1}$$

and

$$\text{RTM:} \begin{cases} q_1' = \dfrac{\beta(1-q_2)}{2\beta+\theta(1-\beta)} \\ q_2' = \dfrac{\beta(1-q_1)}{2\beta+\theta(1-\beta)} \end{cases}. \tag{A.2}$$

The Nash equilibria are still given by Eqs. (9) and (18) for EB and RTM, respectively. Then, by using the same techniques of Sections 3, 4 and 5 it is easy to show that:

$$\text{EB:} \begin{cases} F = \dfrac{[\beta+\theta(1-\beta)][2+\beta+\theta(1-\beta)]}{[1+\beta+\theta(1-\beta)]^2} > 0 \\ TC = F > 0 \\ H = \dfrac{2+[\beta+\theta(1-\beta)][2+\beta+\theta(1-\beta)]}{[1+\beta+\theta(1-\beta)]^2} > 0 \end{cases}, \tag{A.3}$$

and

$$\text{RTM:} \begin{cases} F = \dfrac{[\beta+\theta(1-\beta)][3\beta+\theta(1-\beta)]}{[2\beta+\theta(1-\beta)]^2} > 0 \\ TC = F > 0 \\ H = \dfrac{2\beta^2+[\beta+\theta(1-\beta)][3\beta+\theta(1-\beta)]}{[2\beta+\theta(1-\beta)]^2} > 0 \end{cases}. \tag{A.4}$$

It is clear that each of the three conditions in Eqs. (A.3) and (A.4) are always fulfilled. Then, the Nash equilibria Eqs. (9) and (18) are stable when players have a complete knowledge of the market with naïve expectations.

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[4] We thank an anonymous reviewer for pointing this out.

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