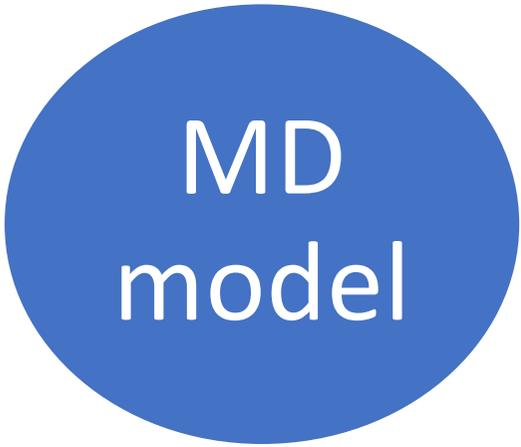


Maintaining dragging in a dynamic geometry environment: the interplay between a psychological model and task design

Anna Baccaglini-Frank, University of Pisa

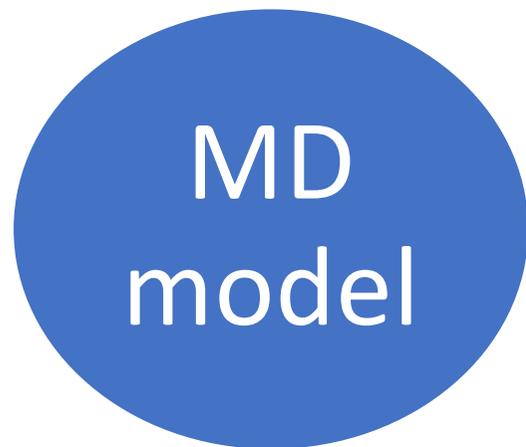




MD
model

Mathematical learning:

- Conjecturing in Euclidean Geometry
- Conditionality between premise and conclusion
- Argumentation and proof
- Abduction
- Geometric prediction

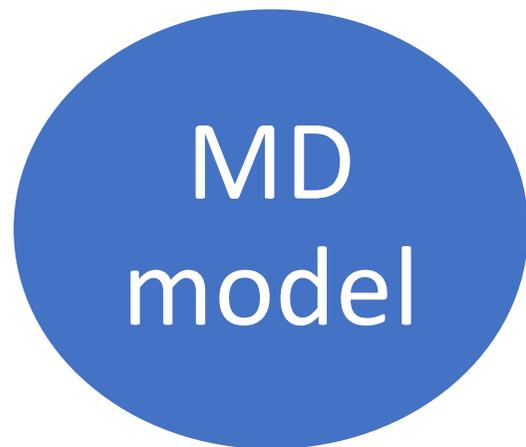


Mathematical learning:

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Educational task design:

- Introduction of dragging modalities and tasks to foster development of the scheme
- Sequences of tasks that make use of the scheme to foster new mathematical learning



Learning with technology:
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Dragging modalities introduced

Wandering Dragging

Maintaining dragging

Dragging with active trace

Dragging Test

Open problems designed

The construct “open problem” refers to

a problem or a question given in a form that does not reveal its solution or answer.

Frequently the solution process asks for the generation of conditionality (or a conditional statement) after a physical or mental exploration of the problematic situation.

A conjecture can be asked for explicitly.

Conjecturing open problems

A conjecturing open problem is an open problem in which the solver is explicitly asked to produce *a statement* about the relationships between the elements of a figure or between its properties.

“What configuration occurs... when...?”

“What relationship can you identify between... and ...?”

“Into which kinds of figures can... be transformed?”

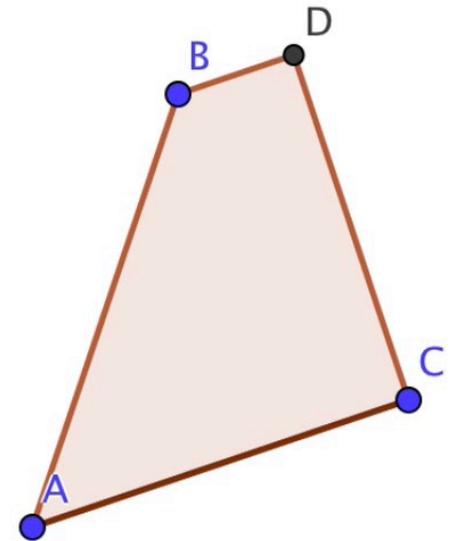
“Formulate a conjecture.”

Conjecturing open problems

A conjecturing open problem is an open problem in which the solver is explicitly asked to produce *a statement* about the relationships between the elements of a figure or between its properties.

“Let A, B, C be three points, construct the parallel line l to AC through B . Construct D as the foot of the perpendicular to l through C . Consider the quadrilateral $ABCD$.

Make conjectures about the kinds of quadrilateral it can become, describing all the ways in which it can become a certain type of quadrilateral.”



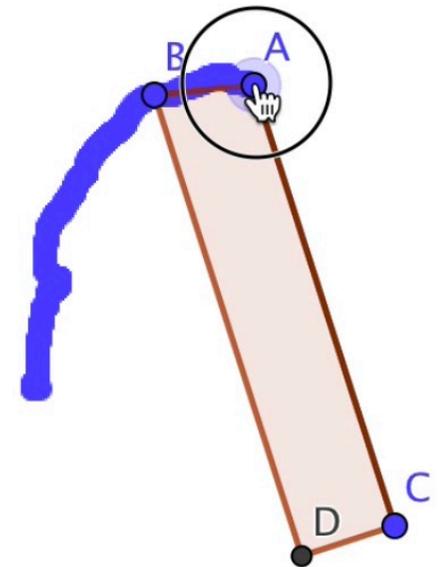
Key components in the MD-conjecturing process

A belongs to C_{BC} (circle
with diameter BC)

Direct control

ABCD rectangle

Indirect control

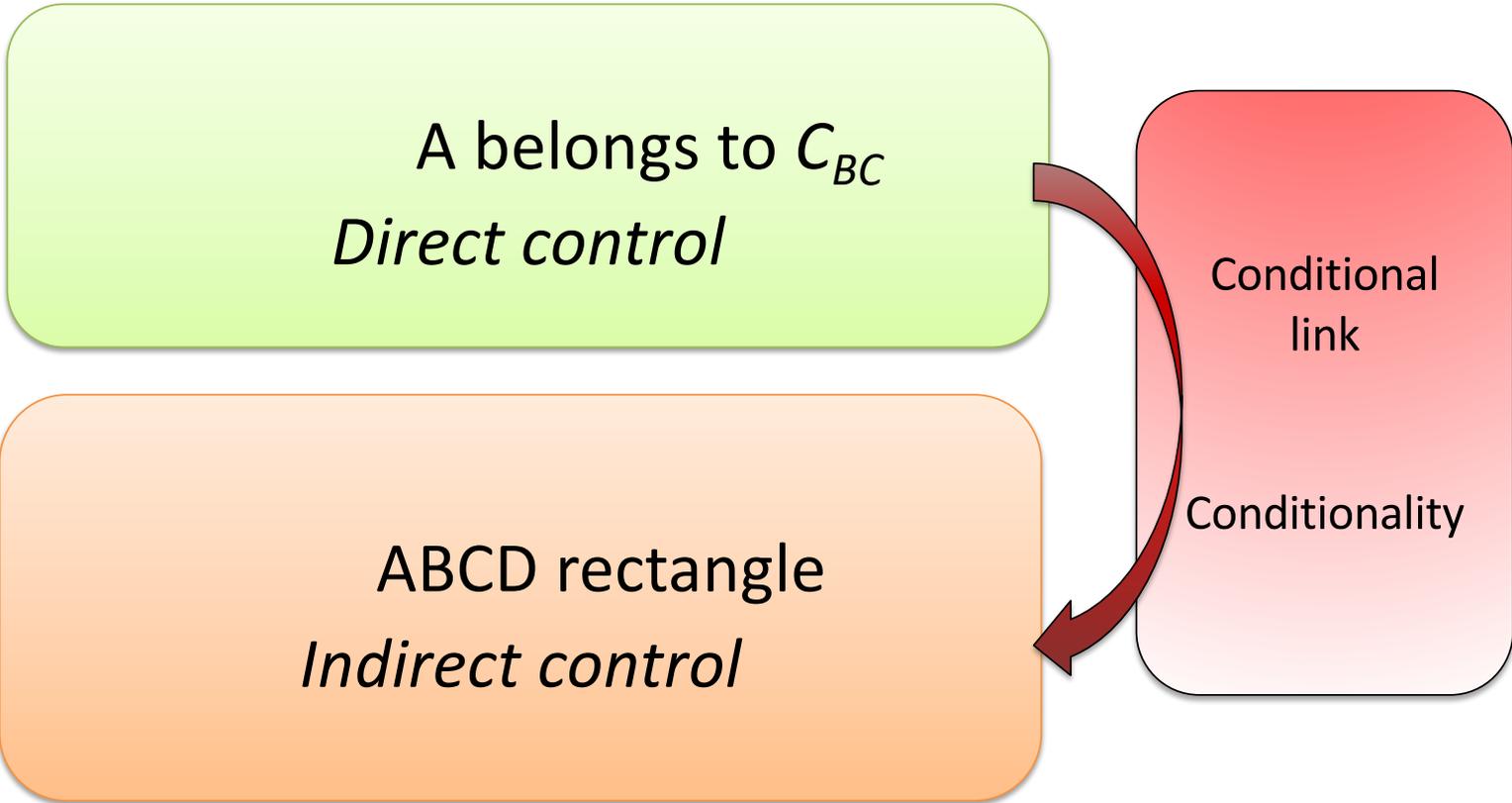


It's a bit like...



But I can only act on the chains, making them move through pulleys
Direct control

I want to lower the draw bridge
Indirect control

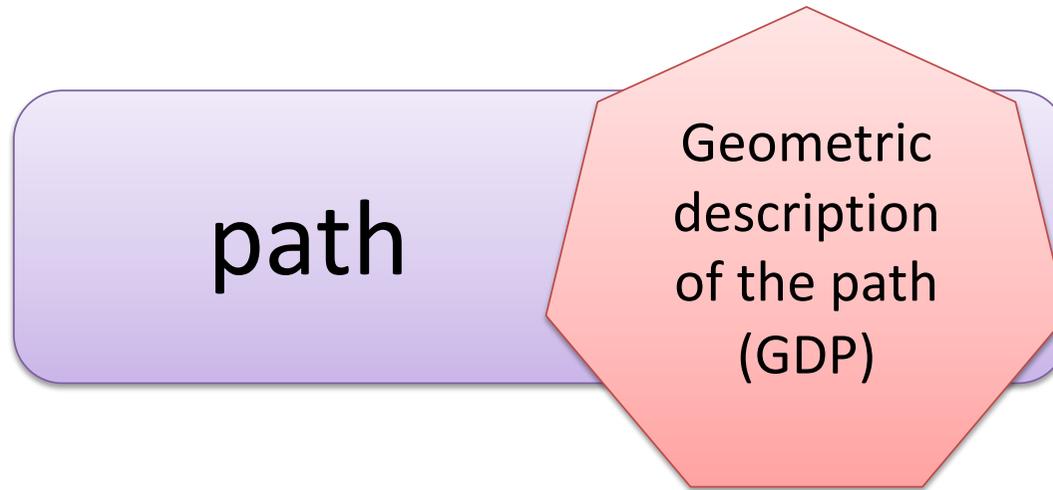


The MD model

(implicit) Task: identify an interesting configuration and try to induce it as an invariant

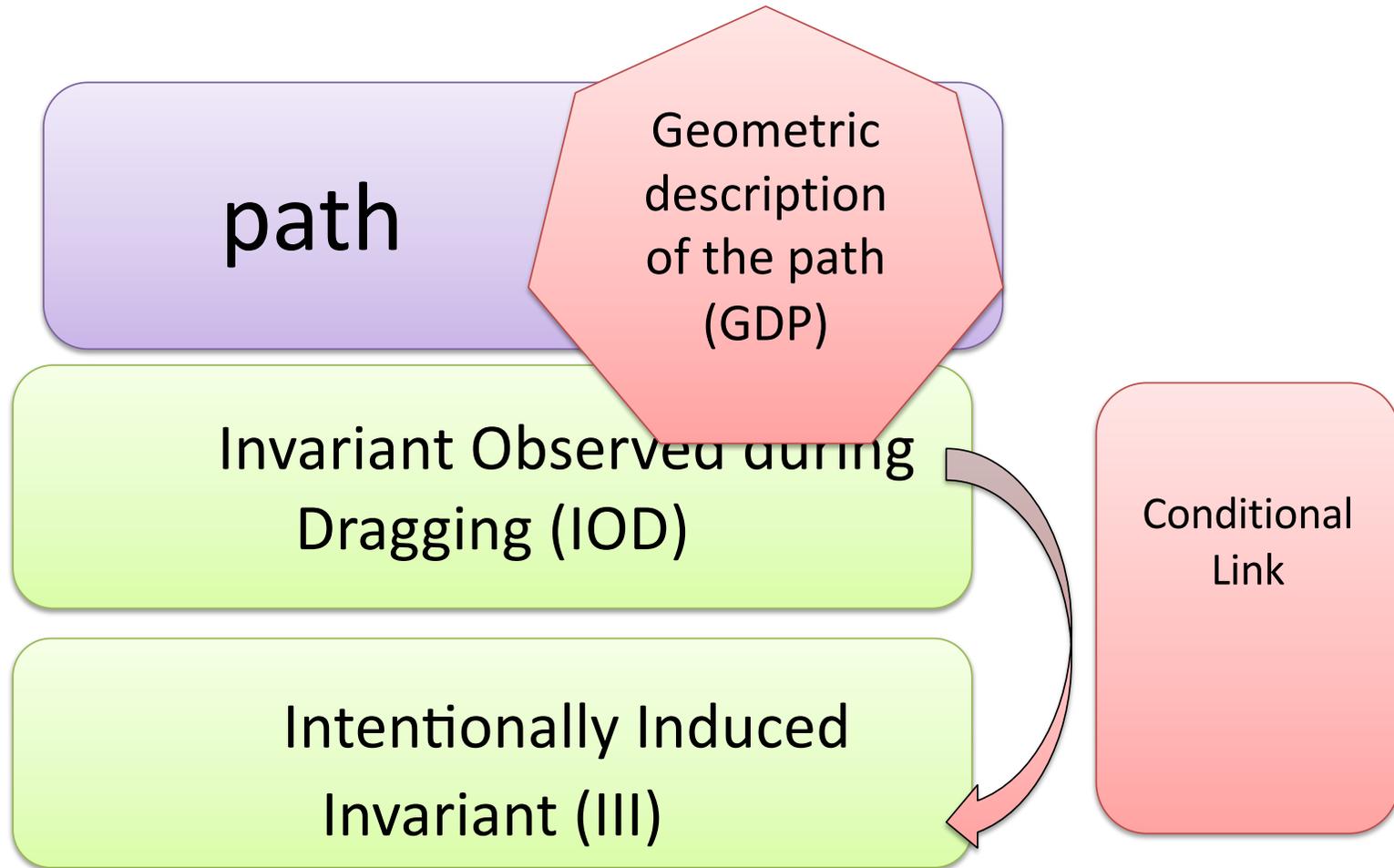
Intentionally Induced
Invariant (III)

(implicit) task: maintain the interesting configuration (III) during dragging;
search for a “condition” for the interesting configuration;

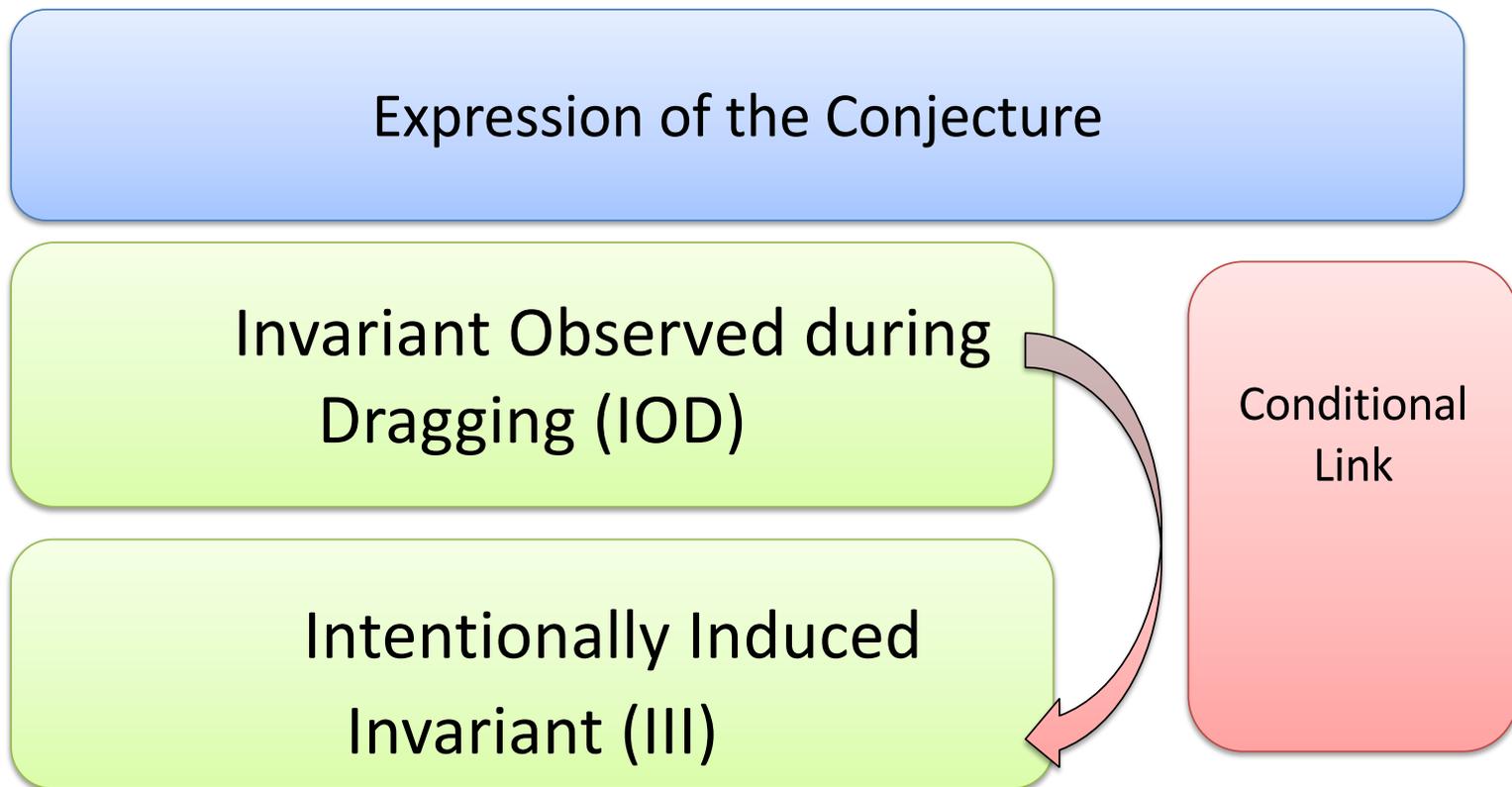


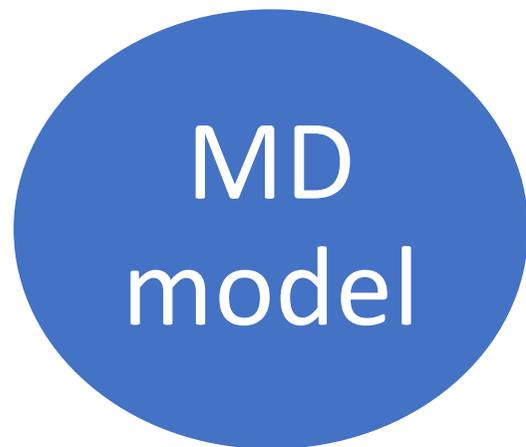
Intentionally Induced
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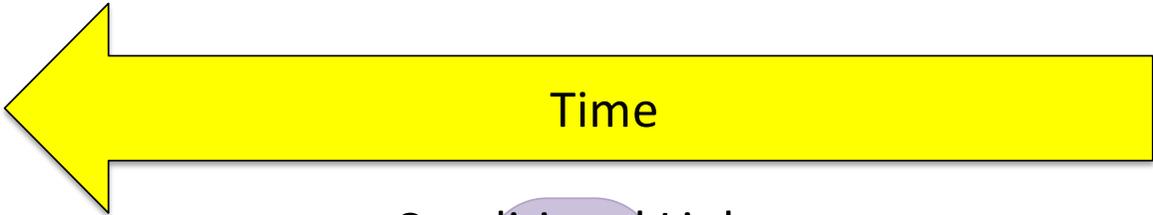
(implicit) task: Verify the conditional link and express the conjecture



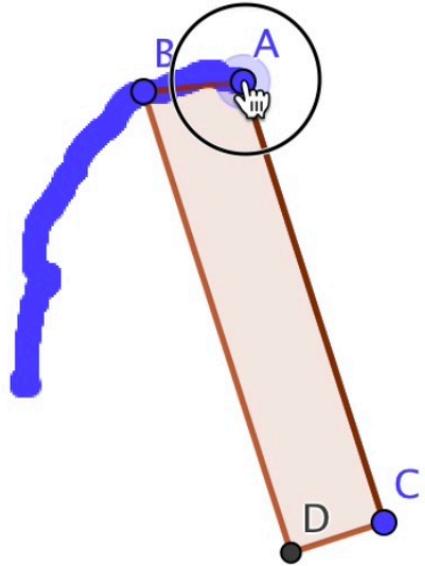


Learning with technology:
relationships between phenomenological
experiences and mathematical theory

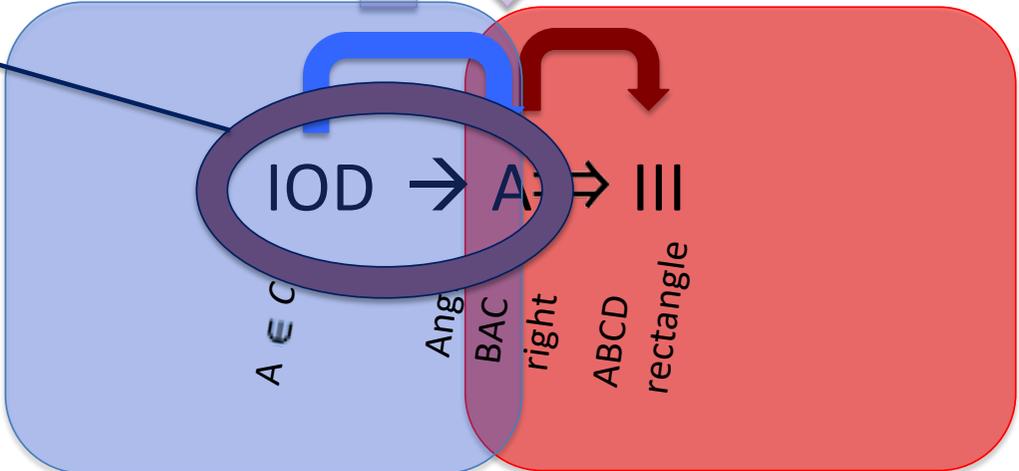
- Mathematical learning:
- Conjecturing in Euclidean Geometry
 - Conditionality between premise and conclusion



Instrumented abduction

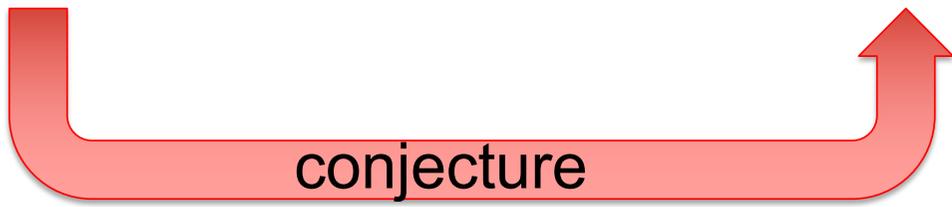


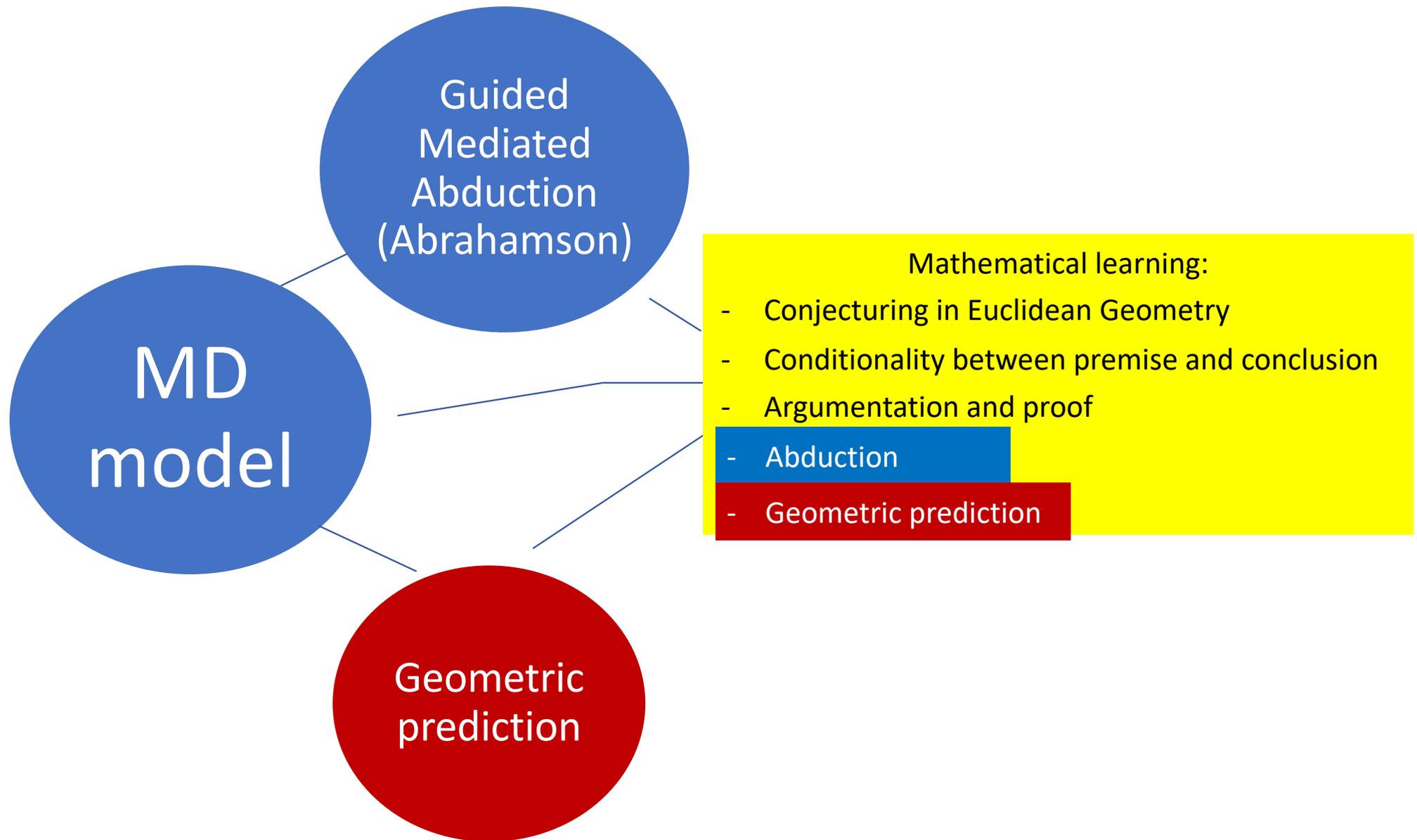
Conditional Link



phenomenological domain of Dynamic Geometry

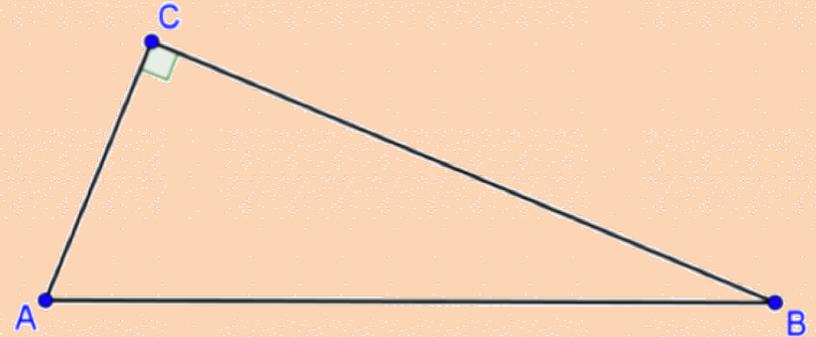
theoretical domain of Euclidean Geometry



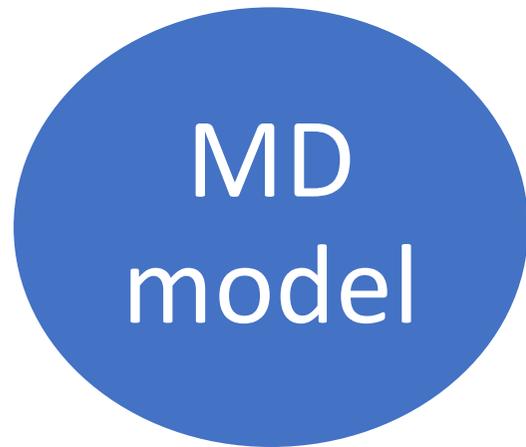


From an ongoing doctoral dissertation on geometric prediction...

Consider the right triangle in the figure.
A and B are fixed.
The length of AB has to stay the same.



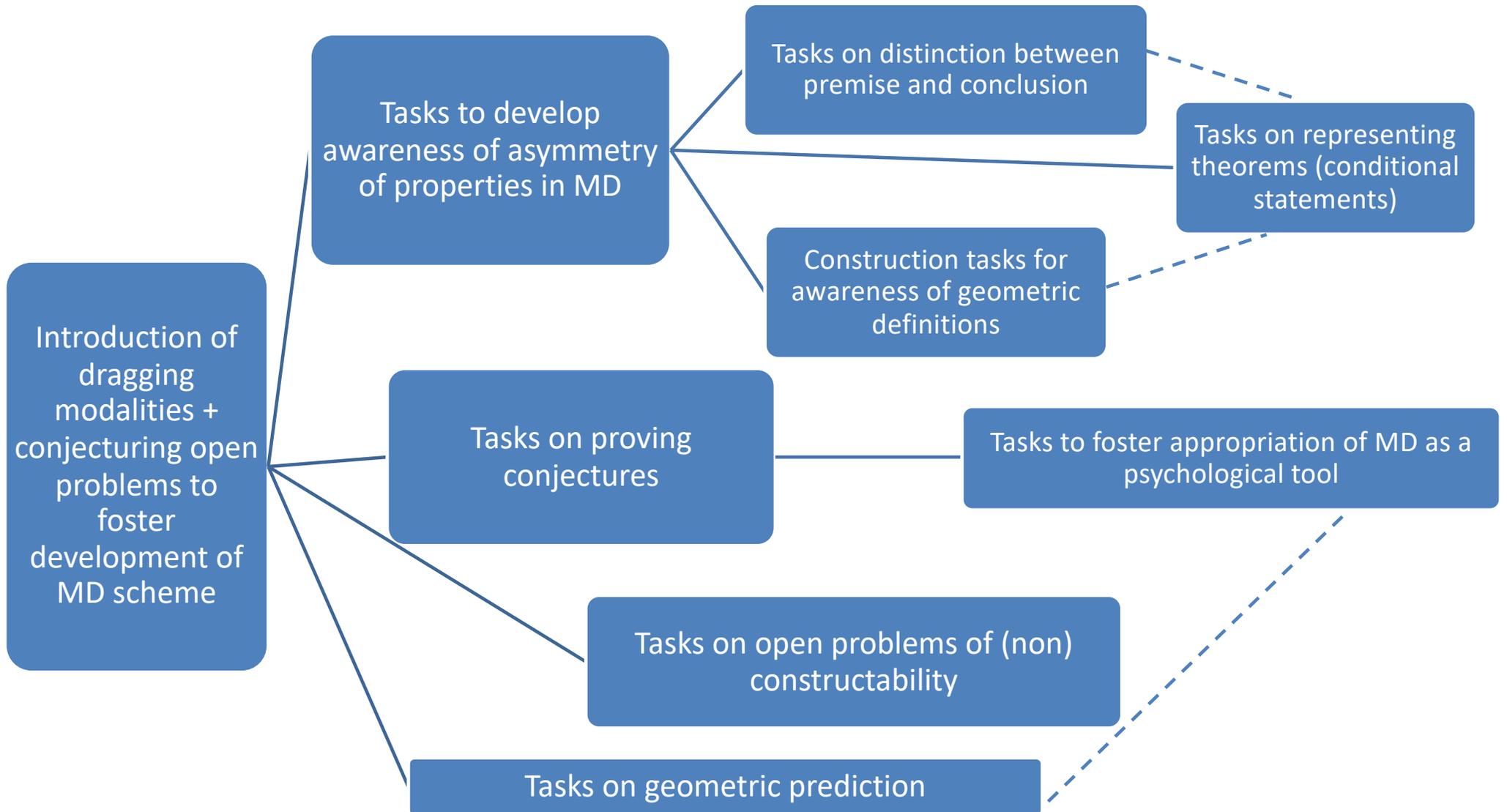
What can you say about the vertex with the right angle?
Make a prediction: can C be in other positions? If so, why? If not, why not?



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A few (commented) references

Early work on students' (spontaneous) dragging schemes:

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