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TWG24 made its first appearance as a new Thematic Working Group at CERME10, focusing on representations of mathematical concepts or mathematical objects because of their constituting an "integral part of the doing of mathematics" (Presmeg, 2002) and thus an important part of teaching and learning mathematics. Indeed, representation has been a crucial topic in research, for instance, in PME groups, in a special issue of ESM, in a special issue of ZDM, in ICME 13 in 2016. In the group's "Call for papers" the term representations referred to thinking tools for doing mathematics encompassing graphs, tables, diagrams, formulas, symbols, texts, concrete models, and, in a broader sense, even gestures, videos, sounds etc.

Keywords: Representation, visualization, imagine, visual-spatial abilities, visual-spatial image.

Introduction

This Thematic Working Group explicitly welcomed papers from a variety of different theoretical approaches and methodological frameworks addressing the role of representations of different types in teaching and learning processes, in particular those involving visualization (considered here as defined by Arcavi (2003)). In TWG24 there were 24 participants (authors, co-authors, and some other participants), from 13 countries (these included Chile, Denmark, Finland, France, Germany, Italy, Mexico, The Netherlands, Portugal, Sweden, Switzerland, Turkey, the UK) with 16 accepted papers and 2 accepted posters. The most part of the 16 papers, were empirical studies (related to primary and secondary school). The 2 posters reporting empirical studies conducted at the primary and secondary school levels. The poster concerning primary school described what students learn in mathematics lessons when different representations of fraction are used; and the poster concerning secondary school described how a variety of multi-sensory activities allowed 14 year old students to familiarise with some pivotal mathematical concepts such as prime and irrational numbers. The structure of the timeslots was designed in order to stimulate interaction and collaboration among participants: all participants were asked to read all papers, and prepare reaction-questions to two papers in particular that had been assigned ahead of time by the TWG leaders. After a 10-minute presentation by the presenting author, the prepared questions were posed and a general discussion was initiated and conducted for 25 minutes: first the authors of the paper would reply to the reaction-questions, then there was a discussion on issues related to the general list of questions designed for TWG24's call for papers. Posters were also allocated a few minutes of presentation time within the working group, and a short follow-up discussion took place after each of them. The last session was completely devoted to summing up the main issues that had emerged from the group discussions. One of these was that certain key words, present in the literature on representations and visualization in mathematics education, were not being used consistently by the participants. Therefore a list was put together with the suggestion for the upcoming CERME of making explicit the definitions used in each study. Among these (in alphabetical order): figure, gesture, mental imagery, metaphor, representation (including the distinction between internal and external), sign, symbol, visualization, visual-spatial abilities, visual-spatial image.

Gestures and representations

The group agreed on the following: gestures can be considered as a way to create temporary external visualizations of internal imagery or structures, to explain or communicate thinking; movement involved in the gesture can connect physical properties and theoretical properties; different kinds of artifacts affording (or fostering) the use of gestures can be involved (such as the movement within dynamic geometry software). The importance of gestures in the context of representations in mathematics education was evident in TWG24, because many of the papers presented included a focus on gestures. Okumus and Hollebrands investigated how middle school students created 3–dimensional objects from 2–dimensional figures using an extrusion method, and they identified students' strategies for forming 3–dimensional objects with a focus on their gestural signs. The paper by Joffredo-Le Brun, Morellato, Sensevy, and Quilio focused on the relation between gestures and (other kinds of) representations (and metaphors), through the analysis of an extract from a lesson proposed in primary school during which the students work on the notion of difference, introduced with the help of several systems of representation. Ferrara and Ferrari also considered the relation between gestures and (other kinds of) representations, presenting the diagrammatic activity of secondary school students exploring motion through graphing technology, which captures a pair of space-time graphs on a single Cartesian plane. Indeed, the use of computers and technology was another transversal theme present in many papers and group discussions.

Technology and representations

TWG24 discussed the issue of how technology can change the dynamics of teaching-learning by offering specific kinds of representations. The paper by Okumus and Hollebrands presented findings from a study conducted during a summer enrichment program, in which students used manipulatives and a dynamic geometry program (Cabri 3D). Miragliotta and Baccaglini-Frank presented analyses of excerpts from a set of activities designed and proposed within the context of a 2D dynamic geometry software (Geogebra) for a group of 9th grade students. Schreiber and Klose focused on the role of artifacts and different forms and modes of representation when learning mathematics at primary school level, through an interactive approach, in which mathematical audio-podcasts were produced. A perspective on teachers'competencies in the context of multimedia-based representations was presented by Ollesch, Grünig, Dörfler and Vogel. Their study described findings from a project in which they used video-vignettes in order to assess the competencies of mathematics teachers for multimedia use in mathematics lessons. Taking a closer look into how technology can change the dynamics of teaching-learning by offering specific kinds of representations, a study by Garcia Moreno-Esteva, White, Wood, and Black showed how eye movement can be tracked and used as a window to cognitive processes involved with use of representations in mathematical activities.

Theoretical frameworks used in the papers and posters presented

Several different theoretical frameworks were referred to in the papers and posters presented: Arzarello's Semiotic Bundle theory (Bini; Robotti); Balacheff's theoretical notion of epistemological validity (Hoyos); Bartolini Bussi and Mariotti's Theory of Semiotic Mediation (Okumus and Hollebrands; Robotti; Schou; Schreiber and Klose); cognitive psychological approaches, applied in the problem solving context, such as Bayes' (Böcherer-Linder and Andreas Eichler); or Vergnaud's framework, (Serrazina and Rodrigues); Duval's registers of representation and theory of apprehension (Miragliotta and Baccaglini-Frank; Robotti; Hoyos, Bini); Enactivism (Ferrara and Ferrari; Soto-Andrade and Diaz-Rojas); Fischbein's Theory of Figural Concepts (Miragliotta and Baccaglini-Frank); Goldin's definition of representation (Sveider); the Joint Action Theory in Didactics (JATD) (Joffredo-Le Brun, Morellato, Sensevy and Quilio); Lakoff and Núñez's conceptual metaphors (Finesilver); Mishra & Koehler's Technological Pedagogical and Content Knowledge (TPACK) (Ollesch, Grünig, Dörfler and Vogel); psychological approaches such as Bruner's approach (Ott; Finesilver); or Ainsworth's approach (Böcherer-Linder and Eichler; Ollesch, Grünig, Dörfler and Vogel); Krutetskii's approach (Olgun and Ader); Tall and Vinner's Concept Image and Concept Definition (Schou);

According to these, the authors developed different kinds of empirical studies: intervention studies (short term and long term studies; with attention to the teacher's role or focused on learners); and observation studies (observing learners in different educational settings; observing teachers; observing classroom processes). In one case, a paper attempted to make some steps forward in elaborating a new theoretical framework emerging at the intersection between cognitive psychology and mathematics education (Miragliotta and Baccaglini-Frank). In another paper, Ferrara and Ferrari conceive mathematical thinking as a place of events instead of objects, and they bring forth inventive and speculative possibilities for learners to encounter and problematize spatio-temporal relationships, rather than seeing them as ways of being mistaken.

Concluding remarks

We conclude this summary with the two questions, from the general list, that seemed to arise the greatest interest of the participants, and sketch out the main comments advanced by the Working Group.

What aspects of the use of different types of representation, imagery and visualization are effective in mathematical problem solving at various levels?

Participants of TWG24 suggested that a representation does not stand alone, and it cannot be separated from how it is used. Thus, it is important to take into account interaction between the individual and the representation (both its external as well as its internal – though difficult to access – component) and between representations and context in which they are used (Joffredo-Le Brun, Hoyos, Schou). Moreover, representations are used within a social context, partly (but not only), for communication of ideas; it is important to encourage learners to express themselves using their own representational strategies, and appreciate multiple representations of information and of their ideas (Finesilver; Olgun and Ader; Robotti; Okumus). Through a careful and appropriate use of representations it is possible to increase positive affect towards mathematics and inclusion (Soto-Andrade and Diaz-Rojas; Robotti). However, there is a tension between the advantages of flexible representation (and specific useful reps) and pushing students to use representations, which do not come naturally to them (Finesilver).

How can teachers help learners to make connections between visual and symbolic representations of the same mathematical notions (mathematical object)?

In response to this question participants of TWG24 suggested that there are certain registers of signs that are considered conventional (by teachers), and others which are less conventional. Indeed, teachers may be less familiar with the various alternative ways of representing, and either not accept alternatives as legitimate (e.g. drawing), or not be conscious of how they are being used (e.g. gestures) (e.g.: Bini; Olgun and Ader; Ollesch, Gruenig, Doerfler and Vogel; Schou). Finally, in various occasions, the group discussed the issue of low achievers and use of representations both by them and by teachers involved in their education processes. These discussions were fueled especially by the papers by Finesilver and by Robotti. In her paper Finesilver drew on qualitative data from problem-solving interviews with very low-attaining secondary school students, focusing on the visuospatial organization of elements in four types of non-standard student-created and co-created representations. She discussed these four types of representations in terms of relationships between representation type, scenario, calculation success, and the students' developing understanding of multiplication and division concepts. On the other hand, Robotti presented a didactical sequence involving the use of various artifacts, introduced by the teacher, to solve tasks on fractions. She analyzed how the representations, fostered by the artifacts, produced by the students, and then picked up by the teacher, contributed to students' development of mathematical meanings around the notion of fraction.

References

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics, 52*(3), 215–241.

Bartolini Bussi, M.G. & Baccaglini-Frank, A. (2015). Geometry in early years: sowing the seeds towards a mathematical definition of squares and rectangles. *ZDM Mathematics Education, 47*(3), 391-405.

Duval, R. (1999). Representation, vision and visualization: Cognitive functions in mathematical thinking. Basic issues for Learning (Plenary address) In F. Hitt & M. Santos (Eds.), *Proceedings of the Twenty-first Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education,* , Vol. I, (pp. 3-26) México.

Presmeg, N.C. (2002). Preface. In F. Hitt (Ed.), *Representations and mathematics visualization*. North American Chapter of the International Group for the Psychology of Mathematics Education, (pp. ix–xvi).

Presmeg, N. C. (2006). Research on visualization in learning and teaching mathematics. In A. Gutiérrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education*, (pp. 205–235). Rotterdam: Sense Publishers.