MEASURING RICE AS A BRIDGE ACROSS DISCRETE AND CONTINUOUS

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In this paper we report on some qualitative results of an explorative study about how a sample of Italian children of ages 5 and 6 evaluates and manages quantities of rice, a substance that can be treated as continuous or discrete. In particular, we are interested in how young children manage the task of comparing quantities, judging whether there is "as much [rice] as" in different piles. Acknowledging the crucial role of artefacts both at a psychological and social level (Vygotsky, 1978), we also offered the children different artefacts to face this task. Taking the Semiotic Mediation perspective, we analyse the strategies and the situated signs developed by the children, identifying those that could later be used by a teacher as pivot signs when constructing the mathematical meanings associated with the process of measurement.

INTRODUCTION

Measurement of different kinds of magnitudes is one of the goals presented in the section "*learning about the world*" in the Italian National Guidelines for the primary school curriculum (MIUR, 2012). Moreover, within the international panorama, measurement is considered to be a necessary and fundamental concept to master when learning to reason mathematically (NCTM, 2000; OCSE, 2010). Measurement is considered a grounding core of mathematics school curricula because of its special role and pervasiveness in so many aspects of the practical and social life. Indeed, measuring quantities is a common action of our daily life, although it requires a culturally sophisticated operation in terms of both action and abstraction. To measure quantities means, first, to choose a unit of measure: that is, to choose a convenient sample to be compared to the quantity to be measured. Then one has to somehow *count* how many times the unit of measure fits in the quantity to be measured, and deal with a reasonable approximation based on the goal of the measurement. This planned action and *its connections with numbers*, has been one of the crucial triggers of the human social and cultural evolution (Aleksandrov, Kolmogorov & Lavrent'ev, 1974).

Since measuring and its links with numbers is a complex cultural practice, in order to introduce it at school, some appropriate pedagogical conditions need to be created. We believe that such pedagogical conditions must stem from and be coordinated with children's naïve skills in counting and quantifying. With this aim we carried out an explorative study, to analyse children's skills, upon their entrance in first grade, when working with a particular substance, rice, which can be treated as intermediate between discrete and continuous. The idea to use rice is related to the evidence coming from the fields of both neuroscience and psychology that at the origin of human numerical insights there is a strong link between the management of continuous, uncountable quantities and large amounts of discrete, countable objects (Piazza, 2010; Gallistel, Gelman & Cordes, 2006).

In this paper we present some qualitative results of our study, focussing on how young children manage the task of comparing quantities, judging whether there is "as much [rice] as" in different piles. Moreover, acknowledging the crucial role of artefacts both at a psychological and social level (Vygotsky, 1978), we offered the children different *artefacts* to choose from and work with if they thought it might help. Here we report on different strategies activated by the children to accomplish the tasks they were given. We analyse these strategies reflecting on which aspects can be potentially used by a teacher in order to build mathematical meanings associated to the measuring process.

THEORETICAL BACKGROUND OF THE STUDY

Historically and pedagogically (natural) numbers are firstly seen as tools for counting. Most approaches in mathematics education prevalently assign, with minor differences, a primitive and dominant role to natural numbers and to the action of counting discrete, countable magnitudes. The management of countable magnitudes usually consists in counting the discrete entities of which it is composed. In general, approaches from the Western tradition tend to introduce natural numbers before measure. For example, Sfard (1991) proposes a reconstruction of the number concept within a process/object dialectics where the counting process constitutes the starting point, whereas the measuring process appears only at a later step, when rational numbers are generated. A different approach was proposed by the Russian psychologist Davydov who placed the experience of measuring continuous quantities as preliminary to the introduction of numbers (Davydov, 1982). In his perspective, counting itself may be conceived as the particular measuring process of sets of discrete objects. Therefore, Davydov suggests that in early education managing continuous quantities should precede the introduction of natural numbers. This crucial idea has found several followers and initiatives all around the world, one among others the project Measure Up (see for example Dougherty & Slovin, 2004).

In our research we are not arguing in favour of assigning priority to either the discrete or the continuous approach, but, instead, we wish to reflect on possible effective strategies of early cultural mediation, in order to create solid links between the discrete and continuous aspects of numbers (in the direction of Iannece, Mellone and Tortora, 2009). This approach seems to be in line with neuroscientific findings about the preverbal mathematical systems. Two systems for numerical quantification, which children are equipped with before symbolic learning (spoken or written language, number systems, symbol systems and so on), have been identified: the object tracking system (OTS or 'parallel individuation'), and the approximate number system (ANS or 'analogue magnitude') (Piazza, 2010). The first system is specialized in recognizing the numerosity of small groups of objects (usually up to four), by "subitizing", while the second one provides "*an analogical representation of quantities, in which numbers are represented as distributions of activation on the mental number line*" (Dehaene, 2001, pp. 10-11). What is especially interesting is that the second system is activated not only for comparing and manipulating continuous quantities, but also for perceiving and processing discrete quantities in an approximate way. Moreover, there is evidence of existing links between these abilities regarding approximate estimation of non-symbolic magnitudes (for example sets of dots, or grains of rice) and approximate evaluation of spatial extension, and the numerical symbols (Piazza, 2010). Moreover recent research (e.g., McMullen, Hannula-Sormunen & Lehtinen, 2014) showed how children's tendency to spontaneously focus on quantitative relations can be used as predictor of their following rational numbers knowledge. During this longitudinal research project a group of children was followed for four years; in particular, these children's ability to evaluate quantities of cake or of rice (their success in non-explicit mathematical tasks) in first grade positively correlated with their abilities to manage fractions three years later (their conceptual knowledge of fractions).

We took inspiration from these studies to design a particular task with the aim of exploring processes related to these existing links between the approximate estimation of large amounts of discrete quantities and symbolic systems.

This study was set up with the goal to gather information on strategies that children, raised in the Italian culture, may produce, based on possible cognitive links between the approximate estimation of large amounts of discrete quantities and symbolic systems. Moreover, we focus this study on a special age range: that of children at the beginning of primary school, when the exposition to symbolic systems has already started, but is not very advanced. We deal with *spontaneous daily concepts* (Vygotsky, 1987) that could be activated by children managing a particular kind of substance, rice, that is *in between* discrete and continuous magnitudes. We also offered the children a variety of artefacts to choose from if they thought any of these would be helpful; so that we could analyse the use of the particular artefact and, then, the potential semiotic activity produced in accomplishing the task. We take the Semiotic Mediation perspective (Bartolini Bussi & Mariotti, 2008), inspired by the Vygotskian theory, adapted by Bartolini Bussi and Mariotti (2008) to mathematics education. The Theory of Semiotic Mediation states that the progressive and intentional introduction of artefacts in educational activities can play a crucial role if teachers use them as "instruments for *semiotic mediation*".

> […] the teacher may guide the evolution towards what is recognizable as mathematics. In our view, that corresponds to the process of relating personal senses (Leont'ev, 1964/1976, p. 244 ff.) and mathematical meanings, or of relating spontaneous concepts and scientific concepts (Vygotsky, 1934/1990, p. 286 ff.) (Bartolini Bussi & Mariotti, 2008, p. 754).

In particular, to analyze the role of an artifact in semiotic mediation processes, Bartolini Bussi and Mariotti (2008) use a set of interpretative tools from Rabardel's instrumentation approach (Rabardel, 1995; Béguin & Rabardel, 2000), according to which a subject, engaged in a goal-directed activity, can build schemes of instrumented action for an artefact. The artefact, together with the *utilization scheme* a subject has developed for using it to accomplish a task, becomes an *instrument*. In Bartolini Bussi and Mariotti's framework, the instrument is a tool with a double potential: it is a tool for the pupil to accomplish a given task, and also a tool for the teacher to use in the task of helping pupils construct mathematical meanings stemming from the meanings that emerged in the situated context. This framework identifies a particular class of situated signs, *pivot signs* (Bartolini Bussi & Mariotti, 2008), that are words, phrases, drawings and gestures developed within the specific context of the task. The crucial feature of these signs is their shared polysemy: they may refer to specific instrumented actions, but also to oral or written language, and to the mathematical domain. Their polysemy makes them usable as a pivot (or hinge) fostering the transition from the context of the specific task and artefact to the context of mathematics. Indeed, children, as in the case of our interviews, use a variety of terms and gestures with situated meanings; among these signs the pivot signs are good candidates for the teacher to use to foster the evolution of the situated signs and meanings towards signs and meanings in the cultural domain of mathematics.

Since the activities analyzed in our study are interviews (that each time involve a researcher and a children) and they were carried out outside the classroom context, they cannot be considered as classroom activities. This is important to remark, because this is a difference with respect to the activities traditionally studied and described within the semiotic mediation framework. Indeed, the idea of *pivot sign* was introduced in the Theory of Semiotic Mediation to study the teacher's use of a particular situated sign when guiding the students in the appropriation of related mathematical signs with culturally accepted and shared mathematical meanings. This is why we label the signs identified in the interviews as *potential pivot sign*, underlining that they have the potential to later become pivot signs in classroom activities. Knowing which signs could be later used as pivot signs can be quite useful for a teacher when designing or carrying out educational activities, and we believe that such knowledge should become part of teachers' Pedagogical Content Knowledge (Shulman, 1986). Therefore it is important to conduct analyses of students' responses, even in a setting like that of our interview, in order to identify such signs.

METHODOLOGY

In order to gain insight into the strategies mobilized by young children to compare and to measure, we chose to work with a substance typically treated as continuous but that can also be treated as many discrete entities, i.e. uncooked rice; and we asked children to judge whether there is "as much as" of a certain quantity of rice. Indeed, we wanted children to be able to propose strategies through which they could treat the substance as continuous, but we were just as interested in observing whether some children (and which ones) would opt for strategies handling the rice discretely, as many grains.

We interviewed 19 6-year-old children from an Italian public school at the beginning of first grade (at the third month from the beginning of the school year). The children had not all had the same preschool experience, and two had not attended preschool at all. During the first months of first grade, the teacher had introduced a variety of counting activities, including the use of the horizontal abacus to count children, and the use of straws for counting and representing small numbers. The class had also been taught to write the symbols of numbers up to 9. The children were interviewed one at a time through clinical interviews (Hunting, 1997), outside the classroom setting, in a social interaction with the interviewing researcher (one of the authors). The experimental set up consisted in the following material and script: the interviewer picked up a bag containing about ½ kg of rice and she poured about 200 grains onto the table lifting the bag and slowly letting the rice pour out. Then she asked the child (phase 1): "Now can you please give me <u>as much</u> as you have in front of you?". When the child had finished to form another pile of rice, the interviewer asked: "How are you sure they are the same [pointing to one pile of rice and then to the other]?" and she offered the children a variety of artefacts to choose from if they thought one would be helpful to answer the question (phase 2).

The approximate number of grains was chosen to allow the counting process for the comparison, but at the same time to make it a difficult and maybe even a discouraging task, in order to allow different strategies to emerge.

As introduced in the previous paragraph, we attribute a crucial role to the use of artefacts: after an answer to the first request, the interviewer offered the children some artefacts to choose from. By introducing these artefacts, we wanted to see if the children already knew or generated utilization schemes related to the chosen artefacts and to the management of continuous substances, or whether they preferred to count. The artefacts chosen were: a spoon, some transparent plastic cups, some straws, a ruler, a pen and paper. The rationale of the choice of these artefacts is the following: i) the spoon could be used in the attempt to measure the rice in spoonfuls, treating it as a continuous substance, possibly recalling utilization schemes developed outside of school; ii) the plastic cups could be useful for comparing the quantities of rice giving them a same practically 2-dimensional shape[1]; iii) the straws were chosen due to a previous study (Mellone, 2008) in which a child had used a straw to "line up" the grains of rice of the pile he had to measure, and then proceeded to measure the rice in "full-straws"; iv) the ruler was offered in case some children felt inclined to use strategies in which they wanted to try to measure a dimension of 2-dimensional or 3-dimensional arrangements of the rice; v) a pen and paper were offered if children wanted to write anything down, for example for helping themselves in the counting process or for drawing something. The interviewer would set all these items in front of the children during each interview, but not insist in having them use anything if they did not immediately want to. We chose to not propose other artefacts, like scales, because we preferred to not involve the concept of weight.

ANALYSES

In the analysis carried out in our study, in addition to the children's strategies and utilization schemes with the artefacts, we intended to identify some *potential pivot signs* produced by these children during the accomplishment of the task. We refer to these signs as potential pivot signs because they have the potential of evolving towards mathematical signs linked to the measurement process; indeed, these signs can be built on in the design of later teaching interventions grounded in the semiotic mediation framework. We will point out all these aspects in the excerpts we chose to present in the next section.

Strategies before introduction of the artefacts

After a first round of analysis of the strategies adopted in the first phase of each interview, we found that a main distinction can be made on whether the children's strategies are oriented towards an evaluation of the amount of rice in terms of numerosity of the grains, surface or volume occupied by the quantities of rice considered.

Four of the fourteen children focus on numerosity activating, at least initially, counting strategies. The children who choose to do this seem quite confident in their counting skills, but only two seem to really be acting in accordance with Gelman and Gallistel's counting principles (1978) and to be using the correct strategies and number words even when dealing with large numbers that had not yet been introduced at school.

> Child 1: These are more [pointing to the larger pile] because I counted them. And these here were one hundred eighty and these here fewer…one hundred…one hundred twenty three.

This child chooses to measure a group of discrete objects (in our case a pile of rice) using as a unit a single discrete object (here the grain of rice) of the group itself and then a counting process. The child uses the comparison between the numbers he found counting as a means to compare the cardinality of the two piles. Here the number words and the comparison made "these are more [...] one hundred eighty [...] these here fewer [...] one hundred twenty three", are "situated signs" because related to

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[1] The about 200 grains of rice form a thin layer in the plastic cup, if the quantities to compare were slightly larger, we expected that the cups might be used to compare the heights of the rice in each cup.

the particular task, but they also refer to a standard measurement strategy potentially extendable to other situations. These could be used as pivot signs in a subsequent teaching intervention about counting discrete quantities.

Three children consider the properties of the piles, seeing them as two-dimensional objects. One child forms little flattened piles and rounds them into similar circular shapes. When the interviewer asks how this is helpful, he answers:

Child 2: This is a lot big and this is big

The "a lot big" [it: "tanto grande"] seems to be deictic, as if it indicates a specific quality of that pile, almost saying that the pile is "very large", and the other "big" is used to express that the second pile has a similar property as the first, but it is a bit smaller. In this sense the child expresses a comparison which in not quite yet a comparison between *measurement*s but it can become the starting point to introduce the relationship between two quantities.

Another child tries to compare the amounts by flattening the piles: he seems to focus on the two-dimensionality of the shapes obtained. In this case the identification of the same "shape", as the same surface occupied, can ground the processes about the area comparison. When asked to explain what he is seeing and doing, he says:

Child 3: I have done like this [he scatters the two piles so that they roughly occupy the same surface, (Fig. 1)] [...] I look at the rice, and that this and this are the same.

Figure 1: child 3 flattening piles of rice.

In another case, a child has trouble explaining what she is evaluating, but her manipulation of the rice and her manual control suggest that she is considering the two-dimensional form. In response to the interviewer's request to explain how she was sure, she says:

Child 4: They are the same because ... uhm …because this one has the amount of this [she highlights the limits of the two piles with her hands] because they are equal, because this is equal to this.

In this case the potential pivot sign are not only the words, but also the child's gesture of limiting the two piles in order to identify congruent shapes.

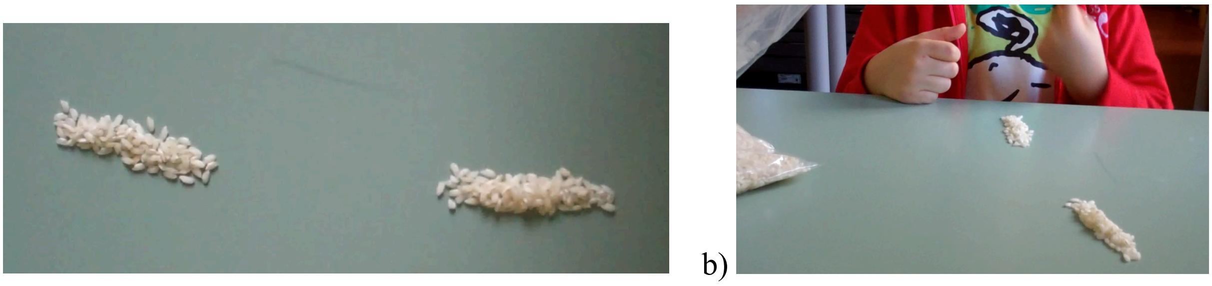
Indeed, the gesture led to the generation of similar shapes for the piles. This could later, through discussion and comparison with other proposed strategies (e.g., that of child 6 showed below), be described as volumes with a same base (2D surface) of which the heights can be compared to establish which is greater.

Five children more or less explicitly refer to the volumes of the piles. After making the two handfuls, a little girl says:

Child 5: This here is fatter [she then rearranges the pile making it more compact].

The control on the rice is essentially visual and tactile; it seems that in making the pile more compact with her hands she *feels* something, but the term "fatter [it: "ha più ciccezza"]" suggests that she is substantially considering both the surface and the height, thus the volume occupied.

Then there are children who show more organized strategies, such as a child who, after forming two elongated shapes (Fig 2), says:



a) b)

Figure 2a,b: child 6 comparing piles of rice using volume.

Child 6: Mine is lower and yours is higher and it means that I have less and there, you there have more.

It would have been sufficient for the child to add something like: "if/since they have the same base area", and her method would have been mathematically exact. We interpret the expression as a potential pivot sign, since it can go beyond the particular task we are considering. Here, the unit of measurement is precisely the selected surface given by the shape of the base: the volume is measured by layers of equal surfaces. Similar strategies that depend on the measurement of volume are used by other children: in particular, one who, after generating the second pile, checks the amount looking at the height.

Child 7: It went a bit up [pointing to the first pile] and also here it went a bit up [...] Wait [he makes the gesture of moving his index finger horizontally from a pile at the other Fig. 3] [...] No, here there is still some missing [he adds 3 or 4 grains and makes the pile compact again] Yes, now I'm sure.

Figure 3: child 7 comparing piles of rice using volume.

Here, with respect to the previous strategy, the idea seems quite similar except for an initial focus of giving piles the same base area. Moreover, we consider the gesture of the finger that moves from one level to another as a potential pivot sign, that is essentially isomorphic to the recognition of the difference in the volumes of water contained in identical cylinders through the level (Fig. 4) proposed by Davydov (1982). Therefore, this sign can be used in the design of future in measurements activities with continuous quantities.

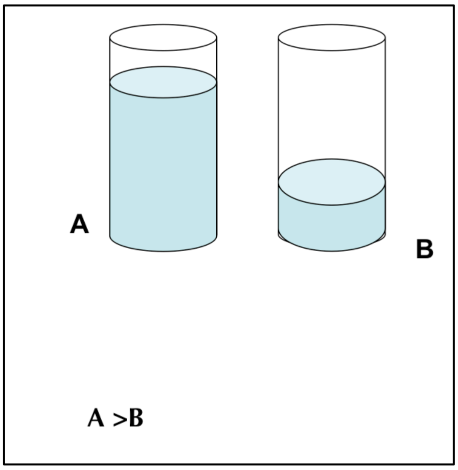


Figure 4: difference in volumes of water in identical cylinders.

Strategies with artefacts

Some children exhibit sophisticated counting abilities during the interview, and successfully compare the piles based on the numerosity of the rice in them. We note that for most of these children the counting strategy seems to be satisfying enough for them to choose not to use the artefacts in the second phase of the interview. On the other hand, the children who choose to count, but have trouble due to difficulties in remembering the number words for the numbers after ten, tend to use new strategies for evaluating the quantities of rice in the second phase of the interviews.

When the interviewer proposed to use artefacts, two children chose the ruler to evaluate **1-dimensional** measures of the rice piles. One of these children had initially decided to count up the rice grains in each pile, but he had then given up; he then decided to use the ruler. To do this he lines up the two piles of rice on the sides of the ruler (Fig. 5) and he says that to make the piles the same he has to add some rice to the second pile. He does this, lining up the piles again and again after each addition of rice. He then says:

Figure 5: 1-dimensional strategy comparing lengths of piles "lined up".

Child 8: I measure it like this. […] I measure, like, I measure if they are the same or not.

A careful analysis of the video indicates that the child is not reading the numbers on the ruler, so the instrumented action scheme is not the conventional one the artifact was constructed for. Also another child uses the ruler to compare the rice quantities without reading the numbers on the artifact: she arranges the rice along the entire length of both sides of ruler, as if she were trying to confirm the fact that the two piles contain the same amount of rice. In this case the ruler seems to be used as an axis of symmetry (Fig. 6).

Figure 6: 1-dimensional strategy comparing lengths of "symmetric" piles.

The word "measure" in these examples is inappropriate with respect to its mathematical meaning, because here it relates to a process of comparison in which no numbers are actually involved. However, in a later whole-class activity the meaning of this word, used as pivot sign, could potentially evolve towards its cultural meaning. For example, the teacher could make explicit the differences between the child's process and the culturally approved process of *measuring*.

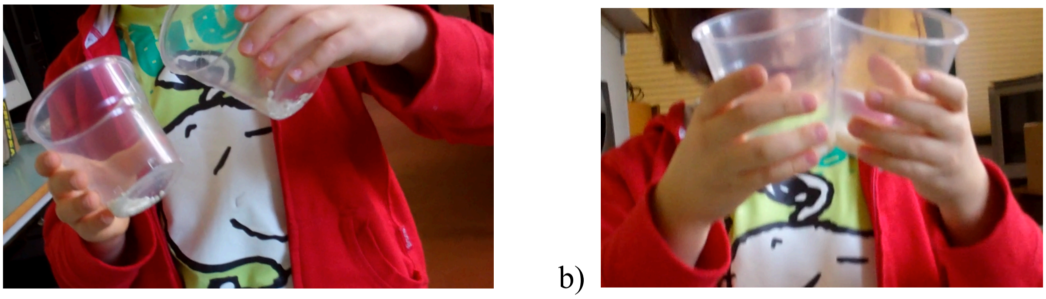
A **2-dimensional** evaluation of the size of the rice piles is accomplished by some of the children through two main different processes: an evaluation of the length of one of the curved sides of the surfaces occupied by the piles, or an evaluation of the surface by using a selected unit of area. We will focus on this second process. One child chooses the straws and picks up a handful (6 of them), which suggests she is not trying to establish a one-to-one correspondence between straws and grains. Then she holds the straws flat over each flattened rice pile (Fig. 7) and says:

Figure 7: 2-dimensional strategy comparing surfaces with straws lined up.

Child 9: They look equally big [it: "grosse", it could also mean "fat"] if I put them like this. […] Like to count we would put thousands, thousands, thousands [it: "migliaia"]…

In this case the straws seem to be used to measure the area. Also in this case the utilization schemes are not those for which the artifact was constructed: the student shows the interviewer that the same number of straws covers both piles, and she seems to be referring to a straw as "a thousand". It is interesting that the word *a thousand* is already present in some children's vocabulary (another girl writes “1000” on paper). Although it is not appropriate for this context because the grains of rice are in the order of hundreds, the word seems to be somewhat connected to an idea of orders of magnitude and to an idea of possibility of using a counting strategy for rice, perceived as a discrete substance.

To evaluate volumes using **3-dimensional** strategies, one child decides to use the ruler to measure the height of the piles of rice, while four children choose the plastic cups. Of these four children one is Child 6, who before being introduced to the artefacts referred to the heights of the piles (Fig. 2b). She chooses the plastic cups and places the two piles of rice one in each cup. The cups turn out to be rather large compared to the quantity of rice considered (the rice barely covers the bottom of the cup). So the child inclines the cups trying to obtain the same degree of inclination (Fig. 8 a-b) and says:



a) b)

Figure 8a-b: 3-dimensional strategy comparing heights of the volumes of rice in cups.

Child 6: These are more because they do not stay [it: “stare”, it could also mean “fit”] and they go farther forward than the others, and the others are fewer because they reach to here.

The child seems to be trying to adapt the artefact to her initial idea by obtaining same surfaces and then comparing levels in the cups: same heights of the piles would indicate equivalent volumes. This idea seems to be expressed in the pivot signs “they go farther forward” or “they reach to here”: these signs have specific contextual connotations, but they are potentially exploitable in a process of semiotic mediation leading to culturally accepted forms of volume evaluation, such as using a graduated measuring cup and using expressions referred to precise levels (frequently identified through numerical values) of the measuring cup.

Since the student says that the piles do not contain the same amount of rice, the interviewer asks her to modify the piles so that they contain the same quantity. Now she adds enough rice to each pile so that the cups properly set down on the table contain rice up to the same height. The interviewer asks her if she is sure that now each pile contains as much rice as the other and the child answers:

Child 6: I am not sure, counting is better, but [I would have to] count up to a thousand and a hundred.

Here we can notice that, because in the task the degree of precision required in the comparison is not clarified, the children can choose the approximation that they consider satisfying or easier to obtain.

Then the interviewer asks the child to show what she is looking at; the child puts the two cups close to each other (Fig. 8) and says: “the difference”. This could also become a pivot sign, in that here it indicates a specific contextual aspect, the difference in height of the rice in the cups, but it can also refer to a general strategy linked to the result of the operation of subtraction. The situation is analogous to the one in Davydov (Fig. 4, in which “difference” has a double meaning: it refers to the result of a comparison and to the result of the operation of subtraction, related not only to the classical meaning of "taking away" but also to the identification of the quantity to add or subtract in order to obtain an equality).

DISCUSSION AND FUTURE PERSPECTIVES

The analyses presented in this paper show different strategies carried out by children at the beginning of first grade when asked to evaluate and manage quantities of rice, a substance that can be treated as continuous or discrete. This led to the identification of a set of situated signs that they could later be used by teachers as pivot signs when constructing the meanings of discrete and continuous measurement of area and volume. As discussed earlier, we label these signs as *potential pivot signs* because they potentially can become pivot signs in semiotic mediation activities (Bartolini Bussi and Mariotti, 2008).

The design of educational intervention during classroom activities is not the focus of the current paper, so it is not included. Indeed, the interviews we conducted serve as a preliminary basis assessing school children's incoming informal knowledge. Building on the results of this research, we plan to design an intervention framed within the theory of semiotic mediation, in which the teacher builds on the children's initial signs identified here, to then lead them to refine their own strategies and develop cultural and more formal approaches and meanings related to *measurement*.

Moreover, in classroom activities using "as much as" can be framed as a problem in a narrative context that could further motivate children to compare the rice quantities (see also the activities with the rice proposed in McMullen, Hannula-Sormunen & Lehtinen, 2014). A possible activity could start by splitting the children into groups of 4-5 components working at each table, and telling a story in which each of them is a farmer who collected a certain amount of rice (a quantity of about 200 grains each). The farmers are frightened by a monster who makes them give it all their rice, which all gets placed in the center of the table in a large bowl (with a picture of a monster on it). All children around the table pour their rice into the monster's bowl. At night time the monster falls asleep and the farmers can go and take their rice back. Taking turns, the children take back "their" rice: some will take more and some less than their original share, so there will be a felt need of finding a way to measure the rice. At this point the teacher could give the children some small measuring containers like transparent cylinders with a diameter of about 4 cm; we expect that some children will use this artefact comparing the levels of the rice in different measuring containers and pointing out these levels, as we noticed when identifying potential pivot signs in the interviewed children's responses. Now the teacher, through appropriate didactical mediation and, possibly, orchestrating a mathematical discussion, could foster the evolution of this sign into a mathematical sign, using it as a pivot between situated meanings and culturally accepted mathematical meanings.

This type of activity can pave the way for gradually more formal reflections on measuring instruments or even, more generally, on the theme of approximation.

Obviously, we could have chosen other artefacts, different from the ones we worked with here; and this may be included in the later activities with rice, always with the awareness that the children may use these artifacts with non-conventional and personal utilization schemes. These sorts of activities should be intertwined with other ones involving numbers, shapes, areas and volumes; together they can be used to broaden the mathematical meanings gradually constructed. With this aim, it could be insightful to conduct deeper analyses of the children's strategies and to identify relationships between the children's strategies before they use the artefacts and the utilization schemes for their chosen artefacts. This direction of research promises to be quite fruitful.

Another development of this study could be to relate the strategies used by the children to some of the neuroscientific models described in recent studies. In this respect, we note that many of the strategies we identified, especially the ones based on comparison of spatial properties of the piles of rice, may be grounded in cognitive abilities that deal with manipulation of non-symbolic magnitudes (for example, sets of dots) and an approximate sense of spatial extension. This direction of research could be quite useful for educational purposes, because, as suggested by Piazza (2010), there is evidence of existing links between these abilities and those involved in managing numerical symbols. Indeed, Piazza argues that cognitive abilities that deal with manipulation of non-symbolic magnitudes and those involved in an approximate sense of spatial extension can lead to the development of neural mappings that strengthen the meaning of the numerical symbols and that consequently strengthen many mathematical skills based on them.

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