THE PROCESSES OF JUSTIFICATIONAL MEDIATION - DEVELOPING AN ANALYTICAL MODEL

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On the poster we present research in progress on developing an analytical tool, to capture the interplay of the reasoning competency from the Danish KOM-framework (2019) and a digital environment, specifically GeoGebra. Competency is "…someone's insightful readiness to act appropriately in response to a specific sort of mathematical challenge in given situations." (Niss & Højgaard, 2019, p. 6). The reasoning competency is associated with situations where students analyze or engage in oral or written mathematical argumentations in various forms from exemplifying to formal proof.

Geraniou and Jankvist (2019) propose to apply the Theory of Instrumental Genesis (TIG) to bridge the theoretical gap between mathematical competencies and students' use of digital technologies. However, relating the TIG to the reasoning competency presents new challenges. It implies that using instruments involves *pragmatic mediation,* concerning the subject's actions on objects and *epistemic mediation* concerning how the subject gains knowledge of objects' properties (Rabardel & Bourmaud, 2003). Moreover, Misfeldt and Jankvist (2019) develop the notion of *justificational mediation* (JM) to describe mediation aiming at establishing truth of mathematical statements; this is done in the context of CAS-assisted proofs in textbooks in upper secondary school. We aim at extending such previous research on JM to other situations, broadening the notion to the context of informal justification processes of early secondary students interacting with GeoGebra.

CONSTRUCTING A MODEL OF JM

Grounded in the TIG the JM model particularly draws on Drijvers, Godino, Font, and Trouche (2013), who describe the TIG through dualities of which we use two. First, the artefact-instrument duality - the process of instrumental genesis as the user turning an artefact into an instrument for accomplishing specific tasks. This duality must have been initiated for students to be able to use GeoGebra in a process of JM, we refer to GeoGebra as an instrument. Second, the scheme-technique duality that relates Vergnaud's (2009) notion of scheme to a student's visible actions with an instrument, by addressing the relationships between gestures (what is said and done) - and schemes (what is thought). Schemes hold conceptual elements as mathematical concepts, expressed formally through definitions and theorems, as well as expressed through the instrument.

Seeing JM as a process that has the objective of changing the status of a claim, we find that the analytical tools provided previously were not sufficient for unraveling the structure of the JM process. So, considering JM a particular process of argumentation, we use Toulmin's model to identify key structural elements, and we develop a model describing such elements when these emerge from the use of an instrument.

Combining Toulmin's model and the TIG

Toulmin's model is commonly used in research of argumentation and proof (Hanna, 2014). Following, we introduce elements from Toulmin's model (2003) and how we interpret it with respect to JM.

The *claim* is a statement uttered with a certain indication of likelihood (*qualifier*) and justified through data, warrant, and backing (Toulmin, 2003). The first utterance of a claim indicates the start of the JM process, aiming to change the qualifier. Change in status can be from likely to more likely, true or false. Status change can be students' restatement of the claim accompanied by a new qualifier. It is reached through generating *data* that for the solver constitutes evidence supporting the claim along with the *warrant* consisting of inference rules connecting the data to the claim (Toulmin, 2003). If the warrant is implicit it must be inferred. We relate this process of *generating data* to the scheme-technique duality (Drijvers et al. 2013), seeing it as the techniques carried out by the student along with utterances and hand-gestures; the *data* are the result of these. The warrant consists of the conceptual elements in schemes. . I consider the *backing* for JM as an explanation of why the warrant is relevant (Simpson, 2015), explaining why warrants are relevant for generating data that will allow the change in the status. Thus, the backing becomes fundamental to the JM process. Currently, I have reached the following formulation as *backing* in JM processes:

> If the claim is true, I can generate data, within the specific instrument, that is consistent with the claim.

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