

Can subsidies rather than pollution taxes break the trade-off between economic output and environmental protection?

Abstract

We build a general equilibrium dynamic model in which individual investors are endowed with “warm-glow” preferences a la Andreoni (1990) so that they feel partly responsible for the pollution content of their portfolio. Through investors’ portfolio choice, firms are induced to engage in costly abatement activities, given that higher pollution also implies a higher cost of capital. In this scenario, we characterize the equilibrium of the economy and investigate, through a fiscal reform analysis, the effects of such tax instruments on the equilibrium scale of the economy, per-capita consumption, pollution abatement and “pollution premium”. We show that an increase of the pollution tax, while reducing pollution, also depresses consumption, the scale of the economy and the pollution premium. On the contrary, an increase of subsidies on abatement activity increases the scale of the economy and can also decrease pollution and the pollution premium and increase per-capita consumption. All our results have relevant testable implications, which we leave for future empirical research.

JEL Classification: D21, D53, G11, H21, H23, M14, Q58.

Keywords: Socially responsible investment, corporate social responsibility, pollution, taxation.

1. Introduction

In this paper we analyse the effects of fiscal policies aimed at reducing pollution in the context of financial markets populated by socially responsible investors. More precisely, we build a general-equilibrium-dynamic model in which individual investors are endowed with “warm-glow” preferences a *la* Andreoni (1990), so that they feel partly responsible for the pollution content of their portfolio and, at the equilibrium, ask for a “pollution premium” in order to hold “dirty assets”. Pollution emerges as a by-product of production activity of profit-maximizing firms, which are also endowed with a costly abatement technology. Finally, the Government levies a tax on firm’s pollution flow and a subsidy on its abatement activity. In this scenario, we characterize the equilibrium of the economy and investigate, through a fiscal reform analysis, the effects of such fiscal instruments on the equilibrium scale of the economy, pollution abatement, per-capita consumption and pollution premium.

The issue we investigate is relevant under several points of view. First of all, concern on natural resource depletion and environmental quality has been spreading throughout the world in recent decades. Several initiatives have been put in place by private organizations aimed at implementing common strategies to integrate social, environmental and governance (ESG) factors into both business reporting and traditional investment risk analysis (see OECD 2017).

Moreover, the mounting public debate on these issues promoted by shareholder activism, mobilization of NGOs and social media, the diffusion of new data from credit rating agencies, coupled with the effects of the recent financial crisis, have increased the demand for more transparent management from financial investors and for more responsible behaviour from the productive sector. All these elements have contributed to the spreading of the well-known phenomenon called “Socially Responsible Investing” or “Sustainable investing”.

According to Eurosif, SRI “is a long-term oriented investment approach, which integrates ESG [i.e. Environmental, Social, Governance] factors in the research, analysis and

selection process of securities within an investment portfolio. It combines fundamental analysis and engagement with an evaluation of ESG factors in order to better capture long term returns for investors, and to benefit society by influencing the behaviour of companies.” (Eurosif 2016, p. 9). Hence, SRI is a process of identifying and investing in companies that meet certain standards of Corporate Social Responsibility (CSR)¹ through such activities and strategies as positive or negative screening, shareholder advocacy, impact and community investing (for more details see GSIA, 2016). SRI has been argued to be a possible instrument to improve environmental quality through a market mechanism.

In this scenario, under the impulse of several international summits (from Kyoto in 1997 to Paris 2015), many public institutions have been implementing or proposing fiscal and regulatory policies aimed at contrasting the worsening of environmental quality and at increasing the ESG-practices (as for the recent initiatives, see OECD 2017 and, for the EU, see Eurosif 2018)

For example, in 1995 the Dutch government has launched the Green Funds Scheme, a tax incentive scheme for investors into green initiatives. In the U.S. there are examples of tax-credit bonds, (bond investors receive tax credits instead of interest payments so issuers do not have to pay interest on their green bond issuances) or tax-exempt bonds.

In light of this recent trend and for its wide potential implications concerning, in particular, the design of pollution abatement policies and of fiscal incentives for green initiatives, in this paper we aim at shedding new light on the effectiveness of fiscal policies in enhancing environmental quality and showing the conditions under which they can also improve the performance of the economy, increase per-capita consumption and reduce the pollution premium.

¹ For a recent review of economic literature on CSR, see Brekke and Pekovic (2018).

The paper is organized as follows: in section 2 we discuss the related literature, in section 3 we specify the model and characterize the equilibrium and its stability; in section 4 we carry out a tax-reform analysis and discuss the results. Section 5 concludes.

2. Related literature

Several scholars have analysed the issue of environmental quality and fiscal incentives from an economic perspective². However, while the literature on pollution taxes has been flourishing, the economic literature on SRI³ and on its consequences on taxation is still embryonic and results are mixed.

For example, Hainkel et al. (2001), adopt a one-period model to show that negative screening on polluting firms by fund managers can induce firms to adopt cleaner technologies in order to avoid higher costs of capital. The positive effects of financial markets on environmental quality are also stressed by Dam (2011), who argues that SRI creates a role for the stock market to deal with intergenerational environmental externalities. The author shows that, although socially responsible investors are short-lived, the forward-looking nature of stock prices, reflecting the warm-glow motive, can help to mitigate the conflict between current and future generations. Dam and Scholtens (2015) develop a model that links SRI and CSR, showing that responsible firms display higher returns on assets, although the overall effect on stock market returns depends on the relative strength of supply and demand side effects.

On the other hand, Barnea et al. (2005) argue that negative screening reduces the incentives of polluting firms to invest so that also the total level of investment in the economy decreases. Dam and Heijdra (2011) analyse the effects of SRI and public abatement on

² The seminal work is Sandmo (1975). See also Cremer et al. (2001) and the survey by Bovenberg and Goulder (2002). More recent works on this subject are Bontems and Bourgeon (2005), Goulder and Parry (2008), Gahvari (2014), Jacobs and De Mooij (2015), Kampas and Horan (2016), Belfiori (2018), Pizer and Sexton (2019).

³ For a survey on the topic, see Renneboog et al. (2008).

environmental quality in a growth model with socially responsible investors and show that SRI behaviour by households partially offsets the positive effects on environmental quality of public abatement policies.

Finally, Vanwalleghem (2017) argues that SRI may have a mixed effect on firms' incentives to remove negative externalities. In fact, whereas SRI screening incentivizes the removal of externalities (as predicted by Heinkel et al. 2001 and confirmed by the empirical work of Hong and Kacperczyk 2009), SRI trading can disincentivise it when traders disagree on the externality removal's cash flow effects.⁴

While providing interesting results, the above-mentioned literature has not analysed the effects of fiscal instruments in presence of SRI.⁵ To bridge this gap, in this paper we specify a continuous-time model, where pollution is a by-product of production, but firms can engage in abatement, reducing net pollution. We model investors' social-responsibility objective through a warm-glow mechanism as in Andreoni (1990) and Dam (2011)⁶. Through investors' portfolio choice, firms are induced to engage in socially responsible activities (abatement), given that higher pollution also implies a higher cost of capital on the capital markets. In this scenario, we carry out a tax-reform analysis to evaluate the effects of fiscal instruments (i.e. taxes on

⁴ Graff Zivin and Small (2005) and Baron (2007) also focus on socially responsible firms and financial markets. However, these are partial-equilibrium and static models, in which social responsibility is concerned with charitable giving and not with abatement of externalities or public bads.

⁵ In a recent paper, Renström et al. (2019) present a normative analysis of second-best taxes in presence of socially responsible investors, although disregarding subsidies to abatement activities. Martín-Herrán, Rubio (2018) carry out an analysis of second-best taxation for a polluting monopoly. Fiscal policies for pollution abatement are discussed by Poyago-Theotoky, (2007, 2010), Ouchida and Goto (2014, 2016), Lambertini et al. (2017), although in partial equilibrium static models with imperfect competition.

⁶ While there is increasing evidence of the very existence of warm-glow preferences (see Andreoni et al. 2017), the exact shape is far from being clear. However, some recent works have produced axiomatizations of the warm glow that can help to characterize its shape (see Evren and Minardi 2017 and the literature therein). In this work, we follow Bernehim and Rangel (2005) when stating that "one can interpret it [the warm-glow] as a reduced form for a variety of mechanisms with starkly differing welfare implications" (p. 63).

pollution and subsidies to abatement activities) on the economy scale of production, on pollution and on the “pollution premium”.

3. The model setup

We assume an infinite horizon economy, populated by H identical households and J identical firms. At date t , individuals’ utility, $u(c(t), p(t))$, is an increasing function of consumption, $c(t)$, and decreasing function of the perceived pollution content, $p(t)$, of their portfolio holdings. The idea is that an individual (as an investor) feels responsibility for the pollution caused by firms in their portfolio (*warm-glow* objective). At each date an individual chooses consumption, portfolio allocation, and savings. We treat government bonds, $b(t)$, as the only clean (pollution free) asset. Firms operate on perfectly competitive markets under constant returns technologies, both in production and abatement activity. They face taxes on pollution flow and subsidies on the amount spent on abatement.

An individual household’s life-time utility, at date 0, is:

$$U(0) = \int_0^{\infty} e^{-\rho t} u(c(t), p(t)) dt \quad (1)$$

with $\rho > 0$ the intertemporal discount rate, and $u_c > 0, u_p < 0, u_{cc}, u_{pp} < 0$. For simplicity we assume that u is additively separable. Labour supply is exogenously given at unity, so total labour supply equals population size, H , (assumed constant). Let $e^j(t)$ denote the number of shares of firm j owned by the individual, \bar{E}^j the total number of shares of firm j , and $\bar{p}^j(t)$ the “pollution content” of firm j (as perceived by the individual), then, as in Dam and Scholtens (2015), the portfolio perceived pollution index $p(t)$ is:

$$p(t) \equiv \sum_{j=1}^J \frac{e^j(t)}{\bar{E}^j} \bar{p}^j(t) \quad (2)$$

We also follow previous literature (e.g. Dam 2011 and Dam and Heijdra 2011), in assuming \bar{p}^j is linear in x^j (as any non-linearity can be captured by u):

$$\bar{p}^j(t) = \gamma \cdot x^j(t) \quad (3)$$

where $x^j(t)$ is the flow of pollution produced by the j^{th} firm.⁷ Notice that $x^j(t)$ is chosen by firm j , through its production and abatement decision, taking into account it can affect its “cleanness rating.” $X(t) = \sum_{j=1}^J x^j(t)$ is the aggregate flow of pollution. At date t , the individual’s wealth is

$$a(t) \equiv b(t) + \sum_{j=1}^J e^j(t) P_e^j(t) \quad (4)$$

where $P_e^j(t)$ the stock-market price of share j . It is convenient to define

$$\omega^j(t) \equiv \frac{e^j(t) P_e^j(t)}{a(t)} \quad (5)$$

as the portfolio share invested in firm j , and $V^j(t) \equiv \bar{E}^j P_e^j(t)$ as the stock market value of firm j . Then the portfolio pollution content is:

$$p(t) \equiv \sum_{j=1}^J \frac{\omega^j(t) a(t)}{V^j(t)} \bar{p}^j(t) \quad (6)$$

⁷ We assume that pollution is observable by investors, so we ignore any monitoring issues. In this respect see Aldashev et al. (2015).

Denoting $r_e^j(t)$ as the return on share j , $r(t)$ as the interest rate on public debt, and $w(t)$ as the wage rate, the individual's budget constraint is⁸:

$$\dot{a}(t) = \sum_{j=1}^J \omega^j(t) r_e^j(t) a(t) + [1 - \sum_{j=1}^J \omega^j(t)] r(t) a(t) + w(t) - c(t) - z(t) \quad (7)$$

where $z(t)$ a lump sum tax. Finally, the returns on shares of firm j are:

$$r_e^j(t) \equiv \frac{\dot{v}^j(t)}{v^j(t)} e^j(t) + \frac{d^j(t)}{v^j(t)} \quad (8)$$

where $\frac{d^j(t)}{v^j(t)}$ is the dividend pay-out ratio and $d^j(t)$ total dividend payments by firm j .

The individual maximizes (1) w.r.t. $c(t)$ and $\omega^j(t)$ subject to (4) and (7). The current value Hamiltonian reads:

$$\Lambda(t) = u(t) + q(t) \dot{a}(t) \quad (9)$$

where $q(t)$ is the shadow price of wealth. The first-order conditions are:

$$u_c(t) - q(t) = 0 \quad (10)$$

$$u_p(t) \frac{\bar{p}^j(t)}{v^j(t)} a(t) + q(t) a(t) [r_e^j(t) - r(t)] = 0 \quad (11)$$

$$q(t) [\sum_{j=1}^J \omega^j(t) r_e^j(t) + (1 - \sum_{j=1}^J \omega^j(t)) r(t)] + u_p(t) \sum_{j=1}^J \frac{\omega^j(t) \bar{p}^j(t)}{v^j(t)} = \rho q(t) - \dot{q}(t) \quad (12)$$

Combining eq. (10) and eq. (11) we have:

⁸ We follow Merton (1971).

$$\frac{u_p(t)}{u_c(t)} \bar{p}^j(t) + V^j(t)[r_e^j(t) - r(t)] = 0 \quad (13)$$

Equation (10) is the usual consumption-optimality consumption, while (13) is the optimal portfolio-choice condition.

We notice that there is a “pollution premium” (the difference between the return on assets and the return on government bonds), which proportional to the pollution content by the firm. This is the compensation required by the household for holding “dirty assets”. Combining (8) and (13) we have:

$$\frac{u_p(t)}{u_c(t)} \bar{p}^j(t) + \dot{V}^j(t) + d^j(t) - r(t)V^j(t) = 0 \quad (14)$$

Next we pre-multiply (11) by $\omega^j(t)$ and sum from $j=1$ to J and use (12) to obtain the simplified the law of motion for the co-state:

$$q(t)r(t) = \rho q(t) - \dot{q}(t). \quad (15)$$

By time-differentiating eq. (10) and exploiting (15), we get the usual consumption-Euler equation:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma(t)} (r(t) - \rho). \quad (15')$$

$$\text{with } \sigma(c) \equiv -\frac{u_{cc}(t)c(t)}{u_c(t)}.$$

2.2. Firms

Firms operate on perfectly competitive markets, producing a homogenous good under identical constant-returns to scale production technologies, using capital and labour. We will then be able to aggregate the firms to obtain a representative firm. We denote firm j 's production function as:

$$y^j(t) = f^j(k^j(t), l^j(t)) \quad (16)$$

where $k^j(t)$ and $l^j(t)$ are physical capital- and labour-inputs, respectively. Following Brock and Taylor (2005), we assume that, at date t , every unit of output generates ε units of pollution, and that pollution can be reduced by abatement activity, $\alpha(t)$. Abatement is a constant returns-to-scale function, increasing in the total scale of firm activity $f(t)$ and of the firm's efforts at abatement, $f^\alpha(t)$. Let abatement level $\alpha(t)$ remove $\varepsilon \cdot \alpha(t)$ units of pollution, we can write total pollution by firm j as:

$$x^j(t) = \varepsilon \cdot f^j(t) - \varepsilon \cdot \alpha(f^j(t), f^{\alpha j}(t)) \quad (17)$$

It is convenient to define $\psi^j(t) \equiv \frac{f^{\alpha j}(t)}{f^j(t)}$ as the fraction of output devoted to abatement. Then by exploiting constant returns-to-scale, we obtain:

$$\frac{x^j(t)}{f^j(t)} = \varepsilon \cdot [1 - \alpha(1, \psi^j(t))] = \varepsilon \cdot [1 - \alpha(\psi^j(t))] \quad (18)$$

with α increasing in ψ^j and, thus, eq. (18) gives $\psi^j(t) = \Psi\left(\frac{x^j(t)}{f^j(t)}\right)^9$, with $\Psi' < 0$, $\Psi'' > 0$.¹⁰

As the government levies taxes, $\tau^x(t)$, on pollution and subsidizes abatement spending $s(t)\Psi f^j = S(t)$, the gross operating profits of firm j are:

$$\pi^j(t) \equiv \left[1 - (1 - s(t))\Psi\left(\frac{x^j(t)}{f^j(t)}\right)\right] f^j(k^j(t), l^j(t)) - w(t)l^j(t) - \tau^x(t)x^j(t) \quad (19)$$

We abstract from corporate bonds¹¹ and assume that the total number of shares remains constant, then new investments, $i^j(t)$, can only be financed via retained earnings, $Re(t)$, i.e. $\pi^j(t) = d^j(t) + Re^j(t)$. The firm's capital accumulation is then:

$$\dot{k}^j(t) = i^j(t) - \delta k^j(t) \quad (20)$$

where δ is the (constant) instantaneous depreciation rate. Then we have:

$$\dot{k}^j(t) = \pi^j(t) - d^j(t) - \delta k^j(t) \quad (21)$$

which together with (19) and (21) becomes:

$$\dot{k}^j(t) = \left[1 - (1 - s(t))\Psi\left(\frac{x^j(t)}{f^j(t)}\right)\right] f^j(k^j(t), l^j(t)) - w(t)l^j(t) - \tau^x(t)x^j(t) - d^j(t) - \delta k^j(t) \quad (22)$$

⁹ From (18), $\alpha(\psi^j(t)) = 1 - \frac{1}{\varepsilon} \cdot \frac{x^j(t)}{f^j(t)}$ gives $\psi^j(t) = \alpha^{-1}\left(1 - \frac{1}{\varepsilon} \cdot \frac{x^j(t)}{f^j(t)}\right) \equiv \Psi\left(\frac{x^j(t)}{f^j(t)}\right)$.

¹⁰ For example, assuming the following form for the abatement technology: $\alpha(f, f^\alpha) = f^{(1-\xi)} f^{\alpha\xi} = \psi^\xi f$, then $\frac{x}{f} =$

$\varepsilon \cdot [1 - \psi^\xi]$ and $\psi = \left(1 - \frac{x}{f \varepsilon}\right)^{\frac{1}{\xi}}$.

¹¹ Corporate bonds would be equivalent to shares in our model, as they would also carry the same pollution premium as shares.

We integrate (14) to obtain:

$$V^j(0) = \int_0^\infty e^{-\int_0^t r(s)ds} \left[d^j(t) + \frac{u_p(t)}{u_c(t)} \bar{p}^j(t) \right] dt \quad (23)$$

which is the firm value at date 0. Finally, using (22) to substitute for $d^j(t)$ in (23), we have:

$$V^j(0) = \int_0^\infty e^{-\int_0^t r(s)ds} \left\{ \left[1 - (1 - s(t))\Psi \left(\frac{x^j(t)}{f^j(t)} \right) \right] f^j(k^j(t), l^j(t)) - w(t)l^j(t) - \tau^x(t)x^j(t) - \delta k^j(t) + \frac{u_p(t)}{u_c(t)} \bar{p}^j(t) - \dot{k}^j(t) \right\} dt \quad (24)$$

Firm j maximizes its value (24) w.r.t. $l^j(t)$ and $x^j(t)$, yielding the first-order condtions:

$$\left[1 - (1 - s(t))\Psi \left(\frac{x^j(t)}{f^j(t)} \right) + (1 - s(t))\Psi' \left(\frac{x^j(t)}{f^j(t)} \right) \frac{x^j(t)}{f^j(t)} \right] f_l^j(t) - w(t) = 0 \quad (25)$$

$$\frac{u_p(t)}{u_c(t)} \frac{\partial \bar{p}^j(t)}{\partial x^j(t)} - (1 - s(t))\Psi' \left(\frac{x^j(t)}{f^j(t)} \right) - \tau^x(t) = 0 \quad (26)$$

As for the optimality condition for $k^j(t)$, $\frac{dV^j(0)}{dk^j(t)} = \frac{d}{dt} \frac{dV^j(0)}{dk^j(t)}$, classical calculus of variation yields:

$$\int_0^\infty e^{-\int_0^t r(s)ds} \left\{ \left[1 - (1 - s(t))\Psi \left(\frac{x^j(t)}{f^j(t)} \right) + (1 - s(t))\Psi' \left(\frac{x^j(t)}{f^j(t)} \right) \frac{x^j(t)}{f^j(t)} \right] f_k^j(t) - \delta \right\} dt = \frac{d}{dt} \int_0^\infty e^{-\int_0^t r(s)ds} dt \Rightarrow \left[1 - (1 - s(t))\Psi \left(\frac{x^j(t)}{f^j(t)} \right) + (1 - s(t))\Psi' \left(\frac{x^j(t)}{f^j(t)} \right) \frac{x^j(t)}{f^j(t)} \right] f_k^j(t) - \delta = r(t) \quad (27)$$

It can be shown that, by substituting (25)-(27) into (24), and exploiting CRS in $f^j(t)$, the maximized firm value is $\max V^j(0) \equiv \bar{E}^j P_e^j(0) = k^j(0)$.

3. Equilibrium and stability

We now characterize the equilibrium of the economy. At each date t , given that all firms are equal, plugging eq. (27) into (15)' yields:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma(c)} \left\{ \left[1 - (1 - s(t))\Psi\left(\frac{x(t)}{f(t)}\right) + (1 - s(t))\Psi'\left(\frac{x(t)}{f(t)}\right)\frac{x(t)}{f(t)} \right] F_K(t) - \delta - \rho \right\}. \quad (28)$$

Moreover, under the assumption that firms are equal, the feasibility constraint stating that private plus investment be equal to aggregate output (recall that we assume a balanced Government budget), reads as.¹²

$$\dot{K}(t) = \left[1 - \Psi\left(\frac{x(t)}{F(t)}\right) \right] F(K(t), H) - c(t)H - \delta K(t) \quad (29)$$

We notice that in equilibrium $\omega^j(t) \equiv \frac{v^j(t)}{H \cdot a(t)}$; which by using (4), yields the perceived pollution function (warm-glow):

$$p(t) \equiv \sum_{j=1}^J \frac{\omega^j(t)a(t)}{v^j(t)} \bar{p}^j(t) = \sum_{j=1}^J \frac{\bar{p}^j(x^j(t))}{H} = \frac{\bar{p}(x(t))}{H} = \gamma \cdot \frac{x(t)}{H}$$

where the second equality follows the linearity of $\bar{p}^j(t)$ in $x^j(t)$, and, hence, $\frac{\partial \bar{p}}{\partial x} = \gamma$. Finally, in equilibrium, eq. (26) becomes:

¹² In fact aggregating over firms we get:

$$\begin{aligned} & \sum_{j=1}^J \left[1 - (1 - s(t))\Psi^j\left(\frac{x^j(t)}{f^j(t)}\right) \right] f^j(k^j(t), l^j(t)) = \\ & = \sum_{j=1}^J \left[1 - (1 - s(t))\Psi\left(\frac{x(t)}{f(t)}\right) \right] f(k(t), l(t)) = \left[1 - (1 - s(t))\Psi\left(\frac{x(t)}{F(t)}\right) \right] F(K(t), L(t)), \text{ with } L(t) = H. \end{aligned}$$

$$\frac{u_p(t)}{u_c(t)} \frac{\partial \bar{p}(t)}{\partial X(t)} - (1 - s(t)) \Psi' \left(\frac{X(t)}{F(t)} \right) - \tau^x(t) = 0 \quad (30)$$

Eqs. (28), (29) and (30) characterise the equilibrium at each date for $(K(t), c(t), x(t))$. From (30) we obtain:

$$X(t) = X(c(t), s(t), \tau^x(t), K(t)) \quad (31)$$

Total differentiation of (31) yields:

$$-\left(R\eta + T \frac{F}{K}\right) \frac{dX}{X} + \left(T \frac{F}{K} \theta\right) \frac{dK}{K} - (R\sigma) \frac{dc}{c} - \left(\frac{X}{K}\right) d\tau^x + \left(\Psi' \frac{X}{K}\right) ds = 0 \quad (31')$$

where

$$T \equiv \left(\frac{X}{F}\right)^2 (1 - s) \Psi'' > 0, R \equiv -\frac{u_p}{u_c} \frac{\partial \bar{p}}{\partial X} \frac{X}{K} > 0, \eta \equiv \frac{u_{pp}}{u_p} p > 0, \theta \equiv \frac{F_K}{F} K > 0$$

Hence, from (31') we get:

$$\frac{\partial X}{\partial K} = \frac{X}{K} \frac{T \frac{F}{K} \theta}{R\eta + T \frac{F}{K}} > 0, \frac{\partial X}{\partial c} = -\frac{X}{c} \frac{R\sigma}{R\eta + T \frac{F}{K}} < 0, \frac{\partial X}{\partial s} = \frac{\Psi' \frac{X}{K} X}{R\eta + T \frac{F}{K}} < 0, \frac{\partial X}{\partial \tau^x} = -\frac{\frac{X}{K} X}{R\eta + T \frac{F}{K}} < 0 \quad (31'')$$

By substituting (31) into (28) and (29), the dynamic system describing the equilibrium path of the economy boils down to the following two equations in $[c(t), K(t)]$:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma(c)} \left\{ \left[1 - (1 - s(t)) \Psi \left(\frac{X(t)}{F(t)} \right) + (1 - s(t)) \Psi' \left(\frac{X(t)}{F(t)} \right) \frac{X(t)}{F(t)} \right] F_K(t) - \delta - \rho \right\}. \quad (32)$$

$$\dot{K}(t) = \left[1 - \Psi \left(\frac{X(t)}{F(t)} \right) \right] F(K(t), H) - c(t)H - \delta K(t). \quad (33)$$

By recognizing that

$$\frac{d\left(\frac{X}{F}\right)}{dK} = \frac{1}{F} \frac{dX}{dK} - \frac{X}{F} \frac{F_K}{F} = \frac{X}{K} \frac{T\theta \frac{1}{K}}{R\eta + T\frac{F}{K}} - \frac{X}{F} \frac{\theta}{K} = -\frac{X}{F} \frac{\theta}{K} \left(\frac{R\eta}{R\eta + T\frac{F}{K}} \right) < 0$$

$$\frac{d\left(\frac{X}{F}\right)}{dc} = \frac{1}{F} \frac{dX}{dc} = -\frac{X}{F} \frac{1}{c} \left(\frac{R\sigma}{R\eta + T\frac{F}{K}} \right) < 0$$

and defining $\beta \equiv -\frac{F_{KK}}{F_K} K$, the Jacobian matrix of system (32)-(33) can be written as:

$$J = \begin{bmatrix} -\frac{F_{KTR}}{R\eta + T\frac{F}{K}} & -\frac{c}{\sigma} \frac{F_K}{K} \left[T\theta \frac{R\eta}{R\eta + T\frac{F}{K}} + \left(1 - (1-s)\Psi + (1-s)\Psi' \frac{X}{F} \right) \beta \right] \\ \Psi' \frac{X}{c} \frac{R\sigma}{R\eta + T\frac{F}{K}} - H & \Psi' \theta \frac{X}{K} \frac{R\eta}{R\eta + T\frac{F}{K}} + (1-\Psi)F_K - \delta \end{bmatrix}$$

The following Proposition contains sufficient conditions for this economy to display saddle-path stability.

Proposition 1: *Sufficient for saddle path stability of the economic system is:*

$$F_K \left(1 - \Psi + \Psi' \frac{X}{F} \right) - \delta > 0$$

Proof: We recall that saddle-path stability requires $Tr(J) > 0$ and $Det(J) < 0$.

The trace of the Jacobian matrix can be written as:

$$Tr(J) = F_K \left[-\frac{T}{R\eta + T\frac{F}{K}} \left(R + \Psi' \frac{X}{K} \right) + \left(1 - \Psi + \Psi' \frac{X}{F} \right) \right] - \delta$$

By eq. (30), $R + \Psi' \frac{X}{K} = -\frac{X}{K} (-s\Psi' + \tau^X)$, which is non-positive if $s, \tau^X \geq 0$. Hence, if $F_K \left(1 - \Psi + \Psi' \frac{X}{F} \right) - \delta > 0$, then $Tr(J) \geq F_K \left(1 - \Psi + \Psi' \frac{X}{F} \right) - \delta > 0$. Next, given that $Tr(J) = J_{11} + J_{22}$ and that $J_{11} < 0$, $Tr(J) > 0$ implies $J_{22} > 0$. Moreover, since $Det(J) = J_{11}J_{22} - J_{12}J_{21}$ and $J_{12} < 0, J_{21} < 0$ and $J_{22} > 0$, then the result $Det(J) < 0$ follows. \square

Notice that the sufficient condition above is equivalent to the corresponding dynamic efficiency condition in standard in Ramsey-Cass-Koopmans models.

4. Tax reforms

In this section we carry out a comparative statics analysis to verify the effects of the tax instruments on the endogenous variables of the model, i.e. the scale of the economy, per-capita consumption, pollution and the pollution premium. In this exercise we assume that the reforms are carried out by keeping the public budget balanced, i.e. any tax change is financed by a corresponding change in individuals' lump sum tax $z(t)$.

Total differentiation of system (31'), (32) and (33) provides:

$$\begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} dc \\ dK \end{bmatrix} = \begin{bmatrix} -\frac{c}{\sigma} F_K \left[\Psi - \Psi' \frac{X}{F} \frac{R\eta}{R\eta + T \frac{F}{K}} \right] ds + \frac{c}{\sigma} F_K \frac{X}{K} \frac{T}{R\eta + T \frac{F}{K}} d\tau^X \\ \frac{(\Psi')^2}{K} \frac{X^2}{R\eta + T \frac{F}{K}} ds - \frac{\Psi'}{K} \frac{X^2}{R\eta + T \frac{F}{K}} d\tau^X \end{bmatrix}$$

Hence, using Cramer's rule we get the following results:

Proposition 2: *An increase of the tax on pollution reduces the capital installed in the economy, while an increase of the subsidy on abatement activity increases the capital installed.*

Proof: As for the effect of the tax on pollution, by defining $A \equiv \frac{c}{\sigma} F_K \frac{X}{K} \frac{T}{R\eta + T \frac{F}{K}} > 0, B \equiv$

$-\frac{\Psi'}{K} \frac{X^2}{R\eta + T \frac{F}{K}} > 0$ and given that $J_{11} < 0$ and $J_{21} < 0$, Cramer's rule provides the following:

$$|J| \frac{dK}{d\tau^X} = \begin{vmatrix} J_{11} & A \\ J_{21} & B \end{vmatrix} = J_{11}B - AJ_{21} > 0. \text{ Given that } |J| < 0, \text{ it follows that } \frac{dK}{d\tau^X} < 0.$$

As for the effect of the subsidy on abatement activity, by defining

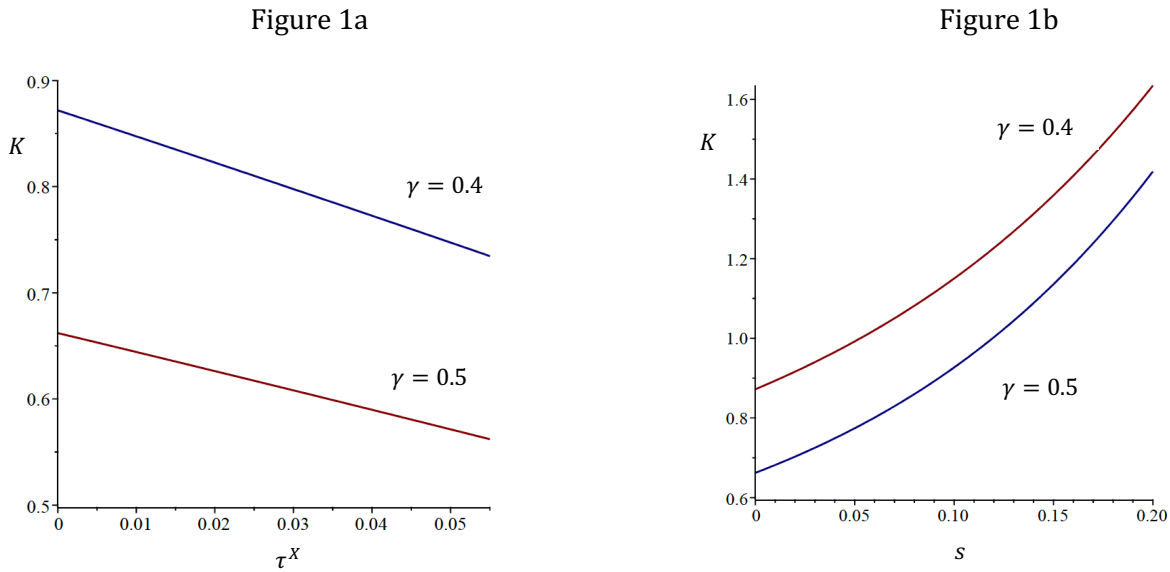
$$a \equiv -\frac{c}{\sigma} F_K \left[\Psi - \Psi' \frac{X}{F} \frac{R\eta}{R\eta + T \frac{F}{K}} \right] < 0, b \equiv \frac{(\Psi')^2}{K} \frac{X^2}{R\eta + T \frac{F}{K}} > 0$$

and given that $J_{11} < 0$ and $J_{21} < 0$, Cramer's rule provides the following: $|J| \frac{dK}{ds} = \begin{vmatrix} J_{11} & a \\ J_{21} & b \end{vmatrix} =$

$J_{11}b - aJ_{21} < 0$. Given that $|J| < 0$, it follows that $\frac{dK}{ds} > 0$. □

The economic rationale behind the results can be stated as follows: the tax on pollution corresponds to a tax on profits, and thus, by reducing the marginal productivity of capital, induces firms to reduce the capital invested. On the contrary, the subsidy on abatement reduces firms' production costs, so that it will provide an incentive to increase the scale of the firm.

Figure 1: Steady state aggregate capital K as a function of the fiscal instruments and warm-glow parameter γ .



Parameters: CIES utility function with parameters $\eta = \sigma = 1$, $F = AK^\alpha L^{1-\alpha}$, $\alpha(f, f^\alpha) = f^{(1-\xi)} f^{\alpha\xi} = \psi^\xi f$; $A = 1, H = 1, \alpha = 0.5, \xi = 0.5, \epsilon = 5, \rho = 0.1, \tau^x = 0, \delta = 0.01$

For illustrative purposes, we present Figure 1, providing a graphical representation of the content of Proposition 2, where we have assumed CIES utility functions with $\eta = \sigma = 1$, Cobb-Douglas production function $F = AK^\alpha L^{1-\alpha}$ and abatement technology as the one already presented in section 2.2 and parameters specified below.

Note that the warm-glow parameter γ , other things being equal, exerts a negative effect on the equilibrium level of capital, in that investors, as γ increases, penalize firms by asking for a higher pollution premium.

As for the effects on pollution, we can summarize the results in the following proposition:

Proposition 3: An increase of the tax on pollution reduces pollution, while an increase of the subsidy on abatement activity can either increase or reduce pollution. However, when $\tau^X = 0$, sufficient for $\frac{dX}{ds} < 0$ is:

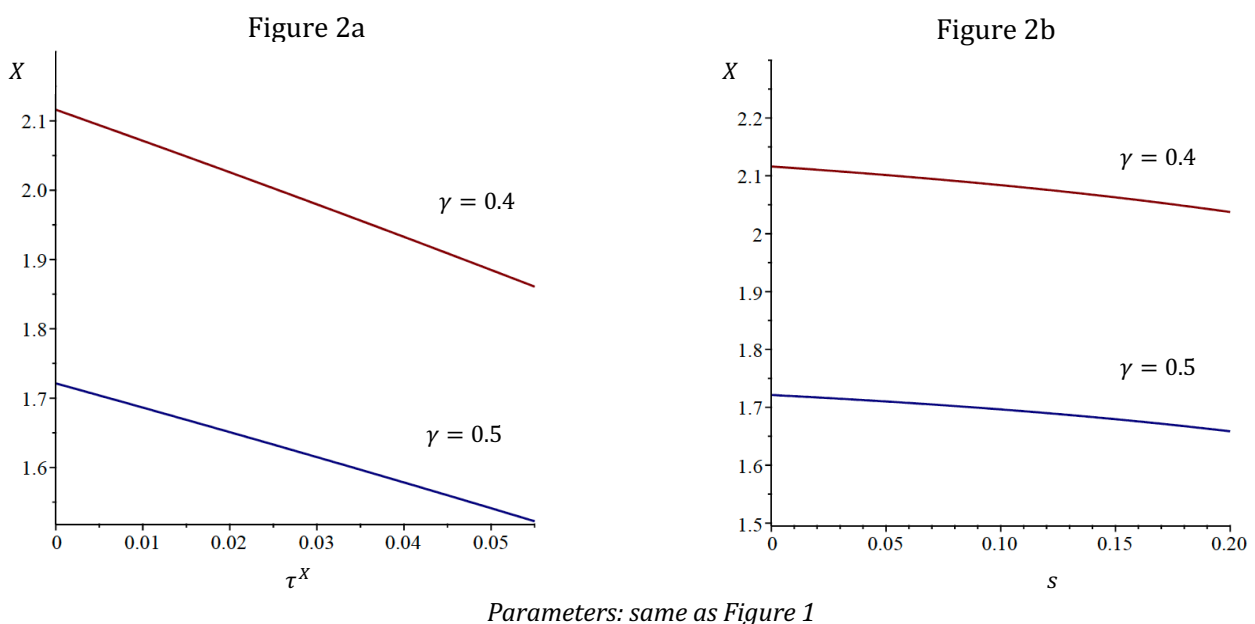
$$s \leq \frac{\rho - \frac{(1-\xi)}{\sigma}}{\phi}.$$

with $\phi \equiv F_K(s = 0)$, i.e. marginal productivity of capital for $s = 0$.

Proof: See Appendix A □

The economic rationale of the results is the following: as for the pollution tax, as expected, an increase of the latter will reduce pollution. As for the subsidy, the latter exerts two opposite effects on pollution: on one hand, by increasing the installed capital, it also tends to increase production and, thus, pollution (which is a by-product of production); on the other hand, it tends to increase the resources that the firms devote to abatement, thus reducing pollution. The final result depends on the relative strength of each effect.

Figure 2: Steady state pollution flow X as a function of the fiscal instruments and the warm-glow parameter γ .



Results of Proposition 3 are summarized in Figure 2, where we assumed the same parameters' specification as the one used in Figure 1.

In this case, as expected, a higher warm-glow parameter, other things being equal, is associated with a lower flow of pollution X at the steady state, due to the fact that investors, as γ increases, penalize firms by asking a higher pollution premium, which, in turn, reduces the scale of the economy, production and, consequently, X .

We now focus on the effects of fiscal instruments on steady state per-capita consumption.

Proposition 4 summarizes our results:

Proposition 4: *An increase of the tax on pollution reduces per-capita consumption, while an increase of the subsidy on abatement activity can either increase or reduce per-capita*

consumption. However, sufficient for $\frac{dc}{ds} > 0$ is $s \leq 1 - \frac{\frac{\beta(\rho+\delta)}{\theta(\frac{\rho}{\rho})}}{(\eta+1-\xi)}$

where $0 < \xi < 1$ is the elasticity of the abatement technology $\alpha(f, f^\alpha)$ with respect to Ψ (i.e. the fraction of total output devoted to abatement activity).

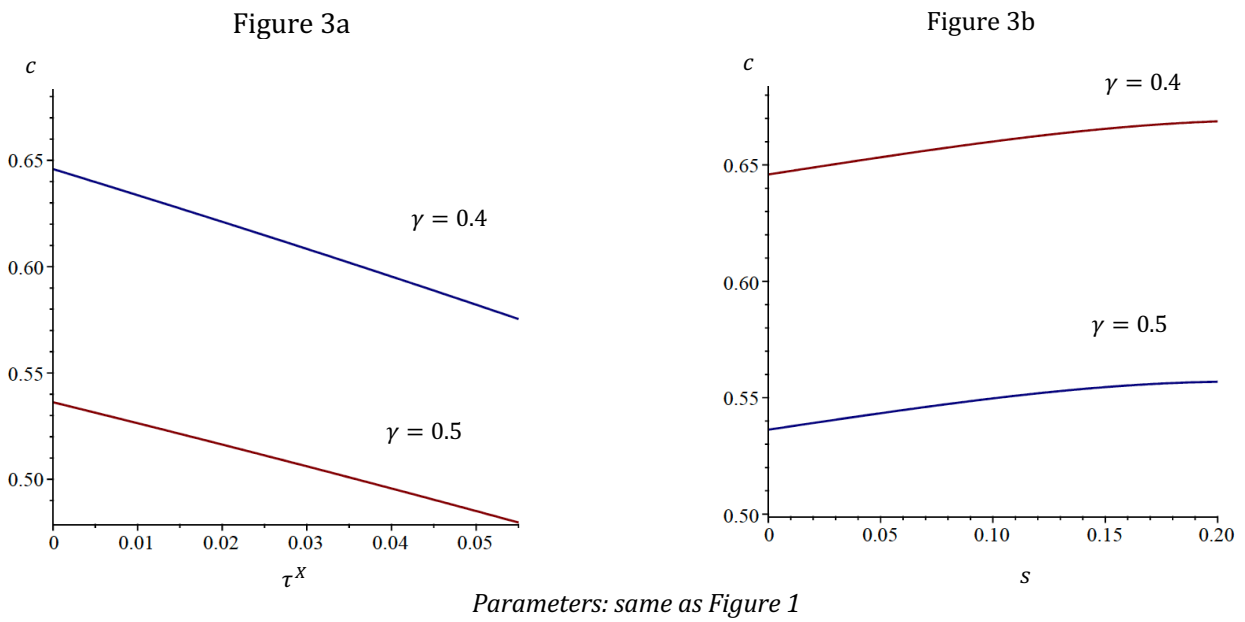
Proof: See Appendix B. □

The economic intuition of the result can be stated as follows: preliminarily, recall that steady state consumption, by eq. (33) is proportional to net-of-abatement product. Given that an increase of the tax on pollution reduces net-of-abatement production, it follows that also consumption decreases. On the other hand, the subsidy can either increase or decrease net-of-abatement production, given that it exerts two opposite effects: on one hand, by increasing the installed capital, it will also increase production and, thus, consumption; on the other hand, it tends to increase the resources that firms devote to abatement, thus reducing net-of-

production and, consequently, consumption. The final result depends on the relative strength of each effect.

Figure 3 provides a graphical representation of the content of Proposition 4, where we assumed the same parameters' specification as the one used in Figure 1.

Figure 3: Steady state per capita consumption c as a function of the fiscal instruments and the warm-glow parameter γ .



In this case, a higher warm-glow parameter, other things being equal, is associated with a lower per-capita consumption c at the steady state, due to the fact that investors, as γ increases, penalize firms by asking a higher pollution premium, which, in turn, reduces the scale of the economy, total production and, consequently, per-capita consumption.

More in general, the sufficient conditions provided in Propositions 3-4 can be further summarised through the following interval for s :

$$0 \leq s \leq \min \left\{ \frac{\rho - \frac{(1 - \xi)}{\sigma}}{\phi}, 1 - \frac{\frac{\beta}{\theta} \left(\frac{\rho + \delta}{\rho} \right)}{(\eta + 1 - \xi)} \right\}$$

within which an increase of such a subsidy produces not only an increase of the installed capital, but also an increase of per-capita consumption and a reduction of pollution.

We now turn to the effects of the fiscal instruments on the pollution premium, which, by eq. (13), at the steady state, is:

$$R \equiv [r_e^j - r] = \frac{u_p}{u_c} \gamma \frac{X}{K} \quad (34)$$

The following Proposition summarizes the results:

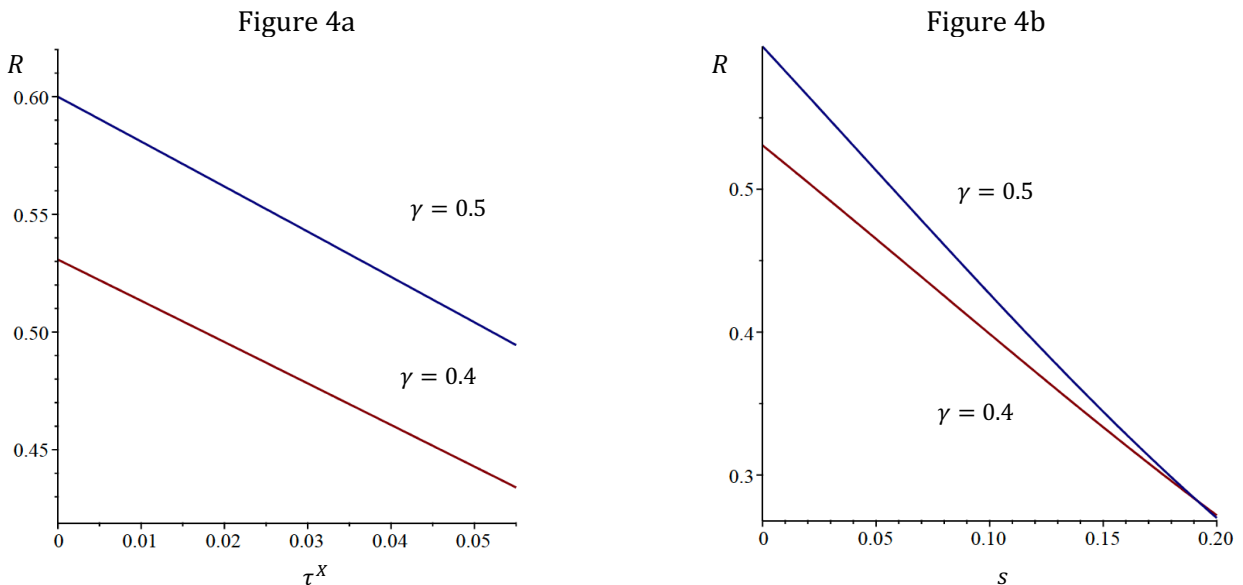
Proposition 5: *An increase of the tax on pollution reduces the pollution premium, while an increase of the subsidy on abatement activity can increase or decrease the pollution premium.*

when $\tau^X = 0$, sufficient for sufficient $\frac{dR}{ds} < 0$ is $(1 + \eta)\theta - 1 \leq 0$.

Proof: See Appendix C. □

Notice that the sufficient condition above holds for values of η (elasticity of marginal (dis)utility of the warm-glow) sufficiently close to 1 and values of θ (the elasticity of output with respect to capital) sufficiently smaller than $\frac{1}{2}$.

Figure 4: Steady state pollution premium R as a function of the fiscal instruments and the warm-glow parameter γ .



Parameters: same as Figure 1

Figure 4 provides a graphical representation of the content of Proposition 5 concerning the effect of both τ^X and s on the pollution premium R , for different values of the warm-glow parameter γ .

Notice that a higher warm-glow parameter, for any level of τ^X , implies a higher pollution premium R , due to the fact that investors, as γ increases, penalize polluting firms by asking for a higher remuneration for holding “dirty assets”. However, the role of γ on the pollution premium can change as s increases. In fact, as shown by Figure 4b, stronger warm glow motive reinforces the effect of s . At $s=0$, an economy with stronger warm glow will have a higher pollution premium. As s increases the pollution premium reduces more rapidly for high-worm glow economies, meaning that eventually for large enough s , the R curves will cross. In any case, even though the pollution premium is reduced by increases in s (public subsidy crowding out worm glow), marginal worm glow is still present, rendering subsidy an effective instrument in reducing X .

To sum up, our analysis shows that the mechanisms of s and τ^X are different.

An increase in τ^X directly affects the firms' incentive to abate (see eq. 26). The firm will equate the marginal cost of abatement to the sum of the pollution tax and the marginal pollution premium (marginal cost of capital in production units).

An increase in τ^X (for every given level of marginal pollution premium) will call for an increase in the marginal abatement cost (i.e. an increase in abatement). Given that pollution declines, the marginal pollution premium will also decline (but not enough to offset the first effect). τ^X also acts as a tax on economic activity, calling for an equilibrium reduction in K . The worm glow motive (higher value of γ) will tend to reduce pollution levels at each level of τ^X , while $\frac{dX}{d\tau^X}$ quantitatively remains roughly the same.

On the other hand, an abatement subsidy hinges on the pollution premium. If there is no warm glow, the firm will always have higher profits by doing no abatement (unless it is subsidized more than 100%). So, s is not effective here.

In the presence of warm glow, the firms equate the (net after subsidy) marginal cost of abatement to the marginal pollution premium. For given marginal pollution premium, an increase in s lowers the marginal abatement cost, calling for an increase in abatement. This will also lower the marginal pollution premium, but under certain conditions, will not offset the first positive effect on abatement. At the same time, s acts as a production subsidy calling for an increase in K in equilibrium. This effect, in turn, reduces the effect of s on X . Therefore, *coeteris paribus*, quantitatively X responds less to changes in s (than it does to changes in τ^X).

5. Conclusions

In this paper we analyzed the effects that fiscal instruments, aimed at reducing pollution, can exert on the scale of the economy, on pollution and on the pollution premium. In particular, we compared two different instruments: a tax on pollution and a subsidy on abatement activity.

We found that the former, besides reducing pollution, depresses also per capita consumption and the capital installed in the economy. As for the subsidy, rather interestingly, we found that, under fairly general assumptions, it decreases pollution and increases per-capita consumption. As for the pollution premium, we also provided very general conditions ensuring that an increase of both pollution taxes and subsidies for pollution abatement generates a reduction of the pollution premium.

Some policy implications follow: in an economy populated by socially responsible investors, pollution abatement, a goal which is on the political agenda of most developed countries and international organizations, is not necessarily at odds with economic performance. In fact, while the subsidy to abatement has smaller quantitative effects on pollution, relative to the tax on pollution, other things equal, it increases steady state

consumption and capital. For these reasons the former fiscal instrument may be politically more feasible than latter, especially in economies characterised by investors with stronger social responsibility motives (warm glow).

Finally, we notice that our results have clear testable implications, which we leave for future research.

References

- Aldashev, G., Limardi, M., & Verdier, T. (2015). Watchdogs of the invisible hand: Ngo monitoring and industry equilibrium. *Journal of Development Economics*, 116, 28-42.
- Andreoni, J. (1990). Impure altruism and donations to public goods: A theory of warm-glow giving. *The Economic Journal*, 100(401), 464-477.
- Andreoni, J., Rao, J. M., Trachtman, H. (2017). Avoiding the ask: A field experiment on altruism, empathy, and charitable giving. *Journal of Political Economy*, 125(3), 625-653.
- Barnea, A., Heinkel, R., Krause, A., (2005). Green investors and corporate investment. *Structural Change and Economic Dynamics* 16 (3), 332–346.
- Baron, D. P. (2007). Corporate social responsibility and social entrepreneurship. *Journal of Economics & Management Strategy*, 16(3), 683-717.
- Belfiori, M. E. (2018). Climate change and intergenerational equity: Revisiting the uniform taxation principle on carbon energy inputs. *Energy Policy*, 121, 292-299.
- Bontems, P., Bourgeon, J. M. (2005). Optimal environmental taxation and enforcement policy. *European Economic Review*, 49(2), 409-435.
- Bovenberg, A.L., Goulder, L.H. (2002). Environmental taxation and regulation. In: Auerbach, A.J., Feldstein, M.S. (eds) *Handbook of public economics*, vol 3. North-Holland, Amsterdam, 1471–1545.
- Brekke, K. A., Pekovic, S. (2018). Why Are Firms Environmentally Responsible? A Review and Assessment of the Main Mechanisms. *International Review of Environmental and Resource Economics*, 12(4), 355-398.

- Brock, W. A., Taylor, M. S. (2005). Economic Growth and the Environment: A Review of Theory and Empirics, in: Philippe Aghion & Steven Durlauf (ed.), *Handbook of Economic Growth*, edition 1, volume 1, chapter 28, pages 1749-1821 Elsevier.
- Cremer, H., Gahvari, F., Ladoux, N. (2001). Second-Best Pollution Taxes and the Structure of Preferences. *Southern Economic Journal*, 68(2), 258-280.
- Dam, L. (2011). Socially responsible investment in an environmental overlapping generations model. *Resource and Energy Economics*, 33(4), 1015-1027.
- Dam, L., Heijdra, B. J. (2011). The environmental and macroeconomic effects of socially responsible investment. *Journal of Economic Dynamics and Control*, 35(9), 1424-1434.
- Dam, L., Scholtens, B. (2015). Toward a theory of responsible investing: On the economic foundations of corporate social responsibility. *Resource and Energy Economics*, 41, 103-121.
- Eurosif (2018). EUROPEAN SRI STUDY 2018.
- Evren, Ö., Minardi, S. (2017). Warm-glow Giving and Freedom to be Selfish. *The Economic Journal*, 127(603), 1381-1409.
- Gahvari, F. (2014). Second-best pigouvian taxation: a clarification. *Environmental and Resource Economics*, 59(4), 525-535.
- Graff Zivin, J., Small, A. (2005). A Modigliani-Miller theory of altruistic corporate social responsibility. *The BE Journal of Economic Analysis & Policy*, 5(1).
- Goulder, L. H., Parry, I. W. (2008). Instrument choice in environmental policy. *Review of environmental economics and policy*, 2(2), 152-174.
- GSIA (2016): *Global Sustainable Investment Review*. Available at: http://www.gsi-alliance.org/wp-content/uploads/2017/03/GSIR_Review2016.F.pdf
- Heinkel, R., Kraus, A., Zechner, J. (2001). The effect of green investment on corporate behavior. *Journal of financial and quantitative analysis*, 36(4), 431-449.
- Hong, H., Kacperczyk, M. (2009). The price of sin: The effects of social norms on markets. *Journal of Financial Economics*, 93(1), 15-36.
- Jacobs, B., & De Mooij, R. A. (2015). Pigou meets Mirrlees: On the irrelevance of tax distortions for the second-best Pigouvian tax. *Journal of Environmental Economics and Management*, 71, 90-108.

- Kampas, A., Horan, R. (2016). Second-best pollution taxes: revisited and revised. *Environmental Economics and Policy Studies*, 18(4), 577-597.
- Lambertini, L., Poyago-Theotoky, J., Tampieri, A. (2017). Cournot competition and “green” innovation: An inverted-U relationship. *Energy Economics*, 68, 116-123.
- Martín-Herrán, G., & Rubio, S. J. (2018). Second-best taxation for a polluting monopoly with abatement investment. *Energy Economics*, 73, 178-193.
- Merton, R. C. (1971). Optimum Consumption and Portfolio-Rules in a Continuous-Time Framework. *Journal of Economic Theory*, 3, 373-413.
- OECD (2017), Investment governance and the integration of environmental, social and governance factors.
- Ouchida, Y., Goto, D. (2014). Do emission subsidies reduce emission? In the context of environmental R&D organization. *Economic Modelling*, 36, 511-516.
- Ouchida, Y., Goto, D. (2016). Environmental research joint ventures and time-consistent emission tax: Endogenous choice of R&D formation. *Economic Modelling*, 55, 179-188.
- Pizer, W. A., Sexton, S. (2019). Distributional impacts of energy taxes. *Review of Environmental Economics and Policy*, rey021, <https://doi.org/10.1093/reep/rey021>
- Poyago-Theotoky, J. A. (2007). The organization of R&D and environmental policy. *Journal of Economic Behavior & Organization*, 62(1), 63-75.
- Poyago-Theotoky, J. A. (2010). Corrigendum to " The organization of R&D and environmental policy"[j. econ. behav. org. 62 (2007) 63-75]. *Journal of Economic Behavior & Organization*, 76(2), 449-449.
- Renneboog, L., Ter Horst, J., Zhang, C. (2008). Socially responsible investments: Institutional aspects, performance, and investor behavior. *Journal of Banking & Finance*, 32(9), 1723-1742.
- Renström, T.I., Spataro, L. Marsiliani, L. (2019): Optimal Taxation, Environment Quality, Socially Responsible Firms and Investors, *International Review of Environmental and Resource Economics*: Vol. 13: No. 3-4, pp 339-373. <http://dx.doi.org/10.1561/101.00000112>
- Sandmo, A. (1975). Optimal taxation in the presence of externalities. *The Swedish Journal of Economics*, 86-98.

Vanwalleghem, D. (2017). The real effects of sustainable & responsible investing?. *Economics Letters*, 156, 10-14.

Appendix A. Proof of Proposition 3

As for the effect of τ^X on pollution, by eq. (31') it follows that $\frac{dX}{d\tau^X} = \frac{X}{K} \frac{1}{R\eta + T\frac{F}{K}} \left(T\frac{F}{K} \theta \frac{dK}{d\tau^X} - R\frac{\sigma}{c} K \frac{dc}{d\tau^X} - X \right)$

and, by Cramer's rule $|J| \frac{dX}{d\tau^X} = \frac{X}{K} \frac{1}{R\eta + T\frac{F}{K}} \left[T\frac{F}{K} \theta (J_{11}B - AJ_{21}) - R\frac{\sigma}{c} K (J_{22}A - BJ_{12}) - X (J_{11}J_{22} - J_{12}J_{21}) \right]$

By expanding J_{11} , J_{21} , A , B and J_{22} and collecting terms we get:

$$|J| \frac{dX}{d\tau^X} = \frac{X}{K} \frac{1}{R\eta + T\frac{F}{K}} \left[T\frac{F}{K} \theta H \frac{\sigma}{c} F_K \frac{X}{K} \frac{T}{R\eta + T\frac{F}{K}} - XHJ_{12} \right] > 0, \text{ so that } \frac{dX}{d\tau^X} < 0.$$

As for the effect of s on pollution, by exploiting eq. (31) it follows that $|J| \frac{dX}{ds} = |J| \frac{\partial X}{\partial K} \frac{dK}{ds} + |J| \frac{\partial X}{\partial c} \frac{dc}{ds} + |J| \frac{\partial X}{\partial s}$. Applying (31''), the expression above can be written as:

$$\left(R\eta + T\frac{F}{K} \right) |J| \frac{dX}{ds} = T\frac{X}{K} \frac{F\theta}{K} |J| \frac{dK}{ds} - \frac{X}{c} R\sigma |J| \frac{dc}{ds} + \Psi' \frac{X^2}{K} |J|. \text{ Using Cramer's rule for } |J| \frac{dK}{ds} \text{ and } |J| \frac{dc}{ds}, \text{ we}$$

obtain: $\left(R\eta + T\frac{F}{K} \right) \frac{|J|}{X} \frac{dX}{ds} = \frac{T}{K} \frac{F\theta}{K} (J_{11}b - aJ_{21}) - \frac{R\sigma}{c} (aJ_{22} - bJ_{12}) + \Psi' \frac{X}{K} (J_{11}J_{22} - J_{12}J_{21})$

After some manipulation and collecting terms the above equation can be written as:

$$\left(R\eta + T\frac{F}{K} \right) \frac{|J|}{X} \frac{dX}{ds} = \frac{T}{K} \frac{F\theta}{K} (J_{11}b - aJ_{21}) + J_{22}RF_K \left(\Psi - \Psi' \frac{X}{F} \right) + H\Psi' \frac{X}{K} J_{12}. \text{ The term } (J_{11}b - aJ_{21}) \text{ is equal}$$

to $F_K \left(\Psi - \Psi' \frac{X}{F} \right) \Psi' X \frac{R}{R\eta + T\frac{F}{K}} - HF_K \frac{c}{\sigma} \left(\Psi - \Psi' \frac{X}{F} \frac{R\eta}{R\eta + T\frac{F}{K}} \right)$. Moreover, recognizing that, by eqs. (27) and (28),

$$J_{12} = -\frac{c}{\sigma} \frac{F_K}{K} \left[T\theta \frac{R\eta}{R\eta + T\frac{F}{K}} + \frac{\rho + \delta}{F_K} \beta \right] \text{ the equation above is:}$$

$$\left(R\eta + T\frac{F}{K} \right) \frac{|J|}{X} \frac{dX}{ds} = J_{22}RF_K \left(\Psi - \Psi' \frac{X}{F} \right) - \frac{c}{\sigma} H\Psi' \frac{X}{K} \frac{F_K}{K} \frac{\rho + \delta}{F_K} \beta + \left[\left(\Psi - \Psi' \frac{X}{F} \right) \Psi' \frac{X}{c} \frac{R\sigma}{R\eta + T\frac{F}{K}} - H\Psi \right] \frac{T}{K} \frac{F\theta F_K c}{K \sigma}.$$

Exploiting the expression for J_{22} and collecting terms the latter equation reads as

$$\left(R\eta + T\frac{F}{K} \right) \frac{|J|}{X} \frac{dX}{ds} = RF_K \left(\Psi - \Psi' \frac{X}{F} \right) \left[F_K \left(1 - \Psi + \Psi' \frac{X}{F} \right) - \delta \right] + \frac{c}{\sigma} \frac{H}{K} \left[-\Psi' \frac{X}{F} \frac{\rho + \delta}{F_K} \beta - TF_K \Psi \right]$$

Next, exploiting the steady state relationships $F_K \left(1 - \Psi + \Psi' \frac{X}{F} \right) - \delta = \rho - s \left(\Psi - \Psi' \frac{X}{F} \right) F_K > 0$ and

$\frac{c}{K} \frac{H}{K} = (1 - \Psi) \frac{F}{K} - \delta$ and collecting terms we obtain:

$$\left(R\eta + T\frac{F}{K} \right) \frac{|J|}{X} \frac{dX}{ds} = RF_K \left(\Psi - \Psi' \frac{X}{F} \right) \left[\rho - s \left(\Psi - \Psi' \frac{X}{F} \right) F_K \right] + \frac{1}{\sigma} \left[(1 - \Psi) \frac{F}{K} - \delta \right] \left[-\Psi' \frac{X}{F} \frac{\rho + \delta}{F_K} \beta - TF_K \Psi \right]$$

that is: $(R\eta + T \frac{F}{K}) \frac{|J|}{X} \frac{dX}{ds} = \Theta + RF_K \left(-\Psi' \frac{X}{F} \right) \left[\rho - s \left(\Psi - \Psi' \frac{X}{F} \right) F_K \right] - \frac{1}{\sigma} \left[(1 - \Psi) \frac{F}{K} \right] TF_K \Psi$

with $\Theta \equiv RF_K \Psi \left[\rho - s \left(\Psi - \Psi' \frac{X}{F} \right) F_K \right] + \frac{1}{\sigma} \left[(1 - \Psi) \frac{F}{K} - \delta \right] \left[-\Psi' \frac{X}{F} \frac{\rho + \delta}{F} \beta \right] + \frac{\delta}{\sigma} TF_K \Psi > 0$. Exploiting the

expressions $R = -(1 - s)\Psi' \frac{X}{K}$, $T \equiv \left(\frac{X}{F} \right)^2 (1 - s)\Psi''$ and the relationship $\frac{\Psi''}{\Psi'} = (1 - \xi) \frac{\Psi'}{\Psi}$, the above

equation becomes:

$$\left(R\eta + T \frac{F}{K} \right) \frac{|J|}{X} \frac{dX}{ds} = \Theta + (1 - s)(\Psi')^2 \frac{X}{K} \frac{X}{F} F_K \left[\rho - s \left(\Psi - \Psi' \frac{X}{F} \right) F_K - \frac{(1 - \xi)}{\sigma} (1 - \Psi) \right]$$

Sufficient for the RHS to be positive (i.e. for $\frac{dX}{ds} < 0$) is: $\rho - s \left(\Psi - \Psi' \frac{X}{F} \right) F_K - \frac{(1 - \xi)}{\sigma} (1 - \Psi) \geq 0$.

Given that $0 < \left(\Psi - \Psi' \frac{X}{F} \right) < 1$ and $0 < (1 - \Psi) < 1$, sufficient for the above inequality to hold is

$\frac{\rho - \frac{(1 - \xi)}{\sigma}}{F_K} \leq s$. Given that $\frac{dK}{ds} > 0$ and $\frac{dF_K}{ds} < 0$, sufficient for the above inequality to hold is

$\frac{\rho - \frac{(1 - \xi)}{\sigma}}{\phi} \leq s$, with $\phi \equiv F_K(s = 0)$, i.e. marginal productivity of capital for $s = 0$. □

Appendix B. Proof of Proposition 4

As for the effect of the tax on pollution, given that $A, B, J_{22}, > 0$ and $J_{12} < 0$, Cramer's rule yields:

$$|J| \frac{dc}{d\tau^X} = \begin{vmatrix} A & J_{12} \\ B & J_{22} \end{vmatrix} = AJ_{22} - BJ_{12} > 0$$

Given that $|J| < 0$, it follows that $\frac{dc}{d\tau^X} < 0$.

As for the effect of the subsidy on abatement activity, by defining given that $J_{11} < 0$ and $J_{21} < 0$,

Cramer's rule provides the following:

$$|J| \frac{dc}{ds} = \begin{vmatrix} a & J_{12} \\ b & J_{22} \end{vmatrix} = aJ_{22} - bJ_{12}$$

whose sign is ambiguous. However, by exploiting the definitions of a, J_{22}, b, J_{12} , the above equation can

be written as:

$$|J| \frac{dc}{ds} = -\frac{c}{\sigma} \frac{F_K}{R\eta + T \frac{F}{K}} \left\{ R\eta \left(\Psi - \Psi' \frac{X}{F} \right) \left[\left(1 - \Psi + \Psi' \frac{X}{F} \right) F_K - \delta \right] + \Psi T \frac{F}{K} \left[(1 - \Psi) F_K - \delta \right] - \left(\Psi' \frac{X}{K} \right)^2 \beta \left[1 - (1 - s)\Psi + (1 - s)\Psi' \frac{X}{F} \right] \right\}.$$

Recognizing that $\frac{\Psi''}{\Psi'} = (1 - \xi) \frac{\Psi'}{\Psi}$, so that $T = \left(\frac{X}{F} \right)^2 (1 - s)(1 - \xi) \frac{(\Psi')^2}{\Psi}$ and $T \frac{F}{K} \Psi = \frac{F}{K} \frac{X}{F} (1 - s)(1 -$

$\xi)(\Psi')^2$ and $\frac{F}{K} = \frac{F_K}{\theta}$, $\left[1 - (1 - s)\Psi + (1 - s)\Psi' \frac{X}{F} \right] F_K - \delta = \rho$, the above expression takes the form:

$$|J| \frac{dc}{ds} = \frac{c}{\sigma} \frac{F_K (\Psi')^2 \frac{FX}{KF}}{R\eta + T \frac{F}{K}} \left\{ -\eta(1 - s) \frac{(\Psi - \Psi' \frac{X}{F})}{-\Psi' \frac{X}{F}} \left[\left(1 - \Psi + \Psi' \frac{X}{F} \right) F_K - \delta \right] - (1 - s)(1 - \xi) \left[(1 - \Psi) F_K - \delta \right] + \frac{\beta}{\theta} (\rho + \delta) \right\}.$$

Given that $|J| < 0$, $-(1-s)(1-\xi)[(1-\Psi)F_K - \delta] < 0$, and $0 < \frac{(\Psi - \Psi' \frac{X}{F})}{-\Psi' \frac{X}{F}} < 1$, sufficient for $\frac{dc}{ds} > 0$ is

$$-\eta(1-s) \left[\left(1 - \Psi + \Psi' \frac{X}{F}\right) F_K - \delta \right] + \frac{\beta}{\theta}(\rho + \delta) < 0. \text{ Given that } \left(1 - \Psi + \Psi' \frac{X}{F}\right) F_K - \delta = \frac{\rho - s(F_K - \delta)}{1-s} > 0,$$

the latter inequality reads as $-\eta \left[\frac{\rho - s(F_K - \delta)}{1-s} \right] + \frac{\beta}{\theta}(\rho + \delta) < 0$, that is $0 \leq s \leq \frac{(\eta+1-\xi) - \frac{\beta}{\theta} \left(\frac{\rho+\delta}{\rho} \right)}{(\eta+1-\xi) \left(\frac{F_K - \delta}{\rho} \right)}$. Finally,

recognizing that, by eq. (32) evaluated at steady state, $1 - \frac{\rho+\delta}{F_K} = (1-s) \left[\Psi - \Psi' \frac{X}{F} \right] > 0$ implies $F_K >$

$\delta + \rho$, the above restrictions can be written as:

$$s \leq 1 - \frac{\frac{\beta}{\theta} \left(\frac{\rho+\delta}{\rho} \right)}{(\eta+1-\xi)} \quad \square$$

Appendix C. Proof of Proposition 5

As for the effect of the tax on the pollution premium R , by total differentiation of logs of (34) we get:

$$\frac{dR}{R} = p \frac{u_{pp}}{u_p} \frac{dp}{p} - \frac{u_{cc}}{u_c} c \frac{dc}{c} + \frac{dX}{X} - \frac{dK}{K} = (1+\eta) \frac{dX}{X} + \sigma \frac{dc}{c} - \frac{dK}{K} \quad (35)$$

Taking (31') and collecting terms we get:

$$\frac{(R\eta + T \frac{F}{K})}{(1+\eta)} \frac{dR}{R} = \left(T \frac{F \theta}{K K} \right) dK + \left(T \frac{F}{K} - R \right) \frac{\sigma}{c(1+\eta)} dc - \left(\frac{X}{K} \right) d\tau^x + \left(\Psi' \frac{X}{K} \right) ds = 0 \quad (36)$$

Focusing on τ^x , exploiting the results of previous Propositions, we can write:

$$|J| \frac{(R\eta + T \frac{F}{K})}{(1+\eta)R} \frac{dR}{d\tau^x} = \left(T \frac{F \theta}{K K} \right) (J_{11}B - AJ_{21}) + \left(T \frac{F}{K} - R \right) \frac{\sigma}{c(1+\eta)} (AJ_{22} - BJ_{12}) - \frac{X}{K} (J_{11}J_{22} - J_{12}J_{21})$$

After some manipulation and collecting terms we can write

$$|J| \frac{(R\eta + T \frac{F}{K})}{(1+\eta)R} \frac{dR}{d\tau^x} = \left(T \frac{F \theta}{K K} \right) H \frac{c}{\sigma} F_K \frac{X}{K} \frac{T}{R\eta + T \frac{F}{K}} + \frac{T}{1+\eta} \frac{X}{K} F_K J_{22} + \frac{\Psi' X}{(1+\eta)} \frac{\sigma X}{c K} J_{12} - \frac{X}{K} H J_{12} > 0$$

and thus, given that $|J| < 0$, $\frac{dR}{d\tau^x} < 0$.

As for the subsidy on pollution abatement, from (35) and (31') we know that

$$\frac{dR}{R} = (1+\eta) \frac{dX}{X} + \sigma \frac{dc}{c} - \frac{dK}{K} = \left[\frac{(1+\eta) \partial X}{X \partial K} - \frac{1}{K} \right] dK + \left[\frac{(1+\eta) \partial X}{X \partial c} + \frac{\sigma}{c} \right] dc + \frac{(1+\eta) \partial X}{X \partial s} ds \quad (35)$$

Substituting from (31'') and rearranging terms we get $\frac{1}{R} \frac{dR}{ds} = \frac{[(1+\eta)\theta - 1] T \frac{F}{K} - R\eta}{K(R\eta + T \frac{F}{K})} \frac{dK}{ds} + \frac{\sigma T \frac{F}{K} - R\eta}{c R\eta + T \frac{F}{K}} \frac{dc}{ds} + \frac{(1+\eta) \Psi' X}{R\eta + T \frac{F}{K}} \frac{X}{K}$

By exploiting previous results on $|J| \frac{dK}{ds}$ and $|J| \frac{dc}{ds}$ we can write the following

$$|J| \frac{(R\eta + T \frac{F}{K})}{R} \frac{dR}{ds} = \left\{ [(1+\eta)\theta - 1] T \frac{F}{K} - R\eta \right\} \frac{1}{K} (J_{11}b - J_{21}a) + \frac{\sigma}{c} \left(T \frac{F}{K} - R\eta \right) (aJ_{22} - bJ_{12}) + (1+\eta) \Psi' \frac{X}{K} (J_{11}J_{22} - J_{12}J_{21})$$

which can also be written as:

$$|J| \frac{(R\eta + T\frac{F}{K})dR}{R} \frac{dR}{ds} = \left\{ [(1+\eta)\theta - 1]T\frac{F}{K} - R\eta \right\} \frac{1}{K} (J_{11}b - J_{21}a) + \left[a\frac{\sigma}{c}T\frac{F}{K} - \frac{\sigma}{c}R\eta a + (1+\eta)\Psi'\frac{X}{K}J_{11} \right] J_{22} \\ + \left[-\frac{\sigma}{c}\left(T\frac{F}{K} - R\eta\right)b - (1+\eta)\Psi'\frac{X}{K}J_{21} \right] J_{12}$$

Recognizing that $\Psi'\frac{X}{K}J_{11} - \frac{\sigma}{c}Ra = RF_K\left(\Psi - \Psi'\frac{X}{F}\right)$ and that $\frac{\sigma}{c}b = \Psi'\frac{X}{K}J_{21} + \frac{H}{R}\Psi'\frac{X}{K}$, the above equation becomes:

$$|J| \frac{(R\eta + T\frac{F}{K})dR}{R} \frac{dR}{ds} = \left\{ [(1+\eta)\theta - 1]T\frac{F}{K} - R\eta \right\} \frac{1}{K} (J_{11}b - J_{21}a) + \left(T\frac{F}{K} + R\eta\right) \frac{\Psi'X}{R} \frac{X}{K} J_{11} J_{22} - \left(T\frac{F}{K} + R\eta\right) \frac{\Psi'X}{R} \frac{X}{K} J_{21} J_{12} \\ - \left(T\frac{F}{K} - R\right) F_K \left(\Psi - \Psi'\frac{X}{F}\right) J_{22} - \frac{\Psi'X}{R} \frac{X}{K} H \left(T\frac{F}{K} - R\right) J_{12}$$

that is, collecting J_{12} and J_{22}

$$|J| \frac{(R\eta + T\frac{F}{K})dR}{R} \frac{dR}{ds} = \left\{ [(1+\eta)\theta - 1]T\frac{F}{K} - R\eta \right\} \frac{1}{K} (J_{11}b - J_{21}a) + F_K J_{22} \left[R\left(\Psi - \Psi'\frac{X}{F}\right) - T\frac{F}{K}\Psi \right] + \Psi'\frac{X}{K} J_{12} \left[(1+\eta)H - \Psi'X\frac{\sigma}{c} \right]$$

Notice that $F_K J_{22} \left[R\left(\Psi - \Psi'\frac{X}{F}\right) - T\frac{F}{K}\Psi \right] = F_K J_{22} \left\{ -\frac{X}{K}(1-s)\Psi' \left[\Psi - \Psi'\frac{X}{F} + (1-\xi)\Psi'\frac{X}{F} \right] - \frac{X}{K}\tau^X \left(\Psi - \Psi'\frac{X}{F} \right) \right\}$, which is positive if $\tau^X=0$, given that $J_{22} > 0$. Moreover, $\Psi'\frac{X}{K} J_{12} \left[(1+\eta)H - \Psi'X\frac{\sigma}{c} \right] > 0$, given that $J_{12} < 0$. Hence, given that $(J_{11}b - J_{21}a) < 0$, sufficient for the RHS to be positive (i.e. $\frac{dR}{ds} < 0$) is

$$T\frac{F}{K} - R \leq 0 \text{ and } [(1+\eta)\theta - 1]T\frac{F}{K} - R\eta \leq 0.$$

If $\tau^X = 0$, the latter inequality reads as

$$[(1+\eta)\theta - 1]\frac{X}{F}(1-\xi)\frac{(\Psi')^2}{\Psi} + \Psi'\eta \leq 0 \text{ or } \frac{[(1+\eta)\theta - 1]}{\eta}(1-\xi)\frac{X}{F}\Psi' + \Psi \geq 0. \text{ Sufficient for the latter}$$

inequality to hold true is $(1+\eta)\theta - 1 \leq 0$. □