

# Statistical Parameters of Steel Rebars of Reinforced Concrete Existing Structures

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Historical and cognitive investigations supported by in-situ and/or laboratory tests are needed for a robust reliability assessment of existing structures. Indeed, an adequate knowledge of material properties and their statistical description is the basis for carrying out accurate reliability analyses and verifications on the investigated structures. In this paper, a procedure for the definition of *pdfs* of mechanical parameters of steel rebars is proposed based on secondary experimental test data. This information is very helpful for the reliability assessment of existing r.c. buildings, where estimation of statistical parameters of mechanical properties of steel reinforcement is very difficult. In fact, it must be highlighted on the one hand that direct information about the examined structure are commonly not sufficient, on the other hand that the number of rebar samples extracted from the structure, if available, is so limited that it does not allow a complete statistical analysis. The first step has been the collection of experimental acceptance tests carried out by Department of Civil and Industrial Engineering of University of Pisa on steel rebars of reinforced concrete (r.c.) structures during the 1960s. The yield strength and the tensile strength are extrapolated for each sample defining a significant database of experimental test results for existing r.c. structures. Then, probability distribution models for the mechanical properties of steel reinforcement have been defined as already done by the authors for concrete strength. A cluster analysis has been carried out based on the Gaussian Mixture Model applying the Expectation-Maximization algorithm to identify homogeneous material classes and their associated *pdfs* of material mechanical parameters. The main advantage of proposed procedure consists in its “blindness”, In fact, not requiring subjective information like pre-classification of data, the methodology is not sensitive to alterations caused by engineering judgement or by inexact identification of declared strength class of the tested samples, due for example to downgraded materials.

*Keywords:* Existing structures, steel bars, yield strength, cluster analysis, experimental tests.

## 1. Introduction

### 1.1 Mechanical parameters and structural reliability of existing reinforced concrete buildings

Reinforced concrete is one of the most widespread building materials for civil engineering structures. This fact led, on the one hand, to the achievement of an important experience in the field of new constructions with an increasing knowledge about material properties, but on the other hand it inevitably implied to deal with outdated buildings, which need substantial refurbishment interventions, both for structural and energy requirements.

For the assessment of existing structures, the evaluation of the statistical properties of the mechanical characteristics of the structural materials is a crucial issue. In order to properly estimate the mechanical characteristics of materials, in addition to an in-depth investigation of historical documents, where useful data can be taken into account for a priori characterization of materials, in-situ or laboratory tests can provide

more reliable results to achieve a higher level of knowledge. Usually, the fundamental mechanical parameters needed to verify the structural resistance of reinforced concrete structures are the compressive strength of the concrete and the yielding strength and sometime the ultimate tensile strength of the steel reinforcement. To determine the strength of the concrete, non-destructive tests such as sclerometric in-situ tests, ultrasonic tests, Sonreb combined tests, or semi-destructive tests, such as core extraction, can be carried out (Beconcini, 2018). Test methods, carried out according to the specific indications suggested by the “Italian Guidelines for the evaluation of in-situ concrete characteristics” (Italian Public Works Council, 2017), provide useful results for estimating the fundamental mechanical parameters of a generic concrete sample. In order to determine the yielding strength and ultimate tensile strength of reinforcing bars, it is suggested to extract a sufficient number of parts of rebars to be analysed in the laboratory by means of specific tests.

However, the in-situ extraction of rebars from existing reinforced concrete structures, is often

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not possible and, in any case, its number is so limited that it does not allow a satisfactory statistical elaboration of test results. Therefore it is necessary to refer on the available design documentation, or on the structural codes adopted at the time of construction, relying on literature data, if any, for the relevant statistical parameters. But, an important indirect source of information for an existing structure can be represented by databases of material acceptance tests obtained on structures coeval to it, commonly stored in the archives of testing laboratory.

Aiming to determine the mechanical parameters of concrete and rebars and to verify the observance to design safety requirements, in Italy material acceptance tests for r.c. structures are mandatory since early 1900s.

In this study, aiming to obtain a proper estimate of the strength of rebars in existing r.c. constructions, a methodology, based on experimental data analysis, is proposed to speed up the knowledge process and to mitigate the use of semi-destructive techniques. A significant database of standard material acceptance tests carried out by the Laboratory of the Department of Civil and Industrial Engineering of the University of Pisa, has been analysed. The research is focused on reinforced concrete structures built in Italy in the 1960s. That period is particularly relevant considering that most of existing r.c. building dated back to it in the framework of the so-called Italian economic boom (Pesenti, 1980). In particular, more than 5000 data have been collected from the Laboratory's archive, concerning geometric characteristics, yielding strength, ultimate tensile strength and ultimate elongation on steel reinforcing bars. A first attempt to perform a statistical elaboration of test results for steel rebars used in Italy can be found in (Verderame et al. 2001). The study was mainly focused on steel rebars belonging to the Aq-category, providing statistical parameters starting from the a priori declared steel grade, as indicated in the test report, but in that way the population resulted extremely scattered.

Aim of the present study is to propose a suitable methodology for the identification of homogenous steel classes populations in the whole dataset of mechanical tests' results, without relying on a preliminary assignment to a given class on the base of information indicated on test reports since this kind of information is on the one hand often unreliable or incomplete, on the other hand heavily affected by the presence of samples of downgraded materials, i.e. coming from production of higher strength classes, but not satisfying the specific requirements.

A blind classification method by unsupervised learning is then proposed by means of a cluster analysis based on Gaussian Mixture Model (GMM). The procedure, which has been already implemented for the identification of concrete classes in (Marsili et al., 2017a) (Croce et al., 2018), is extended to reinforcing steel, focusing on the evaluation of yielding and ultimate strength.

The proposed method offers the opportunity to evaluate probability density functions (*pdfs*) of each steel class, which cannot be obtained with the usual approaches, based on a limited number of experimental test results.

The identified *pdfs* can be always updated through Bayesian procedures by collecting data resulting from experimental tests performed on existing buildings. The updating of statistical mechanical parameters can be achieved by the combination, of the prior information given by the material acceptance tests with the results of destructive or non-destructive in situ tests performed on the considered building.

In this way, a refined estimation of the uncertainty associated to the resistances to be used for the assessment of the structural performance of existing reinforced concrete structures can be achieved, leading to a calibration of the mechanical parameters regulating the structural reliability analysis of an existing building (Nowak, 2003) (Marsili et al., 2017b).

### 1.1 *20th century Italian regulations for reinforced concrete structures*

The analysis and planning of interventions on existing buildings require a deep knowledge of the history of the structure. However, as mentioned above, this is not always possible, both for the incompleteness of documentation available concerning the structure and for the difficulty to estimate the relevant statistical parameters about the mechanical properties of the materials. This may lead to inaccurate evaluation of the structural performance of the building. One of the possible solution is to refer to the minimum values of mechanical properties structure required by the regulations in force at the time of the construction.

The first regulations regarding the design of reinforced concrete in Italy dates back to the early 1900s. In that period numerous Royal Decrees were issued, describing the building materials' requirements as well as the standardized tests to be performed to control quality and other issues. A first phase covers the period 1907-1939. An important regulation is undoubtedly the Royal Decree of 16 November 1939 (Royal Decree, 1939), which remained in force until 1972 and

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introduced for the first time a classification of rebars required for reinforced concrete. In particular, the classification consisted of three distinct types of steel, depending on the carbon content: mild, medium or high strength steel (see Table 1), characterized by different values of the mechanical parameters: yield stress  $f_y$ , tensile strength  $f_t$ , and ultimate elongation on five diameter  $A_{595}$ , and by values of permissible stresses varying between 140 and 200 MPa.

Table 1. Classification of rebars introduced by the Royal Decree of 1939 and the Ministerial Decree of 1957

Decree	Type	Class	Mechanical parameters		
			$f_y$	$f_t$	$A_{595}$
			[MPa]		
R.D. 1939	plain	mild	$\geq 230$	420-500	$\geq 20$
		medium	$\geq 270$	500-600	$\geq 16$
		High strength	$\geq 310$	600-700	$\geq 14$
M.D. 1957	plain	Aq42	$\geq 230$	420-500	$\geq 20$
		Aq50	$\geq 270$	500-600	$\geq 16$
		Aq60	$\geq 310$	600-700	$\geq 14$
		Ribbed			$\geq 12$

Between the Royal Decree (1939) and the Ministerial Decree (1972), several guidelines were issued concerning steel properties, in particular, the Guidelines of September 25, 1948 nr. 2083, June 8, 1953 nr. 1082 and May 20, 1954 nr. 1433 but no major changes were introduced with respect to the Royal Decree.

The Guidelines of May 23, 1957, instead, introduced the use of three steel classes, Aq 42, Aq 50 and Aq 60, corresponding to the previous mild, medium or high strength steel classes. For the first time, some indications were also provided about ribbed bars, allowing mechanical locking between rebars and surrounding concrete.

After that, with reference to plain rebars, the subsequent Ministerial Decree of 30 May 1972 divided plain steels into two categories, FeB22 and FeB32, and established, in the regulatory sense, the definitive transition from plain rebars and ribbed rebars. The Decree of 1972 also marked the transition from a deterministic characterization of steel resistance to a statistical evaluation of mechanical properties, anticipating the definition of limit states by introducing the characteristic values of resistance as reference value for the evaluation of allowable stress to be adopted in the assessments.

In the following years, more specific and detailed regulations were enacted, in the last years also the development of structural Eurocodes. In 2005, all the regulation concerning structural design were collected in an organic law, which led

to the Italian Building Code released in 2008 and updated in 2018.

**2. Database of material acceptance tests from the 1960s**

As already said, the present study focuses on the analysis of the mechanical properties of steel rebars used in r.c. structures in the 1960s. The study has been developed on the basis of material acceptance tests carried out on numerous structures in Italy and collected in the archive of the official Laboratory of the Department of Civil and Industrial Engineering of the University of Pisa. Through the statistical evaluation of the fundamental mechanical parameters of steel rebars, the study aims to reduce the epistemic uncertainties which characterize the analysis of structural reliability. First, a significant database, consisting of about 5000 tensile tests, has been defined for the steel reinforcements used in the years 1961, 1963, 1965 and 1967. Plain and ribbed rebars have been considered, of different types and diameters, collecting data for the yielding strength, the ultimate tensile and the ultimate percentage of elongation. Samples of reinforcing bars came from all over Italy to the Laboratory, which was one of the first official Laboratory in Italy. Test samples, extracted from the materials used during the erection phase, concern beams, columns, slabs and foundations of buildings, bridges, or other civil and industrial engineering works. Different classes were found such as Aq42, Aq50, Aq60, ALE, GS and RUMI, as well as other generic steels.

Specifically, for each sample, the strength class, if present, the diameter  $\Phi$  (mm), the area  $A$  (mm<sup>2</sup>), the yielding force (kN), the ultimate force (kN), the yielding stress  $f_y$  (MPa) and the ultimate tensile stress  $f_t$  (MPa) have been recorded.

Once defined the database, the results of the Laboratory's tests have been analysed to identify homogenous classes and estimate the associated probability distribution functions for the fundamental mechanical parameters. In the following sections, some results concerning the yield stress and the ultimate tensile stress of steel rebars will be illustrated.

**3. Methodology for data Analysis**

The aim of the cluster analysis, based on Gaussian Mixture Model (GMM), is to identify within the overall database homogenous statistical subpopulations, corresponding to various subclasses.

The procedure, which is completely blind, since it does not require preliminary information, allows to evaluate, in particular, the mean values  $\mu$  and the coefficient of variation (COV) of each identified cluster, which can be sound "a priori"

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estimations of the statistical parameters of mechanical properties of steel rebars commonly used in the investigated historical period.

### 3.1 Mixture model basic definition

Gaussian mixture model is a mixture of several Gaussian distributions which represent the distribution of distinct density functions in different Gaussian distribution of vectors already part of a whole population. In some sense it could be interpreted as a method of classification by unsupervised learning (Press et al., 2007), and can be used for twofold purposes: modelling situations in which a single distribution is unable to provide a satisfactory model and/or when the existence of distinct groups for the investigated variable is known a priori. In simple terms MM is a probabilistic model for identifying homogenous subpopulations within an overall population.

According to (MacLachlan and Peel, 2000), let  $Y_1, \dots, Y_n$  a random sample of size  $n$ , where  $Y_j$  is a  $p$ -dimensional random vector with probability density function  $f(y_j)$  on  $\mathbb{R}^p$ . Obviously,  $Y_j$  contains the random variables corresponding to  $p$  measurements made on the  $j$ -th recording of some features on the phenomenon under study.

Considering  $y_j$  as the observed value of the vector  $Y_j$ , we can view  $f(y_j)$  as a density in the case where  $Y_j$  is discrete by the adoption of counting measure.

The probability density function of the random vectors  $Y_j$  can be written in the form

$$f(y_j) = \sum_{i=1}^k \pi_i f_i(y_j) \quad (1)$$

where  $f_i(y_j)$  are the component densities of the mixture and the quantities  $\pi_1, \dots, \pi_k$  are the mixing proportions, or weights.

We shall refer to the density (1) as a  $k$ -component finite mixture distribution.

The number of  $k$  component, which could be unknown (MacLachlan and Peel, 2000), is considered fixed. In this case, an evaluation based on engineering experience let us to define a proper value of  $k$  which allows to speed up the process and minimize the error in cluster convergence. Although the number of  $k$  component is fixed in advance the means and covariances of the sub-distributions are unknown, therefore the aimed outputs is, for each data point, an estimate of the probability that it came from the class  $k$  (Press et al., 2007).

### 3.2 Fitting mixture model

As well it is well described in (Hastie 2008) the expectation maximization (EM) algorithm is a popular tool able to simplify maximum likelihood problems. Therefore, for the definition of the

Gaussian mixture the EM procedure have been used, which follows the following defined steps:

- the *Expectation step (E)*, at which a preliminary assignment of each observation to each model is done;
- the *Maximization step (M)*, where weights, variance and mixing probability are computed starting from the *Expectation step* estimations;
- *E* and *M* steps are iterated increasing the likelihood value until convergence is reached.

## 4. Data analysis: results

The entire dataset of tensile test collected for this study, consists of exactly 5681 samples of steel rebars concerning, as said above, test results obtained in the years 1961, 1963, 1965 and 1967. The 85% of the samples are plain bars, the 12% ribbed bars and the remaining 3% is not labelled as plain or ribbed. The time period under examination represents a transition phase characterized by the first wide application of ribbed bars in reinforced concrete constructions. Indeed, as it is shown in Table 2, an higher percentage of plain rebars is recorded in years 1961, 1963, 1965, while from 1967 the use of ribbed bars' is increasing, being no longer marginal but becoming one-third of the population. It must be highlighted that the results of this last year, to which reference is made, cover only 13% of the entire data collection. The following Table 2, illustrates how the shape subdivision between plain and ribbed steel bars, is distributed along the years.

Table 2. Subdivision by year of the percentages of plain and ribbed bars and probabilistic distribution of yield strength  $f_y$  with relative average values  $\mu$  and coefficient of variation COV.

Year	$\mu$ (N/mm <sup>2</sup> )	COV	Plain	Ribbed
1961(39%)	396	0.19	83%	8%
1963(26%)	403	0.19	88%	10%
1965(22%)	399	0.15	88%	8%
1967(13%)	413	0.14	66%	28%

With regard to the type of analysed steel rebars, it is possible to divide them into ten macro-categories: 47% "not classified" steel, 39% Aq, 5% RUMI, 3% G.S., 3% ALE, 1% star steel, 1% TNT, 0.9% special steel, 0.4% FERROBOX, 0.02% TOR. The category labelled "not classified steel" is used for all cases for which the type of

steel rebars is not described in the test certificates and it is composed mainly by plain rebars.

In each of these macro-categories, in turn, subcategories have been recorded: Aq 42, Aq45, Aq 42/50, Aq 50, Aq 50/60, hard 60/70, ALE, ALE 4400, ARES, RUMI, RUMI LU, RUMI 400, RUMI LU3 or RUMI LU3 4000, RUMI4400, RUMI LU3/4400, RUMI LU3/5000, GS, GS3600, GS4400, GS 4400 ALE, GS4500, GS5000, star steel, star steel 4400, star steel 4500, star steel 5000, TNT 60, RUMI TNT, FERRO BOX 4400, TOR, Thor Aq52/60, special steel.

Three main steel categories can be recognized for a first classification:

- *Not classified steel*, containing all the plain rebars not labelled in a specific strength class;
- *Aq-category*, the whole class containing all the Aq subcategories composed only by plain rebars;
- *Ribbed steel*, containing all the deformed rebars belonging to RUMI, G.S., star steel and “not classified” steel categories.

An ANOVA analysis has been carried out to verify the hypothesis that data are drawn from three different subpopulations. The results are shown in Table 3 and show that the null hypothesis, all groups belonging to the same population, is rejected. Moreover, as it will be shown after, homoscedasticity is not satisfied, as the identified clusters are characterized by different variances. For that reasons, in the author’s opinion, the proposed mixture model is more appropriate than ANOVA linear models, and the efforts to adapt the ANOVA are not justified in the present case, where alternative and sounder model is available.

Table 3. Results of the ANOVA analysis, multiple comparisons.

Groups		Difference between groups means			Null Hypothesis <i>p</i>
		5%	Mean	95%	
Not-Classified	Aq-Category	5.6	9.6	13.6	4.6·10 <sup>-8</sup>
Not-Classified	Ribbed	-86.7	-80.6	-74.4	9.6·10 <sup>-10</sup>
Aq-Category	Ribbed	-96.4	-90.2	-84	9.6·10 <sup>-10</sup>

The first two categories, as observed in Fig. 1, contain approximately 88% of the entire population of test results, including only plain bars, while deformed rebars cover the remaining 12%. The *Aq-category* and *Not classified steel* show a similar distribution with an average value

$\mu=382.3 \text{ N/mm}^2$  and  $COV=0.16$  in the first case and  $\mu=392.4 \text{ N/mm}^2$  and  $COV=0.18$  in the second case. In those years, in fact, the only macro-category of plain rebars recommended by the M.D. of 1957 was the “Aq”, in which the subclasses (Aq42, Aq50, Aq60) corresponded to the steel typologies identified by R.D. 1939 (mild, medium, high strength). *Ribbed bars*, introduced by M.D. of 1957, represents instead only a marginal part of the database, with an average value  $\mu=474.5 \text{ N/mm}^2$  and  $COV=0.10$ .

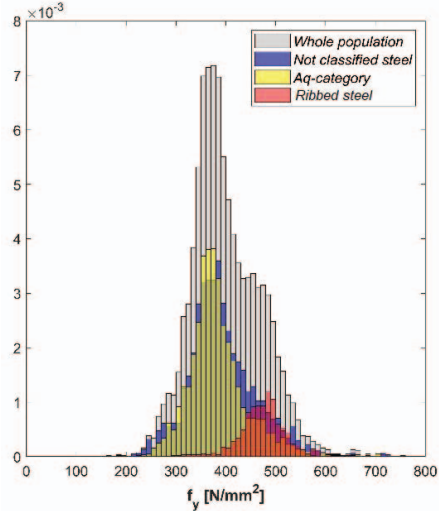


Fig. 1. Histogram of yielding strength of steel rebars for the whole dataset and sub-division in “Not classified steel”, “Aq-category” and “Deformed steel”.

As shown in Fig. 1 for yielding strength  $f_y$ , the two main subpopulations *Not classified steel* (in blue) and *Aq-category* (in yellow) are in part overlapped, making not easy the lecture of the sub-classes. Moreover, the *Ribbed steel* category (in red), although clearly identified as a subpopulation in the descending part of the frequency histogram corresponding to the higher values, appears also overlapped to the higher values pertaining to plain bars. Therefore, the analysis of these results confirms the need to define a robust methodology, which should be able to identify homogenous sub-populations for rebars classification, as illustrated in details in the following.

#### 4.1 Cluster analysis of rebars’ strength

Although for several tested samples steel category is indicated in the test result reports, this information can be not sufficient for an appropriate classification or even misleading, for the reasons explained in the previous section and for the philosophy of regulations in force at that time. In fact, the R.D and D.M. assigned to each

class only a minimum strength level and it is thus usual to find steel rebars belonging to the same class, which overcome the required minimum level with not uniform safety margin. It is also frequent the case when bars with medium or high value of strength are declared belonging to lower classes. Another disclaimer/divergence is represented by the high values of the ratio between ultimate and yielding strength characterizing the mild steel bars.

On the basis of the above remarks, taking into account the significant differences existing between actual experimental results and those expected considering declared or labelled steel classes, it is further confirmed that a more robust and objective classification of steel rebars is necessary, and that a cluster analysis seems to be the more appropriate.

Analysing the whole dataset in the years 1961-1967 by means of a cluster analysis based on Gaussian Mixture Models (GMM) three sub-classes of steel rebars can be identified. The results for yielding and ultimate strength are illustrated in Fig. 2 and 3 respectively. At each class a probability distribution functions (*pdf*) is associated whose relevant statistical parameters, mean value  $\mu$ , standard deviation  $\sigma$  and COV are given in Table 4 together with the 5% percentile for yielding strength.

In Figures 2 and 3, the frequency histogram for the whole population is reported together with the corresponding probability density functions obtained by fitting a Normal distribution (in blue) and a Lognormal distribution (in red), while the sub-classes, identified by means of the GM model implemented with EM algorithm, are illustrated with dashed blue lines.

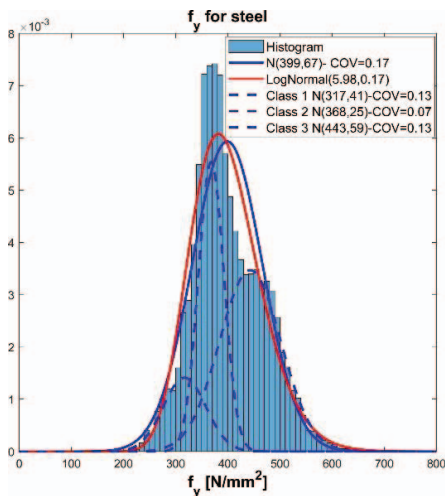


Fig. 2. Results of the cluster analysis for yielding strength  $f_y$ , identification of three different sub-classes.

As we can observe from these preliminary results, the cluster analysis leads to a better evaluation of statistical parameters of steel rebars through the definition of resistance sub-classes. In fact, a reduction of the coefficient of variation associated to each distribution is thus obtained.

Table 4. Results of the cluster analysis for yielding strength  $f_y$ .

	Whole Dataset	Class I	Class II	Class III
$\mu$ (N/mm <sup>2</sup> )	399	316	368	443
$\sigma$ (N/mm <sup>2</sup> )	68	41	26	58
COV	0.17	0.13	0.07	0.13
5%	287	248	325	348

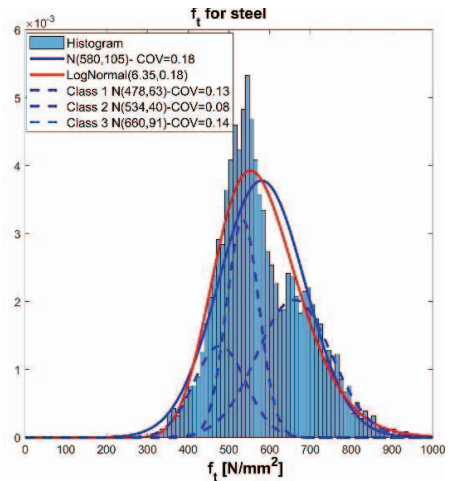


Fig. 3. Results of the cluster analysis for ultimate strength  $f_t$ , identification of three different sub-classes.

## 5. Conclusions

A procedure for the probabilistic evaluation of mechanical properties of steel reinforcing bars in reinforced concrete structure has been presented, with the aim to provide a statistical basis for the evaluation of statistical properties of resistances of steel rebars, to be assumed in reliability assessment of existing r.c. buildings.

The key point of the methodology is a cluster analysis of available dataset of material acceptance tests. The analysis, which is based on the definition of a Gaussian Mixture Model allows to identify sub-classes for steel rebars and the *pdfs* associated to each relevant mechanical parameter.

The procedure has been illustrated considering steel rebars used for r.c. structures built in Italy during the 1960s. The identification of steel classes and their *pdfs* is obtained from the data

analysis without relying on a-priori classification, often not trustworthy enough.

The preliminary results, presented for yielding and ultimate tensile strength, are very promising and provide a useful tool for a blind and objective analysis of large databases of raw test results.

Further research on this topic is envisaged, especially to establish suitable non-destructive tests to make infer on the identified steel classes. It is the intention of the authors to carry on further studies in this direction.

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