

# Shortest Path Problem variants for the Hydro Unit Commitment Problem

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## Abstract

In this paper, we study the deterministic single-reservoir Hydro Unit Commitment Problem. Under some hypotheses, we present a time expanded graph representation for the problem, where, at each time step, nodes correspond to discrete operational points, and arcs refer to possible state changes. We show that our problem reduces to a Constrained Shortest Path Problem, propose and compare different approaches to solve the HUCP, based on mixed integer linear or dynamic programming.

## 1 Introduction

In energy management, unit commitment problems are strategic in day-ahead operations (e.g., [?]). In countries where hydro-generation is abundant (e.g., France, Brazil, Canada) specifically dealing with the optimization of cascaded reservoirs is quite challenging too (e.g., [?]). The reason for this is that quite some modelling detail can be required to accurately model reality. One of the essential difficulties stems from representing the efficiency curves linking the flow rate with the actual amount of generated power.

One particular way of dealing with this difficulty is by disposing of an *a priori* discretization of it. The specific set of points is typically chosen by an operational team in order to have maximal efficiency (highest derivatives).

In this paper we will focus on the deterministic price-taker model with a single reservoir and potentially many discrete operational points for the underlying units, in which larger problems can be decomposed efficiently.

In particular, we propose a path formulation and three approaches to solve the HUCP, namely an integer linear programming formulation, an expanded graph reformulation including the volume dimension, and a monotone reformulation to apply a labelling algorithm. We compare our approaches with a simplification of the model in [?].

## 2 A path based modelling approach

We consider the deterministic single reservoir Hydro Unit Commitment Problem (HUCP) under some hypotheses: i) head effect can be neglected, i.e., we do not consider the non-linear effect of the level of the uphill reservoir on the efficiency ii) the set of operational points (o.p.) for the generated production is discrete. In particular, we focus on a single reservoir composed of several units  $J = \{1, 2, \dots, \bar{n}\}$  that can be either pumps or turbines. Moreover, we order the different units, thus they can be aggregated in a unique unit.

### 2.1 Graph modelling

To obtain the path formulation, the main idea is to model the operational profiles of the aggregated units we are considering by means of a graph that includes all the possible operational points at each time step, see Figure ???. Given  $T$  the number of steps in the time horizon, the graph shows  $T + 1$  periods of time  $0, 1, 2, \dots, T$  and an additional fictive period,  $T + 1$ , that will be used in the sequel. More formally, let us denote by  $G = (N, A)$  the graph of Figure ??, where  $N$  is the set of operational points at each time step and  $A$  is the set of possible arcs between nodes of  $N$ , which are only forward.  $G$  is a weighted Directed Acyclic Graph (DAG). For clarity we present the graph construction for the case in which, at each time step, the operational points are the same. However, it is easy to generalize it to the case in which each time period has different operational points. Supposing to have  $\bar{z}$  operational points at each time step, we will have  $|N| = \bar{z}T + 2$ , i.e.,  $\bar{z}$  nodes for each time step and 2 artificial nodes that represent time steps 0 and  $T + 1$ , respectively. In the following, these two nodes will be also called source  $s = 0$  and destination  $d = T\bar{z} + 1$ , respectively, as they represent the initial and final node of the path representing the operational profile of the unit.

The set of nodes  $N$  can be partitioned as follows:  $\{0\} \cup \bigcup_{t=1}^T N_t \cup \{T\bar{z} + 1\}$  where sets  $N_t = \{(t - 1)\bar{z} + 1, (t - 1)\bar{z} + 2, \dots, t\bar{z}\}$  represent the nodes that correspond to the operational points at period  $t$ , for  $t \in \{1, \dots, T\}$ . Thus, arc  $(i, j) \in A$  if  $i \in N_t$ ,  $j \in N_{t+1}$  and if it is possible to move from the operational point corresponding to node  $i$  to the operational point corresponding to  $j$  without violating the physical constraints of the problem. In particular, all the constraints of the considered problem (see, e.g., [?]) are

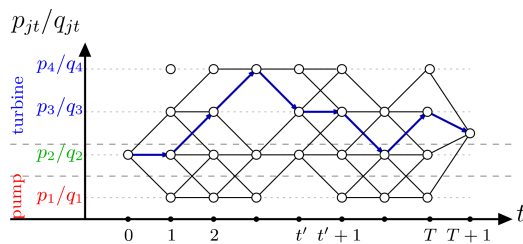


Figure 1:  $G = (N, A)$

included in the graph structure except: i) the bounds on the water volume at each time step, and ii) the target water volume in the reservoir, i.e., the minimum amount of water volume that has to be reached at the end of the time horizon. Finally, a water flow, passed through a turbine or pumped by the pump, is associated with each node and a cost to every edge equal to the difference between the turbine/pump unit startup costs and the power selling profit.

A feasible solution of the HUCP is represented by a path in  $G$  between 0 and  $T + 1$ , see Figure ???. Note that the path consists of exactly  $T + 1$  arcs. Thus, the deterministic single reservoir HUCP, under the assumptions previously mentioned, reduces to a Shortest Path Problem (SPP) from  $s$  to  $d$  with extra constraints, i.e., bounds on water volume of the reservoir.

The Constrained Shortest Path Problem has largely been studied in the literature [?]. The problem can be stated as a minimum-cost path problem subject to one or more resource constraints, a problem widely used in many Branch-and-Price algorithms (see [?]). The (Resource) Constrained Shortest Path Problem ((R)CSPP) constitutes in fact the pricing subproblem of many classical problems.

Different solutions methods have been proposed to solve the (R)CSPP. In all of these approaches, the resource function is assumed to be monotonically decreasing. This is not the case of our water volume constraints. Therefore, these methods cannot be used as defined to solve HUCP, consequently some reformulations are necessary. In the following sections, we propose some approaches to solve the HUCP, namely an integer linear programming formulation, an expanded graph reformulation including the volume dimension, and a monotone reformulation to apply a labelling algorithm.

## 2.2 Integer Linear Programming formulation

Let us consider the weighted directed acyclic graph  $G = (N, A)$  described in Section ??. Let us also introduce the following notation:

- $I_t$  = water inflow in period  $t$  ( $t = 1, \dots, T$ ) [ $\text{m}^3/\text{s}$ ].

- $\Delta t =$  period duration [s].
- $[\underline{V}, \overline{V}] =$  lower and upper bounds on water volume in the reservoir [ $\text{m}^3$ ].
- $V_T =$  target water volume in reservoir at the end of the time horizon [ $\text{m}^3$ ].
- $V_0 =$  initial water volume in reservoir [ $\text{m}^3$ ].
- $Q_i =$  water flow corresponding to node  $i$  ( $i \in N$ ) [ $\text{m}^3/\text{s}$ ], where  $Q_i = 0$  for  $i \in N_{T+1}$ .
- $c_{ij} =$  cost of the arc  $(i, j) \in A$ , depending on the cost of power generated or consumed, on the start up costs of units, and on the pumping cost.

Let  $x_{ij}$  ( $\forall (i, j) \in A$ ) be a binary variable defined as follows:

$$x_{ij} = \begin{cases} 1 & \text{if arc } (i, j) \text{ is part of the selected path} \\ 0 & \text{otherwise.} \end{cases}$$

The short-term single reservoir HUCCP is modelled by integer linear programming, minimizing the cost of the arcs and using classical flow conservation constraints. In addition we add

$$\underline{V} \leq V_0 + \Delta t \sum_{k=1}^t \left( I_k - \sum_{j \in N_k} \sum_{i: (i,j) \in A} Q_j x_{ij} \right) \leq \overline{V} \quad \forall t \in \{1, \dots, T\} \quad (1)$$

$$V_0 + \Delta t \sum_{k=1}^T \left( I_k - \sum_{j \in N_k} \sum_{i: (i,j) \in A} Q_j x_{ij} \right) \geq V_T \quad (2)$$

where constraints (1) model the bounds on the water volume in the reservoir at time step  $t$ , and constraint (2) represents the minimum target water volume to be reached at the end of the time horizon.

### 2.3 Expanded graph reformulation including the volume dimension

The second approach that we propose is a reformulation of the graph  $G = (N, A)$  considering an additional dimension representing the volume. As we mentioned in Section ??, volume constraints are not automatically considered in the graph. Thus, we propose to discretize the volume and to add a volume dimension to the graph, that represents the water volume value in the reservoir at time step  $t$ . In this way, all the possible combinations of volume are considered and all the constraints can be included in the new graph

structure  $G^* = (N^*, A^*)$ , where  $N^*$  is the set of couples of operational points and feasible volume values at each time step and  $A^*$  is the set of possible arcs between nodes of  $N^*$ .  $G^* = (N^*, A^*)$  is a single source single destination DAG, and the set of nodes  $N^*$  is exponential in  $T$ . Finally, we propose to use standard graph techniques to solve the SPP in  $G^* = (N^*, A^*)$ .

Notice, that this approach is equivalent to apply the labelling algorithm (see e.g. [?]) without applying any dominance reduction. The labelling algorithm is one of the most applied state-of-the-art methods for solving the ((R)CSPP). It is based on dynamic programming and consists in applying a label to every node where information on the resource values for partial paths are stored. All the possible partial paths are stored in a pool and iteratively explored. To keep the number of labels as small as possible, it is decisive to perform a dominance step for eliminating unnecessary labels. Assuming that the resource functions are monotonically decreasing, a label  $L1$  is dominated by another label  $L2$  if each resource of  $L1$  is less than or equal to the value of the resource of  $L2$ . In case the monotonicity assumption is not satisfied all the possible partial path are explored, and considering every label as a node of a new graph  $G^\dagger$ , it is evident that  $G^\dagger = G^*$ .

In order to reduce the number of nodes we add the valid constraints

$$v_t + \Delta t \sum_{k=t+1}^T I_k - \Delta t(T-t)Q^- < V_T \quad \forall t \in 1, \dots, T-1 \quad (3)$$

that prune nodes from which the target volume is not obtainable, where  $Q^- = \min_{j \in N} Q_j$  and  $\Delta Q_j = Q_j - Q^-$ .

## 2.4 Monotone reformulation for labelling algorithm

In the third approach, we propose a further reformulation in order to apply the labelling algorithm with dominance reductions, where our resources are volume and cost. We can show that constraints (??)-(??) can be rewritten as follows:

$$\hat{V}_t = \min \left( -V_T + \Delta t \left( \sum_{k=t+1}^T I_k - (T-t)Q^- \right), -\underline{V} \right) \quad \forall t \in 1, \dots, T-1 \quad (4)$$

$$\hat{V}_T = \min (-V_T, -\underline{V}) \quad (5)$$

$$\Delta t \sum_{k=1}^t \sum_{j \in N_k} \sum_{(i,j) \in A} \Delta Q_j x_{ij} \leq V_0 + \Delta t \left( \sum_{k=1}^t I_k - t Q^- \right) + \hat{V}_t \quad \forall t \in 1, \dots, T \quad (6)$$

$$\Delta t \sum_{k=1}^t \sum_{j \in N_k} \sum_{(i,j) \in A} \Delta Q_j x_{ij} \geq V_0 + \Delta t \left( \sum_{k=1}^t I_k - t Q^- \right) - \bar{V} \quad \forall t \in 1, \dots, T. \quad (7)$$

Assuming that at least one o.p. of pumping exist,  $Q^- \leq 0$ . In case there does not exist such a point, reformulation is not required. The resource use modelled by constraints (??)-(??) is monotone additive and increasing. Therefore the domination rule selects labels with the smallest quantity of resources used, i.e., cost and volume. We propose to use a labelling algorithm to solve the reformulation, applying a quick sort algorithm to order the labels. However, also the RHSs of constraints (??) are monotonically increasing, therefore we propose a variant of the classical labelling algorithm, where for every period  $t$ , the domination rule can be applied only if the following additional conditions are satisfied

$$V_0 + \Delta t \sum_{k=1}^t \left( I_k - \sum_{j \in N_k} \sum_{i: (i,j) \in A} Q_j x_{ij} \right) - (T-t) Q^- \leq \bar{V} \quad \forall t \in 1, \dots, T \quad (8)$$

where, selecting always  $Q^-$  in all remaining periods, the lower bound on the volume is satisfied.

### 3 Computational experiments

We tested our approaches and MILP model in [?] on 156 instances generated starting from data from [?]. The instances have combinations of different final volumes, o.p. from 5 to 17, and number of periods from 24 to 96. All experiments are performed on a single machine equipped with an Intel Xeon E5649 processor clocked at 2.53 GHz and 50 GB RAM. We solved the MILPs models with the IBM ILOG CPLEX 12.6 solver with time limit of 1 hour.

In Figure ?? we report performance profile for a simplification of the MILP model in [?] (BDLM), the expanded graph reformulation including the volume dimension (SPP), and the labelling algorithm with dominance reductions (DSPP), for the whole set of instances. We do not include the results for the ILP model of Section (??) because very few instances are solved to optimality within the time limit and this approach appears inefficient compared to the other approaches. The labelling algorithm can solve all instances within 1 minute, while BDLM hits the time limit in 12

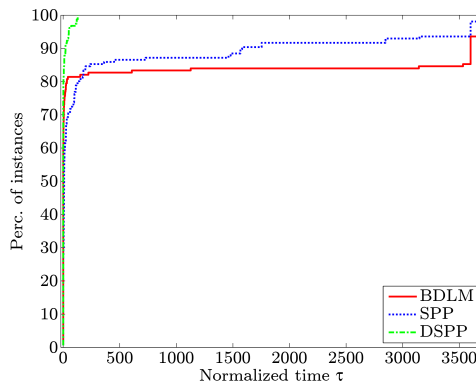


Figure 2: Performance profiles for 156 instances

cases, and the expanded graph approach in 1 case. In addition, the profiles clearly show the significantly better performance of the labelling algorithm compared to the direct solution of the other approaches.

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