



Form factors and decoupling of matter fields in four-dimensional gravity

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ABSTRACT

We extend previous calculations of the non-local form factors of semiclassical gravity in 4D to include the Einstein–Hilbert term. The quantized fields are massive scalar, fermion and vector fields. The non-local form factor in this case can be seen as the sum of a power series of total derivatives, but it enables us to derive the beta function of Newton's constant and formally evaluate the decoupling law in the new sector, which turns out to be the standard quadratic one.

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1. Introduction

The derivation of non-local form factors in the semiclassical theory of massive matter fields on a classical curved background has several interesting applications. The calculation in the higher derivative vacuum sector [1,2] (see also [3]) supports the idea of the gravitational decoupling which is relevant for the graceful exit from the general version of anomaly induced inflation [4–6]. Indeed, this mechanism is not sufficient for deriving the Starobinsky inflation [7,8] from quantum corrections, but one can hope that more detailed study of the gravitational decoupling may be useful for constructing the corresponding field theoretical model [9].

An important application of the effective approach to quantum field theory in curved spacetime is the possible running of cosmological and Newton's constants at low energies, such as the typical energy scale in the late cosmology (which we shall call IR). If such a running takes place, there could be measurable implications in both cosmology (see e.g. [10]) and astrophysics (see, e.g., [11]). Unfortunately, from the quantum field theory side, there is no way to consistently calculate such a running. The reason is that the existing methods of quantum calculations in curved space are essentially based on the expansion of all quantities around the flat space–time. For instance, the normal coordinate expansion and

Schwinger–DeWitt technique are based on the expansions into a power series in the curvature tensor and its covariant derivatives. Such an expansion is not sufficient to establish the physical running of the cosmological and Newton's constants. An observation of such a running requires at least the expansion around space–times of constant nonzero curvature [12], which is not available, except some special cases [13], which are not sufficient to observe the decoupling. In the case when a variation with respect to the scale of the cosmological and Newton's constants does not take place, there would be a discrepancy between the well established running of these constants in the Minimal Subtraction (\overline{MS}) renormalization scheme [14,15] (see [16] for an introduction) and the absence of the non-local form factors for the corresponding terms in the effective action.

The reason why there are no non-local form factors in the zero and second-derivative sectors of the gravitational action can be easily seen from the comparison with the fourth-derivative terms [1]. The non-local form factors can emerge in the square of the Weyl tensor $C_{\alpha\beta\rho\sigma} k_1(\frac{\square}{m^2})C^{\alpha\beta\rho\sigma}$, or in the square of the scalar curvature $R k_2(\frac{\square}{m^2})R$. At the same time it is unclear how to introduce such a form factor for the cosmological constant, because the d'Alembert operator acting on a constant gives zero. Furthermore, if a non-local form factor is inserted into the Einstein–Hilbert action, a function of \square acting on R is equivalent to a sum of the series of surface terms. The simplest solution which was proposed in [1] was to replace the cosmological constant by the non-local expressions

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$$R_{\alpha\beta} \frac{1}{\square^2} R^{\alpha\beta} \quad \text{and} \quad R \frac{1}{\square^2} R, \quad (1)$$

which have the same global scaling as a constant. Similar replacement can be done for the Einstein–Hilbert Lagrangian by using the terms

$$R_{\alpha\beta} \frac{1}{\square} R_{\alpha\beta} \quad \text{and} \quad R \frac{1}{\square} R. \quad (2)$$

The problem with this approach is that the semiclassical form factors can not be derived for the terms (1) and (2) within the existing field theoretical methods. Thus, the interesting cosmological applications of the models based on (1) and (2) which were considered in [17] are as phenomenological as the non-covariant running which is considered in [10,12], and the unique advantage, from the conceptual point of view, is that those are covariant expressions, which are easier to work with. In fact these structures are becoming increasingly of interest even in the context of quantum gravity, in which they might play the role of template to reconstruct the effective action [18].

Recently an alternative approach to the physical running of the inverse Newton's constant has been initiated in [19] which is based on [3]. The consideration was performed for the two dimensional (2D) case and is related to some older works by Avramidi and collaborators [20,21]. The idea is to derive the non-local form factors for the Einstein–Hilbert term, regardless of the fact that the corresponding structures will be total derivatives.¹ There is a serious justification of this approach, but we postpone this part of the discussions for the last section. In what follows we generalize the calculations of [19] to four dimensions (4D) and perform full consideration of the non-local terms. For the sake of completeness we checked all the non-local contributions for higher derivative terms, which are well known from [1,2] and [3]. One of the reasons for this is the detailed discussion of the distinctions and similarities between the form factors for R , $\square R$ and R^2 terms. As we know from previous work (see, e.g., the discussion in [24] with a special emphasize to the role of non-local form factors in massive semiclassical theory), the renormalization of the surface terms results in the finite non-surface contributions, and the explicit form of the non-local surface terms derived here makes our understanding of this relation more detailed.

The outline of the paper is as follows. In Sec. 2 we discuss the structure of the effective action and its renormalization, and construct the necessary equations to observe the gravitational version of the Applequist–Carazzone theorem [25] for the Newton constant in 4D curved space. In Secs. 3, 4 and 5 we give explicit formulas for nonminimally coupled scalars, Dirac spinors and Proca fields respectively. Finally, in Sec. 6 we draw our conclusions, present a general analysis of the results and comment on possible physical interpretations and the prospects of further developments. The two Appendices are included to clarify further the main text, namely in Appendix A we briefly present the heat kernel method which is used for the computations, while in Appendix B we survey the ultraviolet structure of the effective action and its physical implications.

2. Nonlocal effective action

We are interested in the contribution to the vacuum effective action of a set of free massive matter fields which includes n_s non-minimally coupled scalars, n_f Dirac fermions and n_p Proca fields.

The integration of the free matter fields fluctuations on curved background leads to the expression

$$\Gamma[g] = n_s \Gamma_s[g] + n_f \Gamma_f[g] + n_p \Gamma_p[g], \quad (3)$$

in which $\Gamma_s[g]$, $\Gamma_f[g]$ and $\Gamma_p[g]$ denote the individual contributions for a single field of each matter specie. The individual contributions are²

$$\begin{aligned} \Gamma_s[g] &= \frac{1}{2} \text{Tr}_s \ln \left(\Delta_g + \xi R + m_s^2 \right), \\ \Gamma_f[g] &= -\text{Tr}_f \ln \left(\not{D} + m_f \right), \\ \Gamma_p[g] &= \frac{1}{2} \text{Tr}_v \ln \left(\delta_\mu^\nu \Delta_g + \nabla_\mu \nabla^\nu + R_\mu{}^\nu + \delta_\mu^\nu m_p^2 \right), \end{aligned} \quad (4)$$

in which each trace is taken over the appropriate degrees of freedom and Δ_g is defined as positive in Euclidean space. A little work is needed to cast all functional traces in the same form. Squaring the Dirac operator we arrive at the expression

$$\Gamma_f[g] = -\frac{1}{2} \text{Tr}_f \ln \left(\Delta_g + \frac{R}{4} + m_f^2 \right). \quad (5)$$

When dealing with the Proca operator we need to take care of the longitudinal modes, which can be done in at least two equivalent ways [30,31] and results in

$$\Gamma_p[g] = \frac{1}{2} \text{Tr}_v \ln \left(\delta_\mu^\nu \Delta_g + R_\mu{}^\nu + \delta_\mu^\nu m_p^2 \right) - \frac{1}{2} \text{Tr}_s \ln \left(\Delta_g + m_p^2 \right). \quad (6)$$

Now each trace acts on the logarithm of an operator of Laplace-type

$$\Gamma[g] = \frac{1}{2} \text{Tr} \ln \left(\Delta_g + E + m^2 \right) \quad (7)$$

for an appropriate endomorphism E acting on the field's bundle. A standard way to compute traces of Laplace-type operators is to use the heat kernel. We can represent the above trace as an integral over the heat kernel proper time s ,

$$\Gamma[g] = -\frac{1}{2} \text{tr} \int_0^\infty \frac{ds}{s} \int d^4x \sqrt{g} e^{-sm^2} \mathcal{H}(s; x, x), \quad (8)$$

in which we have also separated the original trace into an integration over spacetime and a trace over the internal indices, and introduced the local heat kernel $\mathcal{H}(s; x, x')$ (see Appendix A for a brief explanation regarding the heat kernel technique).

The effective action (8) has ultraviolet divergences, and a simple way to regulate them is through dimensional regularization [26]. For this purpose we continue the leading power $s^{-\frac{d}{2}}$ of the heat kernel to a generic number d of dimensions, and introduce both a reference scale μ to preserve the mass dimension of all quantities when leaving four dimensions and a small parameter $\epsilon = 4 - d$. The result of this substitution is the regularized effective action

$$\Gamma[g] = -\frac{\mu^\epsilon}{2} \text{tr} \int_0^\infty \frac{ds}{s} \int d^4x \sqrt{g} e^{-sm^2} \mathcal{H}(s; x, x). \quad (9)$$

¹ Similar structures have already been explored in the quantum gravity literature [22,23].

² Starting from this section we assume the Wick rotation and all notations are Euclidean. The positively defined Laplacian operator Δ_g is defined in Appendix A and $R_{\mu\nu} = \partial_i \Gamma_{\mu\nu}^i + \dots$. At the same time in all physical discussions we use pseudo-Euclidean notations.

Since all fields are massive the above effective action has no infrared divergences, thanks to the exponential damping factor caused by the mass for large values of s . However, there are ultraviolet divergences which appear as inverse powers of ϵ and require renormalization. We follow the standard practice of subtracting poles of the parameter $\bar{\epsilon}$, which is defined as

$$\frac{1}{\bar{\epsilon}} = \frac{1}{\epsilon} + \frac{1}{2} \ln \left(\frac{4\pi\mu^2}{m^2} \right) - \frac{\gamma}{2} \quad (10)$$

(here γ is the Euler's constant), instead of simply subtracting ϵ poles, exploiting the freedom of the choice of renormalization scheme.

In the process of regularization and renormalization it is often convenient to deal with dimensionless quantities. Keeping in mind that at the moment the energy scales at our disposal are the Laplacian Δ_g and the mass m^2 , we find convenient to introduce the following dimensionless operators

$$z = \frac{\Delta_g}{m^2}, \quad a = \sqrt{\frac{4z}{4+z}}, \quad Y = 1 - \frac{1}{a} \ln \left| \frac{1+a/2}{1-a/2} \right|. \quad (11)$$

With the above definitions we have all the ingredients to discuss the form that the effective action can take up to the second order in a curvature expansion. We have that to this order the most general form can be narrowed down to the sum of a local and a non-local part

$$\Gamma[g] = \Gamma_{\text{loc}}[g] + \frac{m^2}{2(4\pi)^2} \int d^4x \sqrt{g} B(z) R + \frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} \left\{ C^{\mu\nu\alpha\beta} C_1(z) C_{\mu\nu\alpha\beta} + R C_2(z) R \right\}, \quad (12)$$

in which $C_{\mu\nu\rho\theta}$ is the four dimensional Weyl tensor. Since the divergences are local expressions, all dimensional poles are contained in the local part of effective action $\Gamma_{\text{loc}}[g]$. The renormalization can be performed through the introduction of appropriate counter terms and generically results in a renormalized action of the form

$$S_{\text{ren}}[g] = \int d^4x \sqrt{g} \left\{ b_0 + b_1 R + a_1 C^2 + a_2 \mathcal{E}_4 + a_3 \square R + a_4 R^2 \right\}, \quad (13)$$

in which \mathcal{E}_4 is the operator associated to the Euler's characteristic, which is the Gauss–Bonnet topological term in $d = 4$. The renormalized action features the couplings that have to be experimentally determined in order for the theory to be predictive. The couplings include the cosmological constant Λ and the Newton's constant G through the relations $b_0 = 2\Lambda G^{-1}$ and $b_1 = -G^{-1}$. The minimal subtraction (MS) procedure induces a running of all the couplings which is encoded in beta functions that we denote as β_g^{MS} in which g is any of the couplings of (13). In what follows we formulate the beta functions for the parameters b_0 and b_1 , instead of Λ and G .

The minimal subtraction scheme – based one-loop renormalization group flow induced by the beta functions of the couplings of (13) has been known for a long time for all the field types listed in this section. In this work we concentrate instead on the non-local contributions of the effective action. In (12) we have introduced three new covariant functions $B(z)$, $C_1(z)$ and $C_2(z)$ of the rescaled Laplacian z . These functions are known as form factors of the effective action and represent a true physical prediction which comes

from the formalism: in fact one can imagine to pick a specific observable – either from cosmology or from particle physics – and compute it in terms of the form factors themselves [27]. A simple way to understand the physical consequences of the effective action, which is related to the general concept of renormalization group, is to use them to construct new non-local beta functions which are sensitive to the presence of the mass scale m^2 .

Let us first recall that the non-local form factors of the heat kernel of Appendix A, and consequently the non-local contributions to the effective action (12), are obtained for asymptotically flat Euclidean spacetimes in which curvatures are small (schematically $|\nabla^2 \mathcal{R}| \gg |\mathcal{R}^2|$ for any curvature tensor \mathcal{R}) [33]. In practice, the asymptotic flatness offers a special reference frame which can be used to construct meaningful Fourier transformations and in which the expansion in curvatures can be related to the expansion in fluctuations of the metric. In fact, this is precisely the frame in which the form factors are computed in [1–3], even though the final expressions are always presented in a manifestly covariant form. In short, this implies that the Laplace operator Δ_g is in one-to-one correspondence with the square q^2 of a momentum q_μ of the asymptotic frame upon Fourier transformation. This representation is especially useful for the renormalization group applications, where one has to take derivatives with respect to the scale parameter.

The straightforward way to derive the beta functions is to subtract the divergences at the scale $q = |q_\mu|$. For convenience, let us define the dimensionless scale $\hat{q} = q/m$ which is simply q in units of the mass; by definition after the Fourier transform \hat{q} is related to z as $\hat{q}^2 = z$ and the renormalization group flow is parametrized by

$$q \frac{\partial}{\partial q} = \hat{q} \frac{\partial}{\partial \hat{q}} = 2z \frac{\partial}{\partial z}. \quad (14)$$

Let us begin by discussing the renormalization group flow of the terms that are quadratic in the curvatures which has been studied in detail in [1,2]. A simple inspection suggests the non-local generalization of the beta functions of a_1

$$\beta_{a_1} = 2z \frac{\partial}{\partial z} \left[\frac{1}{2(4\pi)^2} C_1(z) \right] = \frac{z}{(4\pi)^2} C_1'(z), \quad (15)$$

in which we indicate the derivative with a prime. The same can be done for the coupling a_4

$$\beta_{a_4} = \frac{z}{(4\pi)^2} C_2'(z). \quad (16)$$

In practice the form factors $C_1(z)$ and $C_2(z)$ play the role of non-local scale-dependent generalizations of the couplings. Since our heat kernel methods work on spaces that are asymptotically flat, we do not have enough information to compute the running of the topological term in this context (although it is still possible to complement this result with standard Seeley–DeWitt methods).

Now let us turn our attention to the couplings of the terms that are linear in the curvature R . On the one hand we have that the renormalized action features two couplings – b_1 and a_3 – but on the other hand there is only a single form factor $B(z)$ acting on R in (12). Naively we are tempted to define a master beta function

$$\Psi = \frac{1}{(4\pi)^2} z \partial_z \left[\frac{B(z)}{z} \right], \quad (17)$$

which we denoted with a new symbol to avoid confusion. The running function Ψ is defined to take into account that in (12) we are measuring a dimensionfull quantity – the coefficient of R – in units of m^2 while instead our rescaling should be done in units

of q^2 , hence the quotient with $z = q^2/m^2$ that restores the right units. While we will find useful to study this object later on, at this stage it is not clear if its renormalization group flow should be associated to the coupling b_1 or to a_3 . Returning to (13) it is easy to see that if $m^2 \gg q^2$ the operator R will dominate over the operator $\square R$, and conversely if $m^2 \ll q^2$ the operator $\square R$ will dominate over the operator R . This implies that in the high energy limit $z \gg 1$ the function Ψ should encode information of β_{a_3} , while in the opposite limit $z \sim 0$ the function Ψ should encode information of β_{b_1} . This property is discussed in more detail later. However, β_{b_1} and β_{a_3} have well known ultraviolet limits which we would like to preserve, associated to $\overline{\text{MS}}$ as we will also see later. We find that the best solution is to define the following beta functions

$$\begin{aligned}\beta_{a_3} &= -\frac{1}{(4\pi)^2} z \partial_z \left[\frac{B(z) - B(0)}{z} \right], \\ \beta_{b_1} &= \frac{m^2}{(4\pi)^2} z \partial_z [B(z) - B_\infty(z)].\end{aligned}\quad (18)$$

The first equation is implied by the comparison with (13) and includes the removal of the constant part that should be attributed to b_1 . In the second equation we subtract the dominating $\square R$ effect from the running of $B(z)$ in the form of $B_\infty(z)$ which is the leading logarithmic asymptotic behavior for $z \simeq \infty$ of $B(z)$ itself. The leftover terms of the subtraction is thus identified with the running of the operator R and hence the coupling b_1 . In the practical computations instead of subtracting the leading logarithm at infinity, we will subtract instead the combination

$$a(1 - Y) \simeq \ln(z), \quad (19)$$

which is shown to be valid for $z \gg 1$ using the definitions (11). General features of the definitions (18) and their ultraviolet properties are discussed in more detail in Appendix B.

In the next section we present explicit results for the form factor and the beta function for the Einstein–Hilbert term. The full set of the form factors and the expressions for all the non-local beta functions in the fourth-derivatives sector, and also the corresponding $\overline{\text{MS}}$ beta functions can be found in the papers [2,3]. All results will be collected in a mini review to appear shortly [28]. However, there are still some general properties that we can discuss here in anticipation. For all the couplings and all the beta functions we can show that there are sensible ultraviolet $z \gg 1$ and infrared $z \sim 0$ limits. Each beta function satisfies the additional property

$$\beta_g = \beta_g^{\overline{\text{MS}}} + \mathcal{O}\left(\frac{m^2}{q^2}\right) \quad \text{for} \quad q^2 \gg m^2, \quad (20)$$

where g is any of the couplings. Furthermore, all the renormalization group running the subject to the effect of decoupling towards the infrared, meaning that when q^2 goes below the m^2 threshold fluctuations stop propagating and have no effect on the quantum physics anymore. We have that

$$\beta_g = \mathcal{O}\left(\frac{q^2}{m^2}\right) \quad \text{for} \quad q^2 \ll m^2, \quad (21)$$

which is the practical evidence of the Appelquist–Carazzone theorem in four dimensional curved space.

Finally, it is interesting to observe the practical implications of the discussion on the function $\Psi(z)$. As argued above, the limits $m^2 \ll q^2$ and $m^2 \gg q^2$ should see the operators $\square R$ and R dominating the running $\Psi(z)$ respectively. For all the matter types that we consider we have the following two limits

$$\Psi = \begin{cases} -\beta_{a_3}^{\overline{\text{MS}}} & \text{for } q^2 \gg m^2 \\ \frac{m^2}{q^2} \beta_{b_1}^{\overline{\text{MS}}} & \text{for } q^2 \ll m^2 \end{cases} \quad (22)$$

which reflect the previous consideration. Notice that while the ultraviolet limit can be straightforwardly proven on the basis of the definitions of $\beta_{a_3}^{\overline{\text{MS}}}$ and Ψ , the infrared limit is much less trivial. Notice also that the infrared limit does not sharply decouple, because it grows with the square of the mass, but this is to be expected since we are measuring a massive quantity in units of q for $q \rightarrow 0$. To get rid of the divergence it is sufficient to switch to measuring the same quantity in units of m in the infrared.

3. Nonminimally coupled scalar field

The effective action of the nonminimally coupled scalar field can be obtained specifying the endomorphism $E = \xi R$ in the non-local heat kernel expansion and then performing the integration in s [29]. We find the local contributions of the regularized action to be

$$\begin{aligned}\Gamma_{\text{loc}}[g] &= \frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} \left\{ -m^4 \left(\frac{1}{\epsilon} + \frac{3}{4} \right) - 2m^2 \left(\xi - \frac{1}{6} \right) \frac{1}{\epsilon} R \right. \\ &\quad + \frac{1}{3} \left(\xi - \frac{1}{5} \right) \frac{1}{\epsilon} \square R - \frac{1}{60\epsilon} C_{\mu\nu\rho\theta} C^{\mu\nu\rho\theta} \\ &\quad \left. - \left(\xi - \frac{1}{6} \right)^2 \frac{1}{\epsilon} R^2 \right\}.\end{aligned}\quad (23)$$

The minimal subtraction of the divergences of local contributions induces the following $\overline{\text{MS}}$ beta functions for the terms with up to one curvature

$$\begin{aligned}\beta_{b_0}^{\overline{\text{MS}}} &= \frac{1}{(4\pi)^2} \frac{m^4}{2}, & \beta_{b_1}^{\overline{\text{MS}}} &= \frac{1}{(4\pi)^2} m^2 \left(\xi - \frac{1}{6} \right), \\ \beta_{a_3}^{\overline{\text{MS}}} &= -\frac{1}{(4\pi)^2} \frac{1}{6} \left(\xi - \frac{1}{5} \right).\end{aligned}\quad (24)$$

The non-local part of the effective action includes the following form factor

$$\begin{aligned}\frac{B(z)}{z} &= -\frac{4Y}{15a^4} + \frac{Y}{9a^2} - \frac{1}{45a^2} + \frac{4}{675} \\ &\quad + \left(\xi - \frac{1}{6} \right) \left(-\frac{4Y}{3a^2} - \frac{1}{a^2} + \frac{5}{36} \right),\end{aligned}\quad (25)$$

while $C_1(z)$ and $C_2(z)$ confirm the results reported in [1]. Using our definitions the non-local beta functions of the couplings associated to the curvature R are

$$\begin{aligned}\beta_{b_1} &= \frac{z}{(4\pi)^2} \left\{ \frac{2Y}{5a^4} - \frac{2Y}{9a^2} + \frac{1}{30a^2} - \frac{aY}{180} + \frac{a}{120} + \frac{Y}{24} - \frac{1}{40} \right. \\ &\quad \left. + \left(\xi - \frac{1}{6} \right) \left(\frac{2Y}{3a^2} + \frac{aY}{6} - \frac{a}{4} - \frac{Y}{2} + \frac{1}{2} \right) \right\}\end{aligned}\quad (26)$$

and

$$\begin{aligned}\beta_{a_3} &= \frac{1}{(4\pi)^2} \left\{ -\frac{2Y}{3a^4} + \frac{Y}{3a^2} - \frac{1}{18a^2} - \frac{Y}{24} + \frac{7}{360} \right. \\ &\quad \left. + \left(\xi - \frac{1}{6} \right) \left(-\frac{2Y}{a^2} + \frac{Y}{2} - \frac{1}{6} \right) \right\}.\end{aligned}\quad (27)$$

The Eqs. (26) and (27) provide all necessary ingredients to study the Appelquist–Carazzone theorem of both parameters. Plots of these beta functions are given in Fig. 1.

As we have explained in the Introduction, the most interesting is the decoupling theorem for the running of the Newton's constant which is related to the inverse of $b_1 = -G^{-1}$. The non-local

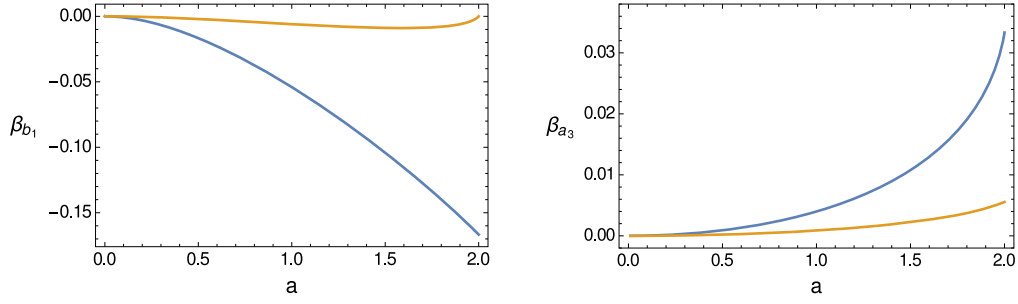


Fig. 1. Plots of the beta functions β_{b_1} and β_{a_3} rescaled by a factor $(4\pi)^2$ that are induced by a single scalar field for the values $\xi = 0$ (blue) and $\xi = \frac{1}{6}$ (yellow) as a function of the variable a defined in (11). The plot ranges from the IR at $a=0$ ($q^2 \ll m^2$) to the UV at $a=2$ ($q^2 \gg m^2$). The effects of the Appelquist–Carazzone theorem are seen on the left where the beta functions become zero. The beta function β_{b_1} for the special conformal value $\xi = \frac{1}{6}$ is zero also in the UV.

beta function of the couplings b_1 and a_3 in units of the mass have the two limits

$$\beta_{b_1} = \begin{cases} \frac{1}{(4\pi)^2} \left(\xi - \frac{1}{6} \right) + \frac{1}{(4\pi)^2} \left\{ \left(\frac{3}{5} - \xi \right) - \xi \ln \left(\frac{q^2}{m^2} \right) \right\} \frac{m^2}{q^2} \\ \quad + \mathcal{O} \left(\frac{m^2}{q^2} \right)^2 & \text{for } q^2 \gg m^2, \\ \frac{1}{(4\pi)^2} \left(\frac{4}{9} \xi - \frac{77}{900} \right) \frac{q^2}{m^2} + \mathcal{O} \left(\frac{q^2}{m^2} \right)^{\frac{3}{2}} & \text{for } q^2 \ll m^2 \end{cases} \quad (28)$$

and

$$\beta_{a_3} = \begin{cases} -\frac{1}{6(4\pi)^2} \left(\xi - \frac{1}{5} \right) + \frac{1}{(4\pi)^2} \left\{ \frac{5}{18} - 2\xi + \left(\xi - \frac{1}{6} \right) \ln \left(\frac{q^2}{m^2} \right) \right\} \frac{m^2}{q^2} \\ \quad + \mathcal{O} \left(\frac{m^2}{q^2} \right)^2 & \text{for } q^2 \gg m^2, \\ \frac{1}{(4\pi)^2} \frac{1}{840} (3 - 14\xi) \frac{q^2}{m^2} + \mathcal{O} \left(\frac{q^2}{m^2} \right)^2 & \text{for } q^2 \ll m^2. \end{cases} \quad (29)$$

The last expressions show standard quadratic decoupling in the IR for both parameters, exactly as in the usual QED situation [25] and as for the fourth derivative non-surface gravitational terms [1,2]. In the high energy limit (UV) we meet the usual $\overline{\text{MS}}$ beta function plus a small correction to it.

4. Dirac field

The effective action of the minimally coupled Dirac fields requires the specification of the endomorphism $E = R/4$. The final result turns out to be proportional to the dimension d_γ of the Clifford algebra and hence to the number of spinor components. We do not set $d_\gamma = 4$, but choose instead to leave it arbitrary so that the formulas can be generalized to other spinor species easily. We find the local regularized action to be

$$\Gamma_{\text{loc}}[g] = \frac{d_\gamma}{2(4\pi)^2} \int d^4x \sqrt{g} \left\{ m^4 \left(\frac{1}{\bar{\epsilon}} + \frac{3}{4} \right) + \frac{m^2}{6\bar{\epsilon}} R \right. \\ \left. - \frac{1}{60\bar{\epsilon}} \square R - \frac{1}{40\bar{\epsilon}} C_{\mu\nu\rho\theta} C^{\mu\nu\rho\theta} \right\}. \quad (30)$$

The minimal subtraction of the $1/\bar{\epsilon}$ divergences induces the following $\overline{\text{MS}}$ beta functions

$$\beta_{b_0}^{\overline{\text{MS}}} = -\frac{d_\gamma}{(4\pi)^2} \frac{m^4}{2}, \quad \beta_{b_1}^{\overline{\text{MS}}} = -\frac{d_\gamma}{(4\pi)^2} \frac{m^2}{12}, \quad (31)$$

$$\beta_{a_3}^{\overline{\text{MS}}} = \frac{d_\gamma}{(4\pi)^2} \frac{1}{120}.$$

The non-local part of the effective action includes the following form factor

$$\frac{B(z)}{z} = d_\gamma \left\{ -\frac{7}{400} + \frac{19}{180a^2} + \frac{4Y}{15a^4} \right\}, \quad (32)$$

while $C_1(z)$ and $C_2(z)$ agree with [1]. The non-local beta functions are

$$\beta_{b_1} = \frac{d_\gamma z}{(4\pi)^2} \left\{ -\frac{2Y}{5a^4} + \frac{Y}{6a^2} - \frac{1}{30a^2} - \frac{aY}{120} + \frac{a}{80} - \frac{1}{60} \right\}, \quad (33)$$

$$\beta_{a_3} = \frac{d_\gamma}{(4\pi)^2} \left\{ \frac{2Y}{3a^4} - \frac{Y}{6a^2} + \frac{1}{18a^2} - \frac{1}{180} \right\}.$$

Likewise the scalar case the non-local beta functions of b_1 and a_3 have the two limits

$$\beta_{b_1} = \begin{cases} -\frac{d_\gamma}{(4\pi)^2} \frac{1}{12} - \frac{d_\gamma}{(4\pi)^2} \left[\frac{7}{20} - \frac{1}{4} \ln \left(\frac{q^2}{m^2} \right) \right] \frac{m^2}{q^2} + \mathcal{O} \left(\frac{m^2}{q^2} \right)^2 \\ \quad \text{for } q^2 \gg m^2; \\ -\frac{d_\gamma}{(4\pi)^2} \frac{23}{900} \frac{q^2}{m^2} + \mathcal{O} \left(\frac{q^2}{m^2} \right)^{\frac{3}{2}} & \text{for } q^2 \ll m^2. \end{cases}$$

$$\beta_{a_3} = \begin{cases} \frac{d_\gamma}{(4\pi)^2} \frac{1}{120} + \frac{d_\gamma}{(4\pi)^2} \left\{ \frac{2}{9} - \frac{1}{12} \ln \left(\frac{q^2}{m^2} \right) \right\} \frac{m^2}{q^2} + \mathcal{O} \left(\frac{m^2}{q^2} \right)^2 \\ \quad \text{for } q^2 \gg m^2; \\ \frac{d_\gamma}{(4\pi)^2} \frac{1}{1680} \frac{q^2}{m^2} + \mathcal{O} \left(\frac{q^2}{m^2} \right)^2 & \text{for } q^2 \ll m^2. \end{cases} \quad (34)$$

Once again, there is a standard quadratic decoupling in the IR for both parameters, while in the UV we find the $\overline{\text{MS}}$ beta function and a sub-leading correction.

5. Proca field

The minimally coupled Proca field could be understood as a four-components vector field, but one of these components is subtracted through a single scalar ghost, so it has effectively three degrees of freedom in four dimensions. The local regularized action is

$$\Gamma_{\text{loc}}[g] = \frac{1}{2(4\pi)^2} \int d^4x \sqrt{g} \left\{ -m^4 \left(\frac{3}{\bar{\epsilon}} + \frac{9}{4} \right) - \frac{m^2}{\bar{\epsilon}} R \right. \\ \left. + \frac{2}{15\bar{\epsilon}} \square R - \frac{13}{60\bar{\epsilon}} C_{\mu\nu\rho\theta} C^{\mu\nu\rho\theta} - \frac{1}{36} R^2 \right\}. \quad (35)$$

The minimal subtraction of the $1/\bar{\epsilon}$ poles induces the following $\overline{\text{MS}}$ beta functions

$$\beta_{b_0}^{\overline{\text{MS}}} = \frac{1}{(4\pi)^2} \frac{3m^4}{2}, \quad \beta_{b_1}^{\overline{\text{MS}}} = \frac{1}{(4\pi)^2} \frac{m^2}{2}, \quad (36)$$

$$\beta_{a_3}^{\overline{\text{MS}}} = -\frac{1}{(4\pi)^2} \frac{1}{15}.$$

The non-local part of the effective action includes the following form factors

$$\frac{B(z)}{z} = \frac{157}{1800} - \frac{17}{30a^2} - \frac{4Y}{5a^4} - \frac{Y}{3a^2}, \quad (37)$$

and $C_1(z)$ and $C_2(z)$ reproduce [1]. The non-local beta functions are

$$\beta_{b_1} = \frac{z}{(4\pi)^2} \left(\frac{6Y}{5a^4} - \frac{Y}{3a^2} + \frac{1}{10a^2} + \frac{aY}{15} - \frac{a}{10} - \frac{Y}{8} + \frac{7}{40} \right) \quad (38)$$

$$\beta_{a_3} = \frac{1}{(4\pi)^2} \left\{ -\frac{2Y}{a^4} - \frac{1}{6a^2} + \frac{Y}{8} - \frac{1}{40} \right\}.$$

The beta functions of b_1 and a_3 have the two limits

$$\frac{\beta_{b_1}}{m^2} = \begin{cases} \frac{1}{(4\pi)^2} \frac{1}{2} + \frac{1}{(4\pi)^2} \left(\frac{4}{5} - \ln \left(\frac{q^2}{m^2} \right) \right) \frac{m^2}{q^2} + \mathcal{O} \left(\frac{m^2}{q^2} \right)^2 & \text{for } q^2 \gg m^2; \\ \frac{1}{(4\pi)^2} \frac{169}{900} \frac{q^2}{m^2} + \mathcal{O} \left(\frac{q^2}{m^2} \right)^{\frac{3}{2}} & \text{for } q^2 \ll m^2. \end{cases}$$

$$\beta_{a_3} = \begin{cases} -\frac{1}{(4\pi)^2} \frac{1}{15} - \frac{1}{(4\pi)^2} \left\{ \frac{7}{6} - \frac{1}{2} \ln \left(\frac{q^2}{m^2} \right) \right\} \frac{m^2}{q^2} + \mathcal{O} \left(\frac{m^2}{q^2} \right)^2 & \text{for } q^2 \gg m^2; \\ -\frac{1}{(4\pi)^2} \frac{1}{168} \frac{q^2}{m^2} + \mathcal{O} \left(\frac{q^2}{m^2} \right)^2 & \text{for } q^2 \ll m^2. \end{cases} \quad (39)$$

We can observe that for the Proca field there is the same quadratic decoupling for both couplings, and the same $\overline{\text{MS}}$ beta function plus a small correction in the UV.

6. Conclusions

We computed the covariant non-local form factors of the Euclidean effective action of nonminimal scalars, Dirac spinors and Proca fields up to the second order of the curvature expansion on asymptotically flat space. The calculations were performed by means of heat kernel method for the massive quantum fields and an arbitrary external metric. We checked explicitly that the results for the fourth derivative terms confirmed the previous ones derived by [1–3] which were obtained by both Feynman diagrams and heat kernel method as presented in the paper of Barvinsky and Vilkovisky [33]. We used the results for the effective action to find suitable beta functions which arise from the subtraction of the divergences at a physical momentum scale q^2 . These beta functions are special because they display two important limits: in the ultraviolet they reproduce the universal results coming from the minimal subtraction of the poles of dimensional regularization, while in the infrared (IR) limit $q^2 \ll m^2$ they exhibit a quadratic decoupling, as expected from the Applequist–Carazzone theorem. The decoupling can be observed for both inverse Newton constant and for a_3 . With respect to the global scaling the $\square R$ -term is the same as the R^2 term. It is well known that the finite contribution for the R^2 term is linked to the divergences of the $\square R$ -term, while the finite nonlocal contribution for the surface $\square R$ term has smaller relevance than the one for the second derivative term.

The main new result of our work is the non-local form factors for the Einstein–Hilbert term, which has the form $k(\square)R$. For the non-zero mass m of the quantum field such a form factor can be expanded into power series in the ratio \square/m^2 and thus it represents a power series of total derivatives. If we forget that the total derivatives do not contribute to the equations of motion, these form factors show typical quadratic decoupling in the IR limit $q^2 \ll m^2$. The same effect can be observed from both form factors in the effective action and from the “physical” beta functions defined in the Momentum Subtraction scheme of renormalization.

The relevant question is whether there is a manner to construct a physical application for the results for the total derivative terms. In this respect we can note that the total derivative terms may be relevant in the case of manifolds with boundaries. In the theoretical cosmology there are objects of this type called domain walls, and it would be interesting to consider the implications of our results in this case. Even more simple is the situation in cosmology. One can regard the cosmological spacetime of the expanding universe as a manifold with boundary (horizon) which has a size defined by the inverse of Hubble parameter. Taking this into account, the natural interpretation is that we have, for the Einstein–Hilbert term, the decoupling in the form of identification

$$\frac{q^2}{m^2} \rightarrow \frac{H^2}{m^2}. \quad (40)$$

Indeed, the quadratic decoupling for the inverse Newton constant in the IR is not what we need for the phenomenological models of quantum corrections in cosmology [10] or astrophysics [11]. Using the approach of [10] one can easily see that in this case the energy conservation law will tell us that the cosmological constant does not show any significant running in the IR. This is the result which some of the present authors could not achieve in [12]. In our opinion, however, this conclusion can not be seen as final, since it is based on the qualitative and phenomenological identification of the scale (40). Nevertheless, one can expect that the study based on surface terms can be useful in the further exploration of this interesting subject.

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Appendix A. The non-local expansion of the heat kernel

In this Appendix we briefly present the non-local expansion of the heat kernel [3,32,33]. Consider a Laplace-type operator

$$\mathcal{D} = \Delta_g + E \quad (A.1)$$

which acts on a generic tensor bundle equipped with a connection over a Riemannian manifold which has Euclidean metric $g_{\mu\nu}$. We introduced the Laplacian Δ_g which is defined as the negative of the square of the covariant derivative $\Delta_g = -\nabla^2 = -g^{\mu\nu} \nabla_\mu \nabla_\nu$ and a local endomorphism E which acts multiplicatively.

The (local) heat kernel $\mathcal{H}(s; x, x')$ is defined as the solution of the initial value problem

$$(\partial_s + \mathcal{D}_x) \mathcal{H}(s; x, x') = 0, \quad \mathcal{H}(s; x, x') = \delta(x, x'), \quad (A.2)$$

in which $\delta(x, x')$ is the covariant Dirac delta. The heat kernel allows us to give a covariant representation to traces of functions of the Laplace-type operator \mathcal{D} , and specifically allows us to compute

the 1-loop effective action $\Gamma[g]$. The dimensionally regularized effective action is³

$$\Gamma[g] = \frac{1}{2} \text{Tr} \ln (\Delta_g + E + m^2) = -\frac{\mu^\epsilon}{2} \text{Tr} \int_0^\infty \frac{ds}{s} e^{-sm^2} \mathcal{H}(s). \quad (\text{A.3})$$

The trace of the local heat kernel admits a curvature expansion that to the second order is

$$\begin{aligned} \text{Tr} \mathcal{H}(s) = & \frac{1}{(4\pi s)^{d/2}} \int d^4x \sqrt{g} \text{tr} \left\{ \mathbf{1} + sG_E(s\Delta_g)E + sG_R(s\Delta_g)R \right. \\ & + s^2 R F_R(s\Delta_g)R + s^2 R^{\mu\nu} F_{Ric}(s\Delta_g)R_{\mu\nu} \\ & + s^2 E F_E(s\Delta_g)E + s^2 E F_{RE}(s\Delta_g)R \\ & \left. + s^2 \Omega^{\mu\nu} F_{\Omega}(s\Delta_g)\Omega_{\mu\nu} \right\} + \mathcal{O}(\mathcal{R})^3, \end{aligned} \quad (\text{A.4})$$

in which $\mathcal{O}(\mathcal{R})^3$ represents all possible non-local terms with three or more curvatures [32,33]. The functions whose argument is Δ_g are known as form factors of the heat kernel: they act on the curvatures of the expansion and should be regarded as non-local functions of the Laplacian. The form factors appearing in the linear terms have been derived in [3] as

$$G_E(x) = -f(x), \quad G_R(x) = \frac{f(x)}{4} + \frac{f(x) - 1}{2x}, \quad (\text{A.5})$$

while those appearing in the quadratic terms can be found in [3, 32,33]. All form factors depend on a single basic form factor which is defined as

$$f(x) = \int_0^1 d\alpha e^{-\alpha(1-\alpha)x}. \quad (\text{A.6})$$

All the form factors admit well-defined expansions both for large and small values of the parameter s , since s is dual to the energy of the fluctuations the non-local expansion is a suitable tool to explore the effective action from high- to low-energies

Appendix B. Comments on the UV structure of the effective action

The local and non-local contributions to the effective action (12) are not fully independent, but rather display some important relations which underline the properties described in Sect. 2. Let us concentrate here on the renormalization of a generic operator $O[g]$ on which a form factor $B_O(z)$ acts. (The explicit example that appears in the text would be to take R as the operator and $B(z)$ as the corresponding form factor.) For small mass $m^2 \sim 0$ we notice that the regularized vacuum action is always of the form

$$\begin{aligned} \Gamma[g] \supset & -\frac{b_0}{(4\pi)^2 \bar{\epsilon}} \int d^4x O[g] + \frac{1}{2(4\pi)^2} \int d^4x B_O(z) O[g] \\ & = -\frac{b_0}{2(4\pi)^2} \int d^4x \left[\frac{2}{\bar{\epsilon}} - \ln(-\nabla^2/m^2) \right] O[g] + \dots \end{aligned} \quad (\text{B.1})$$

in which the dots hide subleading contributions in the mass and b_0 is a pure number related to the renormalization of the operator. The above relation underlines an explicit connection between the

coefficient of the $1/\bar{\epsilon}$ pole and the leading ultraviolet logarithmic behavior of the form factor [34,35].

The subtraction of the pole requires the introduction of the renormalized coupling g_0

$$S_{\text{ren}}[g] \supset \int g_0 O[g], \quad (\text{B.2})$$

which in the $\overline{\text{MS}}$ scheme will have the beta function

$$\beta_{g_0}^{\overline{\text{MS}}} = \frac{b_0}{(4\pi)^2}. \quad (\text{B.3})$$

Following our discussion of Sect. 2 we find that if we subtract the divergence at the momentum scale q^2 coming from the Fourier transform of the form factor we get the non-local beta function

$$\beta_{g_0} = \frac{z}{(4\pi)^2} B'_{g_0}(z). \quad (\text{B.4})$$

Using (B.1) it is easy to see that in the ultraviolet limit $z \gg 1$

$$B(z) = b_0 \ln(z) + \dots, \quad (\text{B.5})$$

from which it is easy to see in general that the ultraviolet limit of the non-local beta function coincides with the $\overline{\text{MS}}$ result

$$\beta_{g_0} = \beta_{g_0}^{\overline{\text{MS}}} + \dots \quad \text{for } z \gg 1. \quad (\text{B.6})$$

In the above discussion we have however always implicitly assumed that the operator $O[g]$ is kept fixed upon actions of the renormalization group operator $q\partial_q = 2z\partial_z$. Suppose now that the operator $O[g]$ is actually a total derivative of the form

$$O[g] = \square O'[g] = -\Delta_g O'[g], \quad (\text{B.7})$$

in which we introduce another operator $O'[g]$, which itself needs to be renormalized with a coupling $g_{0'}$ and a local term $g_{g_{0'}} \int O'[g]$. If we now act with $q\partial_q$ and keep $O'[g]$ instead of $O[g]$ the renormalization group flow will acquire an overall scaling term due to the above relation. A solution to this problem is to manually remove such scaling and define

$$\beta_{g_{0'}} = -\frac{1}{(4\pi)^2} z \partial_z \left[\frac{B_O(z)}{z} \right]. \quad (\text{B.8})$$

This definition ensures that the $\overline{\text{MS}}$ beta function of the coupling of the total derivative $\square O'[g]$ is correctly reproduced in the ultraviolet limit of the non-local beta function. This is seen in the main text in the definition of Ψ (17).

As a final step to this analysis we point out that there are situations in which the definitions of (B.4) and (B.8) might need to coexist because both the operator O' and its total derivative require renormalization. In the main text, and in particular in the definition (18) we have adopted the strategy of subtracting the leading behaviors at large and small energies to ensure the correct scaling properties of the renormalization group running.

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³ Notice that we use the formal notation $\mathcal{H}(s) = e^{-s\mathcal{D}}$ from which it follows that the heat kernel is given by the matrix values of this operator, i.e. $\mathcal{H}(s; x, x') = \langle x | \mathcal{H} | x' \rangle$.

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