Vehicle interactions and fatigue assessment of bridges

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Summary

Fatigue damage of bridge details is often increased in consequence of vehicle interactions. The paper deals in a very general way with the problem of vehicle interactions. Interactions due to simultaneity are solved in the framework of the queueing theory, while time independent interactions are taken into account on the basis of rainflow or reservoir method concepts. Two numerical examples illustrate the practical application of the procedure.

1. Introduction

The fatigue assessment of bridges requires the knowledge, for each detail sensitive to fatigue, of the stress history $\sigma(t)$, representing the relationship between the time t and the stress σ induced by the vehicles crossing the bridge.
In ENV 1991 - 3 [1] five fatigue load models are given: models 1 and 2 are intended to be used whether the fatigue life is unlimited and constant stress amplitude fatigue limit is given, models 3 and 4 are intended to be used for fatigue life assessment with reference to S-N curves given in design Eurocodes, while model 5, using actual traffic data, is the most general one.
In general, for fatigue verifications, fatigue model 2 gives more accurate results than fatigue model 1 as well as fatigue model 4 gives more accurate results than fatigue model 3, provided that the simultaneous presence of several lorries on the bridge can be neglected. On the contrary, when the interaction of several lorries is relevant, fatigue models 2, 3 and 4, and, if the recorded data refer only to individual vehicles, fatigue model 5 as well, can be used only if supplemented by additional data. Clearly, the field of application of these fatigue models could be sensibly enlarged through the definition of general methods allowing to take into account the simultaneous presence of several vehicles on the same lane and/or on several lanes.
In the present paper the interaction between several vehicles is solved theoretically in the framework of the queueing theory, considering the bridge as a service system, with or without waiting queue, and the stochastic processes as Markov processes, so that the number of lorries crossing the bridge simultaneously can be determined.
The case of several vehicles simultaneously present on the same lane is solved first, considering the bridge as a single channel system with waiting queue, in which the waiting time, depending on the number of requests in queue, and the number of requests in queue itself are limited; subsequently, the case of vehicles simultaneously present in several lanes is solved in an

analogous way considering the bridge as a multiple channel system without waiting queue. Applying these procedures, a modified load spectrum, lonely vehicles spectrum, depending on traffic flow and on dimensions of the influence surface is obtained, whose members are single vehicles and vehicle convoys travelling alone, in such a way that the complete stress history can be considered as a random assembly of the individual stress histories induced by each member of this load spectrum. Finally, a general procedure for the evaluation of the stress spectrum is given, starting from the individual stress histories and using the reservoir or the rainflow method, taking into account the possibility that maximum and minimum stresses are induced by different individual members of the load spectrum.
Two numerical examples show the practical application of the method.

2. Simultaneous transit of several lorries

Let the load spectrum consisting in a set of q types of lorries and be N_{ij} the annual flow of the i-th vehicle of the on the j-th lane. The total flow on the j-th lane is then $N_j = \sum_{i=1}^{q} N_{ij}$.
When the characteristic length L of the influence line increases, the simultaneous presence of several lorries on the same and/or in several lanes must be taken into account. Under the hypotheses that the vehicle flow follows a Poisson distribution and that the transit time Θ on L is exponentially distributed, the stochastic processes can be represented as Markov processes [2], the bridge can be then considered as a service system and the problem of the simultaneous transit of several lorries can be solved applying of the queueing theory [3], [4].

2.1. Simultaneous transit of lorries on the same lane

In order to evaluate the probability P_n that n lorries are simultaneously travelling on L, the bridge can be considered as a single channel system with waiting queue, in which the waiting time, depending on the number of requests in queue, and the number of the request in the queue itself are limited. In fact, because there is a minimum value for the time interval T_s between two consecutive lorries, the waiting time for the i-th vehicle in queue is given by $T_i = \Theta - i \cdot T_s$ and the number of requests in queue is limited to $w = \mathrm{int}\left(\Theta \cdot T_s^{-1}\right) - 1$.
Under the assumption that each T_i is distributed with an exponential law whose parameter is $\varphi_i = T_i^{-1}$, the problem can be solved in a closed form (see [3] and [4]). The probability P_n to have n vehicles on the lane, i.e. n-1 requests in queue, is then given by

$$P_n = \left(\frac{\delta}{\alpha}\right)^n \cdot \left\{1 + \frac{\delta}{\alpha} + \sum_{i=2}^{w}\left[\delta^i \cdot \left(\alpha \cdot \prod_{s=1}^{i-1}\left(\alpha + \sum_{j=1}^{s}\varphi_j\right)\right)^{-1}\right]\right\}^{-1}$$, for n=0, 1, or by

$$P_n = \left\{\frac{\delta^n}{\alpha} \cdot \left[\prod_{s=1}^{n-1}\left(\alpha + \sum_{j=1}^{s}\varphi_j\right)\right]^{-1}\right\} \cdot \left\{1 + \frac{\delta}{\alpha} + \sum_{i=2}^{w}\left[\delta^i \cdot \left(\alpha \cdot \prod_{s=1}^{i-1}\left(\alpha + \sum_{j=1}^{s}\varphi_j\right)\right)^{-1}\right]\right\}^{-1}$$, for $2 \le n \le w$,

where δ represents the lorry flow density and $\alpha = \Theta^{-1}$. The annual number of interactions between n vehicles i_1, .., i_n on the j-th lane can be then obtained substituting these formulae in the general expression $N_{(i_1, i_2, \ldots, i_n), j} = \frac{P_n}{1-P_0} \cdot \frac{N_j}{n} \cdot \frac{\prod_{k=1}^{n} N_{i_k j}}{\sum_{q^n} \left(\prod_{s=1}^{n} N_{i_{t_s} j} \right)}$, where $\sum_{q^n}$ indicates the sum over all the possible choices with repetitions of n elements between a collection of q.

In the practice, the problem is reduced to the simultaneous presence of two lorries r and t, so that it is $P_0 = \left[1 + \frac{\delta}{\alpha} \cdot \left(1 + \frac{\delta}{\alpha + \varphi_1} \right) \right]^{-1}$, $P_2 = \frac{\delta^2}{\alpha \cdot (\alpha + \varphi_1)} \cdot \left[1 + \frac{\delta}{\alpha} \cdot \left(1 + \frac{\delta}{\alpha + \varphi_1} \right) \right]^{-1}$ and the annual number of interactions results $N_{(r,t),j} = \frac{N_{rj} \cdot N_{tj} \cdot \delta}{(\delta + \alpha + \varphi_1) \cdot \sum_{q^2} \left(\prod_{s=1}^{2} N_{i_{t_s} j} \right)} \cdot \frac{N_j}{2}$. When a single vehicle model is given this formula simplifies further in $N_{(1,1),j} = \frac{N_j \cdot \delta}{2 \cdot (\delta + \alpha + \varphi_1)}$.

2.2. Simultaneous transit of lorries on several lanes

Under the aforesaid hypotheses, the simultaneous transit of lorries on several lanes can be solved in an analogous way considering the bridge as a multiple channel system without waiting queue where new requests are refused if all channels are occupied. The probability P_k to have simultaneously vehicles on k lanes, i.e. k occupied channels, can be then obtained solving a system of the Erlang type [3], [4], [5] so that it is $P_k = \frac{\mu^k}{\alpha^k \cdot k!} \cdot \left(\sum_{i=0}^{m} \frac{\mu^i}{\alpha^i \cdot i!} \right)^{-1}$, $0 \le k \le m$, being μ the density of the total flow N^* and $\alpha = \Theta^{-1}$. Substituting in the general formula $N_{i_1 h_1, i_2 h_2, \ldots, i_k h_k} = \frac{P_k}{1-P_0} \cdot \left(\prod_{j=1}^{k} \frac{N_{i_j h_j}}{N_{h_j}} \right) \cdot \frac{N^*}{k} \cdot \frac{\prod_{j=1}^{k} N_{h_j}}{\sum_{\binom{m}{k}} \left(\prod_{s=1}^{k} N_{h_{t_s}} \right)}$, where $\sum_{\binom{m}{k}}$ is the sum over all the possible choices of k elements between a collection of m, it is possible to derive the annual number of interactions of k lorries, i_1 on the h_1-th lane,....., i_k on the h_k-th lane, $N_{i_1 h_1, i_2 h_2, \ldots, i_k h_k} = \frac{\frac{\mu^k}{\alpha^k \cdot k!}}{\sum_{j=1}^{m} \frac{\mu^j}{\alpha^j \cdot j!}} \cdot \left(\prod_{j=1}^{k} \frac{N_{i_j h_j}}{N_{h_j}} \right) \cdot \frac{N^*}{k} \cdot \frac{\prod_{j=1}^{k} N_{h_j}}{\sum_{\binom{m}{k}} \left(\prod_{s=1}^{k} N_{h_{t_s}} \right)}$. As said before, often only the case of two lorries r and t simultaneously present on h-th and j-th lanes is relevant, so that it is $P_2 = \frac{\mu^2}{2 \cdot \alpha^2} \cdot \left(\sum_{i=0}^{2} \frac{\mu^i}{\alpha^i \cdot i!} \right)^{-1}$ and $N_{rh,tj} = \frac{N_{rh} \cdot N_{tj}}{N_h \cdot N_j} \cdot \frac{\mu^2}{2 \cdot \alpha^2} \cdot \left(\sum_{i=1}^{2} \frac{\mu^i}{\alpha^i \cdot i!} \right)^{-1} \cdot \frac{N_h + N_j}{2}$, or, simply, when a single vehicle is considered, $N_{h,j} = \frac{\mu^2}{2 \cdot \alpha^2} \cdot \left(\sum_{i=1}^{2} \frac{\mu^i}{\alpha^i \cdot i!} \right)^{-1} \cdot \frac{N_h + N_j}{2}$.

2.3. Evaluation of the time independent load spectrum

In conclusion, using the procedures described in points 2.1. and 2.2., it is possible to obtain a suitably modified load spectrum, the *lonely vehicles spectrum* (l. v. s.), whose components are individual vehicles and vehicle convoys travelling alone on the bridge.
Generally, the evaluation of the l. v. s. requires the application of both procedures: the simultaneous presence of several lorries on the same lane is considered first, so that it is possible to obtain for each lane a new load spectrum, formed by individual vehicles and vehicle convoys travelling alone on the lane, to be used to solve the multilane case.

3. Time independent interactions

When the l. v. s. is known, it is possible to consider the complete stress history as a random assembly of the individual stress histories induced by each member of the l. v. s. itself. But, unfortunately, as it is well known, the stress spectrum depends on the cycle counting method and cannot be determined, in general, as a pure and simple sum of the individual stress spectra. In fact, it can happen that the maximum and minimum stresses are induced by different members of the l. v. s., so that it is necessary to consider time independent interactions too. When the reservoir or the rainflow methods are employed, the problem can be solved in the general case [3], [4]. The demonstration of the procedure is out of the scope of the present paper and it will be reported only the main results.
Using reservoir or rainflow counting methods it can be proved that two individual stress histories σ_{A_i} and σ_{A_j} interact if and only if $\max\sigma_{A_i} \leq \max\sigma_{A_j}$ and $\min\sigma_{A_i} \leq \min\sigma_{A_j}$ or $\max\sigma_{A_i} \geq \max\sigma_{A_j}$ and $\min\sigma_{A_i} \geq \min\sigma_{A_j}$. If the couples of interacting histories are sorted in such a way that the corresponding $\Delta\sigma_{max}$ are in descending order , it is possible to evaluate the number of the combined stress histories as well as the residual numbers of each individual stress history in a very simple recursive way.
In general, an individual stress history can interact with several others, so that the number of combined stress histories N_{cij}, obtained as h-th combination of the stress history σ_{A_i} and as k-th combination of the stress history σ_{A_j} is given by $N_{cij} = \frac{{}^{(h-1)}N_i \cdot {}^{(k-1)}N_j}{{}^{(h-1)}N_i + {}^{(k-1)}N_j}$, where ${}^{(h-1)}N_i$ and ${}^{(k-1)}N_j$ are the number of the individual stress histories σ_{A_i} and σ_{A_j} not yet combined and being ${}^{(0)}N_i = N_{A_i}$ and ${}^{(0)}N_j = N_{A_j}$ the number of repetitions of σ_{A_i} and σ_{A_j} induced by the l.v.s.. The actual number of the individual stress history σ_{A_i} which don't combines with other stress histories is given by ${}^{(p)}N_i = {}^{(0)}N_i - \sum_{k \neq i} (N_{ik} + N_{ki})$, being the sum extended to all the stress histories σ_{A_k} which are able to combine with σ_{A_i} itself.
In this way it is possible to derive, in conclusion, a new modified load spectrum whose

members, represented by the original individual vehicles, by the vehicle convoys determined according to point 2. and by their time independent combinations, are interaction free, so that it can be defined as interaction-free vehicle spectrum (i. v. s.).

4. Numerical examples

In order to illustrate some practical applications of the formulae derived before, two simple exercises are developed in the following. The first one concerns the evaluation of the maximum length of a single lane for which the presence of several lorries can be disregarded, while the second one shows how the λ-factors for the multilane effect can be calibrated, in view of the fatigue assessment of steel bridges, using fatigue model 3 of ENV 1991 - 3. In the exercises a slope of the S-N curve m=5 is considered, while the flow rates are evaluated assuming 280 working days per year.

4.1. Evaluation of the critical length of one single lane

Let L the span of a simple supported beam, it is required to evaluate, for the bending moment at midspan, the value of L for which the interactions on a single lane become significant. Numerical calculations are developed referring to fatigue model 3 (single vehicle model) of ENV 1991 - 3, considering four different annual flows: $N_1=2.5\times10^5$; $N_2=5.0\times10^5$; $N_3=1.0\times10^6$; $N_4=2.0\times10^6$.

Let v=13.889 m/sec the lorry speed and $T_s=1.5$ sec the intervehicle interval; the annual numbers of interacting vehicles, determined using the formulae of point 2.1., depending on the annual flow and on the span L, are summarized in table 1.

Using these results and taking into account the relative positions of the two lorries along the lane, the equivalent stress range $\Delta\sigma_{eq}$ has been found.

The ratios $\lambda^*=\Delta\sigma_{eq}/\Delta\sigma_1$, being $\Delta\sigma_1$ the equivalent stress range determined disregarding interactions, are reported in table 2 (for L=40 m and L=50 m it results $\lambda^*\approx1$).

	N_1	N_2	N_3	N_4
L=40 m	1190	4729	18566	71605
L=50 m	1690	6670	25987	98813
L=60 m	2165	8515	32940	123618
L=75 m	2858	11177	42796	157689
L=100 m	3978	15423	58110	208240

Table 1 - Number of interactions (1 lane)

	N_1	N_2	N_3	N_4
L=60 m	1.002	1.004	1.007	1.013
L=75 m	1.007	1.014	1.027	1.047
L=100 m	1.013	1.025	1.045	1.076

Table 2 - λ^* values (1 lane)

The results, which appear in good agreement with the numerical simulations, show that the critical length is generally equal to 100 m, unless for high flows, when it reduces to 75 m.

4.2. Calibration of λ-factor for multilane effect

The formulae derived in point 2.2. are used to show how to calibrate of the λ-factor for the multilane effect of fatigue load model 3 [1].

In this case as well, reference is made to the bending moment at midspan of a simple supported

beam. Varying parameters are the span L and the vehicle flow, $N_1=2.5\times10^5$; $N_2=5.0\times10^5$; $N_3=1.0\times10^6$; $N_4=2.0\times10^6$, which is considered to be the same on each lane.
The annual number of interactions, found with v=13.889 m/sec, is reported in table 3.

	N_1	N_2	N_3	N_4
L=10 m	1846	7331	28901	112358
L=20 m	3666	14450	56179	212764
L=30 m	5458	21367	81966	303028
L=50 m	8967	34626	129532	458712
L=75 m	13213	50200	182480	617280
L=100 m	17312	64766	229356	746264
L=150 m	25100	91240	308640	943390
L=200 m	32383	114678	373132	1086953

Table 3 - Interacting vehicles (2 lanes)

	N_1	N_2	N_3	N_4
L=10 m	1.156	1.162	1.174	1.197
L=20 m	1.162	1.174	1.197	1.234
L=30 m	1.168	1.186	1.217	1.264
L=50 m	1.180	1.207	1.250	1.310
L=75 m	1.194	1.230	1.283	1.351
L=100 m	1.207	1.250	1.310	1.381
L=150 m	1.230	1.283	1.351	1.423
L=200 m	1.250	1.310	1.381	1.450

Table 4 - λ-factors (2 lanes)

Taking into account the interactions as well as all the relative positions of the two lorries, the equivalent stress ranges $\Delta\sigma_{eq}$ have been determined under the assumption that the lanes have the same influence. Being $\Delta\sigma_1$ the equivalent stress range induced by one lane flow only, the required λ-factors, reported in table 4, are given by $\Delta\sigma_{eq}/\Delta\sigma_1$. The reference value for λ, which corresponds to zero interactions, is 1.149.
The results demonstrate that λ is a quasi-linear function of $\Theta \cdot N$, which can be expressed as $\lambda = 1.149 \cdot \left(1.03 + 0.01 \cdot \frac{L \cdot N}{v \cdot 10^6}\right)$, where L is in m and v in m/sec.

5. Conclusions

The interaction between the vehicles belonging to a load spectrum is solved in general way, taking into account all types of interactions, depending or not on the time. The solutions given in sequence in the paper allow to attain, through a step by step procedure, to an interaction-free vehicle spectrum (i. v. s.), formed by vehicles or vehicle convoys which cannot interact. The solutions of two simple but important problems show the practical application of the methods outlined in the paper.

References

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