

Berezinskii-Kosterlitz-Thouless transitions in two-dimensional lattice $\text{SO}(N_c)$ gauge theories with two scalar flavors

Claudio Bonati,¹ Alessio Franchi,² Andrea Pelissetto,³ and Ettore Vicari¹

¹*Dipartimento di Fisica dell'Università di Pisa and INFN, Pisa, Italy*

²*Dipartimento di Fisica dell'Università di Pisa, Pisa, Italy*

³*Dipartimento di Fisica dell'Università di Roma Sapienza and INFN, Roma, Italy*

(Dated: January 5, 2021)

We study the phase diagram and critical behavior of a two-dimensional lattice $\text{SO}(N_c)$ gauge theory ($N_c \geq 3$) with two scalar flavors, obtained by partially gauging a maximally $\text{O}(2N_c)$ symmetric scalar model. The model is invariant under local $\text{SO}(N_c)$ and global $\text{O}(2)$ transformations. We show that, for any $N_c \geq 3$, it undergoes finite-temperature Berezinskii-Kosterlitz-Thouless (BKT) transitions, associated with the global Abelian $\text{O}(2)$ symmetry. The transition separates a high-temperature disordered phase from a low-temperature spin-wave phase where correlations decay algebraically (quasi-long range order). The critical properties at the finite-temperature BKT transition and in the low-temperature spin-wave phase are determined by means of a finite-size scaling analysis of Monte Carlo data.

I. INTRODUCTION

Abelian and non-Abelian gauge symmetries appear in various physical contexts. For instance, they are relevant for the theories of fundamental interactions [1–3] and in the description of some emerging phenomena in condensed matter physics [2, 4, 5]. The main features of these theories, such as the spectrum, the phase diagram, and the critical behavior at thermal and quantum transitions, crucially depend on the interplay between global and local gauge symmetries.

These issues have been recently investigated in several two-dimensional (2D) lattice gauge models, considering: (i) the multicomponent lattice Abelian-Higgs model [6], characterized by a global $\text{SU}(N_f)$ symmetry ($N_f \geq 2$) and a local $\text{U}(1)$ gauge symmetry; (ii) the multiflavor lattice scalar quantum chromodynamics [7], characterized by a global $\text{SU}(N_f)$ symmetry and a local $\text{SU}(N_c)$ gauge symmetry; (iii) lattice $\text{SO}(N_c)$ gauge models with $N_f \geq 3$ real scalar flavors [8], characterized by a non-Abelian $\text{O}(N_f)$ global symmetry. In agreement with the Mermin-Wagner theorem [9], all these 2D lattice gauge models do not have finite-temperature transitions. A critical behavior is only observed in the zero-temperature limit: for $T \rightarrow 0$ the correlation length increases exponentially, as in the 2D $\text{O}(N)$ σ model with $N \geq 3$ and in the 2D CP^{N-1} model with $N \geq 2$ [2]. The interplay of global non-Abelian symmetries and local gauge symmetries determines the large-scale properties of the system in the zero-temperature limit, and therefore, the field theory realized in the corresponding continuum limit.

The results for the above-mentioned lattice gauge models support the following general conjecture, originally put forward in Ref. [7]. The universal features, i.e., the universality class, of the asymptotic low-temperature behavior of lattice gauge models is the same as that of the 2D field theories defined on the symmetric spaces [2, 10] that have the same global symmetry. According to this

conjecture, the zero-temperature critical behavior of multiflavor Abelian-Higgs models and lattice scalar chromodynamics with N_f scalar flavors belongs to the universality class of the 2D CP^{N_f-1} model, as both models have the same global $\text{SU}(N_f)$ symmetry. Analogously, lattice $\text{SO}(N_c)$ gauge theories with $N_f \geq 3$ real scalar flavors have the same critical behavior as RP^{N_f-1} models [11] with the same global $\text{O}(N_f)$ symmetry. These predictions have been numerically verified in Refs. [6–8]. We note that all cases considered so far involve systems with global non-Abelian symmetries, which are not expected to show finite-temperature transitions in two dimensions [9].

In this paper we investigate 2D lattice non-Abelian gauge models that undergo a finite-temperature transition, and show that also in this case the conjecture holds. We consider a 2D lattice $\text{SO}(N_c)$ gauge model with two real scalar flavors, obtained by partially gauging a maximally $\text{O}(2N_c)$ symmetric scalar theory. For $N_c \geq 3$ this model is characterized by a global Abelian $\text{O}(2)$ symmetry (for $N_c = 2$ the global symmetry group turns out to be $\text{SU}(2)$ [8], which is non-Abelian, and therefore we only expect a zero-temperature critical behavior). If the general conjecture extends to systems with global Abelian symmetries, we expect this model to have the same critical behavior as the $\text{O}(2)$ -invariant XY lattice model. Therefore, for $N_c \geq 3$, 2D lattice $\text{SO}(N_c)$ gauge models with two scalar flavors may undergo a finite-temperature Berezinskii-Kosterlitz-Thouless (BKT) transition [12–20], between the high-temperature disordered phase and a low-temperature spin-wave phase characterized by quasi-long range order (QLRO) with vanishing magnetization. We recall that BKT transitions are characterized by an exponentially divergent correlation length ξ at a finite critical temperature. Indeed, ξ behaves as $\xi \sim \exp(c/\sqrt{T - T_c})$ approaching the BKT critical temperature T_c from the high-temperature phase.

To verify the general conjecture for the lattice $\text{SO}(N_c)$ gauge theory with two scalar flavors, we present finite-size scaling (FSS) analyses of Monte Carlo simulations for several $N_c \geq 3$. We anticipate that the numerical results confirm the presence of a low-temperature QLRO phase, separated by a BKT transition from the high-temperature disordered phase. These results extend the validity of the conjecture to 2D lattice non-Abelian gauge theories with global Abelian symmetries.

The paper is organized as follows. In Sec. II we define the lattice $\text{SO}(N_c)$ gauge model with scalar fields, and the gauge-invariant observables that we consider in our numerical study. We also describe the FSS analysis we use to investigate the phase diagram and to determine the nature of the critical behavior. Sec. III reports the numerical results for $N_c = 3, 4, 5$. We show that QLRO holds in the low-temperature phase and that the transition between the high-temperature and the low-temperature QLRO phase is a BKT one, as in the standard XY model. Finally, in Sec. IV we summarize and draw our conclusions.

II. 2D LATTICE $\text{SO}(N_c)$ GAUGE MODELS

We consider a multicolor lattice $\text{SO}(N_c)$ gauge model defined on square lattices of linear size L with periodic boundary conditions. It is obtained [21] by partially gauging a maximally $O(M)$ symmetric model with $M = N_f N_c$, defined in terms of real unit-length matrix variables $\phi_{\mathbf{x}}^{af}$, with $a = 1, \dots, N_c$ and $f = 1, \dots, N_f$ (we will refer to these two indices as *color* and *flavor* indices, respectively), such that $\text{Tr} \phi_{\mathbf{x}}^t \phi_{\mathbf{x}} = 1$. Using the Wilson approach [1], we introduce gauge variables associated with each link of the lattice. The Hamiltonian reads [21]

$$H = -N_f \sum_{\mathbf{x}, \mu} \text{Tr} \phi_{\mathbf{x}}^t V_{\mathbf{x}, \mu} \phi_{\mathbf{x}+\hat{\mu}} - \frac{\gamma}{N_c} \sum_{\mathbf{x}} \text{Tr} \Pi_{\mathbf{x}}, \quad (1)$$

where $V_{\mathbf{x}, \mu} \in \text{SO}(N_c)$, $\Pi_{\mathbf{x}}$ is the plaquette operator

$$\Pi_{\mathbf{x}} = V_{\mathbf{x}, 1} V_{\mathbf{x}+\hat{1}, 2} V_{\mathbf{x}+\hat{2}, 1}^t V_{\mathbf{x}, 2}^t. \quad (2)$$

We set the lattice spacing equal to 1, so that all lengths are measured in units of the lattice spacing. The plaquette parameter γ plays the role of inverse gauge coupling. The partition function reads

$$Z = \sum_{\{\phi, V\}} e^{-\beta H}, \quad \beta \equiv 1/T. \quad (3)$$

Note that, for $\gamma \rightarrow \infty$, the link variables $V_{\mathbf{x}}$ become equal to the identity modulo gauge transformations. Thus, one recovers the $O(M)$ -symmetric nearest-neighbor M -vector model, which does not have a finite-temperature transition and becomes critical only in the zero-temperature limit [2, 22].

For $N_c \geq 3$ the global symmetry group of model (1) is $O(N_f)$. For $N_c = 2$ the global symmetry is actually

larger [21], since the model can be exactly mapped onto the two-component lattice Abelian-Higgs model, which is invariant under local $U(1)$ and global $SU(N_f)$ transformations. Therefore, for $N_f = N_c = 2$ the model has a zero-temperature critical behavior belonging to the universality class of the CP^1 field theory [6], which is equivalent to that of the nonlinear $O(3)$ σ model. In the following we consider only the case $N_c \geq 3$.

For $N_f = 2$ and $N_c \geq 3$ the theory is characterized by a global Abelian $O(2)$ symmetry. The conjecture we have discussed in the introduction suggests therefore that the two-flavor gauge model has a finite-temperature transition analogous to that occurring in 2D $O(2)$ -invariant spin models, which undergo a BKT transition from the disordered phase to a low-temperature QLRO phase [2]. As we shall see, this conjecture is supported by the numerical results.

To determine the nature of the transitions, we will perform a FSS analysis [22–25] of the numerical data. We focus on the correlations of the gauge-invariant bilinear operator

$$Q_{\mathbf{x}}^{fg} = \sum_a \phi_{\mathbf{x}}^{af} \phi_{\mathbf{x}}^{ag} - \frac{1}{2} \delta^{fg}. \quad (4)$$

Note that, for $N_f = 2$, $Q_{\mathbf{x}}$ has only two independent real components. We consider the two-point function

$$G(\mathbf{x} - \mathbf{y}) = \langle \text{Tr} Q_{\mathbf{x}} Q_{\mathbf{y}} \rangle, \quad (5)$$

where the translation invariance of the system has been taken into account. We define the susceptibility $\chi = \sum_{\mathbf{x}} G(\mathbf{x})$ and the correlation length

$$\xi^2 = \frac{1}{4 \sin^2(\pi/L)} \frac{\tilde{G}(\mathbf{0}) - \tilde{G}(\mathbf{p}_m)}{\tilde{G}(\mathbf{p}_m)}, \quad (6)$$

where $\tilde{G}(\mathbf{p}) = \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} G(\mathbf{x})$ is the Fourier transform of $G(\mathbf{x})$, and $\mathbf{p}_m = (2\pi/L, 0)$. We also consider universal RG invariant quantities, such as the Binder parameter U

$$U = \frac{\langle \mu_2^2 \rangle}{\langle \mu_2 \rangle^2}, \quad \mu_2 = \frac{1}{V^2} \sum_{\mathbf{x}, \mathbf{y}} \text{Tr} Q_{\mathbf{x}} Q_{\mathbf{y}}, \quad (7)$$

where $V = L^2$ (note that $\chi = V \langle \mu_2 \rangle$), and the ratio

$$R_{\xi} \equiv \xi/L. \quad (8)$$

In the FSS limit we have (see, e.g., Ref. [6])

$$U(\beta, L) \approx F_U(R_{\xi}), \quad (9)$$

where $F_U(x)$ is a universal scaling function that completely characterizes the universality class of the transition. In particular, universality is expected at BKT transitions and in the whole low-temperature spin-wave phase, see, e.g., Refs. [16, 17, 20, 26–28].

Because of the universality of relation (9), we use the plots of U versus R_{ξ} to identify the models that have the

same universal behavior. If the estimates of U for two different systems fall onto the same curve when plotted versus R_ξ , the transitions in the two models belong to the same universality class. Therefore, we will compare the FSS curves for the lattice $\text{SO}(N_c)$ gauge model with the analogous ones for the 2D XY model. If the data for the two models have the same scaling behavior, we will conclude that the gauge model undergoes a BKT transition as the XY model. The same strategy was employed in Refs. [6–8], to characterize the asymptotic zero-temperature behavior of 2D lattice gauge models with non-Abelian global symmetry group.

III. NUMERICAL RESULTS

A. The conjecture for systems with $\text{O}(2)$ global symmetry

We wish to verify numerically the general conjecture originally put forward in Ref. [7]. In the present case it predicts that, for any $N_c \geq 3$, the lattice model with Hamiltonian (1) with two flavors undergoes a transition analogous to that of the paradigmatic 2D $\text{O}(2)$ invariant XY model defined by the Hamiltonian

$$H_{XY} = - \sum_{\mathbf{x}, \mu} \text{Re} \psi_{\mathbf{x}}^* \psi_{\mathbf{x}+\hat{\mu}}, \quad (10)$$

where $\psi_{\mathbf{x}}$ are complex phase variables, $|\psi_{\mathbf{x}}| = 1$, associated with each site of the square lattice. This model undergoes a BKT transition at $\beta_c = 1.1199(1)$ [16, 19], with a low-temperature phase that shows QLRO with vanishing magnetization.

The correspondence can be justified using the arguments presented in Ref. [8]. If the conjecture holds, the lattice model (1) with N_f scalar flavors should be related to the 2D RP^{N_f-1} model, defined by the Hamiltonian

$$H_{\text{RP}} = -t \sum_{\mathbf{x}, \mu} (\varphi_{\mathbf{x}} \cdot \varphi_{\mathbf{x}+\hat{\mu}})^2, \quad (11)$$

where $\varphi_{\mathbf{x}}$ is a unit-length N_f -component real field. Indeed, the RP^{N-1} space is a symmetric space that has the same global $\text{O}(N_f)$ symmetry. The model has also a local \mathbb{Z}_2 symmetry, which effectively appears because the order parameter $Q_{\mathbf{x}}$ is invariant under the local \mathbb{Z}_2 transformations $\phi_{\mathbf{x}} \rightarrow s_x \phi_{\mathbf{x}}$, $s_x = \pm 1$. In the RP^{N-1} model the order parameter is

$$q_{\mathbf{x}}^{fg} = \varphi_{\mathbf{x}}^f \varphi_{\mathbf{x}}^g - \frac{1}{N_f} \delta^{fg}, \quad (12)$$

which is the counterpart of $Q_{\mathbf{x}}^{fg}$ defined in the lattice $\text{SO}(N_c)$ gauge theory. In the two-flavor case, $N_f = 2$, one can easily show that, for the computation of \mathbb{Z}_2 gauge-invariant quantities, the RP^1 model can be mapped onto the XY model. Under this mapping, the order parameter $q_{\mathbf{x}}^{fg}$ (which has only two independent real components)

is mapped onto the complex field $\psi_{\mathbf{x}}$ of the XY model. Therefore, the critical behavior of the correlation function of the operator $Q_{\mathbf{x}}$, defined in Eq. (5), is expected to correspond to that of the two-point function

$$G_{XY}(\mathbf{x}, \mathbf{y}) = \langle \psi_{\mathbf{x}}^* \psi_{\mathbf{y}} \rangle, \quad (13)$$

in the XY model. Using G_{XY} , one can then define the correlation length ξ , the Binder parameter U , and the ratio R_ξ , using again Eqs. (6), (7), and (8), respectively.

B. Monte Carlo simulations

In the following we report numerical results for the 2D lattice $\text{SO}(N_c)$ gauge theories with two scalar flavors, cf. Eq. (1). We consider square lattices of linear size L with periodic boundary conditions. To update the gauge fields we use an overrelaxation algorithm implemented *à la* Cabibbo-Marinari [29], considering the $\text{SO}(2)$ subgroups of $\text{SO}(N_c)$. We use a combination of Metropolis updates and microcanonical steps [31] in the ratio 3:7. In the Metropolis update, link variables are randomly generated, and then accepted or rejected by a Metropolis step [30], with an acceptance rate of approximately 30%. For the scalar fields a combination of Metropolis and microcanonical updates is used, with the Metropolis step tuned to have an acceptance rate of approximately 30%. Errors are estimated using a standard blocking and jackknife procedure, to take into account autocorrelations, which are expected to increase roughly as L^2 . Typical statistics of our runs, for a given value of the parameters and of the size of the lattice, are approximately 10^7 lattice sweeps (in a sweep we update once all lattice variables). For the larger lattice sizes the autocorrelation times of the observables considered were of order 10^4 sweeps at most, even at T_c , thus obtaining a sufficiently large number of independent measures.

C. The low-temperature spin-wave phase

To gain evidence of the existence of a low-temperature QLRO phase, we show that spin-wave relations hold asymptotically for sufficiently low temperatures. The spin-wave theory is expected to describe the critical behavior of the XY model along the line of fixed points that runs from $T = 0$ up to the BKT point T_c . Conformal field theory, see, e.g., Ref. [32], exactly provides the large- L limit of the two-point function in the spin-wave model. In particular, it allows us to compute the universal asymptotic relation between the ratio R_ξ and the exponent η . Results for square lattices with periodic boundary conditions are reported in Refs. [19, 27] (see, in particular, the formulas reported in App. B of Ref. [27]). The exponent η characterizes the temperature-dependent power-law decay of the two-point function in the QLRO phase

$$G(\mathbf{x}) \sim |\mathbf{x}|^{-\eta(T)}. \quad (14)$$

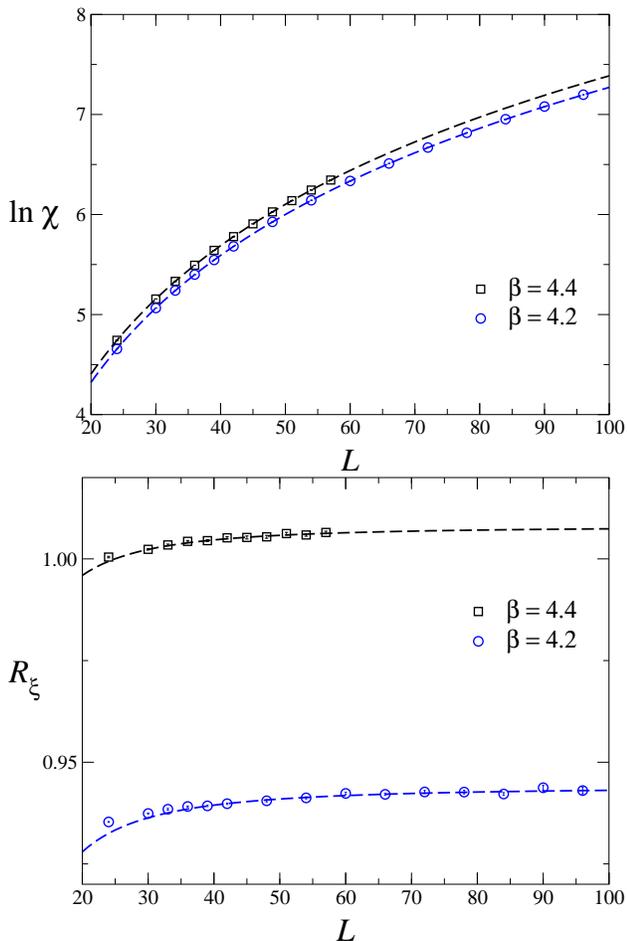


FIG. 1: Data of R_ξ (bottom) and $\ln \chi$ (top) in the low-temperature spin-wave phase of the model with $N_c = 3$ and $\gamma = 0$, at $\beta = 4.2$ and $\beta = 4.4$. The dashed lines are obtained by fitting the data (results for the smallest lattice sizes have been discarded) to the Ansätze (17) and (18).

Alternatively, we can define it by considering the large- L behavior of the susceptibility

$$\chi(L, T) \sim L^{2-\eta(T)}. \quad (15)$$

In the QLRO phase, $\eta(T)$ varies from $\eta(T_c) = 1/4$ to $\eta(T \rightarrow 0) \rightarrow 0$, and R_ξ from $R_\xi(T_c) = 0.750691\dots$ to $R_\xi(T \rightarrow 0) \rightarrow \infty$.

We recall that, at T_c , the RG theory appropriate for the BKT transition predicts the asymptotic large- L behavior [19, 20, 27]

$$R_\xi(L, T_c) = R_\xi(T_c) + \frac{C_{R_\xi}}{w(L)} + O(w^{-2}), \quad (16)$$

$$w(L) = \ln \frac{L}{\Lambda} + \frac{1}{2} \ln \ln \frac{L}{\Lambda},$$

where Λ is a model-dependent constant, and $R_\xi(T_c)$ and C_{R_ξ} are universal. Using the spin-wave theory, one obtains $R_\xi(T_c) = 0.750691\dots$ and $C_{R_\xi} = 0.212431\dots$. Analogous results can be obtained for the Binder parameter U [26], in particular $U(T_c) = 1.018192(6)$.

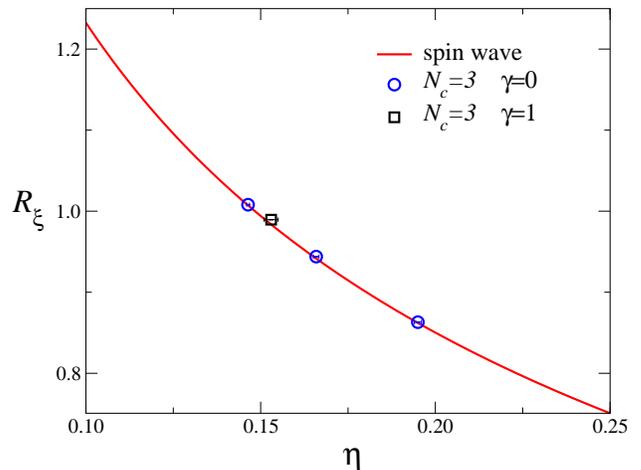


FIG. 2: Plot of the large- L extrapolations of R_ξ versus η (computed from the finite-size behavior of the susceptibility χ) for the lattice SO(3) gauge model. We report results for $\gamma = 0$ and $\beta = 4.0, 4.2, 4.4$, and for $\gamma = 1, \beta = 4.4$. We also report the universal asymptotic large- L curve (full line) computed in the spin-wave theory, for a system with square geometry and periodic boundary conditions [19, 27].

To study the low-temperature behavior, we have performed simulations for $N_c = 3$ at values of β such that $R_\xi > R_\xi(T_c)$, using periodic boundary conditions. We have determined the large- L extrapolations of R_ξ and η , by fitting the data of χ and R_ξ at fixed β to the Ansätze

$$\ln \chi(L) = a + (2 - \eta) \ln L + bL^{-\varepsilon}, \quad (17)$$

$$R_\xi(L) = R_\xi + aL^{-\varepsilon}, \quad (18)$$

respectively, where ε is the exponent associated with the expected leading corrections [27, 33]:

$$\varepsilon = \text{Min}(2 - \eta, \omega), \quad \omega = 1/\eta - 4 + O[(1/\eta - 4)^2]. \quad (19)$$

For $N_c = 3$ and $\gamma = 0$ the quality of the fits can be assessed from the results shown in Fig. 1. Fits to Eqs. (17) and (18) are very good, as it also supported by the values of χ^2/dof (χ^2 is here the sum of the fit residuals and dof is the number of degrees of freedom of the fit), which are smaller than 1, if a few results for the smallest lattice sizes are discarded. For $N_c = 3$ and $\gamma = 0$ we obtain the large- L extrapolations $\eta = 0.195(1), 0.1659(8), 0.1464(4)$, and $R_\xi = 0.8630(5), 0.9439(2), 1.0080(2)$, for $\beta = 4.0, 4.2, 4.4$, respectively. We have also performed a detailed study for $\gamma = 1$ and $\beta = 4.4$. We obtain $\eta = 0.153(2)$ and $R_\xi = 0.9895(3)$. Note that ω , see Eq. (19), is known precisely only for η close to $1/4$. In the fits we use ω as obtained from Eq. (19), and therefore ω gives the leading correction-to-scaling exponent for $\eta \gtrsim 0.17$. In such cases, to estimate the error due to the uncertainty on ω , we checked the variation of the results of the fits when varying ω in around the approximation obtained from Eq. (19), within a reasonable interval of about 10%.

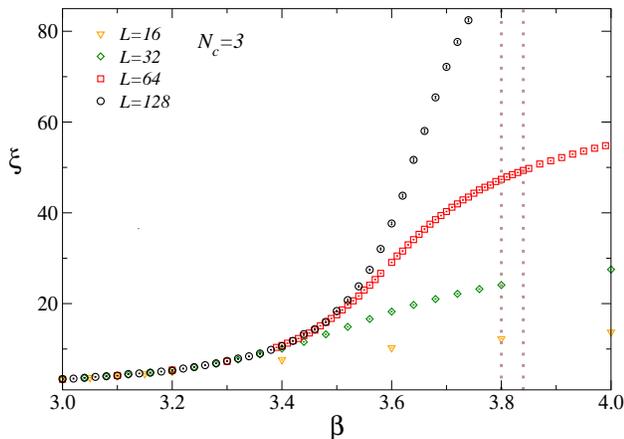


FIG. 3: Estimates of the correlation length ξ versus β for the lattice $SO(3)$ gauge model (1) with $\gamma = 0$, for several values of L , up to $L = 128$. When the results for different values of L agree, they can be considered as good approximations of the infinite-volume correlation length, within errors. The vertical lines indicate the interval of values of β in which the BKT transition occurs.

This has allowed us to estimate how η and R_ξ vary with changes of ω . Such variation has been included in the final error.

In Fig. 2 we plot R_ξ versus η together with the universal curve computed in the spin-wave theory. The results for R_ξ and η are in excellent agreement with the spin-wave predictions. This shows the existence of a low-temperature phase with QLRO, analogous to that occurring in the XY model.

D. FSS at the BKT transition

In Sec. III C we showed that the $SO(3)$ gauge model has a low-temperature phase with the same features of the low-temperature phase of the XY model. Now, we focus on the finite-temperature transition that ends the high-temperature phase, to check whether the FSS behavior is the same as that observed at the BKT transition of the XY model.

To begin with, in Fig. 3 we show the estimates of the correlation length ξ for $N_c = 3$ and $\gamma = 0$. They show a sudden increase around $\beta \gtrsim 3.5$, as expected in the presence of a finite-temperature BKT transition. To characterize the nature of the transition, we plot the Binder parameter U versus the ratio R_ξ . In the FSS limit data should belong to a curve that only depends on the universality class. In Fig. 4 we report our numerical results for $N_c = 3$ and for three values of γ , which are $\gamma = 0, \pm 1$. In all cases, data appear to approach a universal FSS curve with corrections that decrease quite rapidly with the size. We also report data for the 2D XY model, that have been obtained by standard Monte Carlo simulations for lattice sizes $L = 100, 200$. They are apparently suffi-

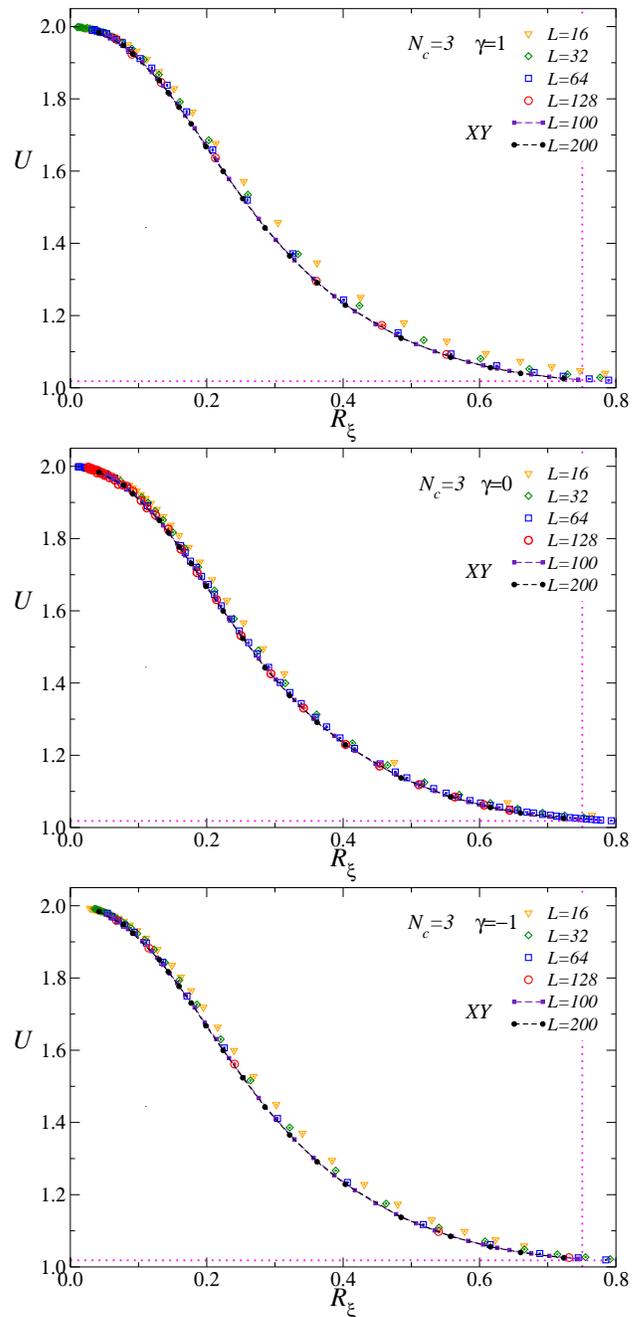


FIG. 4: We plot data of U versus R_ξ for $N_c = 3$, $\gamma = 1$ (top), $\gamma = 0$ (middle), and $\gamma = -1$ (bottom). We report analogous data for the 2D XY model (10). We observe a nice agreement, supporting the conjecture that the lattice $SO(N_c)$ gauge model with two scalar flavors undergoes a finite-temperature BKT transition for generic values of γ . The horizontal and vertical lines indicate the universal values of U and R_ξ at the BKT transition, i.e. $U(T_c) = 1.018192(6)$ and $R_\xi(T_c) = 0.750691\dots$, respectively [19, 26].

cient to provide a good approximation of the asymptotic FSS behavior (the differences between the $L = 100$ and $L = 200$ scaling curves are very small and hardly visible in Fig. 4). It is quite clear that the data for the gauge model fall on top of the XY scaling curve, confirming that the transition has the same universal features: the gauge $SO(3)$ model undergoes a BKT transition as the XY model. Analogous results are obtained for $N_c = 4$ and $N_c = 5$, as shown in Fig. 5, where we report data for $\gamma = 0$. In both cases, the data for the gauge model converge toward the FSS curve of the XY model.

We note that the approach to the asymptotic FSS behavior (9) is apparently quite fast in all lattice models considered, including the 2D XY model. In particular, the scaling corrections for the lattice $SO(N_c)$ gauge models appear to effectively decrease roughly as L^{-1} in the limited range of L that we consider, up to $L = 128$. At BKT transitions, logarithmic corrections are generally expected [16, 17, 20, 26, 27]. However, our range of values of L is too small to allow us to detect logarithmic changes of the estimates. In the range we consider power-law corrections effectively dominate. Significantly larger sizes are needed to allow us to perform fits that include both logarithmic and power-law corrections. Even though our analyses are not sensitive to the slowly-decaying logarithmic corrections, we can argue that the systematic error they induce is small (we only refer here to the behavior of U versus R_ξ ; we are not claiming that logarithmic corrections are always negligible). Indeed, the coefficients of the logarithmic corrections are not universal, and therefore we expect different logarithmic corrections in the XY model and in the gauge models we consider here. Thus, assuming that all models have a common universal asymptotic behavior, we can infer the size of the logarithmic correction by looking at the differences between the results obtained in the different models. As apparent from Figs. 4 and 5, differences are tiny, indicating that these elusive corrections play little role here.

Accurate estimates of the critical BKT temperatures are hard to obtain, since their determination is generally affected by logarithmic corrections, see Eq. (16). The problem of the logarithmic corrections can be overcome by the so-called matching method put forward in Refs. [16, 17, 19] (see also Refs. [27, 28] for applications to some 2D quantum lattice gas models). Here, we do not pursue this analysis further, since we are not particularly interested in obtaining precise estimates of the critical temperatures. We only mention some rough estimates of the transition temperatures obtained by looking at the β -values where $R_\xi(\beta, L) \approx R_\xi(T_c) = 0.750691\dots$. For $N_c = 3$ we find $\beta_c \approx 3.82$ for $\gamma = 0$, $\beta_c \approx 3.77$ for $\gamma = 1$, and $\beta_c \approx 3.92$ for $\gamma = -1$. Moreover, we estimate $\beta_c \approx 4.80$ for $N_c = 4$ and $\beta_c \approx 5.76$ for $N_c = 5$, at $\gamma = 0$.

In conclusion, the FSS analysis has allowed us to determine the nature of the finite-temperature transitions occurring in the lattice $SO(N_c)$ gauge model (1) with two flavors. For $N_c = 3, 4, 5$ we find that the transition be-

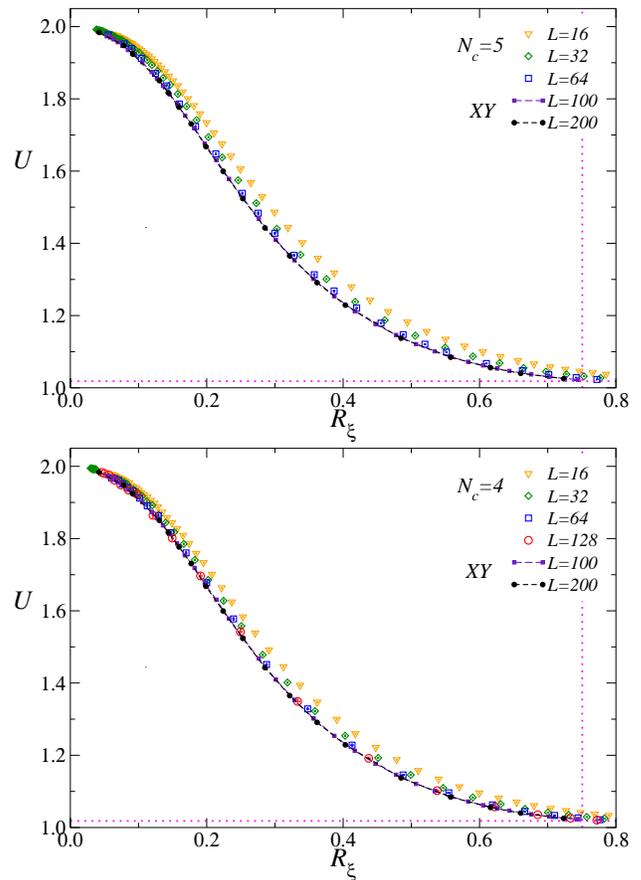


FIG. 5: Plot of U versus R_ξ for $N_c = 4$ (bottom) and $N_c = 5$ (top), at $\gamma = 0$. We also report data for the 2D XY model (10). The FSS curve of the XY model is clearly approached by the data for the lattice $SO(N_c)$ models with increasing L . The horizontal and vertical lines indicate the BKT values $U(T_c) = 1.018192(6)$ and $R_\xi(T_c) = 0.750691\dots$, respectively [19, 26].

longs to the BKT universality class, as in the classical XY model. This occurs at least in an interval of values of γ around the infinite gauge-coupling value $\gamma = 0$.

IV. CONCLUSIONS

We have studied a class of 2D lattice non-Abelian $SO(N_c)$ gauge models with two real scalar fields, defined by the Hamiltonian (1). Such lattice gauge models are obtained by partially gauging a maximally $O(2N_c)$ -symmetric multicomponent real scalar model, using the Wilson lattice approach. For $N_c \geq 3$, the resulting theory is locally invariant under $SO(N_c)$ gauge transformations and globally invariant under Abelian $O(2)$ transformations. This study extends previous work on 2D models with a local gauge invariance and a global non-Abelian symmetry, [6–8], in which a critical behavior can only be observed in the zero-temperature limit. In the models considered here, instead, the global Abelian $O(2)$

symmetry may allow finite-temperature BKT transitions between the disordered phase and the low-temperature QLRO phase.

The universal features of the transitions have been determined by performing FSS analyses of Monte Carlo data. We present results for the two-flavor lattice $SO(N_c)$ gauge models (1) with $N_c = 3, 4, 5$. They show that these systems undergo a finite-temperature BKT transition that separates the disordered phase from the low-temperature phase. Moreover, we have verified that the low-temperature phase is characterized by spin waves, analogously to the standard XY model.

These results provide additional evidence in favor of the conjecture that the critical behavior of 2D lattice gauge models, defined using the Wilson approach [1], belongs to the universality class of the field theories associated with the symmetric spaces that have the same

global symmetry. This conjecture assumes that gauge correlations are not critical and decouple in the critical limit. Therefore, the conjecture may fail when the gauge correlations are critical, giving rise to a more complex behavior. A similar phenomenon has been observed in the three-dimensional lattice Abelian-Higgs model with non-compact gauge fields, see, e.g., Ref. [34] and references therein.

We finally mention that the interplay between global and gauge symmetries has also been studied in three dimensional models, see Refs. [21, 34–36].

Acknowledgement. Numerical simulations have been performed on the CSN4 cluster of the Scientific Computing Center at INFN-PISA.

-
- [1] K. G. Wilson, Confinement of quarks, *Phys. Rev. D* **10**, 2445 (1974).
- [2] J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, fourth edition (Clarendon Press, Oxford, 2002).
- [3] S. Weinberg, *The Quantum Theory of Fields*, (Cambridge University Press, 2005).
- [4] S. Sachdev, Topological order, emergent gauge fields, and Fermi surface reconstruction, *Rep. Prog. Phys.* **82**, 014001 (2019).
- [5] P. W. Anderson, *Basic Notions of Condensed Matter Physics*, (The Benjamin/Cummings Publishing Company, Menlo Park, California, 1984).
- [6] C. Bonati, A. Pelissetto and E. Vicari, Two-dimensional multicomponent Abelian-Higgs lattice models, *Phys. Rev. D* **101**, 034511 (2020).
- [7] C. Bonati, A. Pelissetto, and E. Vicari, Universal low-temperature behavior of two-dimensional lattice scalar chromodynamics, *Phys. Rev. D* **101**, 054503 (2020).
- [8] C. Bonati, A. Franchi, A. Pelissetto, and E. Vicari, Asymptotic low-temperature critical behavior of two-dimensional multiflavor lattice $SO(N_c)$ gauge theories, *Phys. Rev. D* **102**, 034512 (2020).
- [9] N. D. Mermin and H. Wagner, Absence of ferromagnetism or antiferromagnetism in one- or two-dimensional isotropic Heisenberg models, *Phys. Rev. Lett.* **17**, 1133 (1966).
- [10] E. Brézin, S. Hikami, and J. Zinn-Justin, Generalized non-linear σ -models with gauge invariance, *Nucl. Phys. B* **165**, 528 (1980).
- [11] C. Bonati, A. Franchi, A. Pelissetto, and E. Vicari, Asymptotic low-temperature behavior of two-dimensional RP^{N-1} models, *Phys. Rev. D* **102**, 034513 (2020).
- [12] J. M. Kosterlitz and D. J. Thouless, Ordering, metastability and phase transitions in two-dimensional systems, *J. Phys. C: Solid State* **6**, 1181 (1973).
- [13] V. L. Berezinskii, Destruction of Long-range Order in One-dimensional and Two-dimensional Systems having a Continuous Symmetry Group I. Classical Systems, *Zh. Eksp. Theor. Fiz.* **59**, 907 (1970) [*Sov. Phys. JETP* **32**, 493 (1971)].
- [14] J. M. Kosterlitz, The critical properties of the two-dimensional xy model, *J. Phys. C* **7**, 1046 (1974).
- [15] J. V. José, L. P. Kadanoff, S. Kirkpatrick, and D. R. Nelson, Renormalization, vortices, and symmetry-breaking perturbations in the two-dimensional planar model, *Phys. Rev. B* **16**, 1217 (1977).
- [16] M. Hasenbusch, M. Marcu, and K. Pinn, High precision renormalization group study of the roughening transition, *Physica A* **208**, 124 (1994).
- [17] M. Hasenbusch and K. Pinn, Computing the roughening transition of Ising and solid-on-solid models by BCSOS model matching, *J. Phys. A* **30**, 63 (1997).
- [18] J. Balog, Kosterlitz-Thouless theory and lattice artifacts, *J. Phys. A* **34**, 5237 (2001).
- [19] M. Hasenbusch, The two dimensional XY model at the transition temperature: a high precision numerical study, *J. Phys. A* **38**, 5869 (2005).
- [20] A. Pelissetto and E. Vicari, Renormalization-group flow and asymptotic behaviors at the Berezinskii-Kosterlitz-Thouless transitions, *Phys. Rev. E* **87**, 032105 (2013).
- [21] C. Bonati, A. Pelissetto, and E. Vicari, Three-dimensional phase transitions in multiflavor scalar $SO(N_c)$ gauge theories, *Phys. Rev. E* **101**, 062105 (2020).
- [22] A. Pelissetto and E. Vicari, Critical phenomena and renormalization group theory, *Phys. Rep.* **368**, 549 (2002).
- [23] M. E. Fisher and M. N. Barber, Scaling theory for finite-size effects in the critical region, *Phys. Rev. Lett.* **28**, 1516 (1972).
- [24] M. N. Barber, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J. L. Lebowitz (Academic Press, New York, 1983), Vol. 8.
- [25] V. Privman ed., *Finite Size Scaling and Numerical Simulation of Statistical Systems* (World Scientific, Singapore, 1990).
- [26] M. Hasenbusch, The Binder cumulant at the Kosterlitz-Thouless transition, *J. Stat. Mech.: Theory Expt.* P08003 (2008).
- [27] G. Ceccarelli, J. Nespolo, A. Pelissetto, and E. Vicari, Universal behavior of two-dimensional bosonic gases at Berezinskii-Kosterlitz-Thouless transitions, *Phys. Rev. B*

- 88**, 024517 (2013).
- [28] F. Delfino and E. Vicari, Dimensional crossover of Bose-Einstein condensation phenomena in quantum gases confined within slab geometries, *Phys. Rev. A* **96**, 043623 (2017).
- [29] N. Cabibbo and E. Marinari, A New Method for Updating SU(N) Matrices in Computer Simulations of Gauge Theories, *Phys. Lett.* **119B**, 387 (1982).
- [30] N. Metropolis, A. W. Rosenbluth, M. N. Rosenbluth, A. H. Teller, and E. Teller, Equation of state calculations by fast computing machines, *J. Chem. Phys.* **21**, 1087 (1953).
- [31] M. Creutz, Overrelaxation and Monte Carlo Simulation, *Phys. Rev. D* **36**, 515 (1987).
- [32] P. Di Francesco, P. Mathieu, and D. Senechal, *Conformal Field Theory* (Springer Verlag, New York, 1997).
- [33] M. Hasenbusch, A. Pelissetto, and E. Vicari, Multicritical behaviour in the fully frustrated XY model and related systems *J. Stat. Mech.: Theory Expt.* P12002 (2005).
- [34] C. Bonati, A. Pelissetto, and E. Vicari, Lattice Abelian-Higgs models with noncompact gauge field, arXiv:2010.06311.
- [35] C. Bonati, A. Pelissetto and E. Vicari, Phase diagram, symmetry breaking, and critical behavior of three-dimensional lattice multiflavor scalar chromodynamics, *Phys. Rev. Lett.* **123**, 232002 (2019); Three-dimensional lattice multiflavor scalar chromodynamics: interplay between global and gauge symmetries, *Phys. Rev. D* **101**, 034505 (2020).
- [36] A. Pelissetto and E. Vicari, Multicomponent compact Abelian-Higgs lattice models, *Phys. Rev. E* **100**, 042134 (2019).