

“Green” managerial delegation theory

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Abstract This article develops a non-cooperative game with managerial quantity-setting firms in which owners choose whether to delegate output and abatement decisions to managers through a contract based on emissions (conventionally denoted as “green” delegation, GD) instead of sales (sales delegation, SD), and the government levies an emissions tax to incentivise firms’ emission-reduction actions. First, it compares the Nash equilibrium outcomes between GD and SD and then contrasts them also with profit maximisation (PM). A plethora of Nash equilibria emerges, especially in the case GD versus PM (the “green delegation game”), depending on the public awareness toward environmental quality, ranging from the coordination game to the “green” prisoner’s dilemma. Second, though the contract under GD incentivises managers for emissions the environmental damage is lower than under SD. This is because the optimal tax more than compensates the incentive for emissions. These findings suggest that designing GD contracts paradoxically favours environmental quality.

Keywords “Green” managerial delegation; Abatement; Emissions tax; Cournot duopoly

JEL Classification H23, L1, M5, Q58

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1. Introduction

According to a report issued by KPMG (2017), the majority (67%) of firms belonging to the G250 Fortune Index (a list of the largest 250 multinationals) has revealed targets to cut carbon emissions.¹ These companies are generally characterised by the separation between ownership and control, which is delegated to managers. However, on the one hand, only 25% of them link their own targets to climate goals that have been fixed by national governments, regional authorities, or the United Nations, such as The Paris Agreement. On the other hand, family firms constitute another pillar of the production sector in several European, Asian, and Latin American countries (see García-Ramos and García-Olalla, 2011; Chrisman *et al.*, 2014; Singal and Gerde, 2015). In this case, the family group crucially affects the ownership, governance, management, and objectives and strategies of the firm. Regardless of their nature of public listed/private held companies, empirical works have highlighted that family firms widely report environmental disclosure and might focus more on environmental performance than non-family business (Berrone *et al.*, 2010; Dekker and Hasso, 2016; Arena and Michelon, 2018; Zhu and Lu, 2020). This suggests the opportunity for intervention to readjust the firms’ targets with those of environmentally responsive social planners.

At this point, some question arises: in strategic settings such as oligopolistic (duopolistic) markets, is it advantageous for owners to include an incentive based on emissions (instead of

¹ On June 30, 2020, Brett Redman (chief executive of AGL Energy Ltd., Australia’s largest carbon emitter) declared that, parallel to traditional shareholder return measures, the company will partially link executive long-term bonuses to emissions cuts (Reuters.com, 2020). Likewise, on September 10, 2020, Mike Henry (chief executive of the BHP Group, the world’s biggest listed miner) announced that the company will focus on cutting its operational emissions by 30% by the 2030 financial year from 2020 levels and will link executive bonuses to its progress (Reuters.com, 2020). For the sake of precision, AGL Energy Ltd. is currently included in the G2000 Fortune Index, while BHP was included in the G250 Fortune Index from 2005 to 2016, while it currently ranks #261 in the G500 Fortune Index.

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sales) within a managerial scheme when the existing dirty technology cause pollution externalities? What are the effects of this contract on environmental quality? What kind of delegation contract endogenously emerge in a game-theoretic context?

This contribution focuses precisely on these points. To address these questions, the article concentrates on the managerial delegation theory framed in strategic competitive markets and introduces a contract based on a combination of profits and emissions instead of profits and sales, as instead was done by the pioneering works of Vickers (1985), Sklivas (1987), and Fershtman and Judd (1987). Though this contract incentivises managers for emissions, we define it in a conventional way as “green” delegation (GD).²

Indeed, though the GD contract proposed in this article may apparently work in an anti-ecological direction, incentivising the manager to increase employment and production (beyond the corresponding values under SD) and reduce abatement (below the corresponding values under SD) at the pre-tax stage, it allows obtaining eco-friendly results at equilibrium (reducing total emissions and environmental damage). This is because by producing more and abating less than the SD manager, the optimal emission tax levied by the government under GD to incentivise firms’ emissions-reduction actions is larger than the optimal emission tax under SD, in turn allowing at the post-tax stage the GD manager to reduce employment, production, aggregate emissions and environmental damage below the corresponding values under sales delegation, eventually acting as the main driving force of the results. This is the essence for defining our contract based on emissions as “green” delegation. However, this transfer of resources from firms to government reduces social welfare compared to the SD scenario, thereby revealing a clear trade-off between employment (production) and welfare.

² This is because, it counter-intuitively allows for the reduction of aggregate emissions and environmental damages below the levels obtained under the standard sales delegation contract (SD) under abatement.

The main features of the GD contract are as follows: the ex-ante (pre-tax stage) amount of output production is larger and pollution abatement lower than the corresponding ex-ante (pre-tax stage) values under the SD delegation with abatement. However, after the government intervention through the setting of an optimal emissions tax, the GD contract allows the ex-post (post-tax stage) amount of output production being lower and pollution abatement larger than under the SD delegation with abatement. This eventually leads the GD scenario to reduce (at the equilibrium) aggregate emissions and environmental damage below the corresponding values under the SD scenario.

The worldwide apprehension in the public opinion has grown due to the climate scientists’ demand for the immediate need to reduce greenhouse gases emissions because of their future potential impacts on local and global political institutions, and economic systems (see “The climate issue”, *The Economist*, 2019, and the Special report on business and climate change “The great disrupter”, *The Economist*, 2020). Therefore, this investigation appears to be well timed, and the subject is surprisingly absent in the industrial economics literature framed in strategic competitive contexts, apart from Poyago-Theotoky and Yong (2019), who the present work relates. In this regard, this contribution considers a Cournot duopoly in which a dirty technology generates pollution externalities and studies the owners’ decision regarding whether to delegate the choice of pollution abatement to managers when the government levies an emissions tax to incentivise firms undertaking emissions-reduction actions.

To conduct the analysis, we make use of a non-cooperative three-stage (logical timing) game augmented by a pre-play stage. At stage zero (*pre-play stage or contract stage*), the owner of each firm compares the nature of the contracts by choosing whether to become a delegated (“green” or sales) firm or a profit-maximiser (PM) under abatement. At stage one (*the government stage*), the government chooses the optimal emission tax to maximise social welfare. At stage two (*the bonus stage*), owners choose the executive remuneration. At stage

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three (*the market stage*), either owners simultaneously choose the optimal levels of output and abatement or delegate them to managers designing a standard sales delegation contract or a delegation contract based on emissions.

In this context, the article studies and compares three different games always considering a firms’ abatement activity and a government’s activity through emissions taxes to incentivise firms’ emission-reduction actions: 1) GD versus PM; 2) SD versus PM; 3) GD versus SD. If products are homogeneous, the GD contract emerges as the unique pure-strategy Pareto inefficient Nash equilibrium (“green” prisoner’s dilemma) of the game GD versus PM (the “green” delegation game) if the public awareness towards environmental quality is sufficiently high (a self-interest/mutual benefit conflict exists to become a “green”-delegated firm). Otherwise, if the public awareness toward environmental quality is sufficiently low, owners are engaged in a coordination game with two symmetric equilibria: firms are either “green” delegated (GD) or profit maximisers (PM), but PM payoff dominates GD (the “green” coordination game). If product are heterogeneous (horizontal differentiation, Appendix C, available online), a richer range of Nash equilibria emerges in the “green” delegation game depending on the degree of public awareness toward the environmental quality (or, alternatively, against the damage caused by industrial production) coming from the environmental damage function.

Additionally, the work shows that the SD strategy under abatement emerges as the unique pure-strategy Nash equilibrium of the game SD versus PM if products are homogenous (confirming the results of the managerial delegation literature, that is Vickers, 1985; Sklivas, 1987; Fershtman and Judd, 1987), whereas showing the emergence of alternative Nash equilibrium outcomes if products are differentiated (in line with Fanti *et al.*, 2017a, 2017b).

Finally, the comparison of the “green” contract based on emissions and the sales contract under abatement reveals that the latter always emerges the unique pure-strategy Nash

equilibrium of the game that can be Pareto inefficient or Pareto efficient depending on the degree of product differentiation.

To sum up, in the “green” delegation game a self-interest/mutual benefit conflict exists to become a “green”-delegated firm or, alternatively, a “green” coordination game can emerge depending on the public awareness toward environmental quality. However, the environmental damage is the same under GD and PM because the difference between the optimal emission tax rate under GD and the optimal emission tax rate under PM is exactly equal to the incentive designed by the owner about the emissions chosen by the manager who is remunerated through an incentive based on emissions. In contrast, in the games in which owners choose whether to delegate under SD versus GD or PM, the sales delegation contract always emerges as the unique pure-strategy Pareto-inefficient Nash equilibrium. However, both the environmental damage and social welfare under SD are larger than the corresponding values under GD or PM. Therefore, comparing SD and GD reveals a clear trade-off between employment (production) and social welfare. These findings suggest that designing a managerial contract based on emissions instead of sales paradoxically favours environmental quality.

The remainder of the paper is organised as follows. Section 2 describes the main ingredients of the Cournot duopoly with homogenous products. Section 3 introduces the “green” delegation game comparing the owners’ endogenous choice to become “green”-delegated (GD) or profit maximisers (PM). Section 4 considers the sales delegation (SD) game by comparing SD and PM under pollution abatement. Section 5 compares GD and SD. Section 6 concludes the paper with an outline of the future research agenda on this issue. The Supplementary Material online includes (i) the literature review, (ii) the welfare analysis (iii) analytical details (Appendices A and B), and (iv) an extension of the basic model with horizontal product differentiation (Appendix C).

2. The model with homogeneous products

Consider a duopoly industry in which firm 1 and firm 2 compete in a Cournot rivalry setting by producing quantities q_1 and q_2 , respectively, whose varieties are perceived as homogenous compared to that of the rival. The existing technology allows firm i ($i = 1,2$) to produce q_i units of output of variety i , causing e_i units of emissions (pollution), where $e_i = q_i - k_i$ (Ulph, 1996), with $0 \leq k_i < q_i$ representing the abatement level for environmental protection coming from a cleaning technology available to each firm, implying that emissions cannot be entirely eliminated.³ The available technology is such that firm i ($i = 1,2$) produces with constant returns to scale production function, that is $q_i = L_i$, where L_i represents the labour force employed by the firm and whose identical (constant) average and marginal cost is $w \geq 0$ (representing the wage per unit of labour) for every unit of output produced. Therefore, firm i 's cost function is $C_i(q_i) = wL_i = wq_i$. In addition, the pollution abatement cost function of firm i is $CA_i(k_i) = \frac{z}{2}k_i^2$, where $z > 0$ is a parameter that scales up/down the total abatement cost and represents an exogenous index of technological progress while measuring, for example, the appearance of a new, cost-effective cleaning technology. Indeed, it measures the degree at which the available technology for pollution abatement impacts the environment. A reduction in z can be interpreted as an improvement in the technological progress in abatement so that abating becomes cheaper. The adoption of a clean technology requires sustaining costs with decreasing returns on investment. This implies that, when firms choose to abate emissions, they always face some costs. One might consider a cleaning technology not directly linked to output,

³ See Appendix A (online) for analytical details also supporting the assumption of an abatement technology facilitating the elimination of pollution entirely, e.g., $0 \leq k_i \leq q_i$, confirming the main results of the article. Note also that computing emission as the difference between industrial production and abatement is in line with Ulph (1996) and differs from Asproudis and Gil-Moltó (2015), who assumed that emissions were represented by $e_i = q_i k_i$, so that $k_i \in [0,1)$ is a fraction of total production in that case.

for example, “the number of the filters in a refinery’s pipe for CO₂ reduction, or ‘scrubbers’ to remove SO₂ from a fuel gas coal fired electric plant” (Asproudis and Gil-Moltó, 2015, p. 169). As k_i represents the pollutant abated per q_i units of output, a larger (resp. smaller) value of k_i corresponds *ceteris paribus* to a more (resp. less) efficient abatement. To preserve analytical tractability and without loss of generality, we set $z = 1$ henceforth.⁴ Industrial production causes environmental damage measured by the index $ED = \frac{g}{2}(e_1 + e_2)^2$, where $g > 0$ is the burden the government attaches to the environmental damage, representing the awareness of the overall society towards a clean environment, and thus against the severe or mild environmental damages generated by industrial production, and the sum $e_1 + e_2$ represents aggregate emissions. An increase in g implies *ceteris paribus* an increase in the extent of the relative weight of the environmental damage as measured by the government, implying that the overall society is more aware of environmental protection. This type of damage function is commonly used in the related literature and assumes that (i) the environmental damage is a convex function of total pollution, and (ii) the damage is exogenous for consumers (see, for example, van der Ploeg and de Zeeuw, 1992; Ulph, 1996). The objective function of the profit-maximising firm i is given by π_i (profits). Differently, if the owner of firm i delegates production and abatement decisions to a manager, the objective function of manager i is given by U_i (manager’s utility). Both the profit function and manager’s utility function will be specified later in this article.

We assume a linear (inverse) market demand given by $p = \alpha - \beta Q$, where α is a positive parameter representing the market size ($\alpha > w \geq 0$), $\beta > 0$ measures the slope of the market

⁴ The second-order conditions of the maximisation problem are satisfied for any $z > 0$.

demand being part of its elasticity, and $Q = q_1 + q_2$ is total supply.⁵ This kind of demand structure comes from the usual specification of quadratic utility for consumers’ preferences, that is $V(q_1, q_2) = \alpha(q_1 + q_2) - \frac{\beta}{2}(q_1 + q_2)^2$, as proposed by Dixit (1979) and subsequently used, amongst others, by Singh and Vives (1984). To maintain analytical tractability (and without loss of generality), we assume $\alpha = 1$, $\beta = 1$, and $w = 0$ henceforth.

The government levies an emissions tax $t \in (0,1]$ per each unit of polluting output to incentivise firms undertaking emissions-reduction actions with the aim of maximising social welfare.⁶ Consequently, the tax base of firm i is $q_i - k_i$ (i.e., the remaining pollution after abatement), and the corresponding tax revenue is $t(q_i - k_i)$.

Definitively, our non-cooperative game has a three-stage (logical timing) structure augmented by a pre-play stage (see the introduction for details), as depicted in Figure 1.

⁵ These inequalities should hold, as the (highest) price consumers are willing to pay for the first unit of goods must be larger than the (lowest) marginal cost firms incur to produce the first unit of goods.

⁶ As we assumed a normalised market demand, $t \in (0,1]$. In the non-normalised case $t \in (0, \alpha]$.

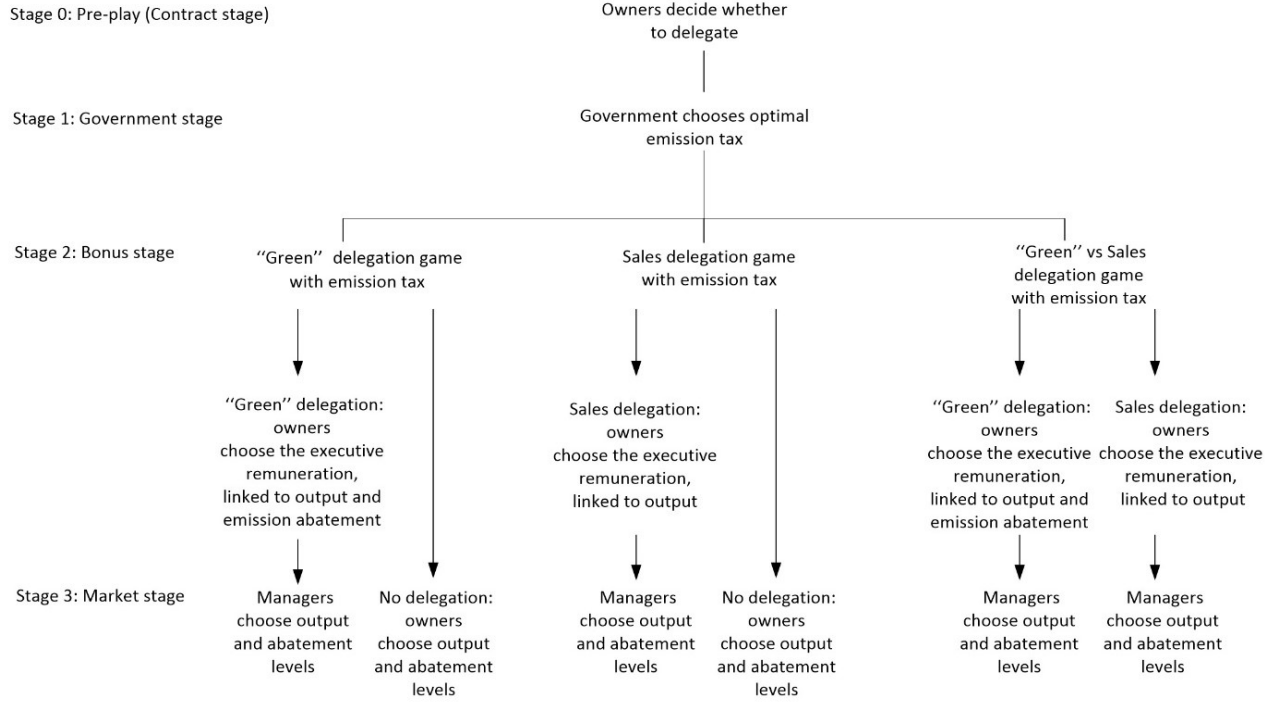


Figure 1. Logical timing of the game.

3. “Green” delegation (GD) versus profit maximisation (PM) with homogeneous products

3.1. The symmetric subgame PM/PM

Consider first the subgame in which the decision about industrial production and abatement is taken by the owner of each firm (i.e., firms are profit maximisers) at the market stage of the game, i.e., the PM scheme works as if the bonus set by owner to the manager were equal to zero. The profit function of firm i is as follows:

$$\pi_i = (1 - q_i - q_j)q_i - t(q_i - k_i) - \frac{k_i^2}{2}, i = 1, 2, i \neq j. \quad (1)$$

At the market stage of the game, owners choose both the optimal amount of output coming from industrial production and abatement to maximise profits. Maximisation of (1) with respect to q_i and k_i yields the following set of first-order conditions:

$$\bar{q}_i^{PM/PM} = \frac{1}{2}(1 - q_j - t), \bar{k}_i^{PM/PM} = k_i^{PM/PM} = t, i = 1, 2, i \neq j, \quad (2)$$

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where the superscript PM/PM stands for profit-maximisation of both firms. The solution of the system of output reaction functions $\bar{q}_i^{PM/PM}$ ($i = 1, 2, i \neq j$) in (2) leads firm i to produce the following pre-tax stage equilibrium quantity of product of variety i as a function of the emission tax t , that is:

$$q_i^{PM/PM} = \frac{1}{3}(1 - t). \quad (3)$$

Using Eq. (3) and the first-order condition $k_i^{PM/PM} = t$ in (2), one can directly obtain the equilibrium expressions for the producer surplus ($PS^{PM/PM}$), consumer surplus ($CS^{PM/PM}$), tax revenues ($TR^{PM/PM}$), and environmental damage ($ED^{PM/PM}$) as a function of the tax rate when both firms are profit maximisers, as reported in Appendix B (online), allowing to obtain social welfare under PM/PM ($SW^{PM/PM} = PS^{PM/PM} + CS^{PM/PM} + TR^{PM/PM} - ED^{PM/PM}$), which is reported in the following lemma:

Lemma 1. The social welfare function under PM/PM as a function of the tax rate is given by the following expression:

$$SW^{PM/PM} = \frac{4}{9} - \frac{2}{9}t - \frac{11}{9}t^2 + g \left(-\frac{2}{9} + \frac{16}{9}t - \frac{32}{9}t^2 \right). \quad (4)$$

At the first stage of the subgame (the government stage), the government levies an emissions tax with the aim of maximising social welfare. The following proposition shows the extent of the welfare maximising tax rate $t^{*PM/PM}$ and the condition under which the optimal tax rate in the subgame PM/PM is positive (the proofs of Lemma 1 and Proposition 1 are in the online Appendix B).

Proposition 1. The welfare-maximising tax rate under PM/PM is given by the following expression, which is positive if and only if $g > 0.125$:

$$t^{*PM/PM} = \frac{8g-1}{32g+11}. \quad (5)$$

Using the optimal tax rate in (5), one obtains the equilibrium quantities of industrial production $q_i^{*PM/PM} = \frac{4(2g+1)}{32g+11} > 0$, abatement $k_i^{*PM/PM} = \frac{8g-1}{32g+11} > 0$ $g > 0.125$, and emissions $q_i^{*PM/PM} - k_i^{*PM/PM} > 0$. Finally, inserting back the optimal tax $t^{*PM/PM}$ into the profit function in (1), one gets equilibrium profits under PM/PM, that is:

$$\pi_i^{*PM/PM} = \frac{192g^2+112g+33}{2(32g+11)^2}. \quad (6)$$

3.2. The symmetric subgame GD/GD

Let us now consider the case in which owners delegate output and abatement decisions to their own managers by designing a delegation contract includes an environmental component based on emissions (“green” delegation), i.e., firms are “green” delegated at the market stage of the game. In a managerial delegation model framed in a strategic competitive context, each firm consists of owners who control the firm and a manager whose decisions are based on an incentive contract that the corresponding owner designs. Managers get a publicly observed contract whose remuneration is $R_i = A + BU_i \geq 0$, where $A \geq 0$ is the fixed salary component of the manager’s compensation, $B \geq 0$ is a constant parameter, and U_i is the manager i ’s utility. Without loss of generality, we set $A = 0$ and $B = 1$ henceforth. Each manager’s compensation structure is proportional to a linear combination of profit and an incentive based on emissions. The objective function of the manager of the polluting firm i takes a form like those introduced by Vickers (1985), Sklivas (1987), and Fershtman and Judd (1987) in their pioneering contribution. The equation is as follows:

$$U_i = \pi_i + b_i e_i, \quad i = 1, 2, \quad i \neq j, \quad (7)$$

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weighting profits and emissions instead of profits and sales. In (7), π_i is the profit function defined in (1), and $b_i > 0$ (< 0) is the incentive (disincentive) designed by the owner regarding quantity and abatement (i.e., emissions) chosen by the manager, which determines actual pollution of firm i , that is $e_i = q_i - k_i$. Therefore, the maximisation of the utility function in (7) by manager i with respect to q_i and k_i at the market stage of the subgame leads to the following set of first-order conditions:⁷

$$\bar{q}_i^{GD/GD} = \frac{1}{2}(1 - q_j - t + b_i), \bar{k}_i^{GD/GD} = k_i^{GD/GD} = t - b_i, i = 1, 2, i \neq j, \quad (8)$$

where the superscript GD/GD stands for the “green” delegation of both managers. The solution of the system of output reaction functions $\bar{q}_i^{GD/GD}$ ($i = 1, 2, i \neq j$) in (8) gives the following pre-tax stage equilibrium output as a function of the emission tax t and the bonuses b_i and b_j , that is:

$$q_i^{GD/GD} = \frac{1}{3}(1 - t + 2b_i - b_j), i = 1, 2, i \neq j. \quad (9)$$

A direct comparison between (2) and (8) reveals that the “green” contract promotes industrial production (as expected) and reduces abatement due to the designing of the executive remuneration at the bonus stage (based on emissions instead of sales), apparently working at first sight as an anti-ecological device at the pre-tax stage compared to the case of profit maximisation ($b_i = 0$).

⁷ Given the non-cost-reducing version of the abatement R&D function (Ulph, 1996; Asproudis and Gil-Moltó, 2015; Buccella *et al.*, 2021), $CA_i(k_i)$, it is indifferent whether managers or owners choose the optimal amount of k_i by maximizing U_i or π_i , respectively, at the market stage of the game. The results of the “green” delegation game might change, instead, upon considering a cost-reducing (product innovation) version of the abatement R&D function like d’Aspremont and Jacquemin (1988, 1990) and Bacchiega *et al.* (2010). We leave this issue for future research.

However, this section will show that the designing of the GD contract by the owner allowing the “green” manager to produce more and abate less than the owner under the PM contract, is such that the welfare-maximising emissions tax levied by the government at the government stage under GD goes beyond the corresponding value under PM. This eventually implies that equilibrium production and abatement at the post-tax stage of the game under GD are the *same* than equilibrium production and abatement at the post-tax stage of the game under PM,⁸ allowing the *same* level of aggregate emissions and environmental damage to be achieved. This policy-driven mechanism – based on a transfer of resources flowing from firms to the public authority – allows the government to perfectly compensate the increased emissions with a larger ad hoc taxation that lowers production and increases abatement under GD. *Indeed, the difference between the optimal emission tax rate under GD and the optimal emission tax rate under PM is exactly equal to the incentive designed by the owner about the emissions chosen by the manager who is remunerated through an incentive based on emissions.* This implies that the GD contract is not anti-ecological compared to the PM contract and there is no trade-off between employment (production) and social welfare in this case (see the welfare analysis in the online Appendix for a thoughtful discussion of this issue).

A relevant effect of the emissions tax is to increase marginal costs and reduce equilibrium profits of the GD strategy below equilibrium profit of the PM strategy. The former, however, will result to be a dominant strategy so that (GD,GD) will be the Pareto inefficient Nash equilibrium of the “green” delegation game if the public awareness towards environmental quality is high enough. Note that we are considering total pollution and the corresponding environmental damage as the main indexes to evaluate the “greenness” of a managerial contract.

⁸ From this standpoint, the results of the “green” delegation game greatly differ from the main finding of the standard managerial delegation theory as output production at the equilibrium under SD with no abatement is higher than output production at the equilibrium under PM with no abatement.

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Thus, the GD contract reduces both below the corresponding values obtained through the SD contract under abatement (see the welfare analysis in the online Appendix for a thoughtful discussion of the post-tax equilibrium values of environmental damage, emission tax, consumer’s surplus, profits and social welfare under SD/SD, GD/GD and PM/PM).

Substituting (9) and $k_i^{GD/GD} = t - b_i$ in (8) into the profit function in (1) allows us to get the expression of the firm i ’s profit as a function of the managerial incentives b_i and b_j . Therefore, at the bonus stage owner i maximises profits with respect to b_i , which leads to the following reaction functions in the bonus space:

$$\bar{b}_i^{GD/GD} = \frac{1}{13}(1 - t - b_j), i = 1, 2, i \neq j. \quad (10)$$

From (10), it is clear that b_i and b_j are strategic substitutes,⁹ i.e., an increase in executive remuneration in firm j reduces executive remuneration in firm i , and the symmetric equilibrium values of both bonus and abatement of player i as a function of the tax rate are the following:

$$b_i^{GD/GD} = \frac{1-t}{14} \text{ and } k_i^{GD/GD} = \frac{15t-1}{14}. \quad (11)$$

Making use of (11), further substitutions directly yield the equilibrium expressions for the producer surplus ($PS^{GD/GD}$), consumer surplus ($CS^{GD/GD}$), tax revenues ($TR^{GD/GD}$), and environmental damage ($ED^{GD/GD}$) as a function of the tax rate when both firms are “green” delegated, reported in Appendix B (online), allowing to get social welfare under GD/GD ($SW^{GD/GD} = PS^{GD/GD} + CS^{GD/GD} + TR^{GD/GD} - ED^{GD/GD}$), which is reported in the following lemma:

⁹ See Bian *et al.* (2016) Hirose *et al.* (2017) and Hirose *et al.* (2020) for analyses of price competition games with negative externalities and delegation (the first two contributions), and price and quantity competition games with environmental corporate social responsibility (ECSR) played by firms competing in the market. Under quantity (resp. price) competition in delegation games, there exists strategic substitutability (resp. complementary) at the bonus stage.

Lemma 2. The social welfare function under GD/GD as a function of the tax rate is given by the following expression:

$$SW^{GD/GD} = \frac{89}{196} - \frac{5}{98}t - \frac{275}{196}t^2 + g \left(-\frac{18}{49} + \frac{120}{49}t - \frac{200}{49}t^2 \right). \quad (12)$$

At the first stage of the subgame, the government levies an emission tax with the aim of maximising social welfare. The following proposition shows the extent of the welfare maximising tax rate $t^{*GD/GD}$ and the condition under which the optimal tax rate in the subgame GD/GD is positive (the proofs of Lemma 2 and Proposition 2 are in the online Appendix B).

Proposition 2. The welfare-maximising tax rate under GD/GD is given by the following expression, which is positive if and only if $g > 0.020833$:

$$t^{*GD/GD} = \frac{48g-1}{5(32g+1)}. \quad (13)$$

Using the optimal tax rate in (13), one obtains the equilibrium quantities of industrial production $q_i^{*GD/GD} = \frac{4(2g+1)}{32g+11} > 0$, executive remuneration $b_i^{*GD/GD} = \frac{4(2g+1)}{5(32g+11)} > 0$, abatement $k_i^{*GD/GD} = \frac{8g-1}{32g+1} > 0$ for any $g > 0.020833$, and emissions $q_i^{*GD/GD} - k_i^{*GD/GD} > 0$. Finally, inserting back the optimal tax $t^{*GD/GD}$ into the profit function in (1), one gets equilibrium profits under GD/GD, that is:

$$\pi_i^{*GD/GD} = \frac{192g^2+96g+}{2(32g+1)^2}. \quad (14)$$

A direct comparison amongst the relevant equations of the subgames PM/PM and GD/GD allows us to conclude that $\pi_i^{*GD/GD} < \pi_i^{*PM/PM}$, whereas the post-tax equilibrium amount of industrial production and abatement are identical in both cases. Indeed, though the “green”

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manager chooses to abate a smaller amount of pollution in comparison with the profit maximising firm, the pre-tax output under GD/GD is larger than the pre-tax output under PM/PM due to the design of the incentive contract through the manager’s executive remuneration (bonus) chosen by the owner, so that the optimal environmental tax rate is higher under “green” delegation. Indeed, a direct comparison between (5) and (13) allows us to observe $t^{*GD/GD} > t^{*PM/PM}$ for any g . This implies, *ceteris paribus*, that the equilibrium pre-tax consumer surplus and total revenue when designing “green” delegation are larger than in the PM/PM case, thus causing the society under GD/GD to be willing to pay a larger amount of environmental taxes than under PM/PM. However, as the difference between the optimal emission tax rate under GD/GD and the optimal emission tax rate under PM/PM is exactly equal to the incentive designed by the owner about the emissions chosen by the manager, the post-tax equilibrium amounts of production and abatement are the same under both contracts, and profits under GD/GD are lower than profits under PM/PM as the post-tax marginal costs are larger in the former case.

3.3. The asymmetric subgame GD/PM

To determine the owners’ endogenous choice regarding whether to delegate via a “green” contract based on emissions or be a profit maximiser at the pre-play stage (or contract stage), one should evaluate the outcomes of the asymmetric subgame in which one firm, say firm 1, is a “green”-delegated firm ($b_1 \neq 0$), and the rival, firm 2, is still profit maximising ($b_2 = 0$). The optimisation problems of the GD player and the PM player at the market stage provide the first-order conditions as in (8) for firm 1 and as in (2) for firm 2. Solving the system of the output reaction functions, one obtains the quantities produced by both firms as a function of the tax rate and the bonus of the “green” delegated firm 1, that is:

$$\bar{q}_1^{GD/PM} = \frac{1}{3}(1 - t + 2b_1) \text{ and } \bar{q}_2^{GD/PM} = \frac{1}{3}(1 - t - b_1). \quad (15)$$

Substituting (15) and the expression of $k_i^{GD/GD}$ in (8) into the profit function of firm 1, one obtains the expression of firm 1’s profits as a function of the managerial incentive. At the bonus stage, the owner of firm 1 maximises profits with respect to b_1 . This gives the optimal bonus $b_1^{GD/PM} = \frac{1}{13}(1 - t)$, so that the equilibrium expressions of quantity and abatement of firm 1 and firm 2 as a function of the tax rate are as follows:

$$q_1^{GD/PM} = \frac{5}{13}(1 - t), q_2^{GD/PM} = \frac{4}{13}(1 - t), \text{ and } k_1^{GD/PM} = \frac{1}{13}(14t - 1), k_2^{GD/PM} = t. \quad (16)$$

Further substitutions by making use of (16) yield the equilibrium expressions for the producer surplus ($PS^{GD/PM}$), consumer surplus ($CS^{GD/PM}$), tax revenues ($TR^{GD/PM}$), and environmental damage ($ED^{GD/PM}$) as a function of the tax rate when firm 1 is “green” delegated and firm 2 is profit maximising, which are reported in Appendix B (online), thus providing social welfare under GD/PM ($SW^{GD/PM} = PS^{GD/PM} + CS^{GD/PM} + TR^{GD/PM} - ED^{GD/PM}$), which is reported in the following lemma:

Lemma 3. The social welfare function under GD/PM as a function of the tax rate is given by the following expression:

$$SW^{GD/PM} = \frac{76}{169} - \frac{22}{169}t - \frac{223}{169}t^2 + g \left(-\frac{50}{169} + \frac{360}{169}t - \frac{648}{169}t^2 \right). \quad (17)$$

At the first stage of the subgame, the government levies an emissions tax with the aim of maximising social welfare. The following proposition shows the extent of the welfare maximising tax rate $t^{*GD/PM}$ and the condition under which the optimal tax rate in the subgame GD/PM is positive (the proofs of Lemma 3 and Proposition 3 are in the online Appendix B).

Proposition 3. The welfare-maximising tax rate under GD/PM is given by the following expression, which is positive if and only if $g > 0.06111$:

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$$t^{*GD/PM} = \frac{180}{648g+223}. \quad (18)$$

Using the optimal tax rate in (18), one obtains the equilibrium quantities of industrial production $q_1^{*GD/PM} = \frac{90(2g+1)}{648g+223} > 0$, $q_2^{*GD/PM} = \frac{72(2g+1)}{648g+223} > 0$, executive remuneration $b_1^{*GD/PM} = \frac{18(2g+1)}{648g+223} > 0$, abatement $k_1^{*GD/PM} = \frac{144g-29}{648g+223} > 0$ for any $g > 0.20138$, $k_2^{*GD/PM} = \frac{180g-11}{648g+223} > 0$ for any $g > 0.06111$, and emissions $q_1^{*GD/PM} - k_1^{*GD/PM} > 0$ and $q_2^{*GD/PM} - k_2^{*GD/PM} > 0$ for any $0.06111 < g < 2.305$. Finally, inserting the optimal tax $t^{*GD/PM}$ back into the profit function in (1), one gets equilibrium profits under GD/PM for firm 1 and firm 2, respectively:

$$\pi_1^{*GD/PM} = \frac{82944g^2 + 46584g + 12757}{2(648g+223)^2}, \pi_2^{*GD/PM} = \frac{73872g^2 + 37512g + 104}{2(648g+223)^2}. \quad (19)$$

3.4. The owners' endogenous choice: the “green” delegation game (GD versus PM)

By using the firms' profits equations given by the expressions in (6), (14), and (19), one can get the payoff matrix reported in Table 1 about the “green” delegation game (GD versus PM). The endogenous choice of firms' owners between being “green” delegated (through the design of a managerial contract based on emissions instead of sales) and profit maximisers is played at the pre-play stage or contract stage of the game.

To satisfy the technical conditions given by the feasibility constraints and obtain well-identified equilibria in pure strategies for all strategic profiles studied in the subgames presented in the previous sections, the analysis is confined to the following range of the degree of public environmental awareness toward a clean environment (or against the damage caused by industrial production), $0.20138 < g < 2.305$. The former inequality bounds from below the parameter region, requiring that the societal awareness of a clean environment should be high

enough to ensure positive levels of pollution abatement for the managerial firm in the asymmetric subgame, in which the owner of one firm designs a delegation contract based on emissions to its own manager (GD), and the rival is profit maximising (PM) under abatement. The latter inequality bounds from above the parameter region, requiring that the societal awareness toward a clean environment should be low enough (i.e., the emission tax rate should not be fixed at too high a level) to guarantee that emissions are non-negative for the PM firm in the asymmetric subgame. Within this range of values of g , all other relevant feasibility constraints discussed above are satisfied and apply accordingly.

To derive the Nash equilibria of the game, we study the sign of the following set of profit differentials for firm $i = 1, 2, i \neq j$:

$$\Delta\pi_a = \pi_i^{*GD/PM} - \pi_i^{*PM/PM}, \Delta\pi_b = \pi_i^{*PM/GD} - \pi_i^{*GD/GD} \text{ and } \Delta\pi_c = \pi_i^{*PM/PM} - \pi_i^{*GD/GD}. \quad (20)$$

An analytical inspection of the expressions in (20) reveals that $\Delta\pi_a \leq 0$ if $0.20138 < g \leq 0.37$, $\Delta\pi_b < 0$ and $\Delta\pi_c > 0$ for any $0.20138 < g < 2.305$, from which one can get the following result (the proof can be found in the online Appendix B).

Table 1. The “green” delegation game under abatement (payoff matrix): GD versus PM.

Firm 2 → Firm 1 ↓	GD	PM
GD	$\pi_1^{*GD/GD}, \pi_2^{*GD/GD}$	$\pi_1^{*GD/PM}, \pi_2^{*GD/PM}$
PM	$\pi_1^{*PM/GD}, \pi_2^{*PM/GD}$	$\pi_1^{*PM/PM}, \pi_2^{*PM/PM}$

Result 1. If the regulator levies an emissions tax to incentivise firms undertaking emissions-reducing actions, the “green” delegation game GD versus PM produces the following set of Nash equilibrium outcomes in pure strategies. (1) If $0.20138 < g < 0.37$, then there exist two pure-strategy Nash equilibria given by (PM,PM) and (GD,GD), and the PM payoff dominates GD (coordination game). (2) If $0.37 < g < 2.305$, then (GD,GD) is the unique Pareto-

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inefficient SPNE of the game (prisoner’s dilemma), and GD is the dominant strategy (there is a conflict between self-interest and the mutual benefit to become a “green”-delegated firm instead of remaining a profit maximiser).

First, we note that asymmetric equilibria are prevented as the model does not include elements affecting one firm differently from the rival. The economic intuition of the Nash equilibrium outcomes of the “green” delegation game can be carried out following the narrative of the cases detailed in Result 1, starting from low values of the societal awareness towards a clean environmental quality (alternatively, against the – severe or mild – environmental damages caused by industrial production). If $0.20138 < g < 0.37$, there are multiple pure-strategy Nash equilibria in which PM payoff dominates GD. When g is sufficiently low, the optimal tax rates under PM and GD are low. As $t^{*GD/GD} > t^{*PM/PM}$, profits under GD/GD are smaller than profits under PM/PM, though the post-tax equilibrium values of industrial production and abatement are the same in both cases. In addition, in the “green” delegation scenario, some resources should also be devoted to executive remuneration, but the amount of the optimal emissions tax at the equilibrium exactly compensates the reduced abatement of the “green” manager due to the incentives designed by the owner through a contract based on emissions.

The combination of the effects within this range of values of g implies a drop in profits in all (symmetric and asymmetric) strategic profiles following an increase in g , as the negative effects on profits of the reduction in the amount of industrial production overcomes the positive effects due the increase in the market price. However, the percentage reduction in profits is larger under PM than under GD in both symmetric and asymmetric subgames. This is partly because production under GD in the asymmetric subgame is larger than under PM.

In addition, *ceteris paribus*, the overall burden of the emission tax under GD is larger than under PM, and this provides an incentive for firms not to deviate toward “green” delegation if the rival also chooses to be a “green”-delegated firm. No dominant strategies exist in this case. Definitively, the outcome of the game is *a priori* uncertain. In fact, on the one hand, each player is interested in PM if the rival chooses not to be “green” delegated, because everyone would like to play PM. On the other hand, no one is willing to play GD if the rival plays PM, as no one wants to be the sole player to get lower profits. Again, no one would like to make decisions that he might regret, but no one can foresee the rival’s decisions (this is because firms are playing a simultaneous non-cooperative game).

It could happen, however, that the two players make non-consistent choices: if someone chooses to play GD to avoid losing the opportunity to get higher profits and the rival chooses PM to reduce the tax burden, players could end up in a situation in which only one is “green” delegated, representing for both an unsatisfactory outcome, as everyone could be better off with a different choice.

Differently, if decisions were compatible, both players would choose to be profit maximisers if they are willing to reduce the tax burden in a context in which social awareness against pollution is not sufficiently high, thus achieving a Pareto-efficient outcome. Players could also make compatible decisions ensuring a sub-optimal result, and thus they can both choose to be “green” delegated if they are willing to avoid being the sole PM player experiencing the worst outcome.

In this context, however, players are interested in agreeing to be PM. Furthermore, by choosing PM, everyone becomes interested in complying with the non-binding agreement, as no one would have the incentive to deviate towards GD. Though the game is non-cooperative, both players have an incentive to coordinate towards PM by not designing a managerial delegation contract based on emissions.

If $0.37 < g < 2.305$, the comparison between profits under GD and PM supports the strategic profile GD. Indeed, the percentage reduction in profits under PM is larger than under GD as g increases. This is because in the asymmetric subgame GD/PM, the GD firm produces more and abate less than the PM firm due to the designing of the contract based on emissions so that $\pi_1^{*GD/PM} > \pi_2^{*GD/PM}$. The optimal emission tax in the asymmetric subgame GD/PM is set at an intermediate value between the symmetric subgames GD/GD (the highest) and PM/PM (the lowest), so that the government is not able induce the GD firm to sufficiently increase pollution abatement in the asymmetric case. Therefore, though in the asymmetric subgame the GD firm gets a higher tax burden ($t^{*GD/PM}(q_1^{*GD/PM} - k_1^{*GD/PM})$) than the PM firm ($t^{*GD/PM}(q_2^{*GD/PM} - k_2^{*GD/PM})$), profits of the GD firm $\pi_1^{*GD/PM}$ reduce when g increases but they remain higher than profits of the PM rival $\pi_2^{*GD/PM}$ that reduce at a higher rate following an increase in g . As the inequality $\pi_2^{*PM/PM} > \pi_2^{*PM/GD}$ holds for any $g < 0.37$, one gets $\pi_2^{*PM/PM} < \pi_2^{*PM/GD}$ for any for $g > 0.37$. Consequently, the increased awareness toward environmental quality allows GD to become the dominant strategy of the game. This outcome, however, is Pareto inefficient for firms, as they both would jointly deviate toward PM to benefit from a reduction in the tax burden, reduce the marginal costs and get higher profits. Therefore, there is a conflict between self-interest and mutual benefit not to become a “green”-delegated firm.

4. Sales delegation (SD) versus profit maximisation (PM) with homogeneous products

This section aims at studying the endogenous choice played at the pre-play stage (or contract stage) of the game between choosing standard sales delegation (SD) or profit maximisation (PM) augmented with abatement decisions and emissions taxes. The subgame referred to profit maximisation under abatement is the same as the one presented in Section 3.1. Therefore, we

can concentrate on the universal adoption of sales delegation and the asymmetric case in which only one firm becomes sales delegated.

4.1. The symmetric subgame SD/SD

Consider the case in which owners offer a standard output (sales) delegation contract to a manager who must choose both output and abatement levels. The utility function of the manager changes from (7) to take the standard form the managerial delegation literature pioneered by Vickers (1985), Sklivas (1987), and Fershtman and Judd (1987), that is:

$$U_i = \pi_i + b_i q_i, \quad i = 1, 2, \quad i \neq j, \quad (21)$$

Eq. (21) is a linear combination weighting profits and sales, where π_i and b_i are defined in the previous section. The maximisation of the managers’ utility function in (21) at the market stage of the game with respect to q_i and k_i yields the following first-order conditions:

$$\bar{q}_i^{SD/SD} = \frac{1}{2}(1 - q_j - t + b_i), \quad \bar{k}_i^{SD/SD} = k_i^{SD/SD} = t, \quad i = 1, 2, \quad i \neq j, \quad (22)$$

where the superscript SD/SD stands for sales delegation of both managers. Solving the system of output reaction functions $\bar{q}_i^{SD/SD}$ in (22), one gets the same solution as in (9), i.e., $q_i^{SD/SD} = q_i^{GD/GD}$. Substituting (9) and $k_i^{SD/SD} = t$ into (1) allows us to obtain firm i ’s profits as a function of the managerial bonuses b_i and b_j and the tax rate t . Comparison of the expressions in (2), (8) and (22) reveals that at the pre-tax market stage of the game the sales manager under an abatement regime is incentivised to produce the same amount and abate more than the “green” manager (indeed the abated pollutant under the SD regime is the same as in the PM regime at this stage) as the former is incentivised by a contract based on sales and the latter by a contract based on emissions.

This section will also show that the designing of the SD contract by the owner is such that the welfare-maximising emissions tax levied by the government at the government stage in the

SD regime is set at an intermediate level between the optimal tax rate in the GD regime (the highest) and the optimal tax rate in the PM regime (the smallest). This is because production under SD is higher than under PM and abatement under SD is higher than under GD. In addition, the owner under SD will set a bonus larger than under GD as the pre-tax quantities are the same but abatement (which is not part of the contract under SD) is larger in the SD regime. This eventually implies that equilibrium production and abatement at the post-tax stage of the game under SD are the *highest*. The largest production under SD causes however the *largest* levels of aggregate emissions and environmental damage compared to the other two contracts by also implying that the SD firm will incur higher marginal costs and lower equilibrium post-tax profits than the GD firms.

Comparison amongst the crucial values obtained under SD and GD contracts (as will be clear later) implies that at the post-tax stage the working of the policy-driven mechanism – based on a transfer of resources flowing from firms to the public authority – does not allow the government to perfectly compensate the increased emissions of the SD regime as the optimal emissions tax rate is smaller than in the GD regime.

At the bonus stage, firm i 's owner ($i = 1,2$) maximise profits with respect to b_i , obtaining the reaction functions in the bonus space $\bar{b}_i^{SD/SD} = \frac{1}{4}(1 - t - b_j)$, $i = 1,2, i \neq j$. Therefore, the symmetric equilibrium values of the bonus as a function of the tax rate is:

$$b_i^{SD/SD} = \frac{1}{5}(1 - t), i = 1,2, i \neq j. \quad (23)$$

We first note that $b_i^{SD/SD} > b_i^{GD/GD}$. This is because the sales manager is incentivised to produce the same amount and abate more than the “green” manager (who is incentivised by a contract based on emissions rather than sales) at the (pre-tax) market stage of the game. Further substitutions using the expression in (23) and other relevant equations yield the equilibrium expressions for the producer surplus ($PS^{SD/SD}$), consumer surplus ($CS^{SD/SD}$), tax revenues

($TR^{SD/SD}$), and environmental damage ($ED^{SD/SD}$) as a function of the tax rate when both firms are sales delegated under abatement, which are reported in Appendix B (online), allowing one to get social welfare under SD/SD ($SW^{SD/SD} = PS^{SD/SD} + CS^{SD/SD} + TR^{SD/SD} - ED^{SD/SD}$), which is reported in the following lemma:

Lemma 4. The social welfare function under SD/SD as a function of the tax rate is given by the following expression:

$$SW^{SD/SD} = \frac{12}{25} - \frac{4}{25}t - \frac{33}{25}t^2 + g \left(-\frac{8}{25} + \frac{56}{25}t - \frac{98}{25}t^2 \right). \quad (24)$$

At the first stage of the subgame, the government levies an emission tax with the aim of maximising social welfare. The following proposition shows the extent of the welfare maximising tax rate $t^{*SD/SD}$ and the condition under which the optimal tax rate in the subgame SD/SD is positive (the proofs of Lemma 4 and Proposition 4 are in the online Appendix B).

Proposition 4. The welfare-maximising tax rate under SD/SD is given by the following expression, which is positive if and only if $g > 0.07142$:

$$t^{*SD/SD} = \frac{2(14g-1)}{98g+3}. \quad (25)$$

Using the optimal tax rate in (25), one obtains the equilibrium quantities of industrial production $q_i^{*SD/SD} = \frac{14(2g+1)}{98g+3} > 0$, executive remuneration $b_i^{*SD/SD} = \frac{7(2g+1)}{98g+3} > 0$, abatement $k_i^{*SD/SD} = \frac{2(14g-1)}{98g+3} > 0$ for any $g > 0.07142$, and emissions $q_i^{*SD/SD} - k_i^{*SD/SD} > 0$. Finally, inserting back the optimal tax $t^{*SD/SD}$ into the profit function in (1), one gets equilibrium profits under SD/SD, that is:

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$$\pi_i^{*SD/SD} = \frac{4(196g^2+84g+25)}{(98g+33)^2}. \quad (26)$$

Before analysing the asymmetric subgame SD/PM and the endogenous choice at the pre-play stage between choosing to become sales delegated under pollution abatement or remaining a profit maximising firm, it is useful to compare emissions taxes, profits, quantities, abatements, executive remunerations (bonuses), emissions, and environmental damages in the symmetric subgames GD/GD, SD/SD, and PM/PM, as asymmetric Nash equilibrium outcomes are prevented in the different models of the article.

Given the results obtained in the previous sections, it is easy to see that the following sets of inequalities hold (for any g) at the post-tax equilibrium stage: $t^{*GD/GD} > t^{*SD/SD} > t^{*PM/PM}$, $\pi_i^{*PM/PM} > \pi_i^{*GD/GD} > \pi_i^{*SD/SD}$, $q_i^{*SD/SD} > q_i^{*GD/GD} = q_i^{*PM/PM}$, $k_i^{*SD/SD} > k_i^{*GD/GD} = k_i^{*PM/PM}$, $b_i^{*SD/SD} > b_i^{*GD/GD}$, $e_i^{*SD/SD} > e_i^{*GD/GD} = e_i^{*PM/PM}$, and $ED^{*SD/SD} > ED^{*GD/GD} = ED^{*PM/PM}$. This allows us to conclude that the “green” delegation contract works in the eco-friendly direction of reducing emissions and environmental damages compared to the standard sales delegation contract under pollution abatement, whereas also giving firms the opportunity to generate higher profits than the strategic profile SD/SD (this is because marginal costs in the SD scenario are higher than the marginal costs in the GD scenario).

In addition, according to the study of the optimal bonus functions, it follows that the higher the public awareness towards environmental quality, the smaller the incentive to pollute. The relationship between b_i and g is “green” oriented, as polluting production reduces the incentive for the manager to pollute. We want to stress that we initially referred to the “green”-related strategic profile as “GD” but do not know in advance whether the introduction of emissions instead of sales in the manager’s utility is eco-friendly or anti-ecological. However, equilibrium results lie indeed within an eco-friendly trajectory.

4.2. The asymmetric subgame SD/PM

To derive the owners’ endogenous choice of whether to become a sales-delegated firm or remaining a profit maximiser under abatement, one must compute the outcomes of the asymmetric game in which one firm, say firm 1, is sales delegated, $b_1 \neq 0$, and the rival, firm 2, is profit maximising, $b_2 = 0$ (SD/PM). The two optimisation problems at the market stage of the game lead to first-order conditions as in (22) for SD firm 1 and as in (2) for PM firm 2. Solving the system of the output reaction functions, one obtains the output levels as in (15). Substituting these outcomes along with condition $k_1^{SD/PM} = t$ into the profit function of the SD firm, one gets the expression of firm 1’s profits as a function of the managerial incentive. It follows that firm 1’s owner maximises profits with respect to b_1 at the bonus stage, leading to the optimal bonus as a function of the tax rate, that is $b_1^{SD/PM} = \frac{1}{4}(1 - t)$. Therefore, the subgame equilibrium outcomes as a function of the tax rate are as follows:

$$q_1^{SD/PM} = \frac{1}{2}(1 - t), q_2^{SD/PM} = \frac{1}{4}(1 - t) \text{ and } k_1^{SD/PM} = k_2^{SD/PM} = t. \quad (27)$$

Further substitutions by making use of (27) allow us to get the equilibrium expressions for the producer surplus ($PS^{SD/PM}$), consumer surplus ($CS^{SD/PM}$), tax revenues ($TR^{SD/PM}$), and environmental damage ($ED^{SD/PM}$) as a function of the tax rate when firm 1 is sales delegated and firm 2 is profit maximising, which are reported in Appendix B (online), thus providing social welfare under SD/PM ($SW^{SD/PM} = PS^{SD/PM} + CS^{SD/PM} + TR^{SD/PM} - ED^{SD/PM}$), which is reported in the following lemma:

Lemma 5. The social welfare function under SD/PM as a function of the tax rate is given by the following expression:

$$SW^{SD/PM} = \frac{15}{32} - \frac{3}{16}t - \frac{41}{32}t^2 + g\left(-\frac{9}{32} + \frac{33}{16}t - \frac{121}{32}t^2\right). \quad (28)$$

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At the first stage of the subgame, the government levies an emission tax with the aim of maximising social welfare. The following proposition shows the extent of the welfare-maximising tax rate $t^{*SD/PM}$ and the condition under which the optimal tax rate in the subgame SD/PM is positive (the proofs of Lemma 5 and Proposition 5 are in the online Appendix B).

Proposition 5. The welfare-maximising tax rate under SD/PM is given by the following expression, which is positive if and only if $g > 0.0909$:

$$t^{*SD/PM} = \frac{3(11g-1)}{121g+41}. \quad (29)$$

Using the optimal tax rate in (29), one obtains the equilibrium quantities of industrial production $q_1^{*SD/PM} = \frac{22(2g+1)}{121g+} > 0$, $q_2^{*SD/PM} = \frac{11(2g+1)}{121g+} > 0$, executive remuneration $b_1^{*SD/PM} = \frac{11(2g+1)}{121g+} > 0$, abatement $k_1^{*SD/PM} = k_2^{*SD/PM} = \frac{3(11g-1)}{121g+} > 0$ for any $g > 0.0909$, and emissions $q_1^{*SD/PM} - k_1^{*SD/PM} > 0$ and $q_2^{*SD/PM} - k_2^{*SD/PM} > 0$ for any $0.0909 < g < 1.2727$. Finally, inserting the optimal tax $t^{*SD/PM}$ back into the profit function in (1), one gets equilibrium profits under SD/PM for firm 1 and firm 2, respectively:

$$\pi_1^{*SD/PM} = \frac{3025g^2+1738g+49}{2(121g+)^2}, \pi_2^{*SD/PM} = \frac{2057g^2+770g+251}{2(121g+41)^2}. \quad (30)$$

4.3. The owners' endogenous choice: the sales delegation game (SD versus PM)

By using the firms' profits equations given by the expressions in (6), (26), and (30), one can obtain the payoff matrix reported in Table 2 about the sales delegation game (SD versus PM) under abatement. The endogenous choice of firms' owners which should choose to be sales delegated (through the design of a managerial contract based on sales) or profit maximisers is played at the pre-play or contract stage of the game.

To satisfy the technical restrictions given by the feasibility constraints and obtain well-identified equilibria in pure strategies for all strategic profiles studied in the subgames presented in the previous sections, the analysis is confined to the following range of the degree of public environmental awareness toward a clean environment (or against the damage caused by industrial production), $0.125 < g < 1.2727$. The former inequality bounds from below the parameter region requiring that the societal awareness toward a clean environment should be high enough to ensure positive levels of pollution abatement (and the optimal emission tax rate) for both PM firms in the symmetric subgame PM/PM. The latter inequality bounds from above the parameter region requiring that the societal awareness toward a clean environment should be low enough (i.e., the emission tax rate should not be fixed at too high a level) to guarantee that emissions are non-negative for the PM firm in the asymmetric subgame in which the owner of one firm designs a sales delegation contract to its own manager (SD) and the rival is profit maximising (PM) under abatement. Within this range of values of g , all other relevant feasibility constraints discussed above are satisfied and apply accordingly.

To derive the Nash equilibria of the game, we study the sign of the following set of profit differentials for firm $i = 1, 2, i \neq j$:

$$\Delta\pi_a = \pi_i^{*SD/PM} - \pi_i^{*PM/PM}, \Delta\pi_b = \pi_i^{*PM/SD} - \pi_i^{*SD/SD} \text{ and } \Delta\pi_c = \pi_i^{*PM/PM} - \pi_i^{*SD/SD}. \quad (31)$$

An analytical inspection of the expressions in (31) reveals that $\Delta\pi_a > 0$, $\Delta\pi_b < 0$ and $\Delta\pi_c > 0$ for any $0.125 < g < 1.2727$, from which one can get the following result (the proof can be found in the online Appendix B).

Table 2. The sales delegation game under abatement (payoff matrix): SD versus PM.

Firm 2 → Firm 1 ↓	SD	PM
SD	$\pi_1^{*SD/SD}, \pi_2^{*SD/SD}$	$\pi_1^{*SD/PM}, \pi_2^{*SD/PM}$
PM	$\pi_1^{*PM/SD}, \pi_2^{*PM/SD}$	$\pi_1^{*PM/PM}, \pi_2^{*PM/PM}$

Result 2. If the regulator levies an emissions tax to incentivise firms undertaking emissions-reduction actions, the sales delegation game SD versus PM with pollution abatement produces the following Nash equilibrium outcome in pure strategies: (SD,SD) is the unique Pareto inefficient SPNE of the game (prisoner’s dilemma), and SD is the dominant strategy (there is a conflict between self-interest and the mutual benefit to becomes a sales-delegated firm).

Result 2 resembles the standard result of Vickers (1985), Sklivas (1987) and Fershtman and Judd (1987) – VFJS – in which sales delegation is the dominant strategy but firms are entrapped in a prisoner’s dilemma. This suggests that augmenting the sales delegation game with pollution abatement does not change the main message of the sales delegation literature framed in strategic competitive markets if products are homogeneous. In this regard, we pinpoint that considering the endogenous choice of the standard sales delegation game without abatement (i.e., SD versus PM following the VFJS’s approach) leads to the well-known conclusion that the SD contract emerges as the unique pure-strategy Pareto inefficient Nash equilibrium of the game in either cases of homogeneous and heterogeneous products if the manager’s bonus is unilaterally chosen by the owner, though results may dramatically change if owners and managers are engaged in a bargaining to choose executive remuneration (see Fanti *et al.*, 2017b), as the bargaining power interacts with the extent of product differentiation in determining the Nash equilibrium outcomes of the game, which can be very different from the classic VFJS result, according to which the SD contract without abatement is always the dominant strategy and the sales delegation game falls under the paradigm of the prisoner’s dilemma. Unlike this outcome, building on the sales delegation game (SD versus PM) under can greatly modify the Nash equilibrium results of the traditional VFJS literature also in the absence of a bargaining mechanism between owners and managers over managerial contracts

just by assuming horizontal products differentiation, allowing for the existence of a plethora of different Nash equilibrium outcomes depending on the interaction between the degree of product differentiation and the public awareness towards environmental quality (see Appendix C, Figure 6 – available online –, for details).

5. “Green” delegation (GD) versus sales delegation (SD) with homogeneous products

This section investigates a delegation game in which the owners offer to their own managers either a “green” delegation contract based on emissions or a standard sales delegation contract under pollution abatement. The section deals explicitly with the asymmetric subgame GD/SD as the symmetric subgames GD/GD and SD/SD were the subjects of Sections 3.2 and 4.1, respectively.

5.1. The asymmetric subgame GD/SD

The owners’ endogenous choice about the type of delegation contract to be designed in this case needs the evaluation of the payoffs of the asymmetric game in which the owner of one firm, say firm 1, designs a delegation contract based on emissions and the rival, firm 2, designs a sales delegation contract. The two managers’ maximisation problems give the first-order conditions presented in Section 3.2 for the GD firm 1 and in Section 4.1 for the SD firm 2.

Solving the system of the output reaction functions at the market stage of the game, one gets the quantities of industrial production as a function of the managerial incentives as in (9). By substituting these outcomes along with (i) the condition $k_1^{GD/SD} = t - b_1$ into the profit function of firm 1, and (ii) condition $k_2^{GD/SD} = t$ into the profit function of firm 2, one obtains the expressions of the firms’ profits as a function of the managerial incentives. It follows that, at the bonus stage, the owner of firm 1 maximises π_1 with respect to b_1 , and the owner of firm 2 maximises π_2 with respect to b_2 . This generates the system of bonus reaction functions

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$\bar{b}_1^{GD/SD} = \frac{1}{13}(1 - t - b_2)$ and $\bar{b}_2^{GD/SD} = \frac{1}{4}(1 - t - b_1)$, whose solutions lead to the optimal bonuses as a function of the tax rate, that is $b_1^{GD/SD} = \frac{1}{17}(1 - t)$ and $b_2^{GD/SD} = \frac{1}{17}(4 - t)$, so that $b_2^{GD/SD} > b_1^{GD/SD}$ (in line with the results of Sections 3 and 4). Therefore, the equilibrium values of quantities and abatement as a function of the tax rate in this asymmetric subgame are as follows:

$$q_1^{GD/SD} = \frac{1}{17}(5 - t), q_2^{GD/SD} = \frac{1}{17}(8 - t), \text{ and } k_1^{GD/SD} = \frac{1}{17}(18t - 1), k_2^{GD/SD} = t. \quad (32)$$

From the expressions in (32) the inequalities $q_2^{GD/SD} > q_1^{GD/SD}$ and $k_2^{GD/SD} > k_1^{GD/SD}$ (in line with the results of Sections 3 and 4). Further substitutions through the use of (32) allow us to get the equilibrium expressions for the producer surplus ($PS^{GD/SD}$), consumer surplus ($CS^{GD/SD}$), tax revenues ($TR^{GD/SD}$), and environmental damage ($ED^{GD/SD}$) as a function of the tax rate when firm 1 is “green” delegated and firm 2 is sales delegated, which are reported in Appendix B (online), providing social welfare under GD/SD ($SW^{GD/SD} = PS^{GD/SD} + CS^{GD/SD} + TR^{GD/SD} - ED^{GD/SD}$), which is reported in the following lemma:

Lemma 6. The social welfare function under GD/SD as a function of the tax rate is given by the following expression:

$$SW^{GD/SD} = \frac{8}{17} - \frac{2}{17}t - \frac{23}{17}t^2 + g\left(-\frac{98}{289} + \frac{672}{289}t - \frac{1152}{289}t^2\right). \quad (33)$$

At the first stage of the subgame, the government levies an emission tax with the aim of maximising social welfare. The following proposition shows the extent of the welfare maximising tax rate $t^{*GD/SD}$ and the condition under which the optimal tax rate in the subgame GD/SD is positive (the proofs of Lemma 6 and Proposition 6 are in the online Appendix B).

Proposition 6. The welfare-maximising tax rate under GD/SD is given by the following expression, which is positive if and only if $g > 0.0506$:

$$t^{*GD/SD} = \frac{336g-17}{1152g+391}, \quad (34)$$

where $t^{*SD/SD} < t^{*GD/SD} < t^{*GD/GD}$.

Using the optimal tax rate in (34), one obtains the equilibrium quantities of industrial production $q_1^{*GD/SD} = \frac{120(2g+1)}{1152g+391} > 0$, $q_2^{*GD/SD} = \frac{192(2g+1)}{1152g+391} > 0$, executive remunerations $b_1^{*GD/SD} = \frac{24(2g+1)}{1152g+391} > 0$ and $b_2^{*GD/SD} = \frac{96(2g+1)}{1152g+391} > 0$, abatement $k_1^{*GD/SD} = \frac{288g-}{1152g+391} > 0$ for any $g > 0.1423$, $k_2^{*GD/SD} = \frac{336g-}{1152g+391} > 0$ for any $g > 0.0506$, and emissions $q_1^{*GD/SD} - k_1^{*GD/SD} > 0$ for any $0.1423 < g < 3.3541$, $q_2^{*GD/SD} - k_2^{*GD/SD} > 0$. Finally, inserting the optimal tax $t^{*GD/SD}$ back into the profit function in (1), one gets equilibrium profits under GD/SD for firm 1 and firm 2, respectively:

$$\pi_1^{*GD/SD} = \frac{202752g^2+78432g+22753}{2(1152g+391)^2}, \pi_2^{*GD/SD} = \frac{260352g^2+136032g+3715}{2(1152g+391)^2}, \quad (35)$$

where $\pi_2^{*GD/SD} > \pi_1^{*GD/SD}$.

5.2. The owners' endogenous choice: “green” versus sales delegation (GD versus SD)

By using the firms' profits equations given by the expressions in (14), (26), and (35), one can obtain the payoff matrix reported in Table 3 about the game “green” versus sales delegation (GD versus SD). The endogenous choice of firms' owners, which should choose to be “green”-delegated or sales-delegated (through the design of a managerial contract based on emissions or sales) is played at the pre-play or contract stage of the game.

To satisfy the technical restrictions given by the feasibility constraints and obtain well-identified equilibria in pure strategies for all strategic profiles studied in the subgames presented

in the previous sections, the analysis is bounded by the following range of the degree of public environmental awareness toward a clean environment (or against the damage caused by industrial production), $0.1423 < g < 3.3541$. The former inequality bounds from below the parameter region requiring that the societal awareness toward a clean environment be high enough to ensure positive levels of pollution abatement (and the optimal emission tax rate) for the GD firm in the asymmetric subgame GD/SD, i.e., when the owner of one firm designs a “green” delegation contract based on emissions to his own manager (GD) and the owner of the rival firm designs a delegation contract based on sales to his own manager (SD) under pollution abatement. The latter inequality bounds from above the parameter region require that the societal awareness toward a clean environment be low enough (i.e., the emission tax rate should not be fixed at too high a level) to guarantee that emissions are non-negative for the GD firm in the asymmetric subgame. Within this range of values of g , all other relevant feasibility constraints discussed above are satisfied and apply accordingly.

To derive the Nash equilibria of the game, we study the sign of the following set of profit differentials for firm $i = 1, 2, i \neq j$:

$$\Delta\pi_a = \pi_i^{*GD/SD} - \pi_i^{*SD/SD}, \Delta\pi_b = \pi_i^{*SD/GD} - \pi_i^{*GD/GD} \text{ and } \Delta\pi_c = \pi_i^{*SD/SD} - \pi_i^{*GD/GD}. \quad (36)$$

An analytical inspection of the expressions in (36) reveals that $\Delta\pi_a < 0$, $\Delta\pi_b > 0$, and $\Delta\pi_c < 0$ for any $0.1423 < g < 3.3541$, from which one can get the following result (the proof can be found in the online Appendix B).

Result 3. If the regulator levies an emissions tax to incentivise firms undertaking emissions-reducing actions, the “green” versus sales delegation game GD versus SD with pollution abatement produces the following Nash equilibrium outcome in pure strategies: (SD,SD) is the unique Pareto-inefficient SPNE of the game (prisoner’s dilemma), and SD is the dominant strategy (there is a conflict between self-interest and the mutual benefit to becomes a sales-

delegated firm). This gives each owner the unilateral incentive to design a “green” delegation contract based on emissions rather than a delegation contract based on sales when firms adopt a clean technology to abate pollution.

Table 3. “Green” delegation versus sales delegation (payoff matrix): GD versus SD.

Firm 2 → Firm 1 ↓	SD	GD
SD	$\pi_1^{*SD/SD}, \pi_2^{*SD/SD}$	$\pi_1^{*SD/GD}, \pi_2^{*SD/GD}$
GD	$\pi_1^{*GD/SD}, \pi_2^{*GD/SD}$	$\pi_1^{*GD/GD}, \pi_2^{*GD/GD}$

The economic intuition of the Nash equilibrium outcomes of the “green” versus sales delegation game can be carried out following the narrative of the cases detailed in Result 3. Because of the strategic interactions, the design of a standard sales delegation contract to managers arises in equilibrium regardless of the weight the government places on the environmental damage. This is because of a twofold effect: 1) on one hand, by including emissions in the compensation scheme managers tend to behave more aggressively at the pre-tax stage of the game, expanding output, *reducing* abatement and eventually gaining market shares more with a delegation contract based on emissions instead of sales; 2) on the other hand, however, the optimal emission tax levied by the government is higher under GD than under SD as managers tend to produce and pollute more if owners design a contract based on emissions instead of sales. The latter effect more than counterbalances the former so that the post-tax profits are larger in the symmetric GD scenario than in the symmetric SD scenario (irrespective of the value of g).

In addition, as in the asymmetric scenario, the SD firm gets higher profits than the GD firm: SD is the dominant strategy of this game. We note that managers under SD abate more pollution than under GD and pay a lower amount of taxes. However, the SD manager produces more than

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the GD manager so that marginal costs under GD are smaller than under SD and the tax base tends to reduce in the latter case. Owners therefore will improve firms’ profitability if they jointly deviate towards “green” delegation; in fact, by reducing output (and thus emissions), profits will increase, and as the percentage reduction in output gets lower, the percentage increases in the market price gets higher. Therefore, by choosing the standard sales delegation contract, owners are cast into a prisoner’s dilemma and this result is policy-driven in the sense emissions taxation is higher in the GD scenario than in the SD scenario due to the relative size of production and abatement in the two regimes.

We pinpoint here that, by changing the structure of the three games studied in Sections 3-5 and considering the setting of the emissions fee occurring at the first stage (i.e., the regulator moves first), we avoid including the pre-play stage, which confirms the Nash equilibrium outcomes of the article except for the “green” delegation game, in which it emerges only (GD,GD) as the unique Pareto-inefficient Nash equilibrium.

6. Discussion and concluding remarks

This article investigates the owners’ decision of whether to delegate to managers the choice of pollution abatement in a Cournot duopoly with homogeneous goods and pollution externalities. It also assumes the existence of a government whose aim is to maximise social welfare using an emission tax to incentive firms to undertake emissions-reduction actions (Buccella *et al.*, 2021). In doing so, the work develops a three-stage non-cooperative game (solved by backward induction) augmented with a pre-lay stage (or contract stage) during which firms choose whether to be delegated (with contracts based either on emissions or sales) or profit maximisers. Under this choice, first, the government fixes the emission tax, and then owners strategically choose whether to retain the choices of the abatement and output levels or delegate those

decisions to a manager via an incentivising emissions contract or a standard sales contract, after having selected the relative optimal incentive at the bonus stage.

Some findings are worthy to be resumed. Comparing equilibrium profits, firms should prefer a contract based on an incentive weighting emission instead of sales. Rather paradoxically, to have a higher environmental quality it is better to design a contract based on emissions. By contrast, employment, production and social welfare are favoured by a contract based on a weighted sum between profits and sales. Therefore, the article definitively pinpoints a well-known trade-off between employment and environmental quality emerging in the choice of managerial delegation contracts.

The article also extends the “green” delegation game to include horizontal product differentiation, confirming that designing a managerial contract incentivising emissions can indeed allow to achieve an equilibrium outcome working towards environmental protection if played against profit maximisation.

The work makes simplicity its major point, as the models are based on a set of precise assumptions that call for further extensions. In this regard, a first line of research could be devoted to the study of different functional forms about demand and cost functions or the introduction of different government’s emission tax schedules while also considering the issue of corporate social responsibility.

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Compliance with ethical standards

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