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Development of statistical methods to highlight differences between endurance limit of re-rolled steel sheets in an assumed homogeneous population

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Abstract. In this paper, we present the results of the activities carried out to estimate the axial fatigue strength of ASTM E566 Gr. 70 sheet metal. We describe the experimental setup, followed by the collected data. To highlight a possible inhomogeneity in some of the material properties, we estimate the endurance limit for the investigated material using various statistical approaches on the set of test articles. The data analysis was carried out both by considering as a whole the set of test articles and by dividing the set into two groups. As a new analysis technique, we propose a method introducing a bimodal distribution (Gaussian mixture).

1. Introduction

The present paper shows the results of the activities carried out in the framework of a collaboration with Toscotec S.p.A. The paper is divided into four sections. After this short introduction, section 2 is dedicated to the presentation of the investigated material and the adopted testing procedure. In section 3 the experimental results are presented, and the estimations of the material fatigue strength by the Staircase Procedure and the Maximum Likelihood Principle are reported. A new analysis technique is finally proposed, in which the population distribution is a linear combination (mixture) of two Gaussian distributions. In section 4 some concluding remarks are presented and a suggestion for the choice of the endurance limit of the material is proposed.

2. Materials and methods

In this section we describe some of the mechanical properties of the steel under investigation and the geometry of the test articles made of this material. We finally describe how the test campaign was carried out.

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2.1. Material properties

In this work we study the fatigue strength limit of steel SA 516 - Gr. 70. The supplier has sent the material needed to manufacture the test articles in two sheets, called sheet A and sheet B, which are nominally the same material, therefore with the same fatigue strength limit. However, it could have occurred that the two sheets were obtained in two separate castings. Therefore it might be interesting to try to understand if differences can be highlighted by considering the test articles extracted from the two sheets as two different samples.

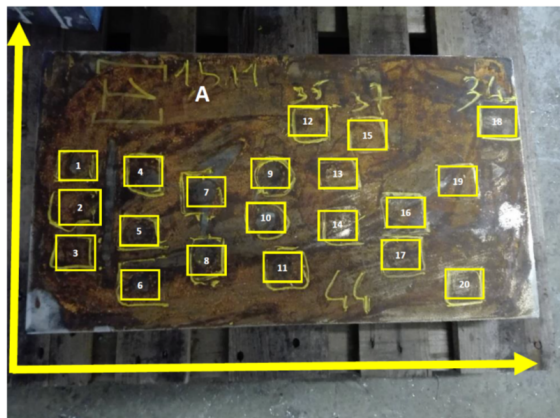


Figure 1. Sheet A, 310 × 600 × 90 (mm). The extraction positions of the test articles are highlighted in yellow.



Figure 2. Sheet B, 410 × 620 × 90 (mm). The extraction positions of the test articles are highlighted in yellow.

Table 1. Main tensile properties of steel SA 516 - Gr. 70

Property	Value
Ultimate Tensile Strength	(520±10) MPa
Yield Strength	(320±10) MPa
Elongation after fracture	(23±3)%
Reduction of area	(67±3)%

The main tensile properties of the material under investigation are listed in table 1. The reported values have been obtained averaging 4 measurements (using test articles extracted from the two sheets). We estimated the dispersion by calculating the standard deviation of these 4 values. The results shown in table 1 do not suggest that a difference can be noted between the mechanical properties of the two sheets, in accordance with the supplier's declaration.

2.2. Description of test articles

The test articles have been extracted from the two sheets through thickness. The extraction positions are highlighted in figures 1 and 2. The geometric dimensions of the test articles are shown in figure 3

We obtained 20 test articles from sheet A (named A1,..., A20) and 19 test articles from sheet B (named B1,..., B19).

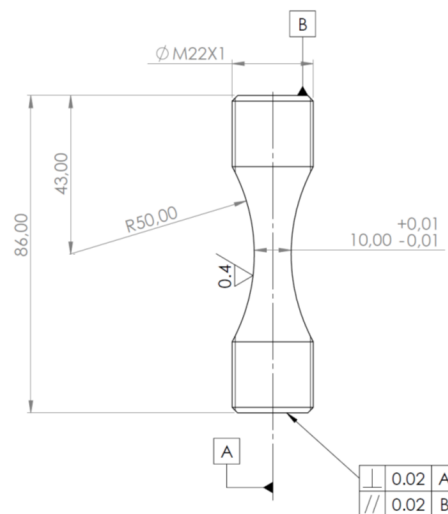


Figure 3. Test article design.

2.3. Experimental setup and test conditions

A campaign of high cycle number fatigue tests was carried out using a resonance testing machine. The test conditions are shown in table 2. The test campaign was conducted following the UNI 3964 standard [1] having the foresight to select test articles from sheet A in the first part of the test campaign and from sheet B in the second and final part. In each of the two parts, the selection was random. This method of conducting the test campaign allows highlighting any differences between the mechanical fatigue behaviour of the test articles extracted from sheet A compared to the ones extracted from sheet B. In fact, the number of test articles present in each of the two parts of the test allows statistically reliable values to be obtained considering separately the data obtained by analyzing the test articles extracted from each of the sheets as well.

Table 2. Test conditions.

Parameter	Value or condition
Loading configuration	Uniaxial tension
Load ratio ($\sigma_{\min}/\sigma_{\max}$)	0
Runout	5×10^6 cycles
Criterion for choosing the stress level $\Delta\sigma$	Staircase procedure
Testing machine	Rumul resonant machine 150 kN
Test frequency	175-176 Hz
Test control	Force
Test temperature	Room temperature
Test stop condition	Frequency drop greater than 1 Hz

The fatigue test facility, with the resonant testing machine, is shown in figure 4. Figure 5 shows the detail of the assembly of the test article in the testing machine.



Figure 4. Fatigue test facility.

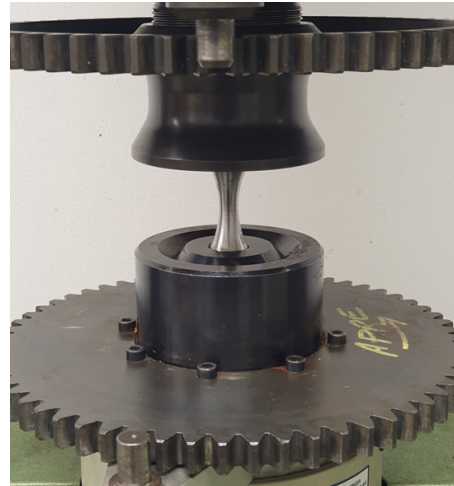


Figure 5. Detail of the test article assembly.

3. Fatigue strength estimation

It is well known that the fatigue strength is not a physical quantity measurable as such. On the contrary, we can obtain the mean value of this quantity (for a statistically homogeneous population) by carrying out a statistical analysis of a binary random variable that indicates the outcome of each of the tests in the campaign (runout/failure). In the present work, we present three methods of statistical analysis. Two of them are applied in two different ways: the test campaign is either considered as a whole or divided into two sub-campaigns.

3.1. Outline of the Staircase Procedure

We briefly recall the data processing method reported in the UNI 3964 standard [1] for determining the fatigue strength. We recall that this method was presented by Dixon and Mood [2] in a famous work in which they show, for the staircase experimental procedure, a smart approximation for the determination of the likelihood maximum. We will describe the “exact” likelihood maximization method in subsection 3.3. In table 3 we list the quantities used in the elaboration of the calculation. The method of analysis that we describe below is applicable in the hypothesis that the sample (set of test articles) is extracted from a Gaussian population distribution.

With the symbols listed in table 3, it is possible to compute the mean value $\Delta\sigma_{50\%}$ of the fatigue strength S as

$$\Delta\sigma_{50\%} = \Delta\sigma_0 + d \left(\frac{A}{N} \pm \frac{1}{2} \right) \quad \begin{array}{l} + \text{ if the less frequent outcome is the runout} \\ - \text{ if the less frequent outcome is the failure} \end{array} \quad (1)$$

Table 3. Nomenclature used in staircase procedure.

Symbol	Nomenclature
d	Staircase stress step
$\Delta\sigma_0$	Lowest stress amplitude for the less frequent event in the test results
N	Number of the less frequent event
i	Stress level index, starting from 0
n_i	Number of samples at the stress level i
A	$\sum i \cdot n_i$
B	$\sum i^2 \cdot n_i$
$\Delta\sigma_{50\%}$	Mean value of the fatigue strength $\Delta\sigma$ (50% survival probability)
$\Delta\sigma_{10\%}$	Fatigue strength corresponding to 10% survival probability
$\Delta\sigma_{90\%}$	Fatigue strength corresponding to 90% survival probability
SD	Standard deviation of $\Delta\sigma$

and the standard deviation SD of this mean value as

$$SD = 1.62d \left(\frac{NB - A^2}{N^2} + 0.029 \right) \quad \text{valid if } \frac{NB - A^2}{N^2} > 0.30. \quad (2)$$

The extremes of the 90% confidence interval for $S_{50\%}$ can be obtained, again assuming the validity of Gaussian population, computing

$$\Delta\sigma_{10\%} = \Delta\sigma_{50\%} + 1.96 \cdot SD \quad \Delta\sigma_{90\%} = \Delta\sigma_{50\%} - 1.96 \cdot SD. \quad (3)$$

3.2. Elaboration of the staircase outcomes

In figure 6 we show the outcomes of the staircase procedure, obtained starting at an initial stress level of 235 MPa and with a staircase stress step $d = 12$ MPa. It should be noted that, although the knowledge of the mechanical properties of the material under examination allowed to establish a reasonable stress value for the first test, three test articles were “wasted” in determining the correct initial stress level (two consecutive different outcomes are needed to start the staircase procedure).

		Actual tests sequence																											
ID		A14	A16	A17	A19	A20	A10	A8	A11	A9	A7	B4	B15	B3	B13	B16	B5	B10	B14	B8	B11	B2	B17	B6	B9	B12	B7	B19	B18
$\Delta\sigma$ (MPa)	271																		x		x								
	259																	.		.		x							
	247																.						x		x				
	235	x		x												.							x		x			x	
	223		.		x										.		.							.		x		.	x
	211					x				x					.														x
	199						x		.		x			.															
	187							.					.																
N (Mcycle)		0.24	5.00	1.37	0.85	3.63	0.88	5.00	5.00	0.98	0.33	5.00	5.00	5.00	5.00	5.00	5.00	5.00	0.11	5.00	0.34	0.33	0.70	5.00	0.23	1.29	5.00	0.32	2.11

Figure 6. Staircase outcomes assuming the sample of test articles as a unique set.

The statistical analysis carried out using equations 1, 2 and 3 provides the results presented in table 4.

Table 4. Fatigue strength assuming the sample of test articles as a unique set.

$\Delta\sigma_{10\%}$ [MPa]	$\Delta\sigma_{50\%}$ [MPa]	$\Delta\sigma_{90\%}$ [MPa]	SD [MPa]	$SD/\Delta\sigma_{50\%}$ [%]
326	228	130	77	33.6

We note that SD is very large when compared with $\Delta\sigma_{50\%}$. Generally, in fact, the ratio $SD/\Delta\sigma_{50\%}$ falls in the interval $[0.080, 0.16]$. We also note that, when the results obtained with this procedure are inserted in the relationship $\Delta\sigma_{50\%} - 6 \cdot SD$ for the determination of the endurance limit, we obtain a negative value. This result would make this material useless in the mechanical design. Observing that all the first seven outcomes carried out using test articles from sheet B were runouts, we analyzed separately the results involving test articles extracted from different sheets. The outcomes are shown in figures 7 and 8. Note that the first six outcomes of the test articles extracted from sheet B cannot be considered in the statistical analysis. We aim to highlight a possible difference between the average values of the mean fatigue strength in the two groups.

Sheet metal A – Actual test sequence											
ID	A14	A16	A17	A19	A20	A10	A8	A11	A9	A7	
$\Delta\sigma$ (MPa)	235	x		x							
	223		*		x						
	211					x				x	
	199						x		*		x
	187							*			
N (Mcycle)	0.24	5.00	1.37	0.85	3.63	0.88	5.00	5.00	0.98	0.33	

Figure 7. Staircase outcomes considering only test articles from sheet A.

Sheet metal B – Actual test sequence															
ID	B10	B14	B8	B11	B2	B17	B6	B9	B12	B7	B19	B18			
$\Delta\sigma$ (MPa)	271		x		x										
	259	*		*		x									
	247						x		x						
	235							*		x			x		
	223										*			x	
N (Mcycle)	5.00	0.11	5.00	0.34	0.33	0.70	5.00	0.23	1.29	5.00	0.32	2.11			

Figure 8. Staircase outcomes considering only test articles from sheet B.

The statistical analyses carried out following equations 1,2,3 for the computation of the fatigue strength limit provides the results presented in tables 5 and 6.

Table 5. Fatigue strength analyzing only test articles from sheet A.

$\Delta\sigma_{10\%}$ [MPa]	$\Delta\sigma_{50\%}$ [MPa]	$\Delta\sigma_{90\%}$ [MPa]	SD [MPa]	$SD/\Delta\sigma_{50\%}$ [%]
248	209	170	31	14.7

Table 6. Fatigue strength analyzing only test articles from sheet B.

$\Delta\sigma_{10\%}$ [MPa]	$\Delta\sigma_{50\%}$ [MPa]	$\Delta\sigma_{90\%}$ [MPa]	SD [MPa]	$SD/\Delta\sigma_{50\%}$ [%]
293	250	207	33	13.3

Comparing the results reported in tables 5 and 6 with the results of table 4, we observe that the average values for the two groups are significantly different from each other and that the standard deviation of each of the two groups is about half the standard deviation obtained when the results from the test articles extracted from the two sheets are analyzed together.

3.3. Fundamentals of the Maximum Likelihood Principle

To enforce the result shown above, we analyzed the same test campaign outcomes using likelihood maximization. Maximum Likelihood Estimators (MLE) must be obtained using a numerical procedure to obtain the values of the arguments at which maximum is attained. In this section, we briefly outline the parameter estimation procedure based on maximization of likelihood (ML). The random variable $\Delta\sigma$, assumed to be Gaussian, cannot be sampled. Given a sample made of n test articles, for the k -th test ($k = 1, \dots, n$), a binary response random variable e_k is sampled: $e_k = 0$ for runout and $e_k = 1$ for failure.

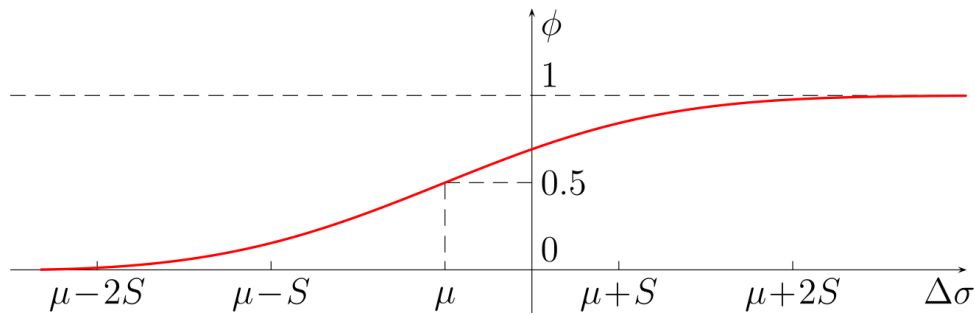


Figure 9. An example of Gaussian cumulative distribution function.

Let μ and S be the mean and standard deviation of the random variable $\Delta\sigma$. If we introduce the cumulative distribution function ϕ (see figure 9), i.e. the probability $\mathbb{P}$ (for given values of μ and S) that the random variable $\Delta\sigma$ is less than the value of $\Delta\sigma_k$ used in the k -th test, as

$$\mathbb{P}(\Delta\sigma < \Delta\sigma_k) = \phi(\Delta\sigma_k|\mu, S) = \frac{1}{\sqrt{2\pi}S} \int_{-\infty}^{\Delta\sigma_k} \exp\left(-\frac{(x-\mu)^2}{2S^2}\right) dx \quad (4)$$

we can define $L(\mu, S)$ as the logarithm of the likelihood function:

$$L(\mu, S) = \sum_{k=1}^n [e_k \log(\phi(\Delta\sigma_k|\mu, S)) + (1 - e_k)(1 - \log(\phi(\Delta\sigma_k|\mu, S)))]. \quad (5)$$

We can obtain the MLEs of μ and S taking the couple of values $\hat{\mu}$ and $\hat{S}$ at which L attains its maximum value.

3.4. Maximum Likelihood estimates and comparison with staircase results

In figure 10 we show the outcomes of the staircase procedure used to obtain the ML estimators. It should be noted that the three initial failure outcomes can be inserted in the calculations, increasing the value of n .

The statistical analysis carried out using maximization of 5 provides the results presented in table 7.

		Elaborated samples																														
ID		A5	A6	A13	A14	A16	A17	A19	A20	A10	A8	A11	A9	A7	B4	B15	B3	B13	B16	B5	B10	B14	B8	B11	B2	B17	B6	B9	B12	B7	B19	B18
$\Delta\sigma$ (MPa)	271	x																				x		x								
	259		x																						x							
	247			x																						x		x				
	235				x		x																									
	223					.		x																								
	211								x				x																			
	199									x																						
	187											.																				
N (Mcycle)		0.23	0.63	0.14	0.24	5.00	1.37	0.85	3.63	0.88	5.00	5.00	0.98	0.33	5.00	5.00	5.00	5.00	5.00	5.00	5.00	0.11	5.00	0.34	0.33	0.70	5.00	0.23	1.29	5.00	0.32	2.11

Figure 10. Outcomes used in MLE calculations, considering the test articles all together.

Table 7. MLE estimators of fatigue strength assuming the sample of test articles as a unique set.

$\Delta\sigma_{10\%}$ [MPa]	$\Delta\sigma_{50\%}$ [MPa]	$\Delta\sigma_{90\%}$ [MPa]	SD [MPa]	$SD/\Delta\sigma_{50\%}$ [%]
295	218	141	60	27.5

We note that the ML estimate of SD is smaller than the Dixon-Mood estimate. A comparison of the estimates of $\Delta\sigma_{50\%}$ and SD obtained with different methods is reported in [3]. The bias of Dixon-Mood SD estimators has been shown in [4] and is not generally considered critical, since it tends to decrease the endurance limit. Notwithstanding this, we observe that, when these results are inserted in the relationship $\Delta\sigma_{50\%} - 6 \cdot SD$ for the determination of the endurance limit, we again obtain a negative value. Again we analyzed separately the results involving test articles extracted from different sheets. The outcomes are shown in figures 11 and 12, for which we can again use all the outcomes.

		Sheet metal A – Elaborated samples													
ID		A5	A6	A13	A14	A16	A17	A19	A20	A10	A8	A11	A9	A7	
$\Delta\sigma$ (MPa)	271	x													
	259		x												
	247			x											
	235				x		x								
	223					*		x							
	211								x				x		
	199									x		*		x	
	187										*				
N (Mcycle)		0.23	0.63	0.14	0.24	5.00	1.37	0.85	3.63	0.88	5.00	5.00	0.98	0.33	

Figure 11. Outcomes used in MLE calculations, considering only test articles from sheet A.

		Sheet metal B – Elaborated samples																	
ID		B4	B15	B3	B13	B16	B5	B10	B14	B8	B11	B2	B17	B6	B9	B12	B7	B19	B18
$\Delta\sigma$ (MPa)	271								x		x								
	259							.		.		x							
	247						.						x		x				
	235					.								.		x		x	
	223				.												.		x
	211		.																
	199		.																
	187	.																	
N (Mcycle)		5.00	5.00	5.00	5.00	5.00	5.00	5.00	0.11	5.00	0.34	0.33	0.70	5.00	0.23	1.29	5.00	0.32	2.11

Figure 12. Outcomes used in MLE calculations, considering only test articles from sheet B.

The statistical analyses carried out maximizing likelihood defined in equation 5 provides the results presented in tables 8 and 9.

Table 8. ML estimators of fatigue strength obtained analysing only test articles from sheet A.

$\Delta\sigma_{10\%}$ [MPa]	$\Delta\sigma_{50\%}$ [MPa]	$\Delta\sigma_{90\%}$ [MPa]	SD [MPa]	$SD/\Delta\sigma_{50\%}$ [%]
228	196	166	25	12.6

Table 9. ML estimators of fatigue strength obtained analysing only test articles from sheet B.

$\Delta\sigma_{10\%}$ [MPa]	$\Delta\sigma_{50\%}$ [MPa]	$\Delta\sigma_{90\%}$ [MPa]	SD [MPa]	$SD/\Delta\sigma_{50\%}$ [%]
285	244	203	32	13.1

Comparing the results reported in tables 8 and 9 with the results of table 7, we observe that also for this statistical technique the average values for the two groups are significantly different from each other. This result confirm that the difference between the two mean values is not due to a limitation of the Dixon-Mood approximation, but is a consequence of a real difference between some of the mechanical properties of the two sheets, only nominally equal.

3.5. Maximum Likelihood estimation of a mixture of Gaussian distributions

We present here an extension of the ML method using, as a distribution function, a mixture of two Gaussians. We define a linear combination ψ of two Gaussian cumulative distribution function with two different mean values μ_1 and μ_2 and (for simplicity) the same standard deviation S for both components as

$$\psi(\Delta\sigma_k|\mu_1, \mu_2, S, \alpha) = \alpha\phi(\Delta\sigma_k|\mu_1, S) + (1 - \alpha)\phi(\Delta\sigma_k|\mu_2, S). \quad (6)$$

where $\alpha \in (0, 1)$ is the relative weight of the two components. In this way, a new objective function (to be maximized) $L'(\mu_1, \mu_2)$ can be defined as

$$L'(\mu_1, \mu_2) = \sum_{k=1}^n [e_k \log(\psi(\Delta\sigma_k|\mu_1, \mu_2, S, \alpha)) + (1 - e_k)(1 - \log(\psi(\Delta\sigma_k|\mu_1, \mu_2, S, \alpha)))] \quad (7)$$

in which, for simplicity, the same standard deviation S has been kept fixed (30 MPa, a reasonable value obtained by previous analyses), and also the weights α and $1 - \alpha$ have been kept fixed as well (0.42 and 0.58, values of the relative numbers of test articulated from the two sheets). The numerical maximization of L' provides $\mu_1 = 191$ MPa and $\mu_2 = 269$ MPa confirming a separation of the two mean values more than twice a reasonable estimation of the standard deviation. These results do not vary much if S and α are varied within a reasonable range of values (20-40 MPa, 0.3-0.6). However, in the current state of research, it is not possible to show reasonable values of the estimate of S using a Gaussian mixture using the ML method because the estimation of S is lowered until it reaches very small and meaningless values. The probability density function obtained with this method is shown in figure 13.

4. Conclusions

In this paper, we have shown the differences that can occur if we introduce the hypothesis that the sample is not distributed like a Gaussian. This hypothesis, which is usually assumed to be true in the analysis of fatigue tests, is very strong and conditions the interpretation of a series of fatigue tests. In staircase tests, 90% confidence intervals for the means show that $\Delta\sigma_{50\%}$ seems to be different for sheets A and B nominally made of the same steel. The standard deviation S is slightly overestimated in the Staircase Procedure with respect to the Maximum Likelihood estimation. A new procedure for the estimation of means of a mixture of two Gaussians confirms the difference between the values of $\Delta\sigma_{50\%}$ for the two sheets. The Staircase method tends to be conservative, and even ML estimates do not differ much. Perhaps the development of

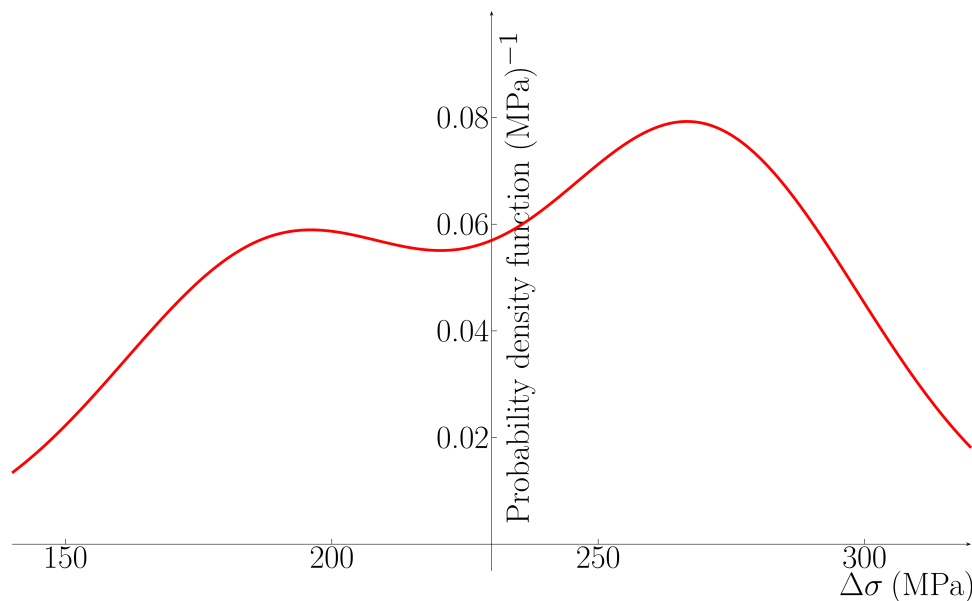


Figure 13. The probability distribution function of the population, modellized as a mixture of gaussian distributions.

new test/analysis methodologies could provide results that improve component design. For the estimation of the endurance limit for this material (to be used in component design), we propose for $\Delta\sigma_{50\%}$ the lowest of the two mean values (209 MPa) and for SD the corresponding value (31 MPa) as obtained from the analysis, carried out according to [1], of test articles extracted from sheet A only, which has the worst mechanical fatigue characteristics. For the endurance limit, we obtain therefore $\Delta\sigma_{50\%} - 6 \cdot SD = 23$ MPa. Although small, it is still a value that can be used in mechanical design. In future developments of this work, we will investigate the characteristics of the Gaussian mixture method also using Monte Carlo simulations, to obtain, for instance, the confidence intervals of the estimates. It should be as well interesting a comparison of MLEs with other estimators or other statistical techniques (Fisher Information, Bayesian analysis).

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