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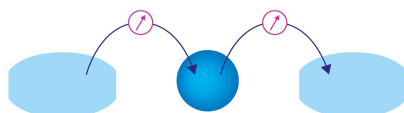
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ABSTRACT

Pseudo-magnetic field (PMF) in deformed graphene has been proposed as a promising and flexible method to quantum-confine electronic states and create gaps in the local density of states. Motivated by this perspective, we numerically analyze various different configurations leading to electronic localization and band flattening in periodically strained graphene. In particular, we highlight the existence of a fine structure in the pseudo-Landau levels confined in large-PMF regions, the emergence of states confined to PMF *nodes* as well as of snake-like orbits. In our paper, we further analyze the importance of the relative rotation and asymmetry of the strain lattice with respect to the atomic lattice and show how it can be used to modulate the PMF periodicity and to create localized orbits far from the strain points. Possible implementations and applications of the simulated structures are discussed.

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I. INTRODUCTION

The impact of the mechanical deformation on the electronic band structure of graphene can be effectively described in terms of a fictitious magnetic field, which is routinely called pseudo-magnetic field (PMF).^{1–3} The PMF offers a non-standard and promising method to tune the band structure of graphene. It can be exploited not only to induce the formation of pseudo-Landau levels (pLLs)^{4–6} but also to trap electrons in pseudo-magnetic quantum dots.^{7,8} Exploiting the pseudo-spin dependent action of PMFs,^{8–10} electron waveguides,¹¹ and valley-filtering devices^{11–14} can be created.

A particularly attractive perspective opened by the periodic PMFs is the possibility to induce a flatband dispersion,^{15,16} where electrons have a quenched kinetic energy and are more prone to form correlated states driven by electron–electron interactions.^{17–21} Recently, periodic strain profiles have been demonstrated in twisted multilayers,^{22,23} providing the cleanest possible implementation of the periodic PMF concept so far. Alternative approaches include the use of non-planar substrates^{24–27} as well as micrometric

polymeric or metallic^{28–31} actuators, which are also used to strain two-dimensional materials different from graphene.^{32,33} The latter methods yield flexible strain profiles and periodicity schemes at the expense of the more invasive action of the micrometric actuators, which can strongly impact the graphene underneath.

Here, motivated by these results, we numerically analyze the different mechanisms and factors leading to quantum confinement of electronic states in periodically strained graphene and to the emergence of flatband dispersion. We show how the relative orientation between the atomic and strain lattices can lead to markedly different PMF patterns and band profiles. In addition, we correlate the emergence of flat dispersions with the electrons localization in the strain lattice. This allows to identify: pLL multiplets in regions of large PMF, delocalized snake-like states propagating in between regions of different PMF signs, and modes confined in the PMF nodes. Furthermore, based on the possibility of controlling the strain on the individual sites, we simulate strain lattices with reduced symmetry and discuss the appearance of edge states, which are localized inside zero-field regions and surround the PMF lobes.

These ring-like states could have possible applications for the creation of electronic waveguides, similarly to what is expected in the case of folded graphene.¹¹

This paper is organized as follows: in Sec. II, we describe the theoretical model which we used in our numerical analysis; in Sec. III, we investigate how the relative rotation between the atomic lattice of graphene and the strain lattice can modulate the resulting PMF pattern; in Sec. IV, we model a periodic stress lattice with two uneven stress sites per supercell and calculate the resulting electronic structure of the system. Finally, the conclusions are drawn in Sec. V.

II. MODEL FOR STRAIN-INDUCED CONFINEMENT

The properties of electrons in deformed graphene near the Dirac points (K and K') can be described through the effective low-energy Hamiltonian^{3,34}

$$\mathcal{H} = v_F \begin{pmatrix} \boldsymbol{\tau}(\mathbf{p} - e\mathbf{A}_{ps}) & 0 \\ 0 & -\boldsymbol{\tau}(\mathbf{p} + e\mathbf{A}_{ps}) \end{pmatrix}, \quad (1)$$

for the four-component spinor $\chi = (\psi_A^K, \psi_B^K, \psi_A^{K'}, \psi_B^{K'})^T$, where A and B stand for the two pseudospin components, v_F is the Fermi velocity, e is the electronic charge, $\boldsymbol{\tau} = (\tau_x, \tau_y)$ is the vector of Pauli matrices, and $\mathbf{p} = -i\hbar(\partial_x, \partial_y)^T$. The term $\mathbf{A}_{ps} = (A_x, A_y)$ can be seen as a vector potential producing a PMF $B_{ps} = \partial_x A_y - \partial_y A_x$. The potential $\mathbf{A}_{ps}$ is directly related to the mechanical deformation of graphene and can be expressed in terms of the strain tensor ϵ ^{1,35} (see the [supplementary material](#)),

$$A_x = \frac{\hbar\beta}{ea} [(\epsilon_{xx} - \epsilon_{yy})\cos(3\theta) - 2\epsilon_{xy}\sin(3\theta)], \quad (2)$$

$$A_y = -\frac{\hbar\beta}{ea} [(\epsilon_{xx} - \epsilon_{yy})\sin(3\theta) + 2\epsilon_{xy}\cos(3\theta)], \quad (3)$$

where β is the Grüneisen parameter ($\beta \approx 2$),³⁶ a is the lattice constant, and θ is the angle between the x-axis and the zigzag direction in the graphene crystal (see Fig. 1). We note that Eqs. (2) and (3) can be used for strains up to 15%.^{8,37}

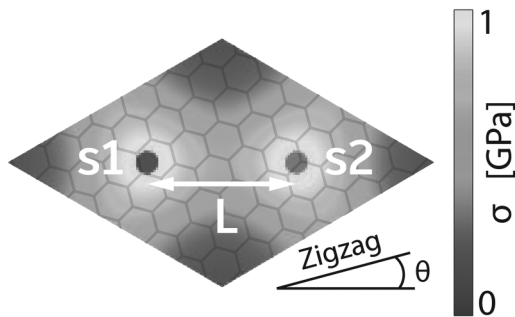


FIG. 1. Sketch of the stress configuration (σ) inside the superlattice primitive cell. The sites $s1$ and $s2$ identify the two stress sites. The underlying honeycomb lattice represents graphene (not drawn to scale).

In our numerical simulation, we generate a periodic strain pattern modeled using a set of Gaussian compressive stress profiles, which provides a first-order description of the effect of a circular actuator directly deposited on graphene (see the [supplementary material](#)). We do not expect relevant effects for small deviations from the circularly symmetric profiles to slightly elliptical ones since the PMF is weakly dependent on this parameter (see the [supplementary material](#)). Stress sites are arranged in an infinite honeycomb superlattice, which allows exploring a large range of confinement scenarios depending on its orientation and asymmetry (see Fig. 1). Given this choice, the whole system can be described simulating two stress regions within the primitive supercell, which coincides with the simulation cell where we solve the eigenvalue equation $\mathcal{H}\chi = E\chi$, for the graphene valley K and the pseudospin A [see Eq. (1)].

The two nonequivalent sites ($s1$ and $s2$) are characterized by two stresses with amplitudes σ_{s1} and σ_{s2} ; in our simulations, we consider two kinds of stress patterns: (i) a symmetric pattern within the primitive cell, i.e., $\sigma_{s1} = \sigma_{s2}$ and (ii) an asymmetric pattern with $\sigma_{s1} \neq \sigma_{s2}$. Note that our system is characterized by different periodicity scales: the periodicity of graphene $a \simeq 2.46$ Å and the periodicity of the mechanical superlattice, with lattice constant $a_\sigma = L\sqrt{3}$, where L is the distance between the two stress sites inside the primitive supercell (see Fig. 2). Experimentally accessible

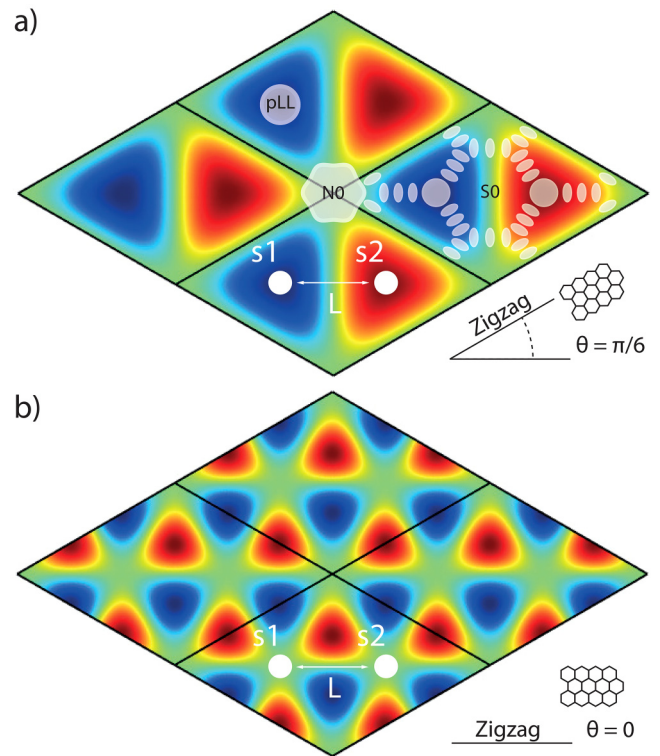


FIG. 2. (a) Primitive cell and pseudo-magnetic field B_{ps} for $\theta = \pi/6$. The sites $s1$ and $s2$ identify the two sublattices of the stress pattern. Schematic sketch of wavefunctions for $N0$, pLL , and $s0$ states, as discussed in the main text. (b) Same sketch of panel (a) for $\theta = 0$.

situations provide a large size difference between the graphene lattice and mechanical superlattice; considering L in a range from few hundred nanometers to few micrometers results in a ratio $a/a_\sigma \simeq 10^{-4} - 10^{-5}$.

Simulations are implemented using a commercial FEM solver (COMSOL Multiphysics). The graphene deformations are calculated by a 2D continuum mechanics solver using $E = 1$ TPa and $\nu = 0.15$ as the Young modulus and Poisson ratio, respectively.³⁸ Being the superlattice periodicity much larger than the periodicity of the graphene crystal, we considered graphene as a continuous and isotropic 2D mechanical material. A first stationary COMSOL simulation was run in order to obtain the strain tensor ε in the superlattice primitive cell (see Fig. 1), using an initial stress of 1 GPa at each stress site $s1$ and $s2$. Our mechanical simulations did not consider the relative alignment of superlattice and graphene crystals θ ; therefore, the orientation angle affects only Eqs. (2) and (3). If the periodicity of the superlattice would be reduced to be comparable with the graphene one, more sophisticated atomistic simulations would be needed to correctly evaluate the strain tensor. Once the strain tensor has been calculated, the pseudo-vector potential $A_{ps}(x, y)$ and the PMF profile $B_{ps}(x, y)$ are obtained according to Eqs. (2) and (3) (see Fig. 2). Finally, the electronic states and dispersions are obtained by diagonalizing the 2D Dirac equation within the First Brillouin Zone of the superlattice.^{1,39} The eigenvalue problem is solved by imposing k -dependent Floquet–Bloch conditions at each side of the superlattice primitive cell.⁴⁰ The periodic condition sets the reciprocal space coordinates within the First Brillouin Zone; the numerical solution of the eigenvalue equation at (k_x, k_y) returns the eigenvalues $E(k_x, k_y)$ from which we build the energy bands. Exploiting the linearity between PMFs and strain,³⁷ the mechanical simulations are performed at a few selected stress values. The value of the PMF is then rescaled to obtain the full set of B_{ps} . The strength of the field is expressed in terms of an adimensional parameter $\xi = L/\ell_B$, where $\ell_B = \sqrt{\hbar/eB_{\max}}$ is the magnetic length and $B_{\max}$ is the maximum $|B_{ps}|$ value. Similarly, energy values will be reported in terms of an adimensional parameter $\tilde{E}$ which is connected to the actual electron energy E by $\tilde{E} = E\sqrt{S}/\hbar v_F$, where S is the surface of the superlattice primitive cell.

To give a more realistic perspective for experiments, we can set hard limits for the value of L which, in turn, allows to evaluate device parameters in absolute units. Considering a fabrication realistic L in a range from 600 nm to 3 μm , we obtain a maximum PMF/strain magnitude of 375 mT/1% and 15 mT/0.2%, respectively. Note that the strain is well within the limit for the use of Eqs. (2) and (3).³⁷ However, we observe that for the largest L value the system energy is relatively small ($\tilde{E} = 50 \rightarrow E = 7$ meV), posing a challenge in a possible device performance if not operated at very low temperature and with defect-free graphene. More reasonable values can be found for $L = 600$ nm, where, for example, the energy of the first pLL (at fixed $\xi = 15$) goes from 5 to 25 meV. Considering that state of the art lithographic facilities can push the system size well below a few hundred nanometers, we concluded that our system complies with experimental observation and its fabrication constraints are ameliorated by operating with low-defect graphene and in a soft-cryogenic environment.

III. ROLE OF ANGULAR ORIENTATION AND IDENTIFICATION OF FLATBAND STATES

The geometry and periodicity of the PMF induced by a periodic stress can be modulated through the relative rotation of graphene with respect to the applied stress pattern, which is parametrized by the angle θ (see Fig. 2). As it can be deduced from the threefold rotational symmetry of Eqs. (2) and (3), $2\pi/3$ rotations leave the pseudo-magnetic field unchanged, while $\pi/3$ -rotations flip its sign. The dependence of angular orientation can be summarized considering the two significant angles $\theta = \pi/6$ and $\theta = 0$. The markedly different PMFs resulting from these cases are displayed in Figs. 2(a) and 2(b), respectively. In the case $\theta = \pi/6$, the field has two lobes with antinodes located at the stress sites $s1$ and $s2$, as displayed in Fig. 2(a). The opposite happens for the case $\theta = 0$ [Fig. 2(b)], where the nodes are located at the stress sites $s1$ and $s2$. By rotating the superlattice, we can shift the PMF in order to have its nodes strongly/weakly overlapping the stress sites. This could become important in an experimental framework where the disturbance at the stress sites makes the underlying graphene inaccessible or significantly perturbed.

For each rotation angle, we use solutions of Eq. (1) to obtain the density of states (DoS) as a function of the PMF field magnitude. This is displayed in Fig. 3(a) for the case $\theta = \pi/6$. As reported in different recent works,^{16,41} the evolution for increasing fields is complex, with a variety of states migrating toward lower and higher energies.

The former ones [see “L0-migrating states” in Fig. 3(a)] converge to the zero-energy level, originating from the $n = 0$ Landau level, which has been observed in graphene in several experiments^{4,5,42} using real magnetic fields.

The blue-shifting set of states (L1, L2, N0, and S0) follow different trends. A general ξ -proportionality can be identified for two sets of states, L1 and L2 [see Fig. 3(a)], which forms from around $\xi \sim 10$ and nearly follows the trend calculated for $n = 1$ and $n = 2$ Landau levels in real magnetic field [white dashed lines in Fig. 3(a)]. This behavior suggests that L1 and L2 states are related to Landau levels originating from the quasi-uniform PMF, which are called pseudo-Landau levels (pLLs). Additionally to the expected pLLs, we identify a low energy peak in the DoS, (N0), which blue-shifts sublinearly with ξ as well as a set of multiple states, (S0), which increase superlinearly with ξ .

The yellow-colored peaks in the DoS of Fig. 3(a) correspond to energy bands, which are flat in the First Brillouin Zone of the supercell [panel (b)]. Note that high symmetry points of the superlattice ($\bar{\Gamma}$, $\bar{K}$, $\bar{K}'$) have been indicated with the bar notation to not confuse them with the ones used for graphene (Γ , K , K'). To show all the different flatband states we described earlier, we have chosen two significant band structures for $\xi = 10, 15$ in order to illustrate S0 states (S0' and S0'') and L0, L1, L2, N0, respectively (for further details, see the supplementary material). The single points in the band structures in Fig. 3(b) have been scaled in size and color-coded using two field-confinement parameters: η^- and η^+ . These are defined as follows:

$$\eta^\pm = \int_S |\psi(\vec{r})|^2 |\tilde{B}(\vec{r})| \Theta[\pm \tilde{B}(\vec{r})] d\vec{r}, \quad (4)$$

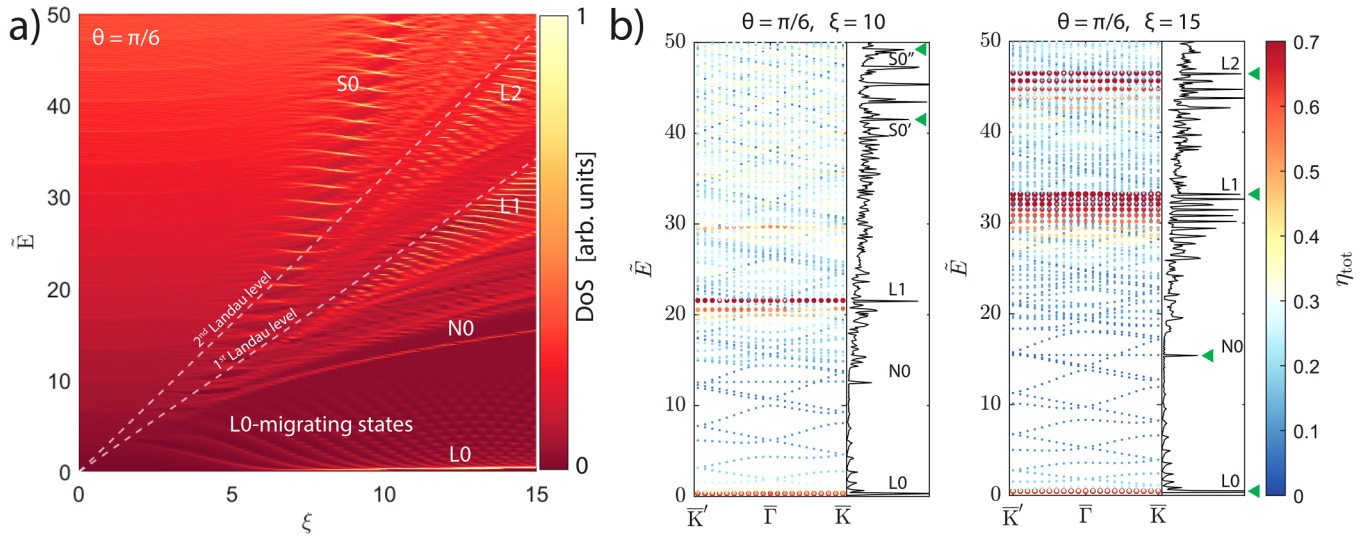


FIG. 3. (a) Normalized density of states as a function of adimensional energy $\tilde{E}$ and PMF (ξ). The white dashed lines are the first and second Landau levels in graphene for real and uniform magnetic fields. (b) Energy band structure and the corresponding density of states [cross section from panel (a)] for $\xi = 10$ (left) and $\xi = 15$ (right). The green triangles in the DoS correspond to the labeled states.

where $\tilde{B}$ is the normalized PMF, Θ the Heaviside function, and S is the surface of the superlattice primitive cell. A wavefunction deeply confined within a PMF lobe will have a large $\eta_{\text{tot}} = \eta^+ + \eta^-$. Conversely, a wavefunction localized within the PMF nodes will have a vanishing η_{tot} . We observe that, combining $\eta^\pm$, the flat states can be operatively classified in terms of their electron confinement, as shown in Fig. 3(b).

We can immediately notice that levels N0 and S0, though being flat and peaked in the density of states, are located outside the field lobes; on the other hand, we observe that the levels L0, L1, and L2 are characterized by peaks in the density of states and high values of η_{tot} (high electron confinement inside the field lobes). The confinement confirms their interpretation as pLLs originating from the PMF. As is the case for the Landau levels in real field, we expect a single degeneracy for level L0; both L1 and L2 present more complex features and by inspecting the energy bands it can be seen that are characterized by a fine structure composed by several subbands approximately equispaced in energy. This particular configuration is due to the non-homogeneous nature of PMFs; the spacing of the pLL subbands can be estimated by considering the number m of iso-field orbits, which enclose an integer magnetic flux (mh/e). The magnetic fields satisfying these relationships, B_m , can be found by solving the following equation:

$$\int_S B_{ps}(\vec{r}) \Theta[B_m - B_{ps}(\vec{r})] d\vec{r} - m \frac{h}{e} = 0. \quad (5)$$

The selected B_m can be plugged in the expression for Landau levels in graphene in a uniform and real magnetic field $E_{n,m} = v_F \sqrt{2e\hbar B_m n}$.³⁴ The results of this numerical evaluation for the first pLL are displayed in Fig. 4(a). The estimated energy

spacing is roughly linear and agrees well with the one for the subbands, with a small discrepancy within 4%. A spatial visualization of the first eight orbits is reported in Fig. 4(b), superimposed to the PMF profile. The good agreement between direct numerical calculation and the results obtained through Eq. (5) can be appreciated in Fig. 4(c), where the $m = 8$ orbit is superimposed with the corresponding wavefunction. Similar results have also been found for the second pLL, L2.

To get some insight into the difference between the pLLs and N0 and S0 states, we plot in Fig. 5 the corresponding wavefunctions. The difference in the eigenstates corresponding to the levels S0, N0, L0, and L1 can be clearly seen. As discussed, the pLLs are confined within the PMF regions, with a characteristic wavefunction length scale given by $\ell_m = \sqrt{\hbar/eB_m}$. Regarding the L0 level, we expect the electronic wavefunction for a given pseudospin to be localized only within the positive or negative PMF region.¹⁵ This is exactly what can be seen in Fig. 5, where the single degenerate L0 state is confined within positive B_{ps} . Changing graphene pseudospin, eigenstate confinement would look exactly the same as in Fig. 5, although it would be confined within a negative B_{ps} region. The levels belonging to L1 are characterized by a double degeneracy, i.e., [L1(I), L1(II)]. One of the two degenerate levels is confined within the positive field region, while the other is confined in the negative one. In addition to the pLLs, geometrical considerations suggest that electrons can build resonances by multiple field-induced scattering, forming Bloch states that are located outside or just partially overlapping with the PMFs. This is what is observed for states N0 and S0. The former is a flat level completely localized within zero-field regions ($\eta_{\text{tot}} \simeq 0.02$).¹⁶ Its peculiar localization is convenient in an experimental framework, being outside the stress sites s1 and s2 which can strongly perturb the electronic

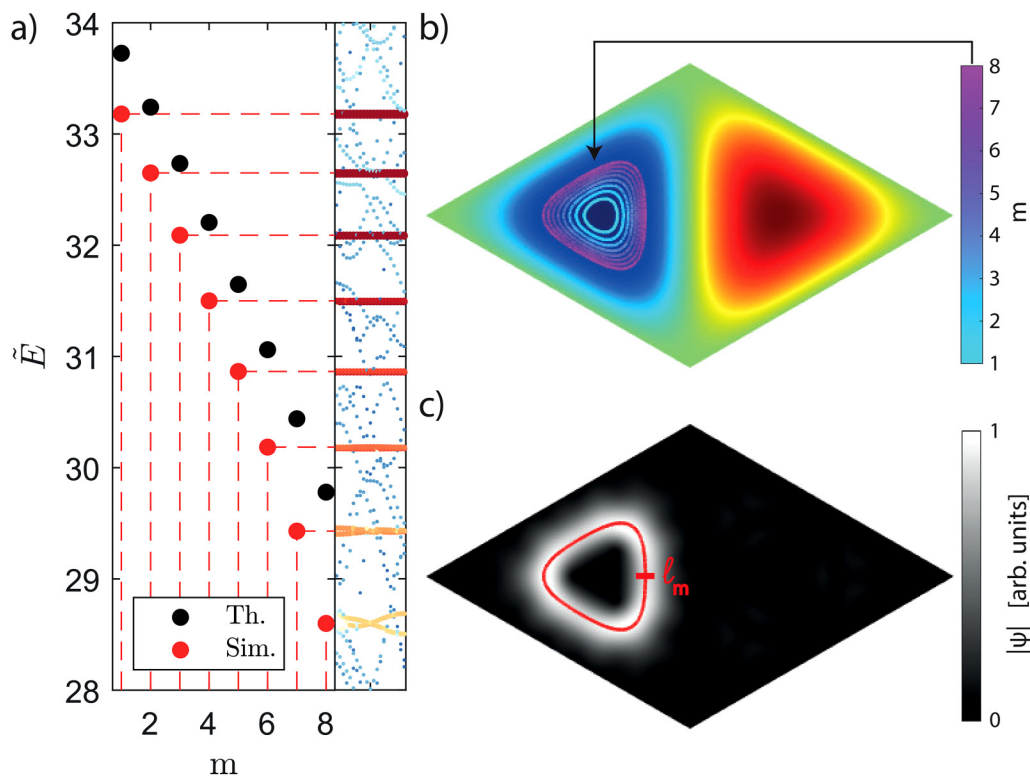


FIG. 4. (a) Energies of the L1 subbands. Black dots correspond to results from Eq. (5), while red dots are obtained from the band structure, whose magnified view is reported in the rightmost panel. (b) Magnetic orbits (from the condition $B_{ps} = B_m$) for L1 and each subband index m . The orbits are superposed to the negative PMF (the result for the positive PMF lobe is analogous). (c) Magnetic orbit (red line) for $n = 1$ and $m = 8$ superposed to the corresponding wavefunction. The red scale bar corresponds to the magnetic length ℓ_m , calculated using B_m .

structure of graphene. The states belonging to the S0 set are particularly interesting; their formation can be understood considering the clockwise (counterclockwise) trajectories of electrons in a positive (negative) magnetic field. The electronic path arranged in an infinite periodic system can form closed loops, as are the eigenfunctions shown in Fig. 5. Interestingly, these “snake states” have the double property of being spatially delocalized while at the same time having flatbands, implying larger electron correlations due to their low group velocities. While the experimental observation of these states could be hindered by the presence of defects or non-homogeneity of the mechanical superlattice, they could act as linking paths for Landau or pseudo-Landau states that could be arranged into high-coherence networks. Looking at the snake states at larger energy, the eigenfunctions increase their localization within a PMF of a set sign and, therefore, have an increasing η^+ (η^-) with a vanishing η^- (η^+), forming a network including only one of the two sublattices of the mechanical superlattice. A sketch of the different flatband states we identified is reported in Fig. 2(a).

In a possible experiment to investigate the states here described, some limitations can arise for states localized in the actuation regions that are mechanically stressed. In most of the local strain engineering techniques,^{28,30} the actuators themselves can strongly perturb the graphene layer or simply make it inaccessible with external out-of-plane

probes such as the ones used in STM²⁹ due to electron irradiation of the actuators. A further tool to address this issue is given by the relative rotation of graphene and the mechanical superlattice. In the analyzed case of $\theta = \pi/6$ rotation, the PMF antinodes coincide with the sites of maximum stress; different rotations could lead to very different situations. In fact, a $\theta = 0$ rotation generates the PMF profile shown in Fig. 2(b), where the stress sites $s1$ and $s2$ coincide with nodes of the PMF. Moreover, the field lobe extension is reduced with respect to the $\theta = \pi/6$ case [Fig. 2(a)]. The reduction of a factor $1/\sqrt{3}$ translates into a shift and reduction of the multiplicity of pLLs, as can be seen in the band structure of Fig. 6(a). A reduced field extension requires a larger energy in order to form a close orbit and give rise to quantization. Significantly, a rearrangement of stress sites can be used to strongly modify the field periodicity and the electronic band structure without modifying the periodicity of the mechanical superlattice. In the $\theta = 0$ case, we observe a clear electron confinement of the L0 and L1 pLL [see Fig. 6(b)], with features similar to the ones reported for $\theta = \pi/6$. N0 states can also be identified: in this case, they have a weak localization within the field region even if most of the wavefunction is confined in the field nodes/underneath the stress sites. The reduced field extension pushes the snake states S0 to lower energies; thus, for the case $\theta = 0$, they can be found at $\xi = 15$ [see an example in Fig. 6(b)].

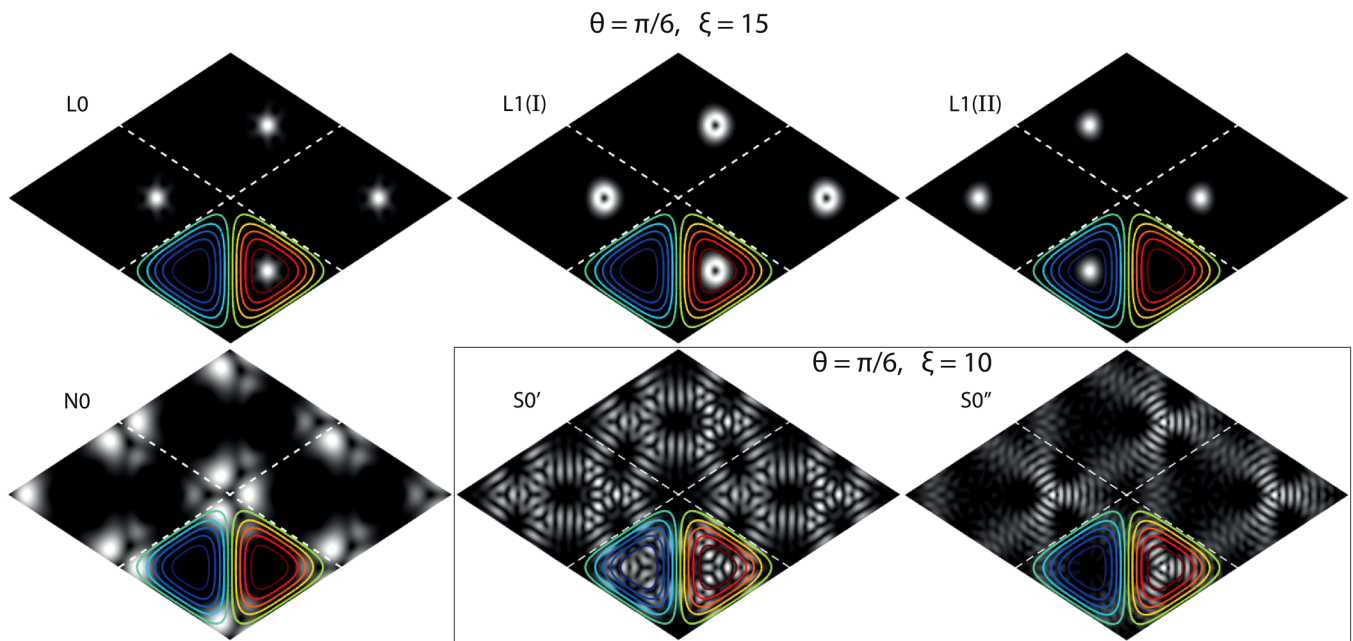


FIG. 5. Eigenstates belonging to the set L0, L1, N0, and S0 [identified from green triangles in Fig. 3(b)] for the case $\theta = \pi/6$ at the $\bar{\Gamma}$ point. The labels (I) and (II) identify each of the two degenerate levels. The contour plot of the pseudo-magnetic field in Fig. 2(a) is superimposed to the bottom primitive cell, for each of the eigenstates. Eigenstates L0, L1(I), L1(II), and N0 are taken at $\xi = 15$ while eigenstates S0' and S0'' are taken at $\xi = 10$ [see green triangles in Fig. 3(b)].

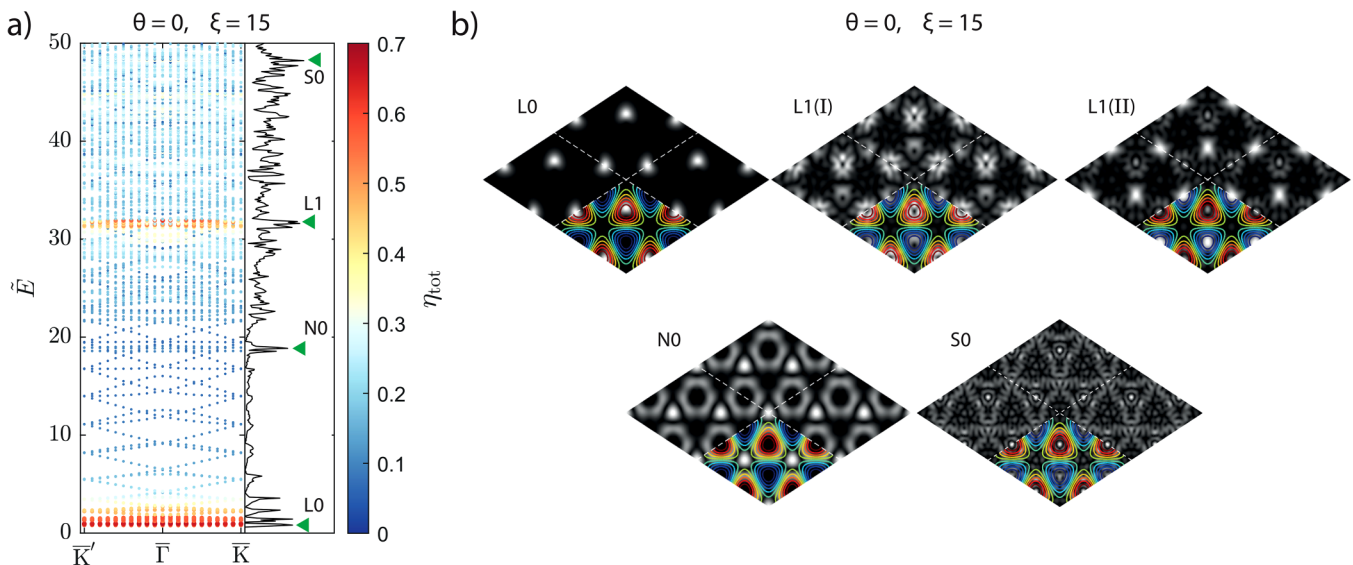


FIG. 6. (a) Energy band structure and the corresponding density of states (cross section) for $\xi = 15$ in the configuration $\theta = 0$. The green triangles in the DoS correspond to the labeled states. (b) Eigenstates for the case $\theta = 0$ at the $\bar{\Gamma}$ point corresponding to the energy levels highlighted by the green triangles [see panel (a)].

Concluding, we identified and classified several kinds of flat-band states in periodically strained graphene. Along with the well known pLLs, where electrons are localized within the lobes of the field regions, we identified states strongly localized within the PMF nodes. A further tuning knob for states localization is given by the graphene and stress lattice relative rotation, which switches the overlap of the stress sites with the PMF nodes/antinodes and changes the overall size of field lobes.

IV. ASYMMETRIC STRESS CONFIGURATION

The possibilities given by infinite strain lattices are not limited by the symmetric honeycomb structure analyzed so far. In the symmetric case, the low-energy electronic bands induced by the superlattice are qualitatively similar to the ones found in graphene layers;

our PMF array *de facto* defines an artificial graphene material. The most recent strain engineering techniques can go one step further, controlling the single superlattice site in a flexible and reconfigurable way. This allows for the creation of more complex superlattice configurations: by changing the stress of a single site, artificial defects can be defined; by modifying the stress of each site belonging to a sublattice, we can configure different artificial materials reminiscent of structures such as hBN,⁴³ which are widely known for having large bandgaps. More complex configurations would see a mix of artificial graphene, defected graphene, and artificial hBN in planar heterostructures, which could lead to advanced electronic transport devices. As a first step in this direction, we show here how the electronic configuration of periodically strained graphene can be modified by increasing the asymmetry of the stress sites belonging to different sublattices. Let us consider the infinite

Asymmetric configuration ($\theta = \pi/6$)

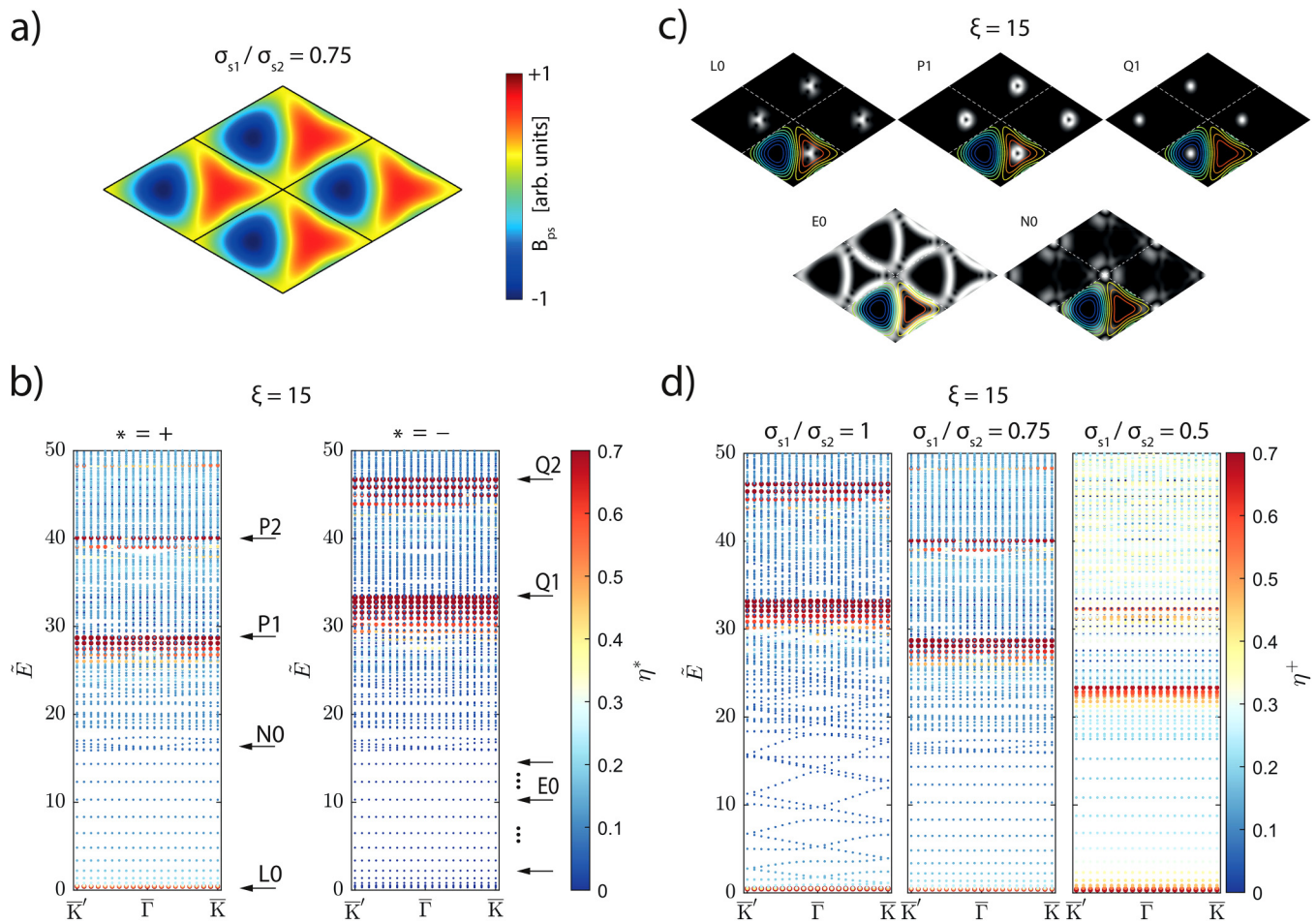


FIG. 7. (a) Pseudo-magnetic field for $\sigma_{s1}/\sigma_{s2} = 0.75$. (b) Electronic band structure ($\xi = 15$) color-coded with η^+ (left) and η^- (right). (c) Eigenstates at the Γ point corresponding to the energy levels highlighted in panel (b). The contour plot of the pseudo-magnetic field in panel (a) is superimposed to the bottom primitive cell, for each eigenstate. (d) Evolution of the band structure for decreasing (from left to right) values of the applied stress at the site s_2 .

periodic stress lattice with a rotation of $\pi/6$, as shown in Fig. 2(a): an asymmetry can be introduced by considering one of the two stress sites as 75% weaker than the other (i.e., $\sigma_{s1}/\sigma_{s2} = 0.75$). The resulting PMF is displayed in Fig. 7(a), and the corresponding band structure has been plotted in Fig. 7(b). The two panels show the same energy dispersion yet with different color codes: the leftmost (rightmost) panel has been colored according to η^+ (η^-), highlighting the states confined in the positive (negative) regions of the PMF. We can recognize most of the flatband states we described so far.

The most striking feature though is the large numbers of a new wide set of states (E0). Energetically localized between L0 and N0, these flat states are separated by full bandgaps; the appearance of bandgaps after breaking the sublattice symmetry of the periodic superlattice supports the picture of considering the symmetric and asymmetric lattices as artificial graphene and hBN, respectively. Moreover, we argue that tunable bandgaps could have applications for the creation of planar artificial heterostructures by controlling the energy level occupation and electron confinement. The states E0 are localized outside the field lobes, forming a ring-like shape network delocalized in the whole crystal, as displayed in Fig. 7(c). For the opposite pseudospin, the wavefunction is qualitatively similar, with the ring-like shape slightly distorted and disconnected vertex. The E0 states closer to L0 gradually localize inside the positive PMF lobe; on the other hand, the ones closer to N0 localize inside the nodes of the PMF [see Fig. 7(c)]. Regarding the other states, the non-degenerate L0 level is still localized only in the positive PMF, with no equivalent state confined within the negative PMF for the pseudospin we are considering.

The asymmetry impacts the $n \geq 1$ PLLs. The pseudospin degeneracy is lifted and two sets of PLLs appear, one confined in the positive field (labeled P) and the other in the negative one (labeled Q). The P and Q states have a net energy difference originating from the different geometries of positive and negative PMFs; in particular, the energy difference $\Delta E = |E_P - E_Q|/E_L$ is about 15% for both $n = 1$ and $n = 2$ PLLs. From Fig. 7(c), we observe that the wavefunctions of $P1$ and $Q1$ are essentially identical to $L1$ (I) and $L1$ (II) shown in Fig. 6, albeit with different eigenenergies.

A further weakening of the magnitude of the stress at one site leads to a continuous tuning of the band structure. Figure 7(d) shows, as an example, a few configurations with increasing degree of asymmetry starting from the symmetric case. The bands have been color-coded according to η^+ , therefore highlighting P levels. An increasing weakening of the site $s1$ produces a red-shift of the P states. This behavior is consistent with the fact that the PMF amplitude in positive regions (site $s2$) becomes smaller than the amplitude in negative ones (site $s1$). Conversely, Q states are almost unchanged, being strongly localized in negative field regions whose amplitude is roughly constant for all considered cases. The continuous tuning of the band structure could represent an important tool for tapering planar PMF heterostructures, in such a way to reduce electronic scattering. A full tuning of the stress sites can eventually lead to creating a superlattice with full triangular symmetry ($\sigma_{s1} = 0$). This configuration has been already studied in the literature,¹⁶ and it results in PMF profiles very similar to the $\theta = \pi/6$ symmetric case discussed before, although with a shift in the position of field nodes and antinodes

(see the [supplementary material](#)). Interestingly, we do not expect any meaningful change of the electronic structure with respect to the symmetric case; however, the latter gives more possibilities in terms of controlling the system parameters, as we have shown by considering the asymmetric tuning and the introduction of a gapped band structure.

V. CONCLUSION

In this paper, we introduce a method to modulate the electronic states in graphene by employing a superlattice of planar micro-actuators.^{28,30} The stress lattice forms PMF patterns whose periodicity and confinement can be controlled by acting on the relative superlattice-graphene twist. The effect of the PMF array creates several flatband states, which have recently been investigated for possible exploitation of their increased electron-electron correlations. On top of the well known PLLs, localized within the PMF regions, we identify flat states lying on the field nodes (N0) as well as snake states (S0) generated from resonant conditions of particular geometric configurations. The possibility to overlap or separate the stress sites and the PMF by controlling the graphene-superlattice rotation can be crucial for the experimental observation of these states in regions of pristine graphene, unaffected by the mechanical actuators. Finally, we show how breaking the sublattice symmetry opens several bandgaps and allows a fine-tuning of the PLL flat states, paving the way to the creation of planar heterostructures defined by different stress superlattices.

Our theoretical results can have a strong impact on graphene devices. The goal of obtaining a graphene-based far infrared laser operating on pseudo-Landau levels^{44–46} will need to take into account their multiplet-splitting in non-uniform fields, such as the one created via actuated strain profiles and discussed in Sec. III. Moreover, the interplay between delocalized snake states and localized PLLs and N0 states can be used to create new concepts for electronic transistors, whose channel conductivity can be controlled by gate-tuning the energy and selecting the appropriate state. Additionally, this kind of electronic device can be reconfigured by locally applying the appropriate strain pattern to exploit localized/delocalized states or bandgaps in the asymmetric strain configuration.

SUPPLEMENTARY MATERIAL

See the [supplementary material](#) for additional information concerning the numerical methods and results.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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